Hybrid Quantum-Classical Algorithms for Sustainable Agriculture

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The Agri-Food Sustainability Challenge

What Goes Wrong Today

- Farmers plan independently based on past prices
- Leads to over- or underproduction
- Causes supply-demand imbalance and price volatility
- Drives food waste and unmet demand

Impact on Sustainability

- Environmental: overuse of land, water, chemicals
- Economic: unstable income, wasted logistics
- Social: unmet demand, food insecurity, unfair profit

Classical Model for Sustainability[1]

Five Objectives

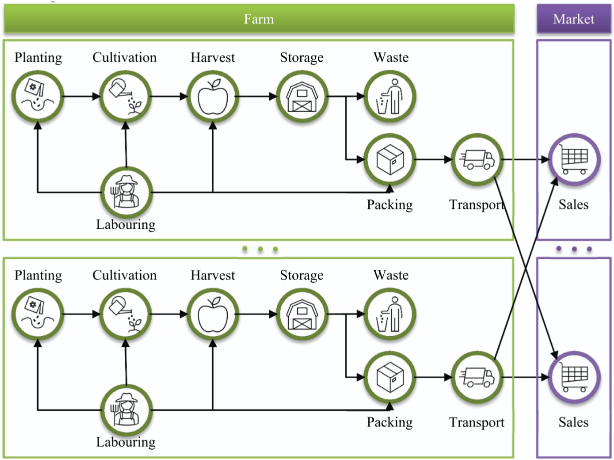
- Maximize supply chain profit (economic)
- Minimize food waste (environmental)
- Minimize unmet demand (social)
- Maximize freshness at sale (social and environmental)
- Minimize economic inequality among farmers (social)

Approach

- Centralized planning across farms and markets
- Multi-objective optimization with tunable weights
- Enables evidence-based decision-making

[1] Esteso, Alemany, and and, "Sustainable agri-food supply chain planning through multi-objective optimisation".

Classical Model Diagram



How Our Approach Supports the SDGs

Reduces unmet demand by aligning supply with actual need.

Minimizes waste and avoids unnecessary overproduction.

How Our Approach Supports the SDGs

Reduces profit inequality between farmers by optimizing equity.

Less waste = fewer emissions; better planning = lower transport and resource use.

Our Approach optimizes for Sustainability

- Maximize economic availability - **Affordability**
- Minimize impact on soil - **Environmental Impact**
- Minimize waste - **Sustainability**
- Provide high food quality - **Nutrition Scores**
- Provide enough variety - **Food Group Constraints**
- Assure farmers are employed - **Social Benefit**

Quantum Meets Sustainability

Problem

- Global food systems must optimize nutrition, cost, and sustainability
- Traditional methods struggle with multi-objective complexity
- optimization that does not take into account the full picture is key to future food security

Vision

- Combine MILP* + quantum techniques for hybrid optimization
- Use Benders' Decomposition to break up the problem
- Apply quantum-inspired and quantum-native solvers

*Mixed Integer Linear Programming

Decision Variables & Parameters

Decision Variables

- $A_{f,c}$: Area (ha) planted, cultivated, and harvested with food c on farm f.
- $Y_{f,c}$: Binary indicator for planting decisions:

$$Y_{f,c} = \begin{cases} 1 & \text{if food } c \text{ is planted at farm } f, \\ 0 & \text{otherwise.} \end{cases}$$

Multi-objective Food Allocation Model

Objective Function

Mixed continuous + binary → MILP

$$Z = \sum_{i \neq \text{env}} w_i \sum_{f \in F} \sum_{c \in C} s_c^{(i)} A_{f,c} - w_{\mathrm{env}} \sum_{f \in F} \sum_{c \in C} s_c^{(\mathrm{env})} A_{f,c} \, . \tag{1}$$

Optimization Goals

- **Maximize:** Nutrition, affordability, sustainability
- **Minimize:** Environmental impact

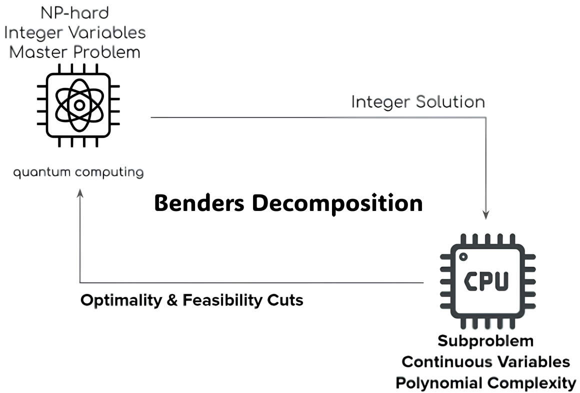
Subject to Constraints:

- Land availability and usage
- Food group diversity requirements

Classical Decomposition: Benders

Strategy

- MILP $\rightarrow$ Master (QUBO) $+$ Subproblem (LP)
- Iterative cuts improve QUBO over time



Hands-on Example: Benders Decomposition

Setup

- 5 crops: c_1, c_2, c_3, c_4, c_5
- Land $L = 10$; Min area $A_c^{\text{min}} = 2$
- Nutritional vector: $N = [3, 5, 2, 4, 1]$
- Environmental vector: $E = [4, 3, 6, 2, 5]$
- Objective:

$$Z = \frac{\sum_c N_c A_c - \sum_c E_c A_c}{\sum_c A_c}$$

Step-by-Step

- Master solution: $Y = [1, 0, 1, 0, 0]$
- Active crops: c_1, c_3
- Subproblem:

$$A_1 + A_3 \leq 10, \quad A_1, A_3 \geq 2$$

- New objective:

$$Z = \frac{-A_1 - 4A_3}{A_1 + A_3}$$

- Smaller problem $\rightarrow$ faster solve

From MILP to QUBO

How to get there?

- Benders splits our MILP into:
  - Binary master problem
  - Continuous subproblem
- The master problem becomes a QUBO.

Why QUBO?

- Standard input for quantum solvers
- Allows interactions and penalties
- “Benders gives structure, QUBO gives gateway to quantum”

What is a QUBO?

QUBO Definition

- QUBO = Quadratic Unconstrained Binary Optimization
- Form: $\min_{x \in \{0,1\}^n} x^T Q x$
- x: binary vector, Q: symmetric matrix

Example

- $Q = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$
- $x = [1, 0] \Rightarrow x^T Q x = 1$

How Does QUBO Work?

Interpretation

- Diagonal terms: cost or reward for choosing x_i
- Off-diagonal terms: interaction between x_i and x_j
- $Q_{ij} > 0$: penalizes both $x_i, x_j = 1$
- $Q_{ij} < 0$: rewards both $x_i, x_j = 1$
- $Q_{ij} = 0$: no effect
- Model synergy or conflict and constraints

Note on Constraints

- QUBO is unconstrained by nature
- Constraints are added as penalty terms
- We'll see how in detail later

Conversion to QUBO

Binary Conversion

The initial binary decision variables make up the vector $\mathbf{y} \in Y$ with length of n. In order to reformulate the master problem into the QUBO formulation, we need to represent the continuous variables $\mathbf{x}$ using binary bits.

$$\mathbf{x} = \underbrace{\sum_{i=-\underline{m}}^{\bar{m}_+} 2^i w_{(i+\underline{m})}}_{\text{Positive Integer+Decimal}} - \underbrace{\sum_{j=0}^{\bar{m}_-} 2^j w_{j+1+\underline{m}+\bar{m}_+}}_{\text{Negative Integer}} = \mathbf{x}(\mathbf{w}) . \tag{2}$$[2]

[2] Zhao, Fan, and Han, *Hybrid Quantum Benders' Decomposition For Mixed-integer Linear Programming*.

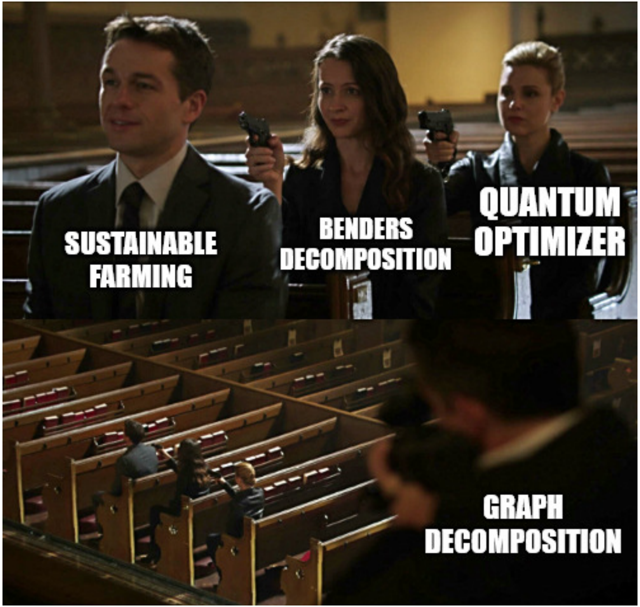
Conversion to QUBO

Penalties

Next, we get the optimal solution by finding the best penalty coefficients of the constraints. Here x_1, x_2 and x_3 are binary variables. P is a user-defined penalty coefficient. [3]

Zhao, Fan, and Han, *Hybrid Quantum Benders' Decomposition For Mixed-integer Linear Programming*.

Constraint	**Equivalent Penalty**
$x_1 + x_2 = 1$	$P(x_1 + x_2 - 1)^2$
$x_1 + x_2 \geq 1$	$P(1 - x_1 - x_2 + x_1 x_2)^2$
$x_1 + x_2 \leq 1$	$P(x_1 x_2)$


SUSTAINABLE FARMING
BENDERS DECOMPOSITION
QUANTUM OPTIMIZER
GRAPH DECOMPOSITION

Objective Conversion

QUBO Objective

$$\min_{\mathbf{x}\in\{0,1\}^n} C(\mathbf{x}) = \max_{\mathbf{x}\in\{0,1\}^n} -C(\mathbf{x}) = \max_{\mathbf{x}\in\{0,1\}^n} \sum_{i,j=1}^{n} (-Q_{ij}) x_i x_j. \tag{3}$$

MaxCut Formalism

- $x_i \rightarrow$ node
- $\mathbf{Q} \rightarrow$ edge weights $w_{i,j}$

$$C(\mathbf{x}) = \sum_{i,j=1}^{\mathrm{V}} w_{ij} x_i (1 - x_j),$$

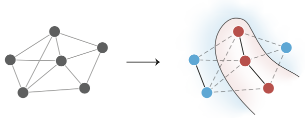


Figure: Left: A problem graph with 6 vertices and 11 equal-weight edges, next to its MaxCut solution.

Quantum Approximate Optimization Algorithm

Hamiltonians

The mapping of the graph to the hamiltonians follows from

$$x_i \to \frac{1}{2}(1 - Z_i). \tag{4}$$

And leads to

$$\hat{H}_C = \frac{1}{2} \sum_{(i,j)\in} w_{ij}(I - Z_i Z_j), \tag{5a}$$

$$\hat{H}_M = \sum_{j\in \mathrm{V}} X_j, \tag{5b}$$

Quantum Solver: QAOA^2 with Graph Partitioning

Scaling Issue

Standard QAOAs require n qubits to solve a MaxCut problem with n vertices.

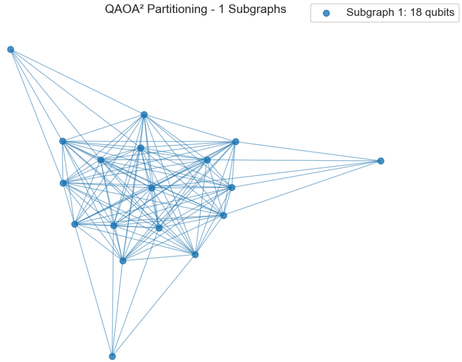
- *Divide*:
  1. Partition the given graph using a **metis** partition from `NetworkX`
  2. QAOAs are exploited to seek optimal solutions $\{x_i\}_{i=1}^{h}$ of these subgraphs in parallel.
- *Conquer*:
  1. The obtained solutions of all subgraphs are merged to obtain the global solution z of G.
  2. Taking into account the connections among h subgraphs, the global solution yields

$$\hat{z} = \arg\max_{z \in \mathcal{Z}} C(z).^4 \tag{6}$$

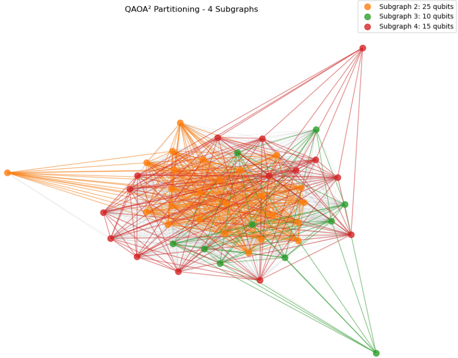
[4] Zhou et al., *QAOA-in-QAOA: solving large-scale MaxCut problems on small quantum machines.*

Graph Partitions

Simple Problem



Full Problem



Results

Problem Setting

(a) Land Availability per Farm

Farm	L_f (ha)
Farm1	50
Farm2	75
Farm3	100

(b) Food Attributes

Food	Nut. Val	Nut. Den	Env. Imp
Wheat	0.7	0.6	0.3
Corn	0.6	0.5	0.4
Rice	0.8	0.7	0.6
Soybeans	0.9	0.8	0.2
Potatoes	0.5	0.4	0.3
Apples	0.7	0.6	0.2

(c) Food Group Classification

Food Group	Constituent Foods
Grains	Wheat, Corn, Rice
Legumes	Soybeans
Vegetables	Potatoes
Fruits	Apples

(d) Optimization Parameters

Category	Parameter	Value
Weights	Nutritional Value	0.25
	Nutrient Density	0.25
	Environmental Impact	0.50

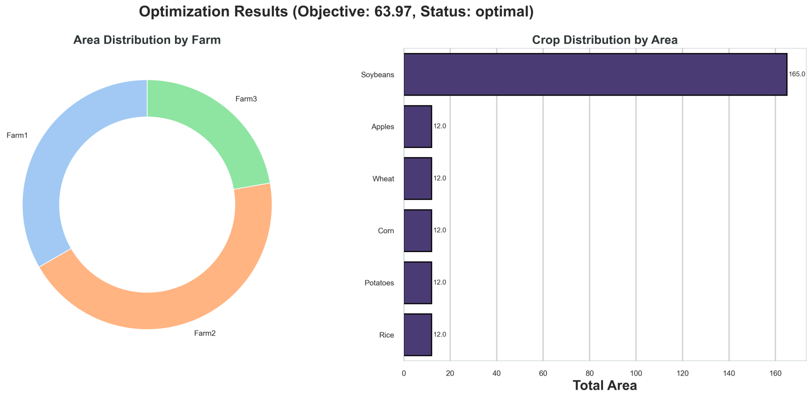
Table 1. Overview of farms, foods, groups, and parameter settings, Simplified Problem

Real World Data **Courtesy of** [5]

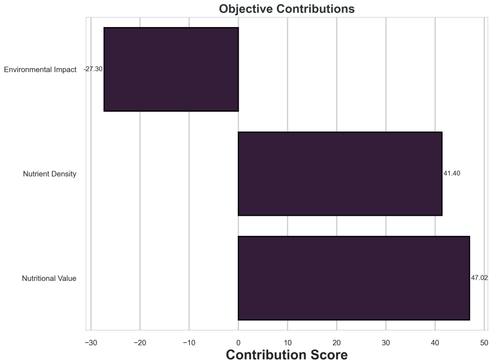
Food Name	Food Group	Nut. Val.	Nut. Den.	Sust.	Env. Imp.	Afford.
Mango	Fruits	0.4468	0.2458	0.0757	0.0044	0.0261
Papaya	Fruits	0.4754	0.2748	0.1778	0.0172	0.0398
Orange	Fruits	0.4707	0.2537	0.1280	0.0083	0.0254
Banana	Fruits	0.4195	0.1963	0.1140	0.0088	0.0801
Guava	Fruits	0.5156	0.3102	0.1791	0.0120	0.0570
Watermelon	Fruits	0.3111	0.0706	0.0833	0.0089	0.0152
Apple	Fruits	0.3710	0.0884	0.0776	0.0045	0.0133
Avocado	Fruits	0.4674	0.2455	0.0511	0.0026	0.0357
Durian	Fruits	0.4516	0.2483	0.0275	0.0016	0.0203
Corn	Starchy staples	0.3908	0.1535	0.1214	0.0113	0.4179
Potato	Starchy staples	0.4782	0.3053	0.1246	0.0113	0.0934
Tofu	Pulses, nuts, and seeds	0.5211	0.3471	0.1052	0.0188	0.1026
Tempeh	Pulses, nuts, and seeds	0.5391	0.3946	0.1115	0.0201	0.2248
Peanuts	Pulses, nuts, and seeds	0.4650	0.4268	0.0546	0.0031	0.2678
Chickpeas	Pulses, nuts, and seeds	0.5153	0.3286	0.1404	0.0125	0.3980
Pumpkin	Vegetables	0.5889	0.4766	0.0579	0.0030	0.0338
Spinach	Vegetables	0.9032	0.9346	0.0859	0.0043	0.0362
Tomatoes	Vegetables	0.5816	0.4394	0.1039	0.0061	0.0387
Long bean	Vegetables	0.5616	0.4127	0.0821	0.0047	0.3634
Cabbage	Vegetables	0.6376	0.5007	0.0791	0.0043	0.0341
Eggplant	Vegetables	0.3967	0.1731	0.0597	0.0035	0.0217
Cucumber	Vegetables	0.4306	0.2272	0.1058	0.0084	0.0188
Egg	Animal-source foods	0.5837	0.4851	0.0343	0.0017	0.0217
Beef	Animal-source foods	0.5968	0.5424	0.0038	0.4468	0.0241
Lamb	Animal-source foods	0.5941	0.5332	0.0088	0.0005	0.0242
Pork	Animal-source foods	0.5840	0.5233	0.0165	0.0008	0.3743
Chicken	Animal-source foods	0.5533	0.4336	0.0249	0.0013	0.0572

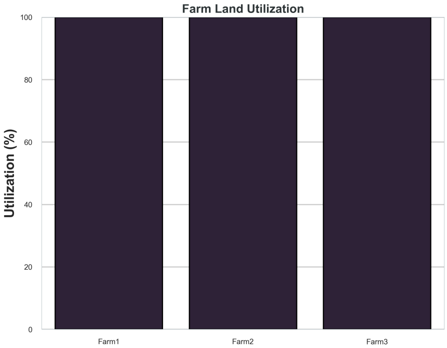
[5] Global Alliance for Improved Nutrition, *Global Alliance for Improved Nutrition (GAIN)*.

Example Solution I

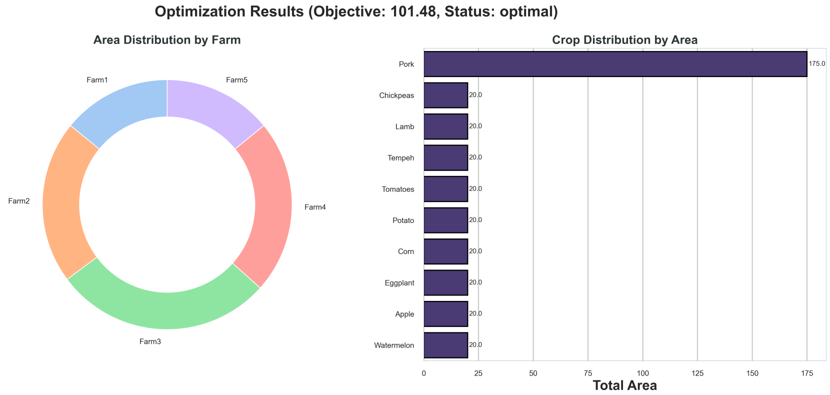


Example Solution II

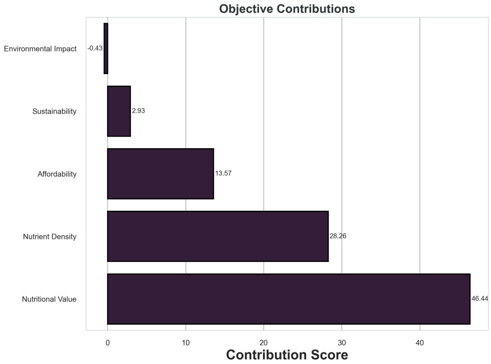


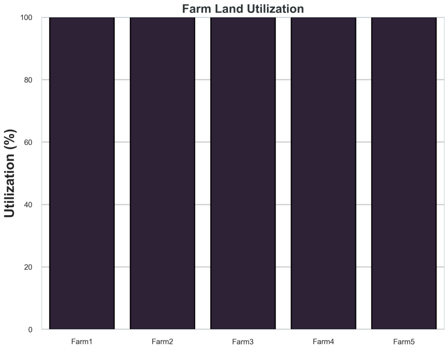


Example Solution III



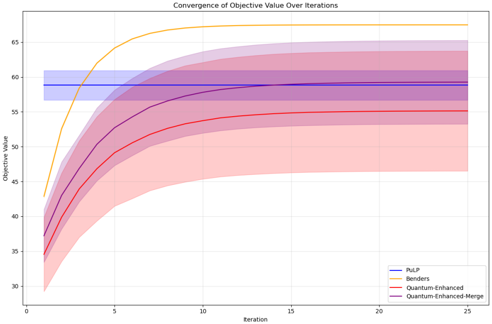
Example Solution IV



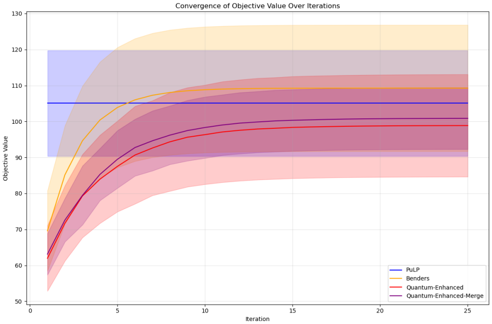


Objective Convergence I

Simple Problem



Full Problem



Objective Convergence II

Simple Problem

Method	Mean	Std. Dev.	Range	Success Rate
PuLP	58.8273	2.1482	[55.6949, 61.5749]	100.0%
Benders	67.4878	0.0412	[67.2471, 67.5000]	50.0%
Quantum-Enhanced	55.1460	8.6992	[17.1000, 67.6000]	80.0%
Quantum-Enhanced-Merge	59.2825	6.0666	[42.1000, 67.3500]	90.0%

Method	Runtime (s)		Memory Peak (MB)	
	Mean	Std. Dev.	Mean	Std. Dev.
PuLP	0.14	0.03	0.17	0.24
Benders	4.73	1.23	0.87	1.02
Quantum-Enhanced	14.95	3.37	134.42	73.79
Quantum-Enhanced-Merge	14.39	3.85	130.55	85.45

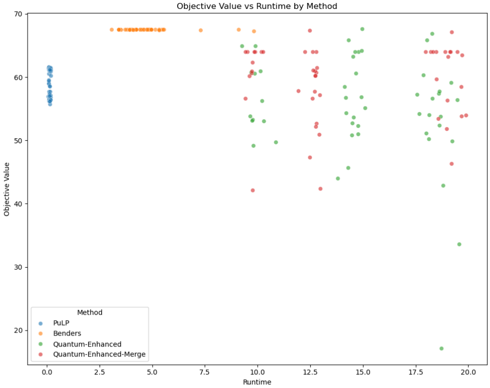
Full Problem

Method	Mean	Std. Dev.	Range	Success Rate
PuLP	105.0812	15.0674	[86.6516, 127.3313]	100.0%
Benders	109.2669	18.0189	[89.0097, 139.4527]	55.0%
Quantum-Enhanced	98.8809	14.6066	[66.2366, 123.0124]	30.0%
Quantum-Enhanced-Merge	100.8892	8.8148	[89.6082, 123.3505]	70.0%

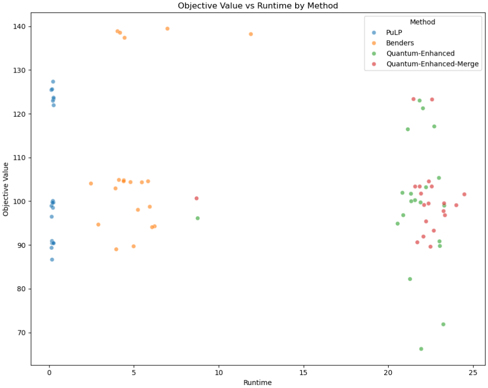
Method	Runtime (s)		Memory Peak (MB)	
	Mean	Std. Dev.	Mean	Std. Dev.
PuLP	0.21	0.05	4.56	3.38
Benders	5.11	1.95	8.87	1.99
Quantum-Enhanced	21.30	3.07	233.56	21.25
Quantum-Enhanced-Merge	21.86	3.20	232.62	22.13

Runtime

Simple Problem



Full Problem



Conclusion and Outlook

Takeaways

- Quantum-enhanced optimization is feasible
- Hybrid classical–quantum models are effective
- Benders offers a quantum-ready interface

What's Next?

- Implementation on Annealer and QPU
- Integration of Market Demand and Labor

Question Time!

References

Esteso, Ana, M.M.E. Alemany, and Angel Ortiz and. "Sustainable agri-food supply chain planning through multi-objective optimisation". In: *Journal of Decision Systems* 33.4 (2024), pp. 808–832. DOI: 10.1080/12460125.2023.2180138. eprint: https://doi.org/10.1080/12460125.2023.2180138. URL: https://doi.org/10.1080/12460125.2023.2180138.

Global Alliance for Improved Nutrition. *Global Alliance for Improved Nutrition (GAIN)*. Accessed: 2025-05-21. 2025. URL: https://www.gainhealth.org/.

Zhao, Zhongqi, Lei Fan, and Zhu Han. *Hybrid Quantum Benders' Decomposition For Mixed-integer Linear Programming*. 2021. arXiv: 2112.07109 [math.OC]. URL: https://arxiv.org/abs/2112.07109.

Zhou, Zeqiao et al. *QAOA-in-QAOA: solving large-scale MaxCut problems on small quantum machines*. 2022. arXiv: 2205.11762 [quant-ph]. URL: https://arxiv.org/abs/2205.11762.