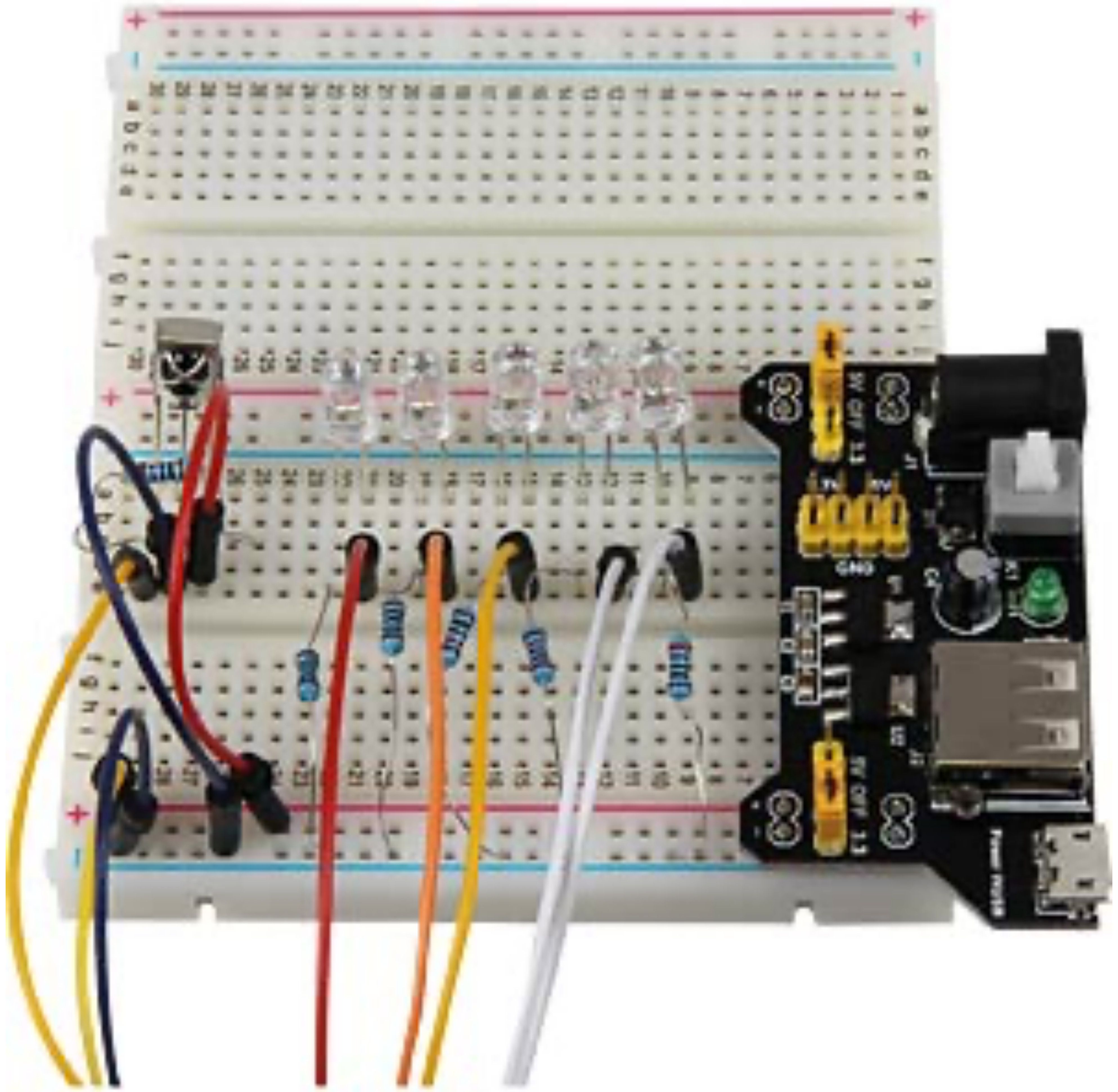
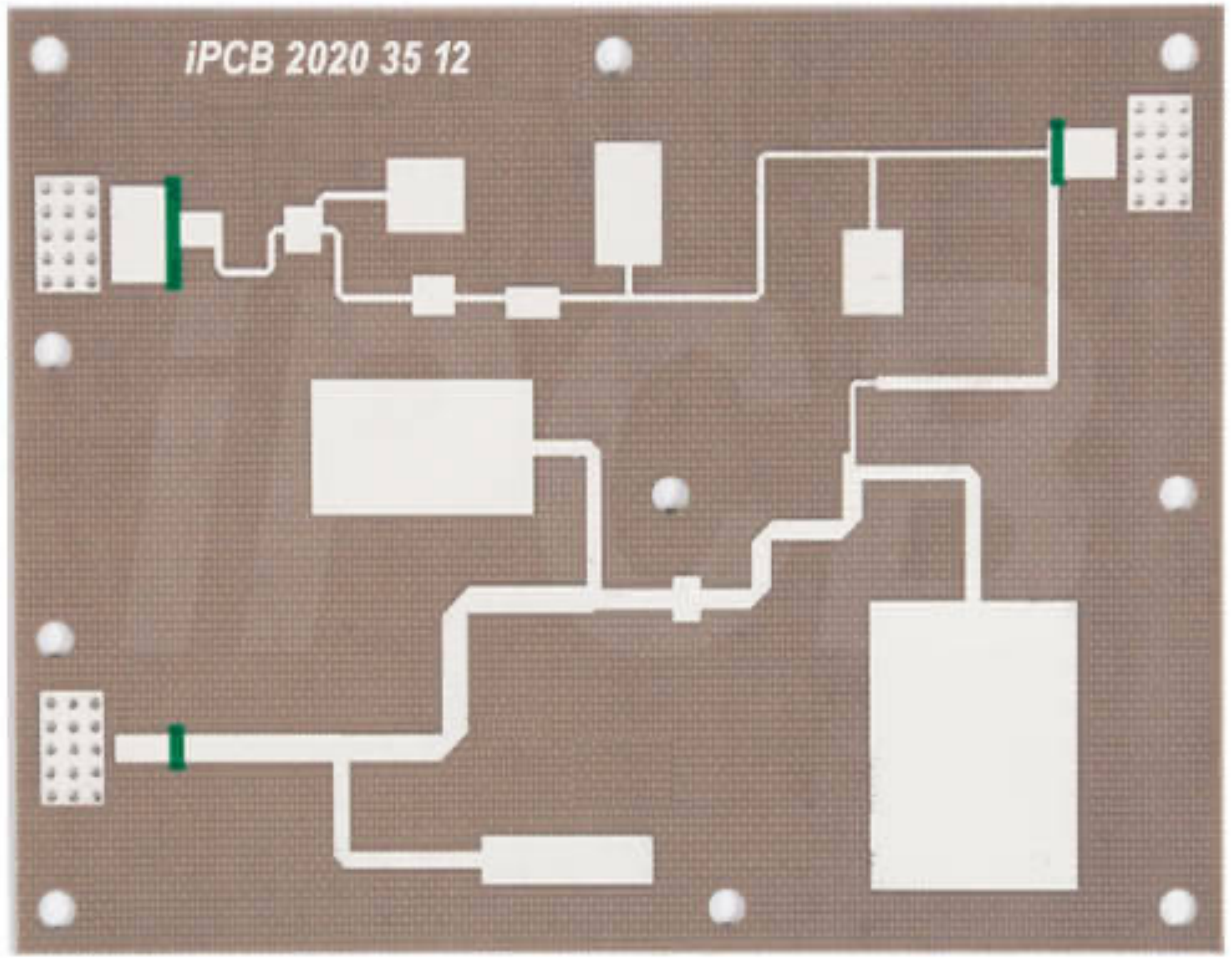
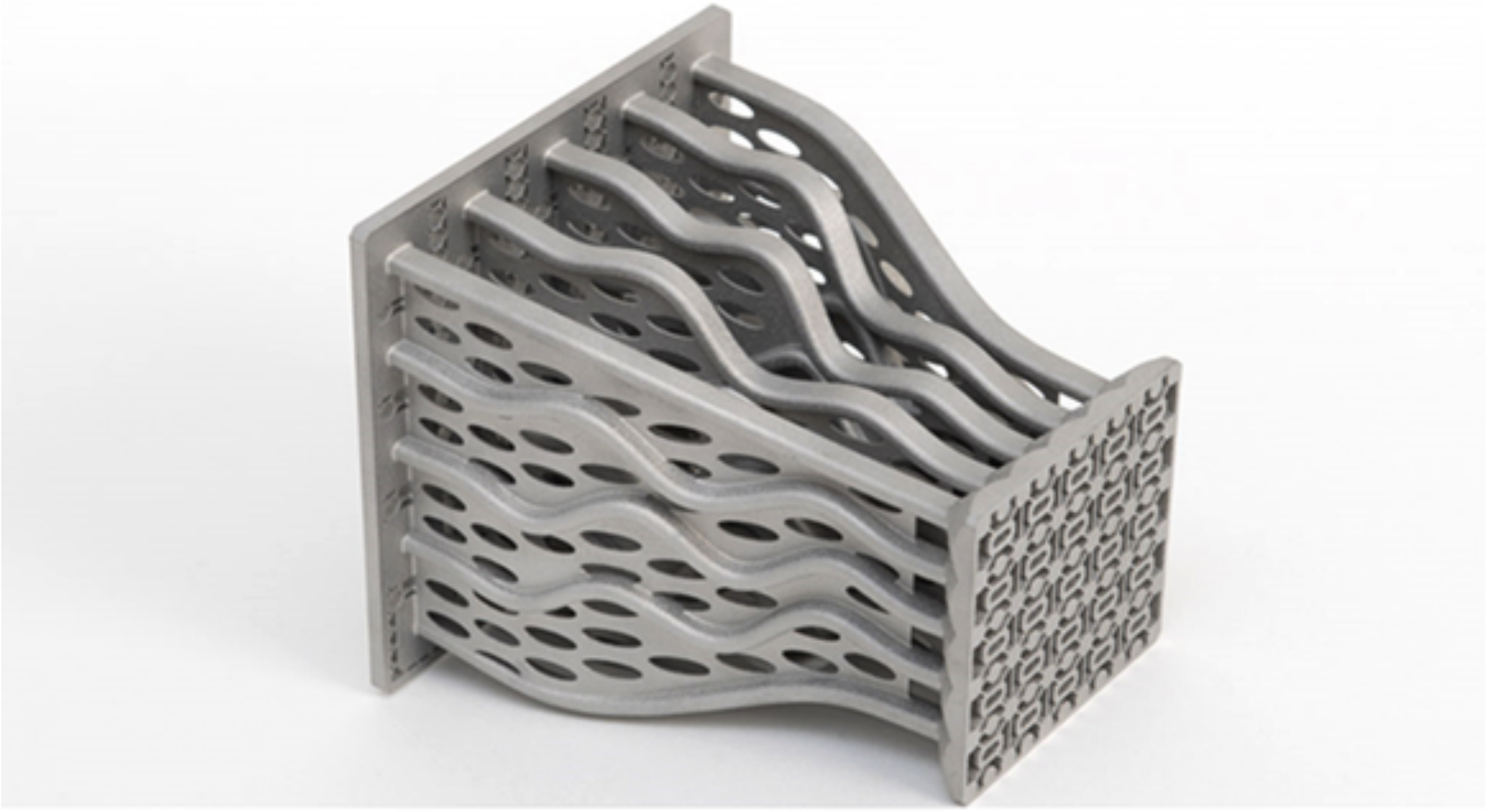
Microwave engineering in physics 2025: key topics & notions

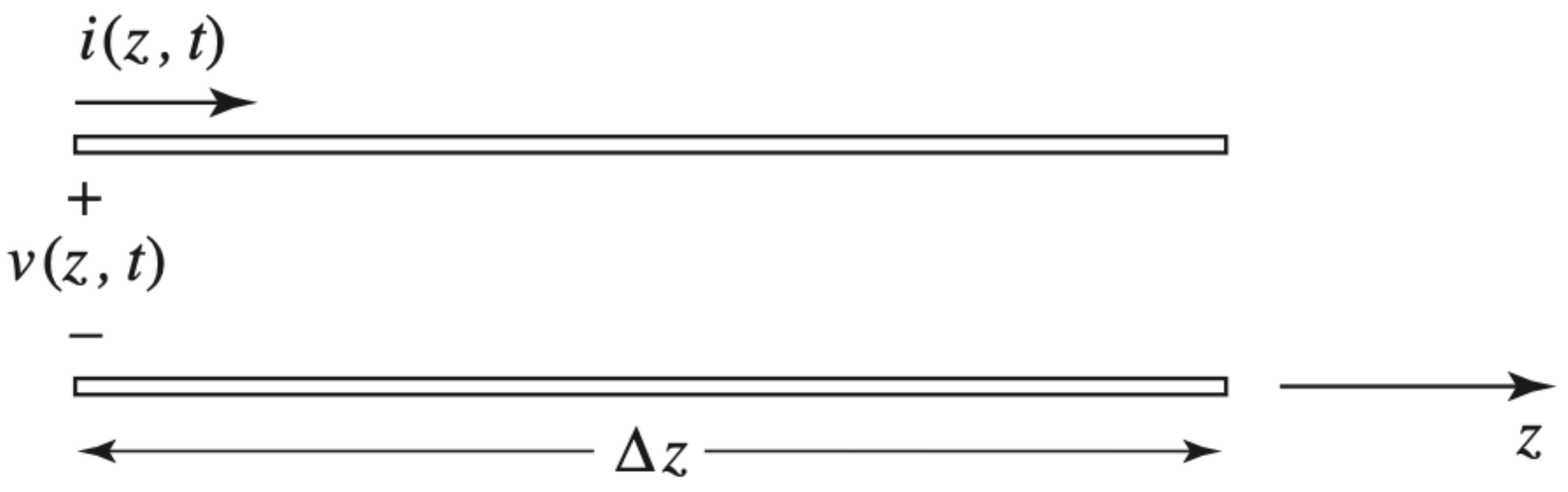
From wires to transmission lines



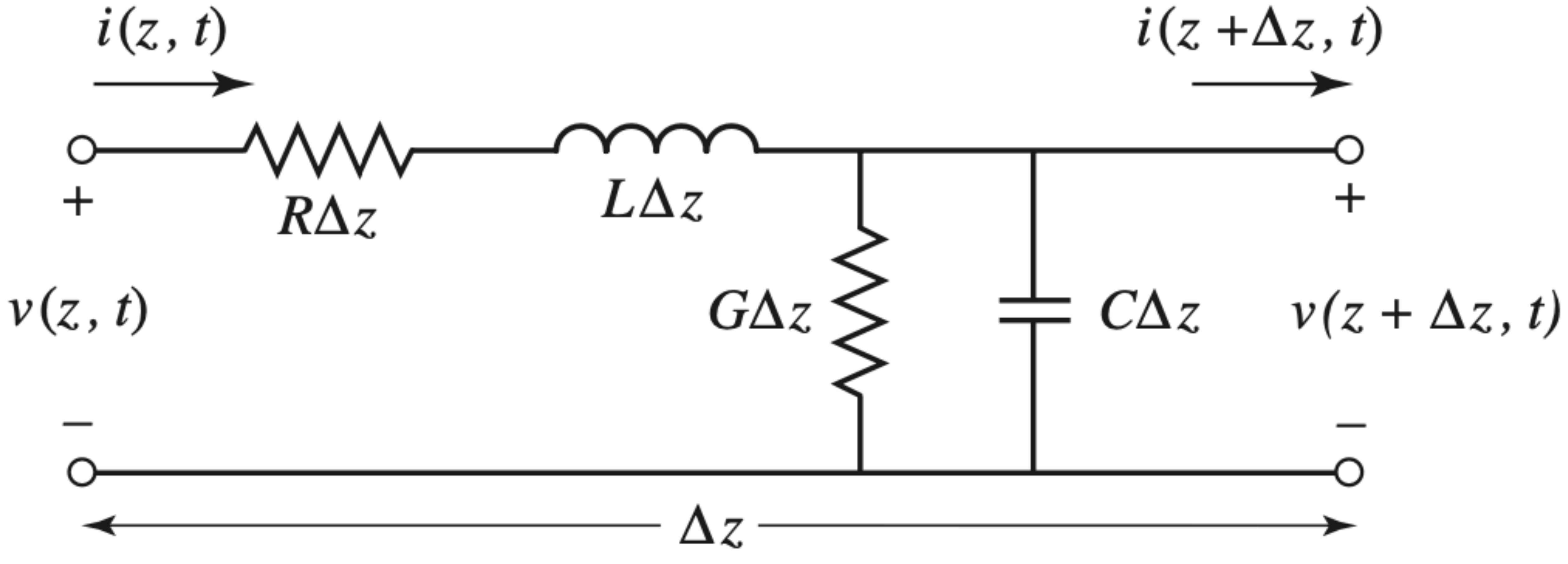




Waves in a TEM transmission lines



(a)



$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z},$$

$$I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z},$$

$$\frac{V_o^+}{I_o^+} = Z_0 = \frac{-V_o^-}{I_o^-}$$

dispersion

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

loss

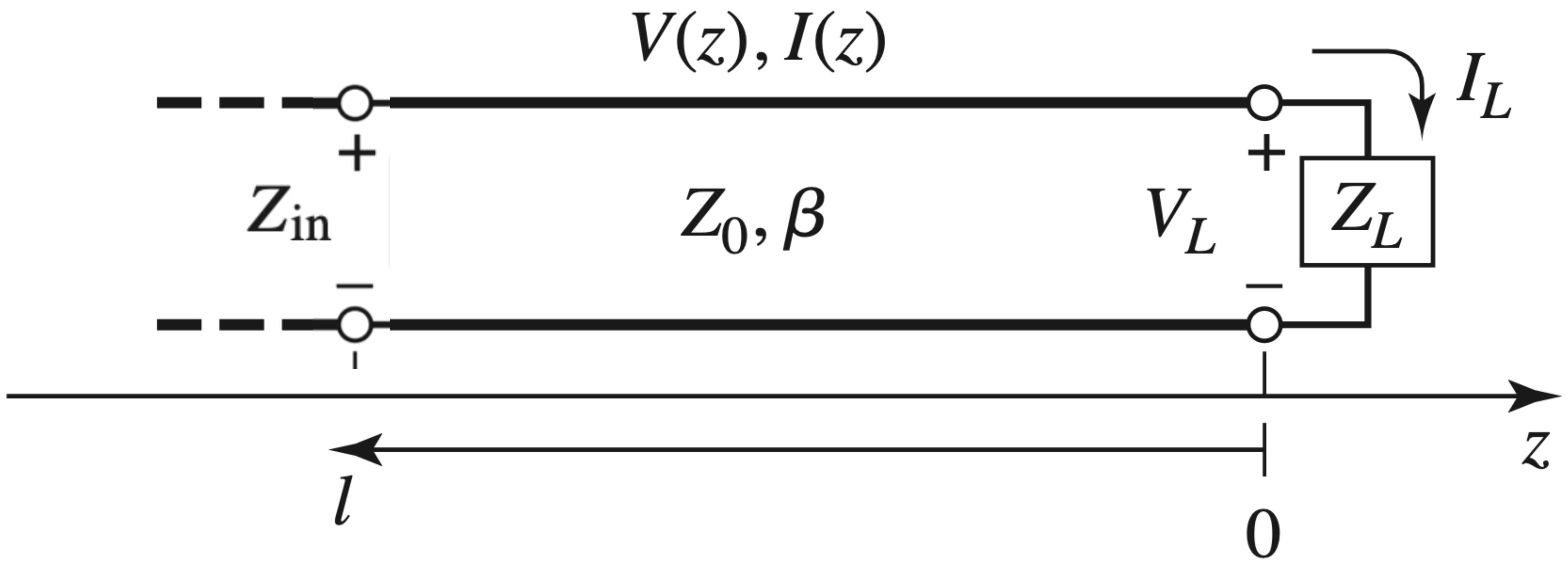
$$Z_0 = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

wave impedance

Frequency dependence of L or C generally leads to dispersion

engineering notation $j = \sqrt{-1}$

Reflectrometry and input impedance



$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$$

Reflection coefficient

$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

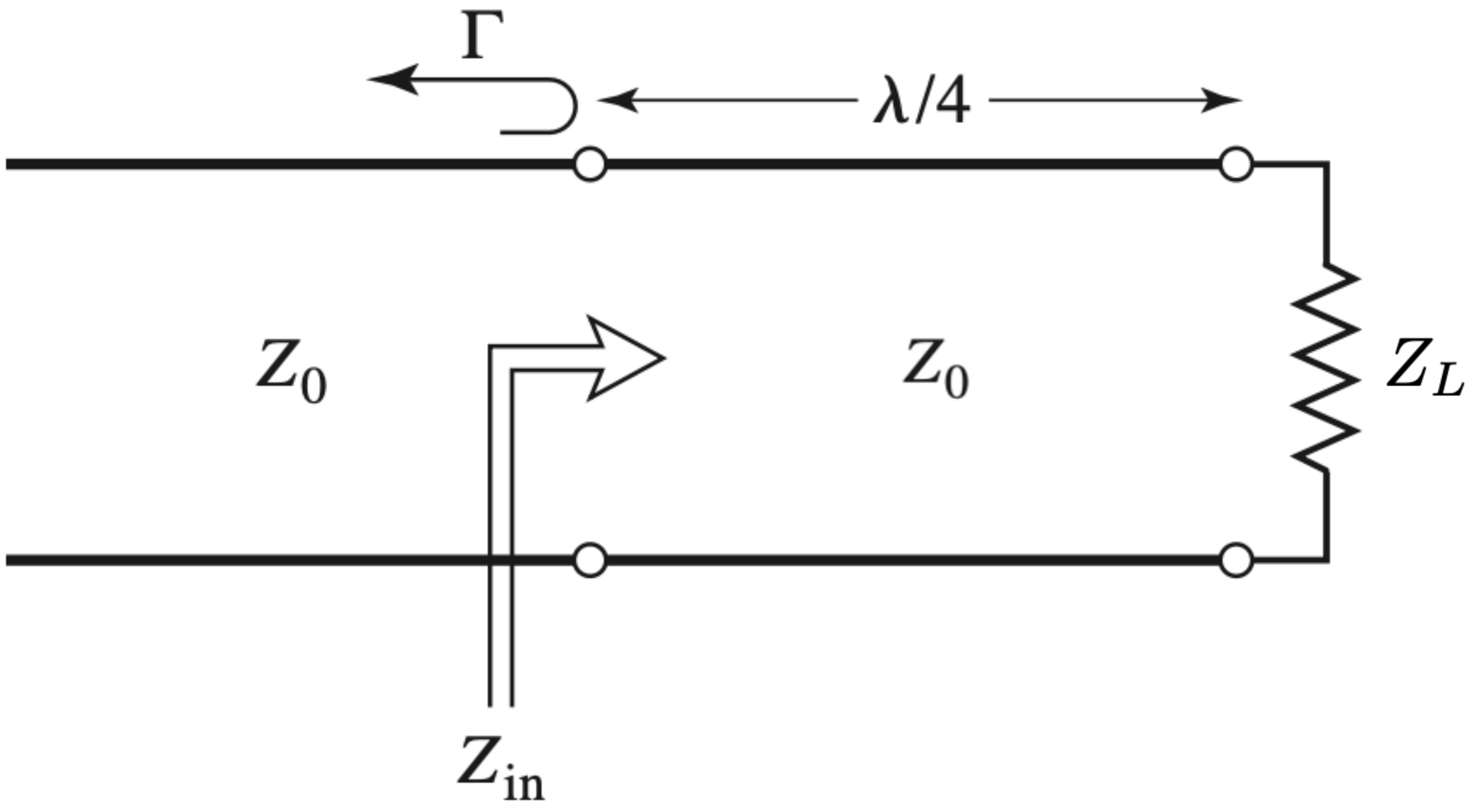
Input impedance

$$Z_{\text{in}} = \frac{V(-\ell)}{I(-\ell)} = \frac{V_o^+ \left(e^{j\beta\ell} + \Gamma e^{-j\beta\ell}\right)}{V_o^+ \left(e^{j\beta\ell} - \Gamma e^{-j\beta\ell}\right)} Z_0 = \frac{1 + \Gamma e^{-2j\beta\ell}}{1 - \Gamma e^{-2j\beta\ell}} Z_0$$

Matching

$$Z_L = Z_0$$

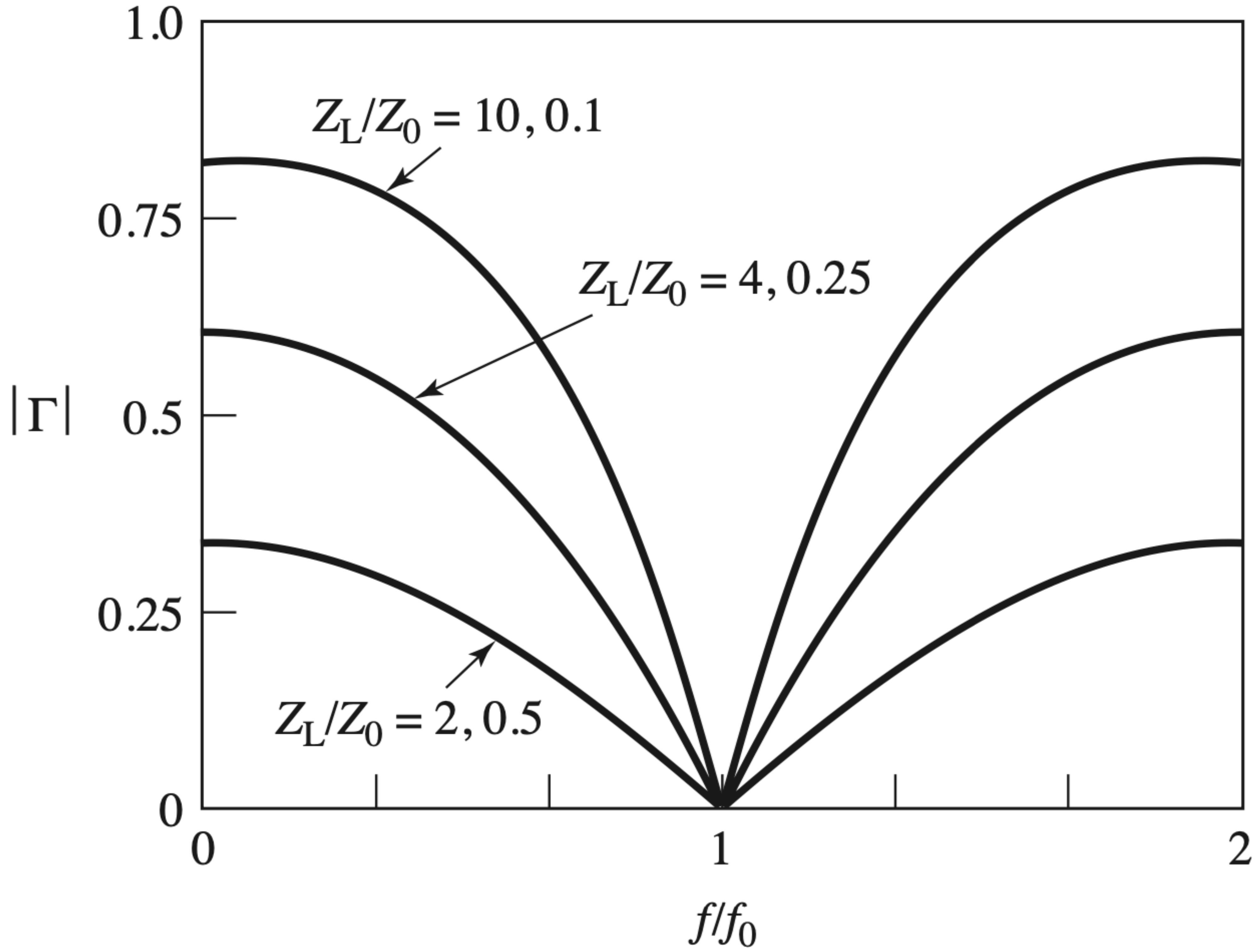
Elementary impedance matching: 1/4-wave transformer



on-resonance

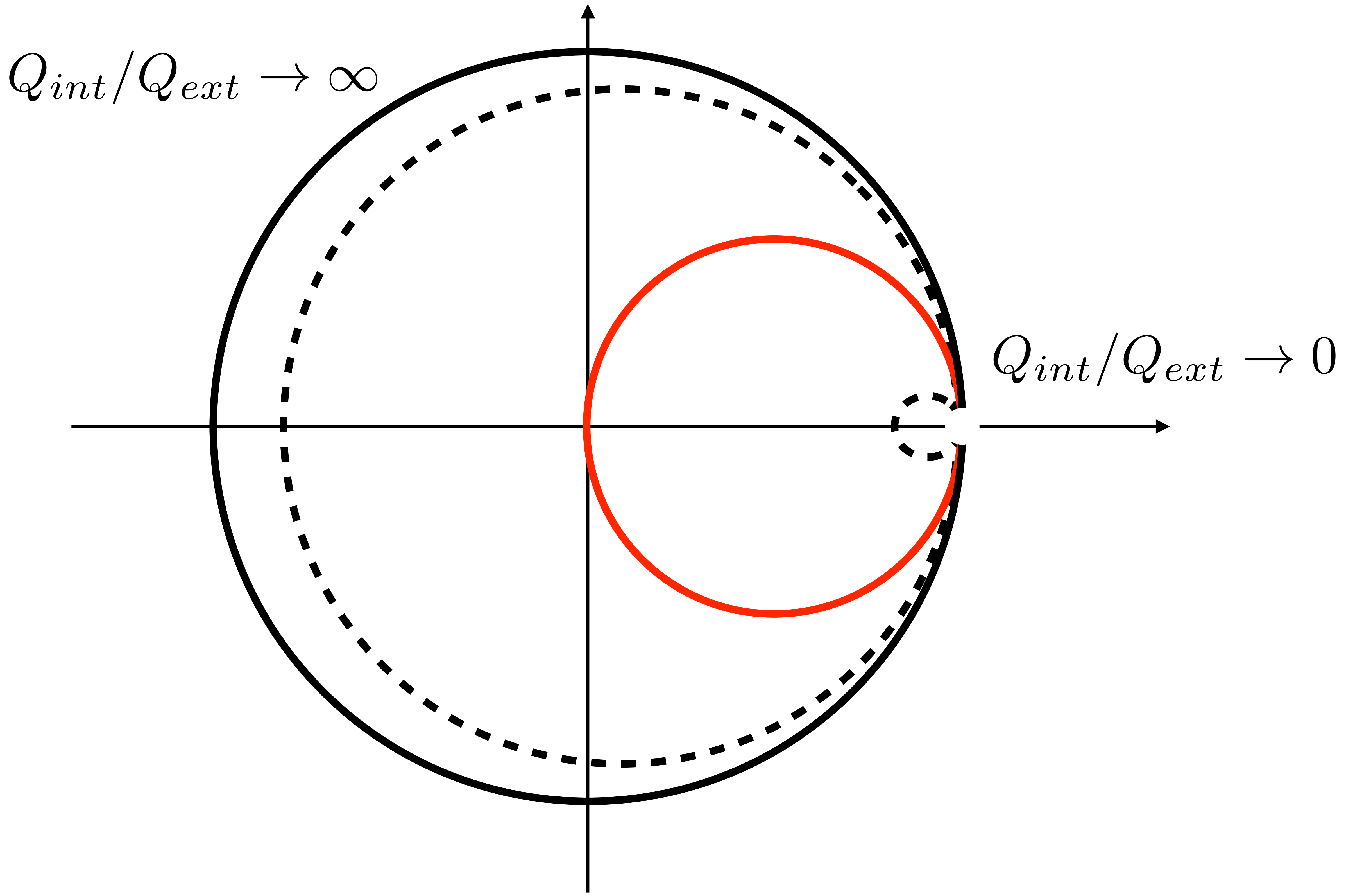
$$Z_{in} = Z_0^2/Z_L$$

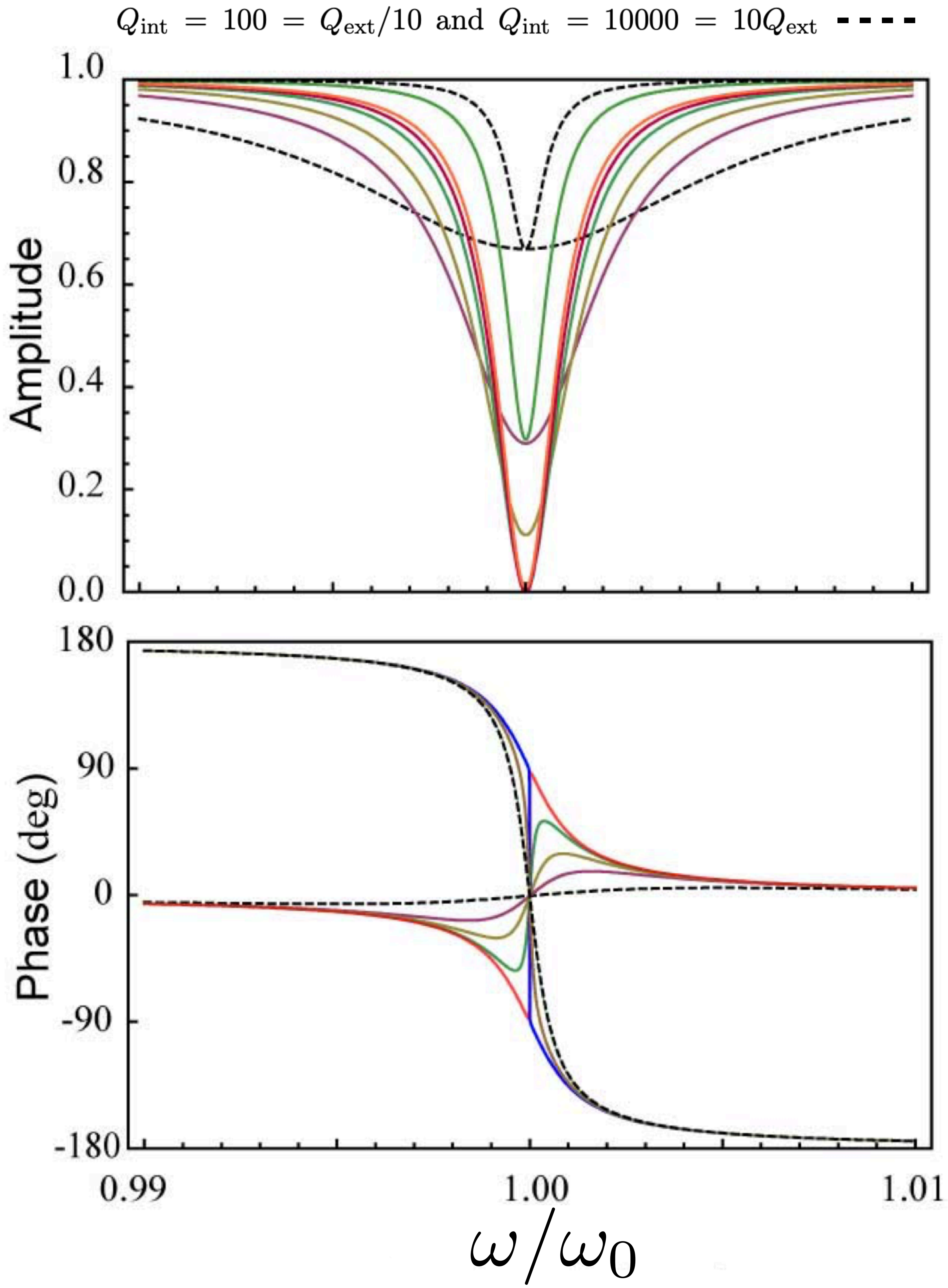
transformation-bandwidth tradeoff



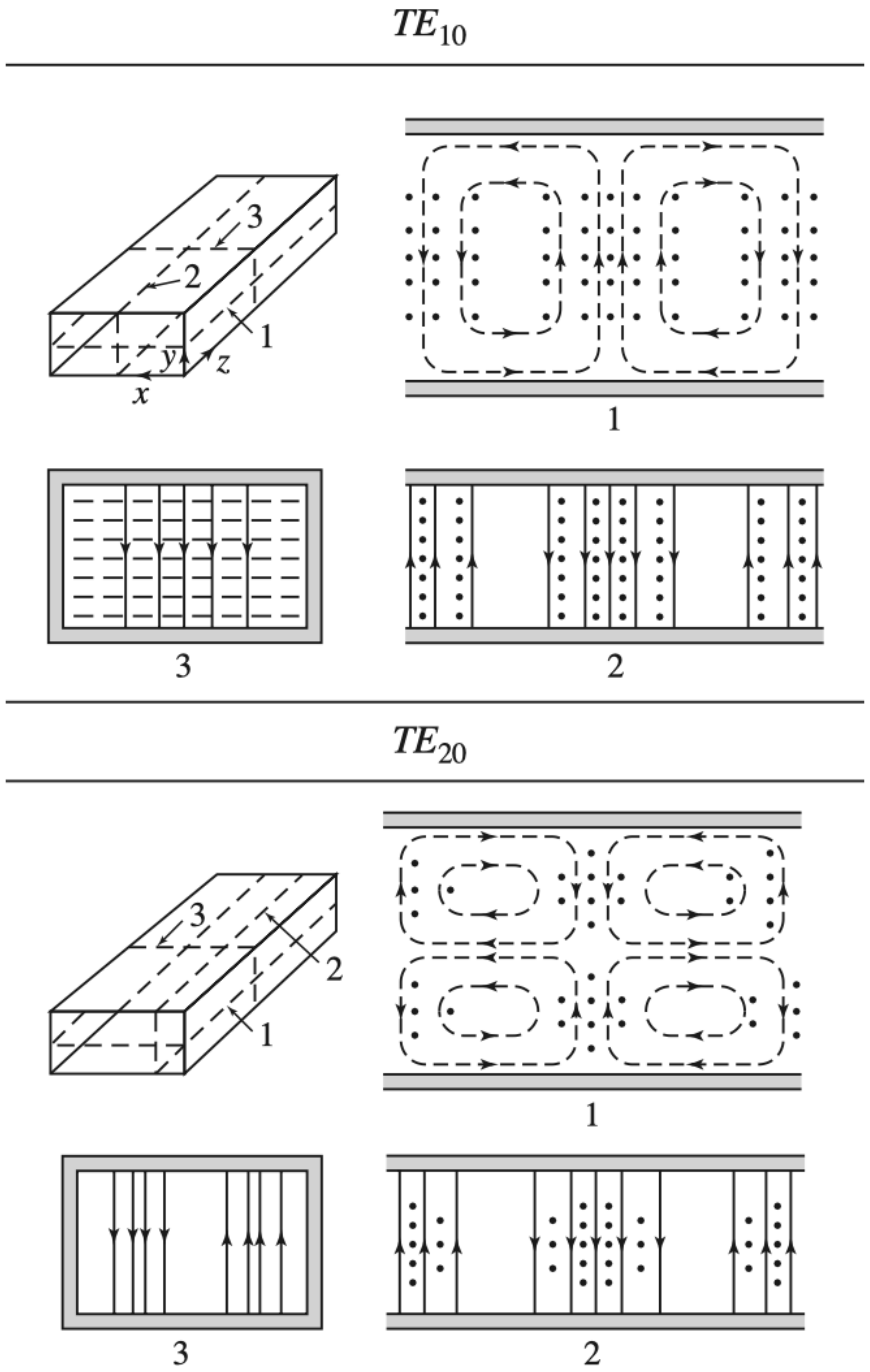
Reflection of a resonator: internal and external loss

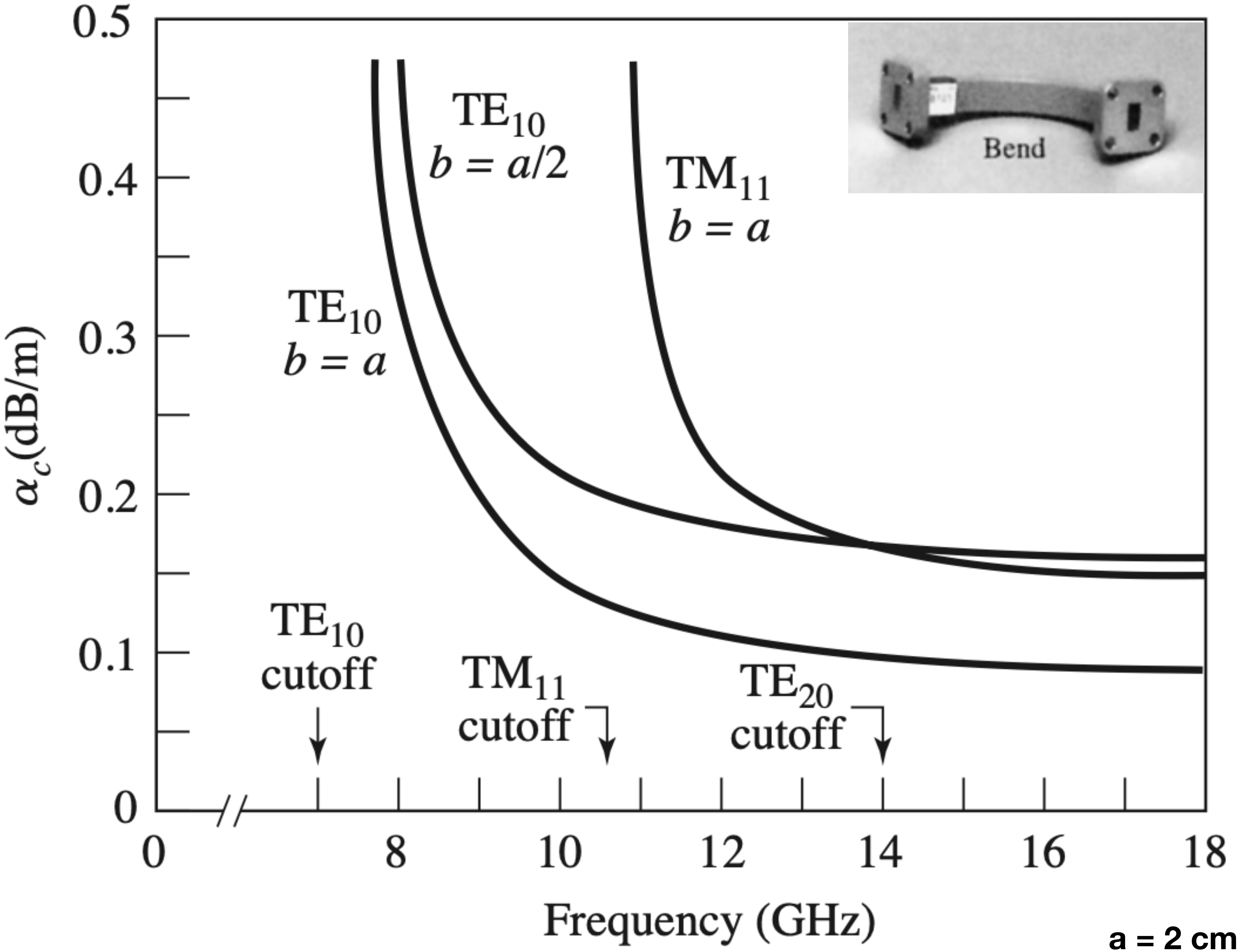
$$\Gamma(\omega, \omega_0) = \frac{2i(\frac{\omega-\omega_0}{\omega_0}) - Q_{\text{ext}}^{-1} + Q_{\text{int}}^{-1}}{2i(\frac{\omega-\omega_0}{\omega_0}) + Q_{\text{ext}}^{-1} + Q_{\text{int}}^{-1}}$$





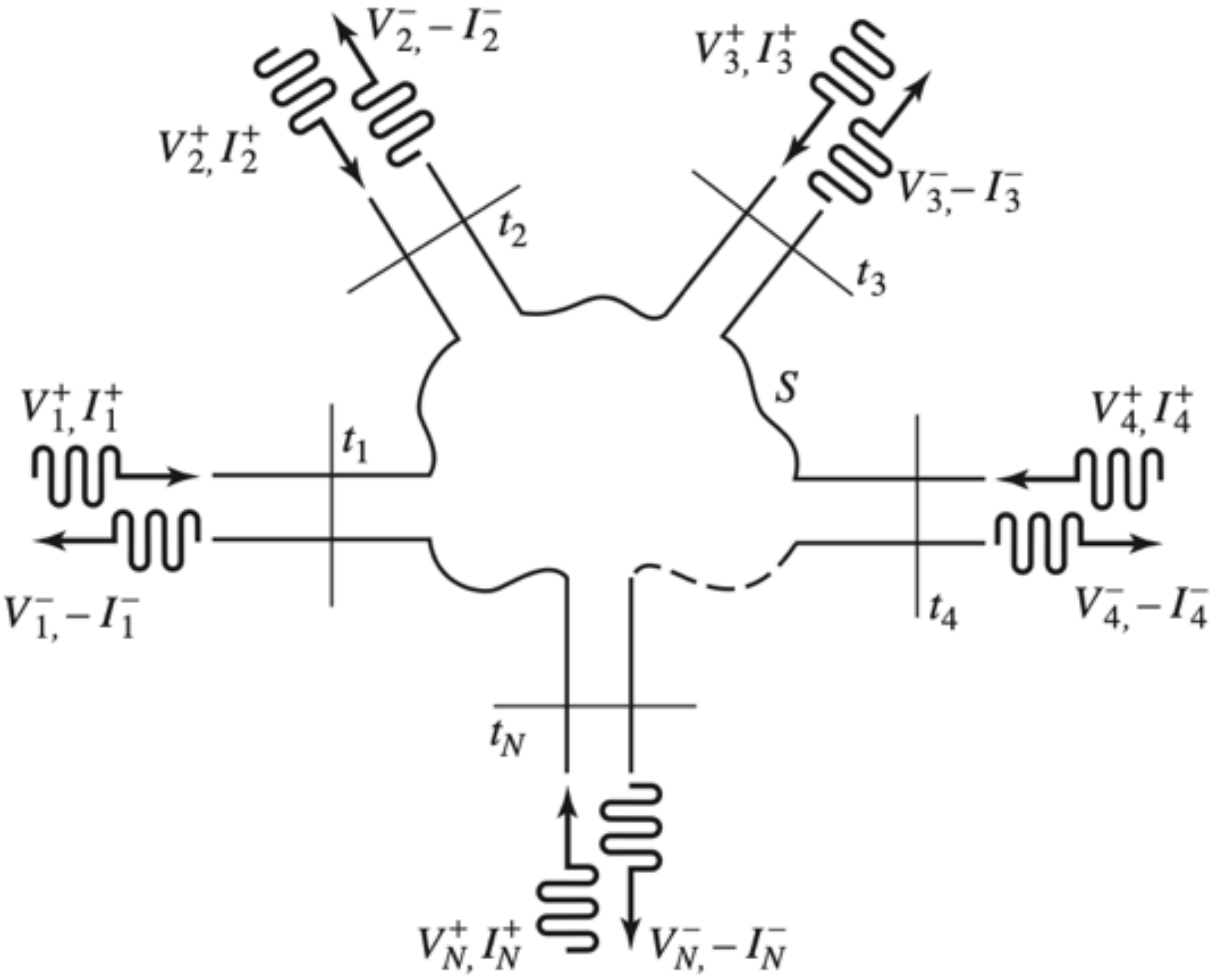
TE/TM waveguides with a propagation cut-off





a = 2 cm

Microwave network analysis



Z-matrix

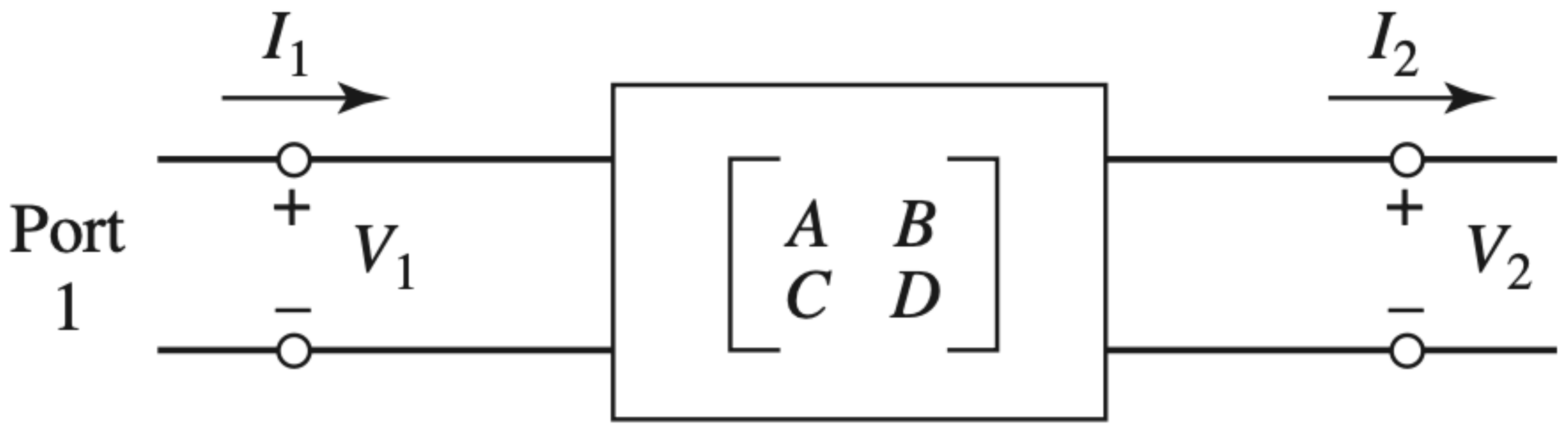
$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & & & \vdots \\ \vdots & & & \vdots \\ Z_{N1} & \cdots & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

S-matrix

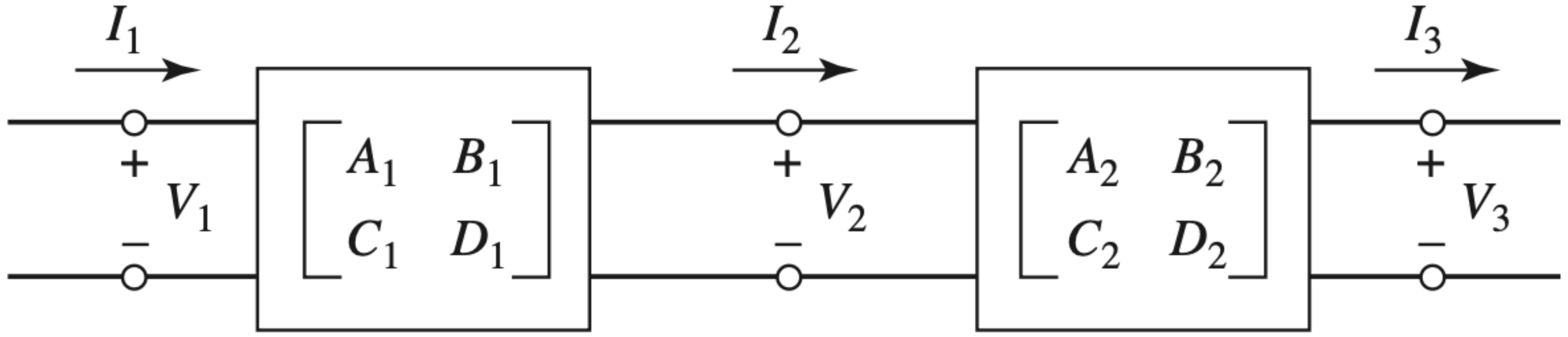
$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & & & \vdots \\ S_{N1} & \cdots & & S_{NN} \\ \vdots & & & \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

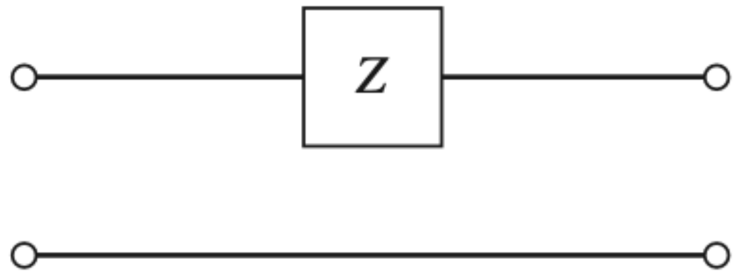
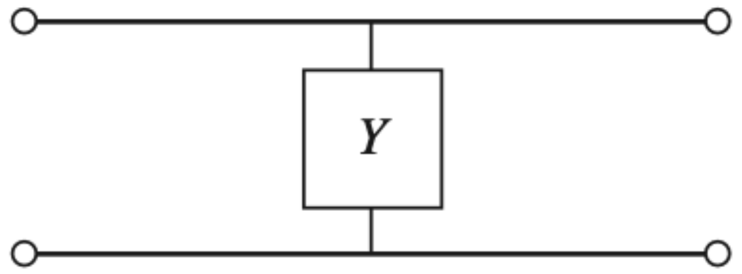
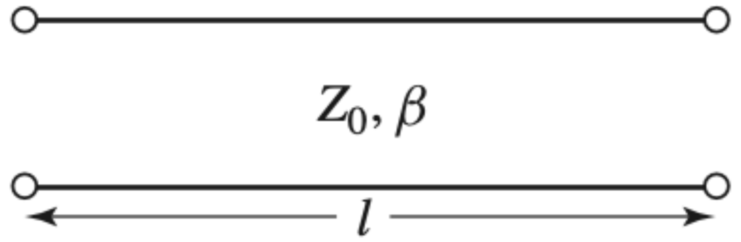
$$\hat{S} = (\hat{Z} + Z_0\hat{I})^{-1}(\hat{Z} - Z_0\hat{I})$$

From T-matrix to S-matrix



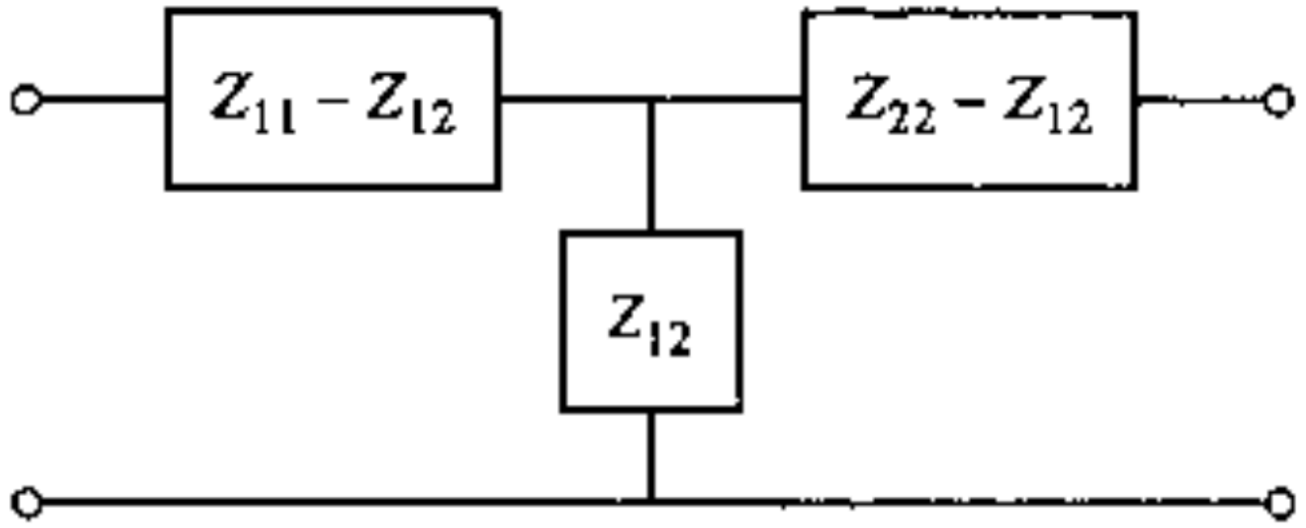
(a)



Circuit	*ABCD* Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos\beta\ell$ $C = jY_0 \sin\beta\ell$	$B = jZ_0 \sin\beta\ell$ $D = \cos\beta\ell$

$$\begin{matrix} S_{11} \\ S_{12} \\ S_{21} \\ S_{22} \end{matrix} = \begin{matrix} \dfrac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} \\ \dfrac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D} \\ \dfrac{2}{A + B/Z_0 + CZ_0 + D} \\ \dfrac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D} \end{matrix}$$

Arbitrary non-reciprocal 2-port circuit



Three-port devices

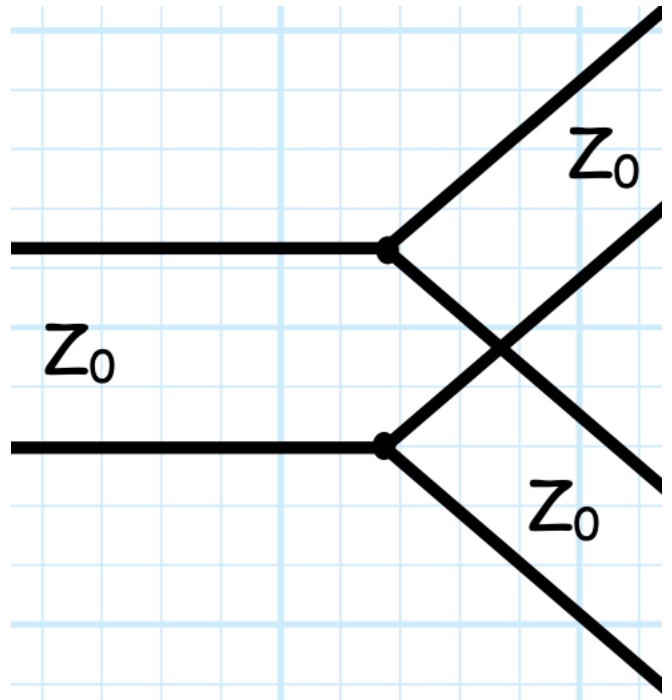
$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Matched $\quad S_{nn} = 0$

Reciprocal $\quad S = S^T$

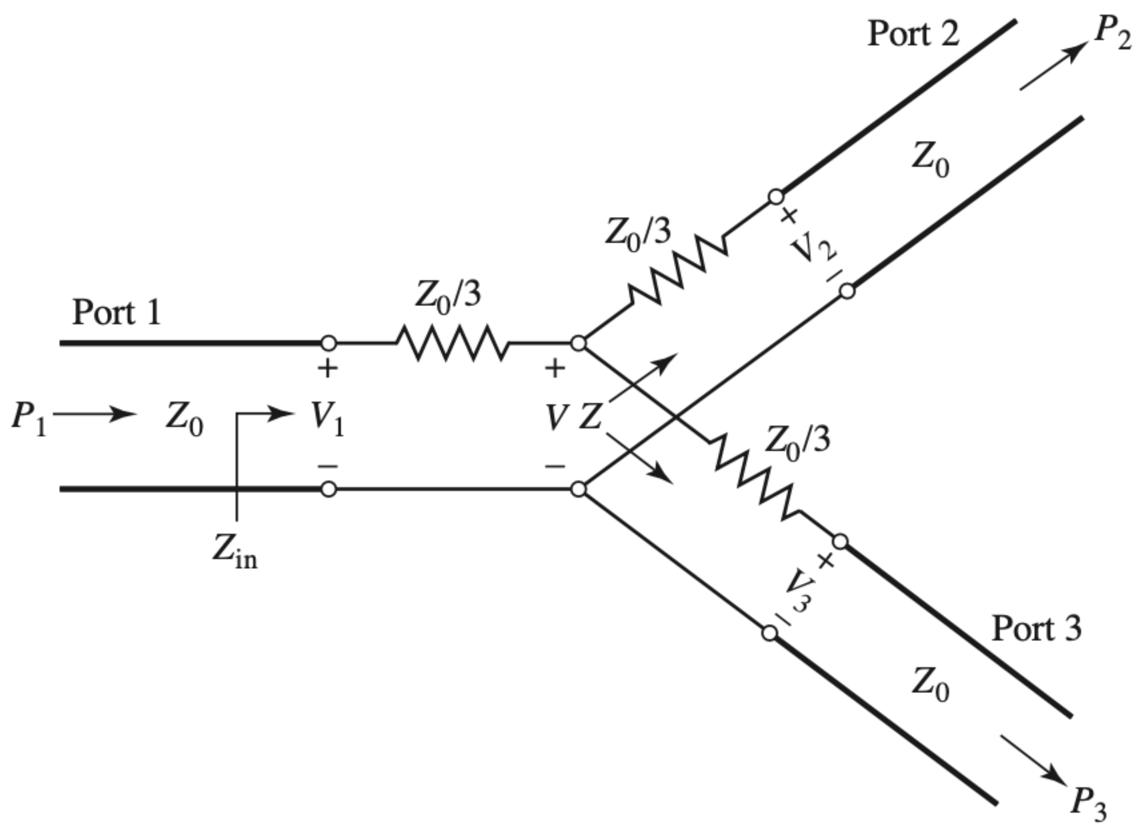
Lossless $\quad S^T S^* = I$

Lossless devider (R,L)



$$\mathcal{S} = \begin{bmatrix} -1/3 & 2/3 & 2/3 \\ 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -1/3 \end{bmatrix}$$

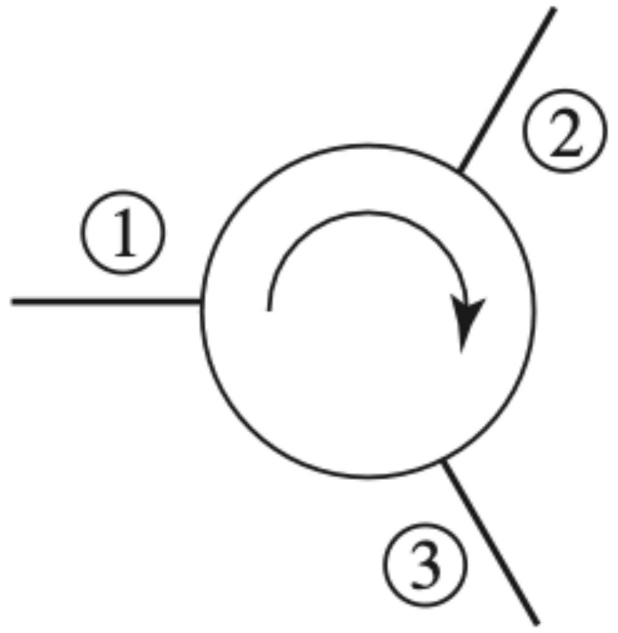
Resistive devider (R,M)



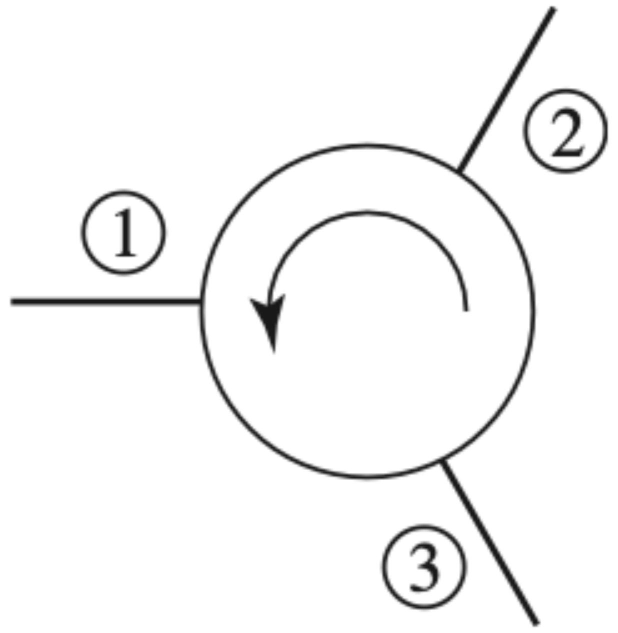
$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Circulator (M, L)

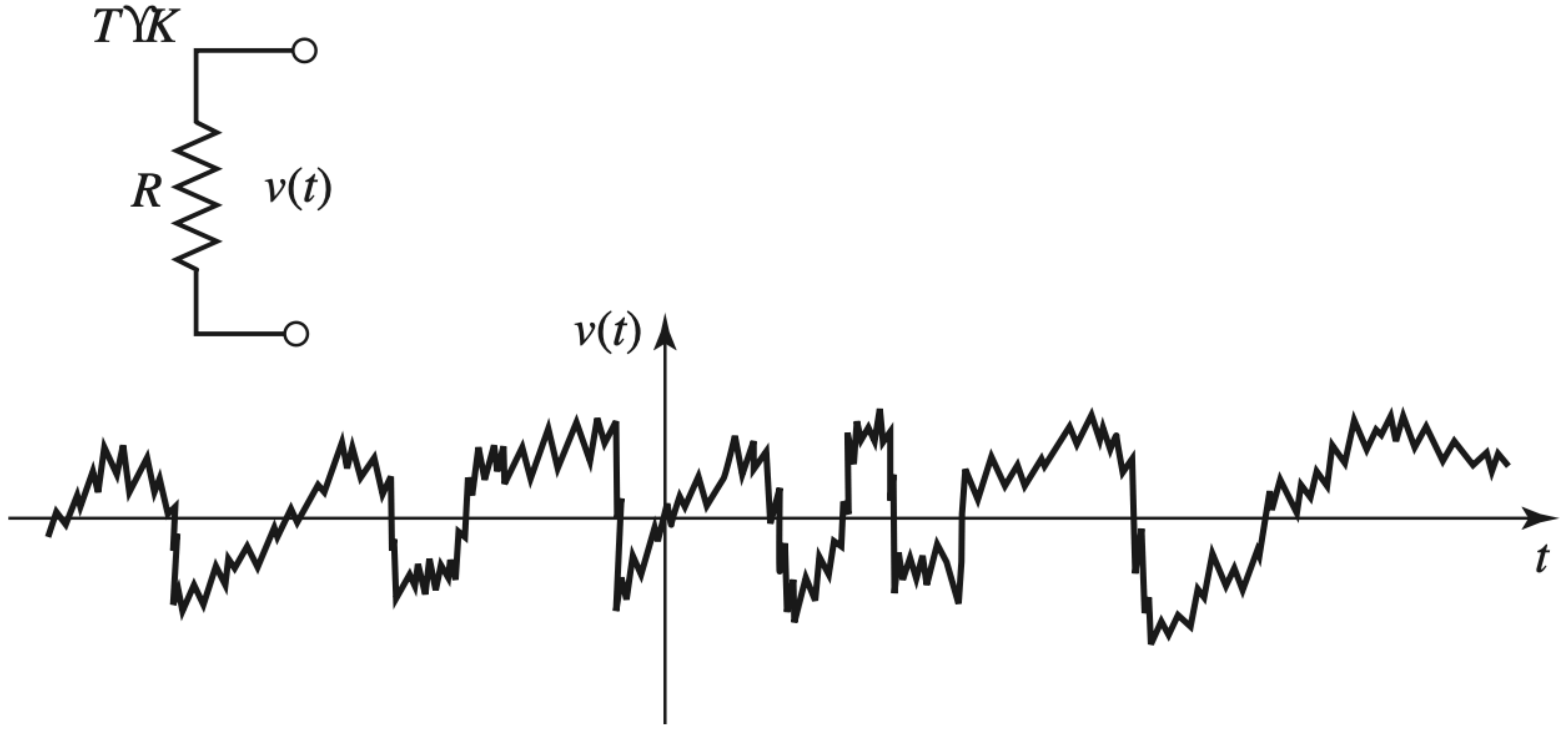
$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

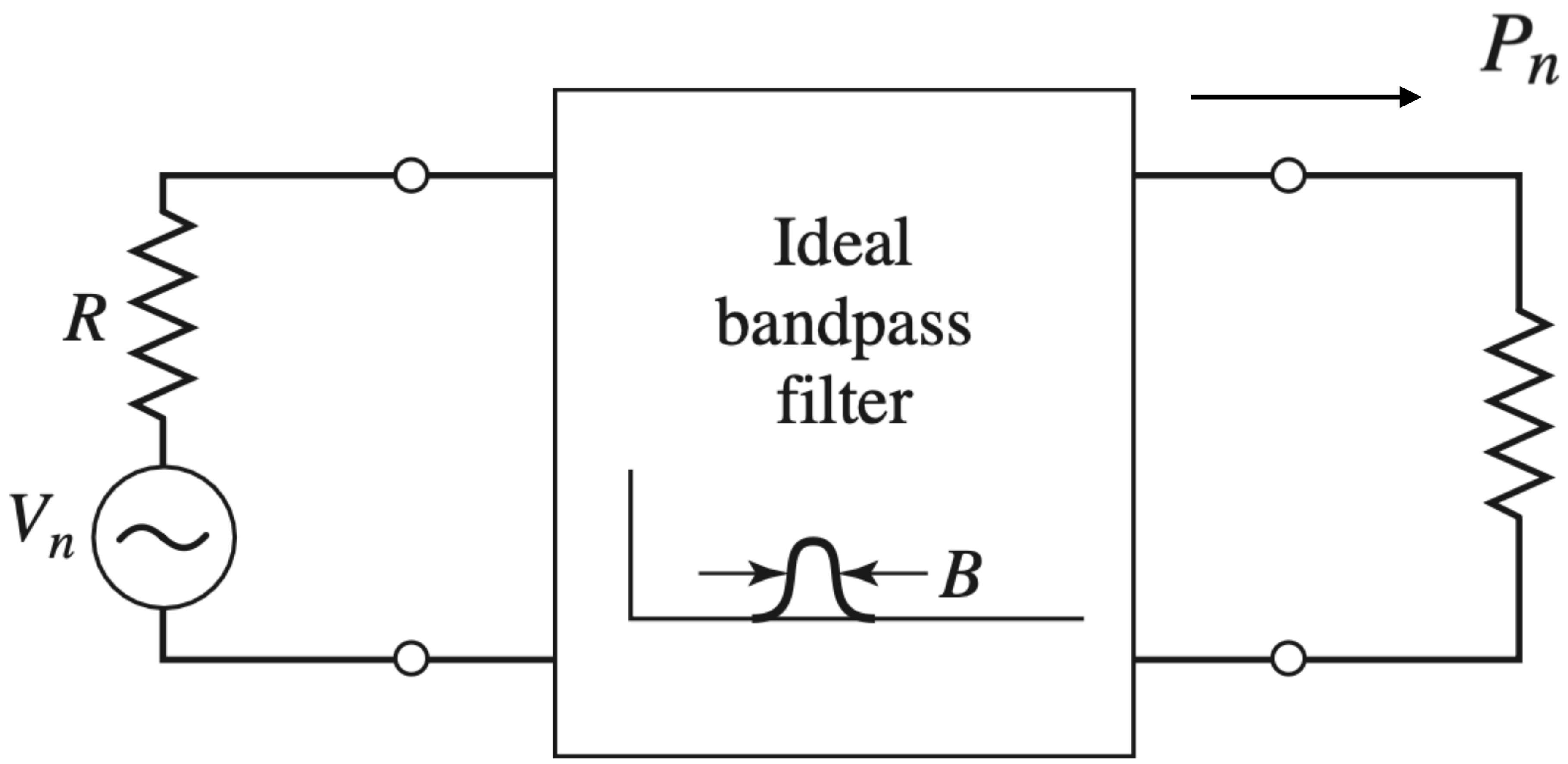


$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



Nyquist-Johnson noise



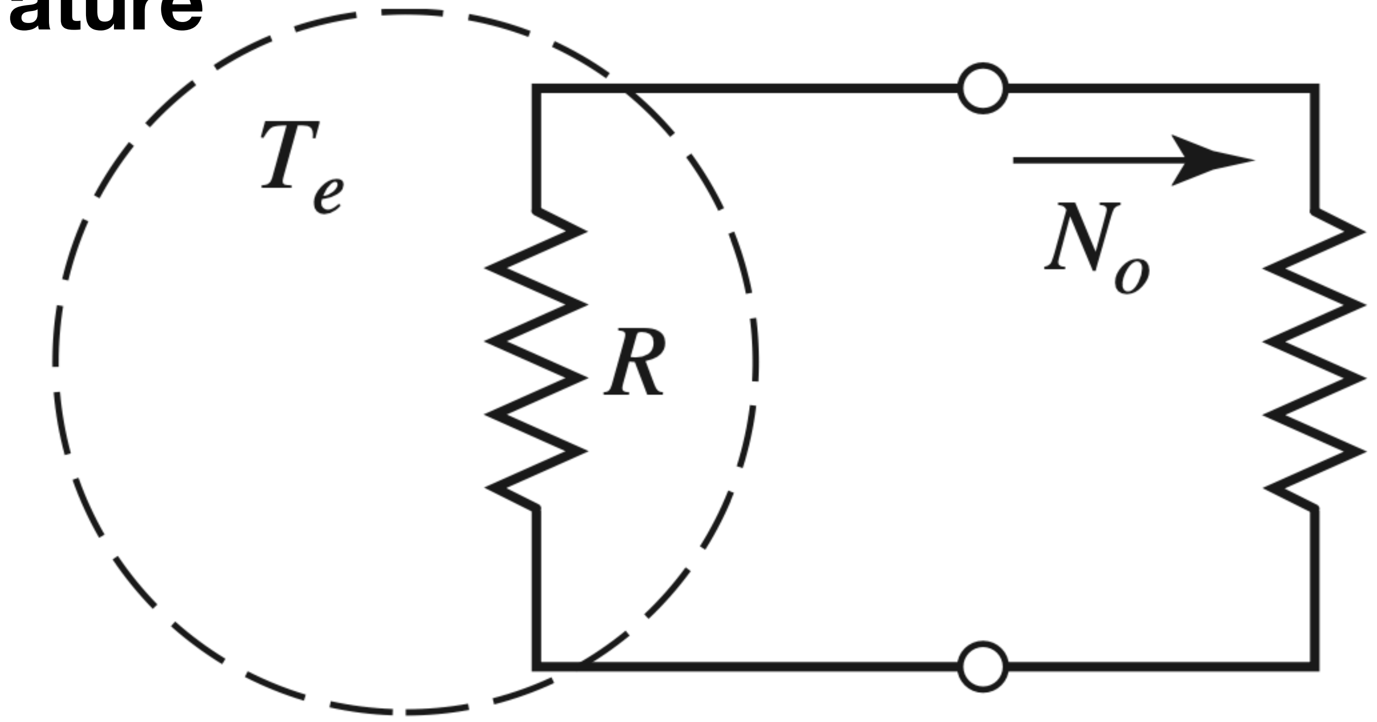


$$V_n = \sqrt{4kTBR}$$

link to stat physics

$$P_n = \left(\frac{V_n}{2R}\right)^2 R = \frac{V_n^2}{4R} = kTB$$

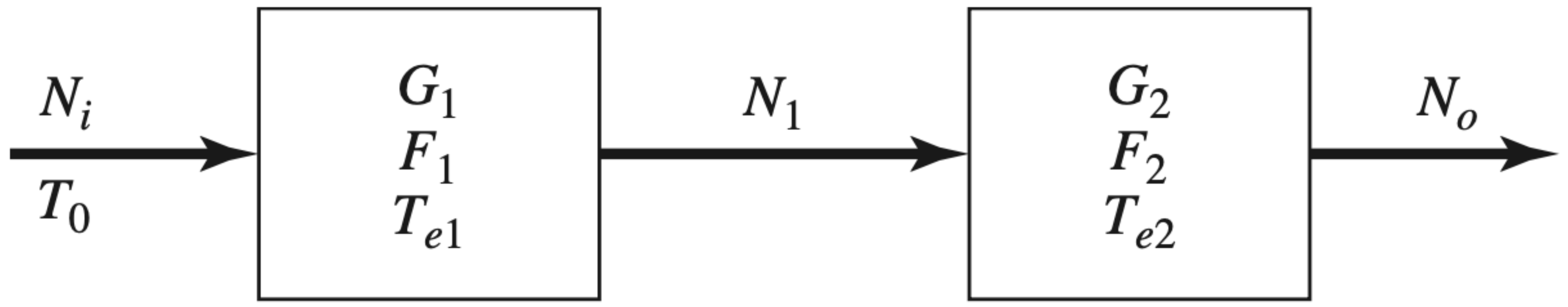
Effective noise temperature



$$T_e = \frac{N_o}{kB}$$

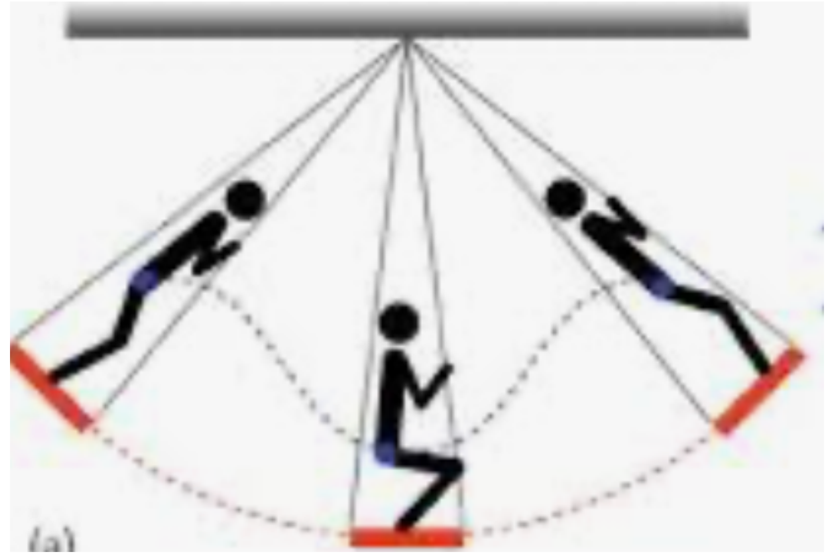
Cascade $$T_{\text{cas}} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \cdots$$

gain of first stage



(parametric) Amplification

M. Devoret lecture @ CdF

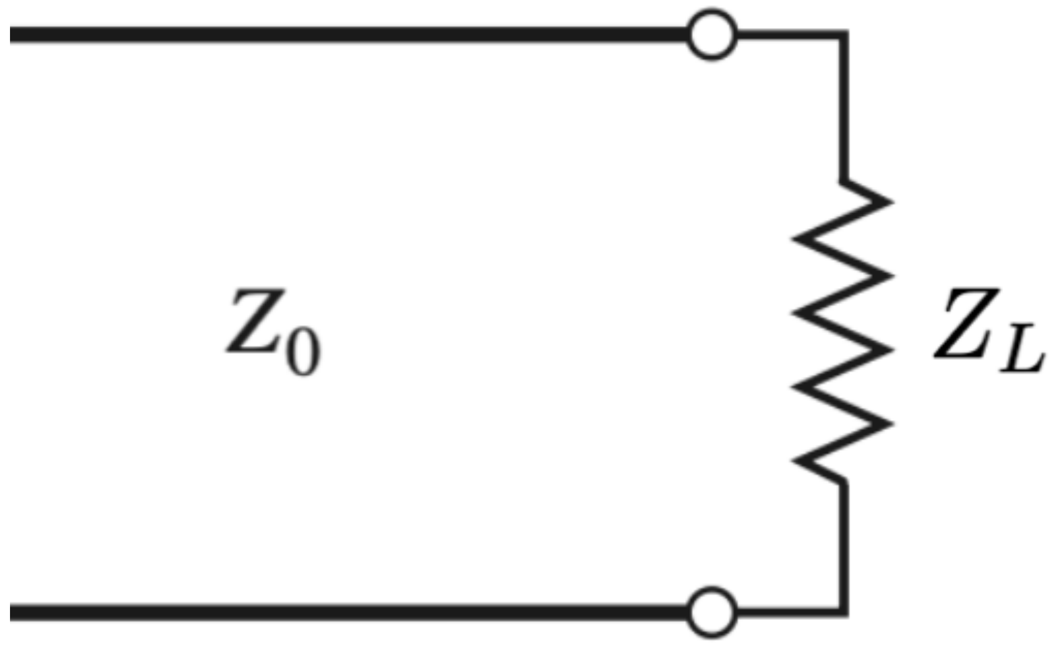


Amplifier

=

negative

load resistance…

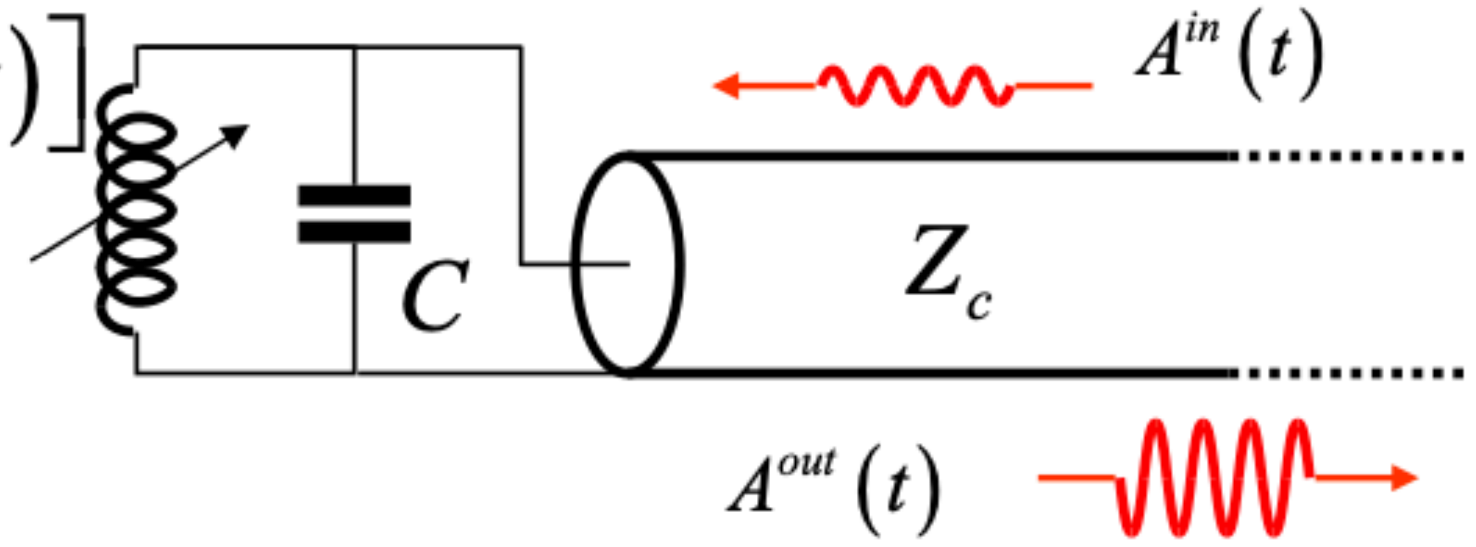


$$|\Gamma| > 1$$

SCATTERING MATRIX OF DPA FOR ARBITRARY ω

$$L^{-1}(t) = L_0^{-1}\left[1 + 2\varepsilon \sin(2\omega_0 t)\right]$$

Stiff pump condition



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Gamma = \frac{1}{2Z_c C}$$

$$\ddot{\Phi} + 2\Gamma\dot{\Phi} + \omega_0^2\Phi - i\omega_0^2\varepsilon\Phi\left(e^{2i\omega_0 t} - e^{-2i\omega_0 t}\right) = 4\Gamma V^{in}(t) \qquad V^{out}(t) = \dot{\Phi} - V^{in}(t)$$

Harmonic balance + RWA $\Rightarrow$

$$i\omega_0\left[\frac{(\omega_0 - \omega)}{i\Gamma} - 1\right]\Phi[\omega] - i\frac{\omega_0^2\varepsilon}{2\Gamma}\Phi[\omega - 2\omega_0] = 2V^{in}[\omega]$$

After a few steps:

$$\begin{bmatrix} A^{out}[+\omega_S] \\ A^{out}[-\omega_S] \\ A^{out}[+\omega_I] \\ A^{out}[-\omega_I] \end{bmatrix} = \begin{bmatrix} r & 0 & 0 & s \\ 0 & r^* & s^* & 0 \\ 0 & s^* & r^* & 0 \\ s & 0 & 0 & r \end{bmatrix} \begin{bmatrix} A^{in}[+\omega_S] \\ A^{in}[-\omega_S] \\ A^{in}[+\omega_I] \\ A^{in}[-\omega_I] \end{bmatrix}$$

$$A = V/\sqrt{Z_c}$$

$$r = \frac{1 + \vartheta^2 + \zeta^2}{(1 - i\vartheta)^2 - \zeta^2}$$

$$s = \frac{2\zeta}{(1 - i\vartheta)^2 - \zeta^2}$$

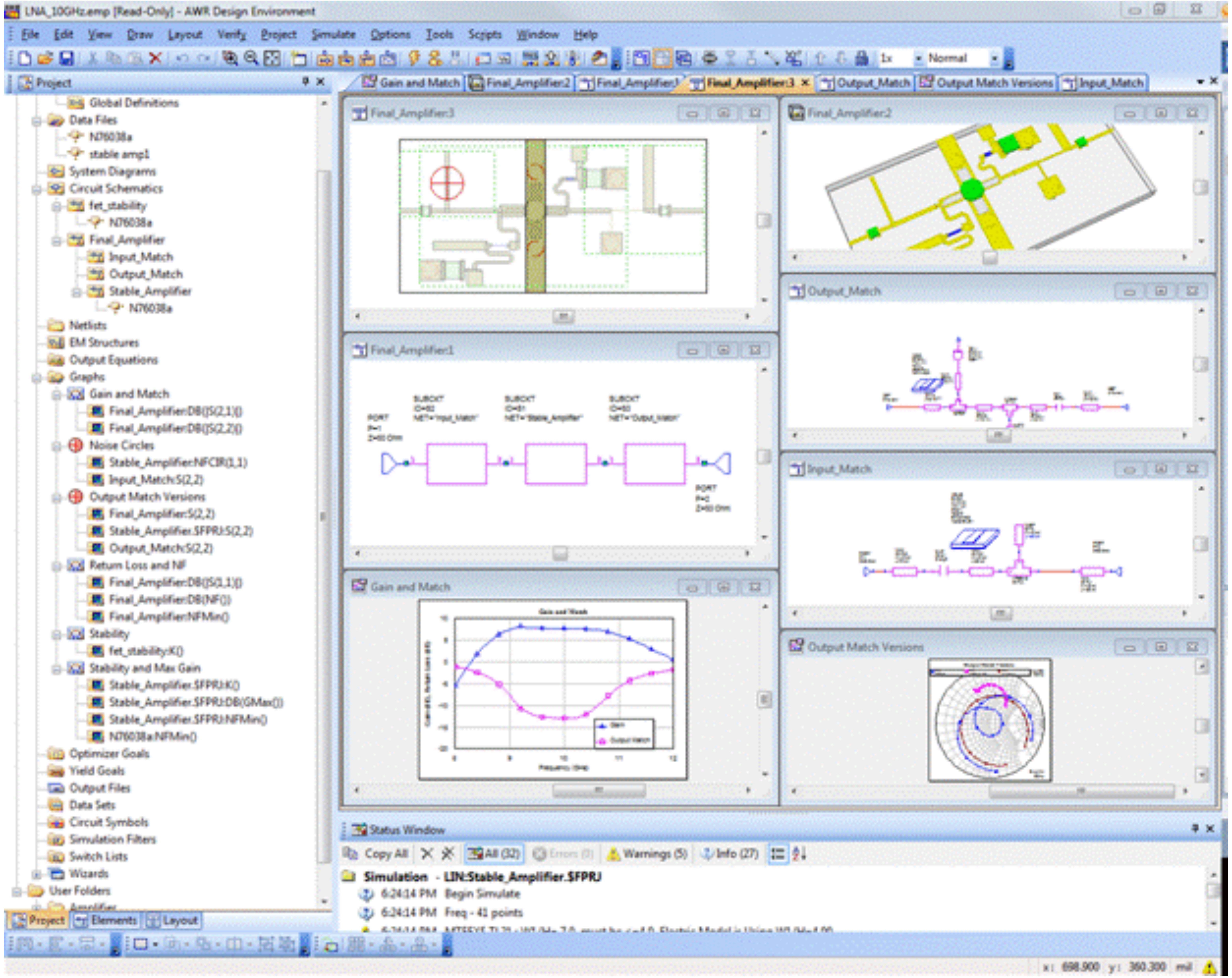
$$\zeta = \varepsilon\frac{\omega_0}{2\Gamma} < 1$$

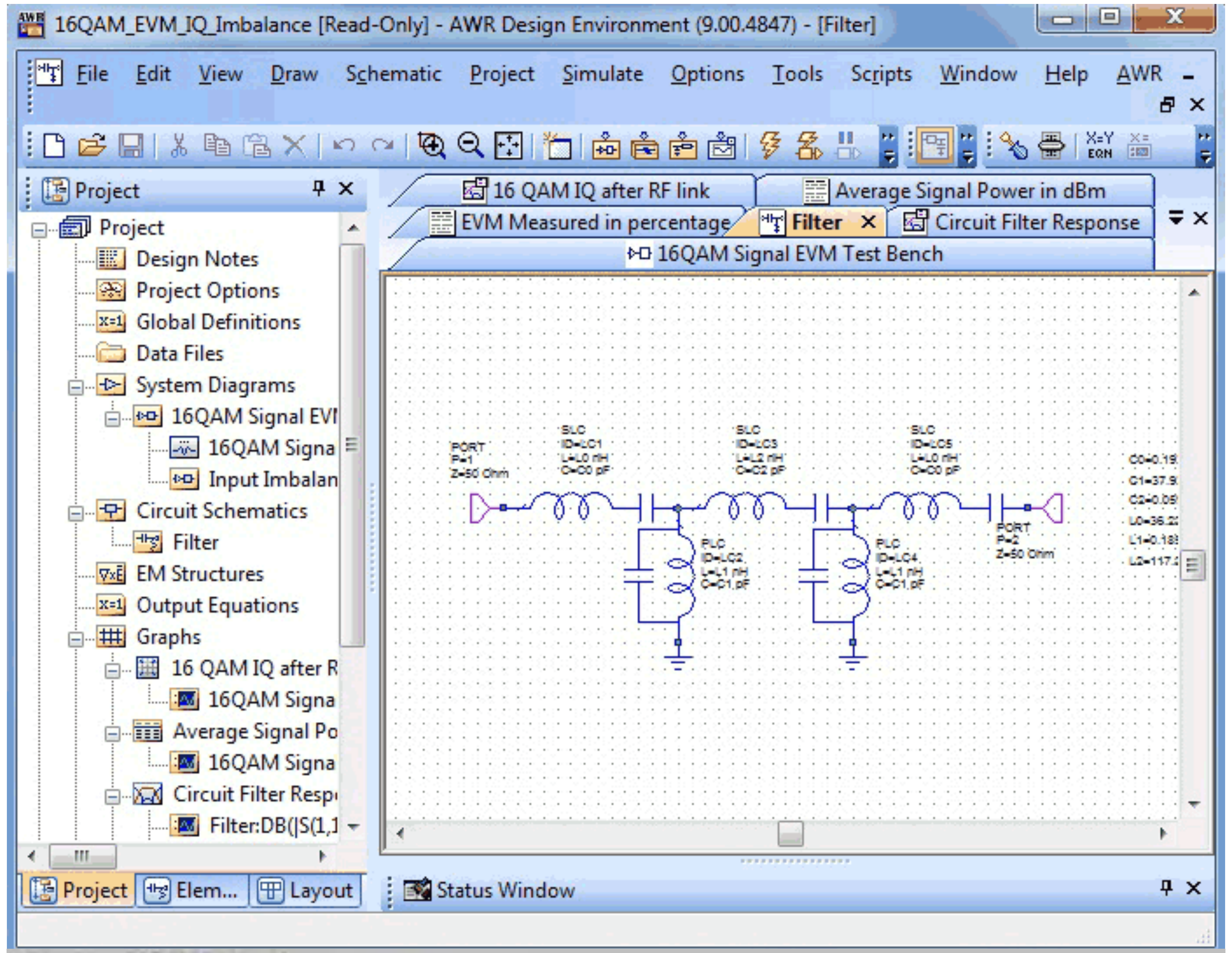
SYMPLECTICITY: $\begin{cases} |r|^2 - |s|^2 = 1 \\ rs^* - r^*s = 0 \end{cases}$

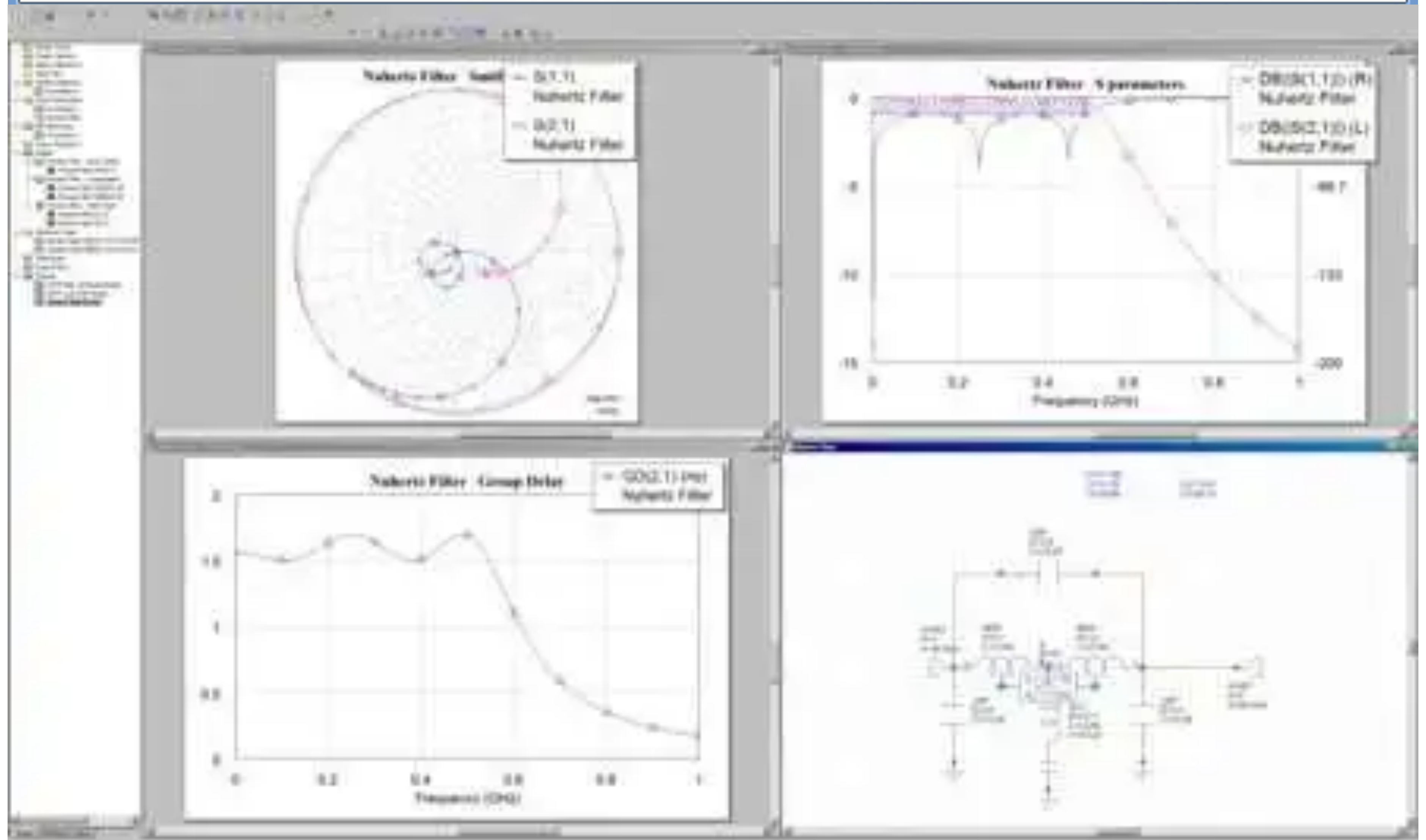
$$\vartheta = \tan\theta = \frac{\omega_S - \omega_0}{\Gamma}$$

08-VI-21c

Microwave office suite for lumped-element simulations







HFSS suite for distributed systems simulations

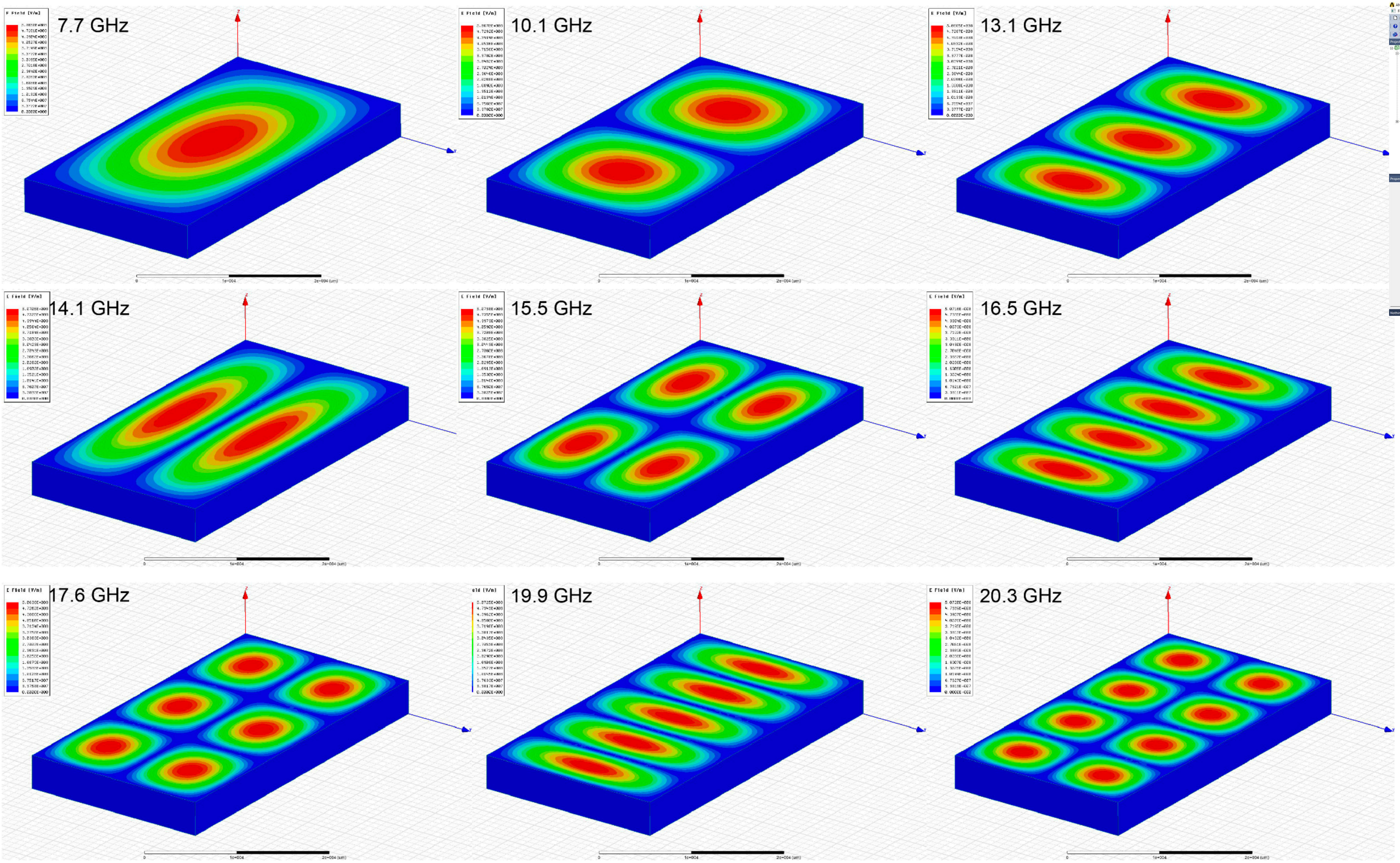
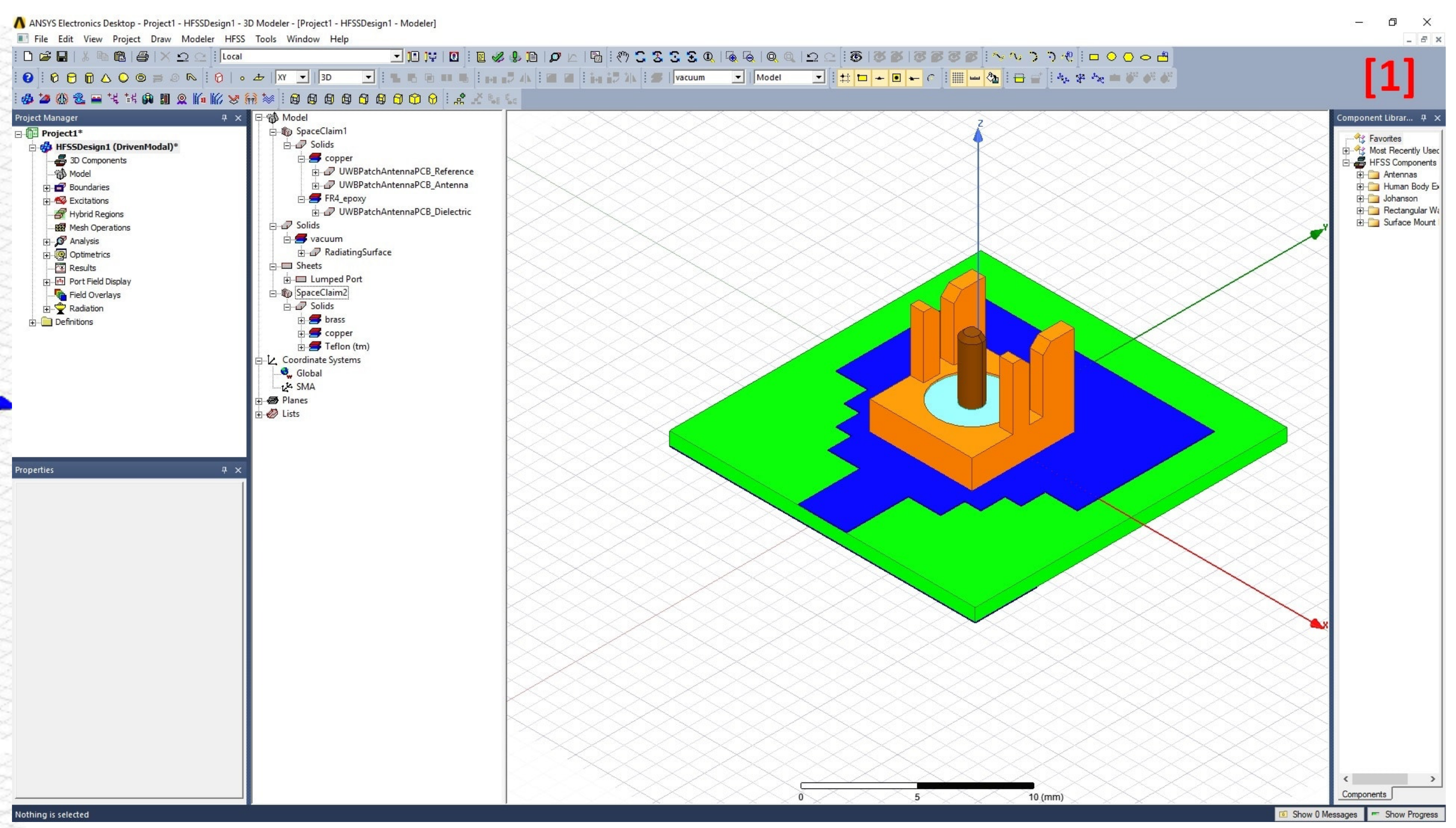
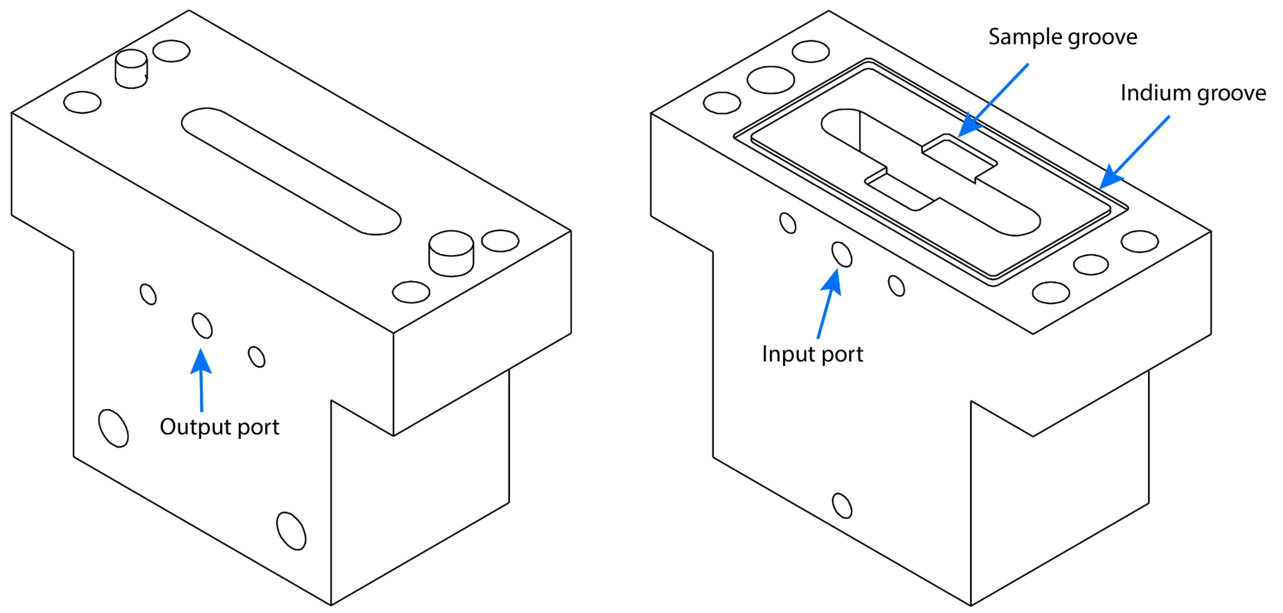
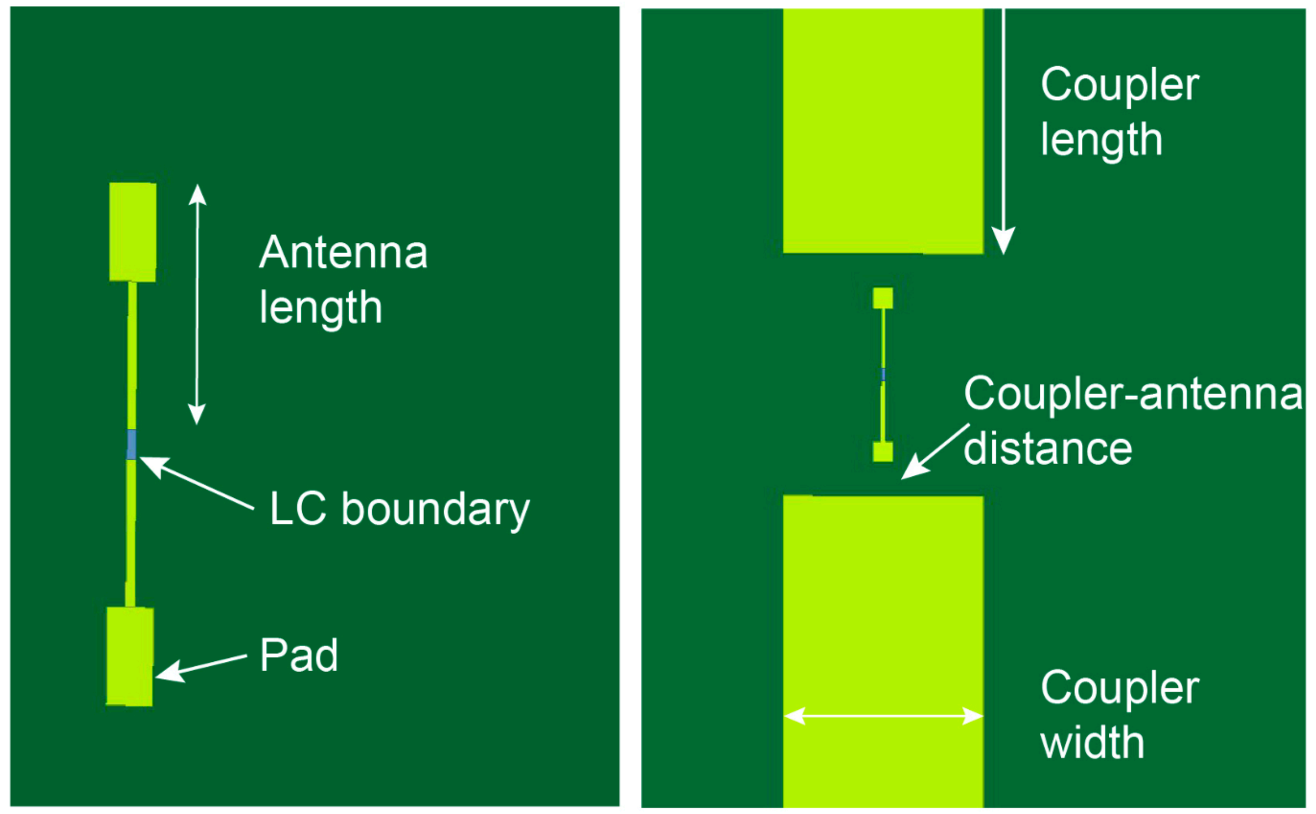


Figure 5.5: Electric field density of resonant modes in a rectangular cavity.



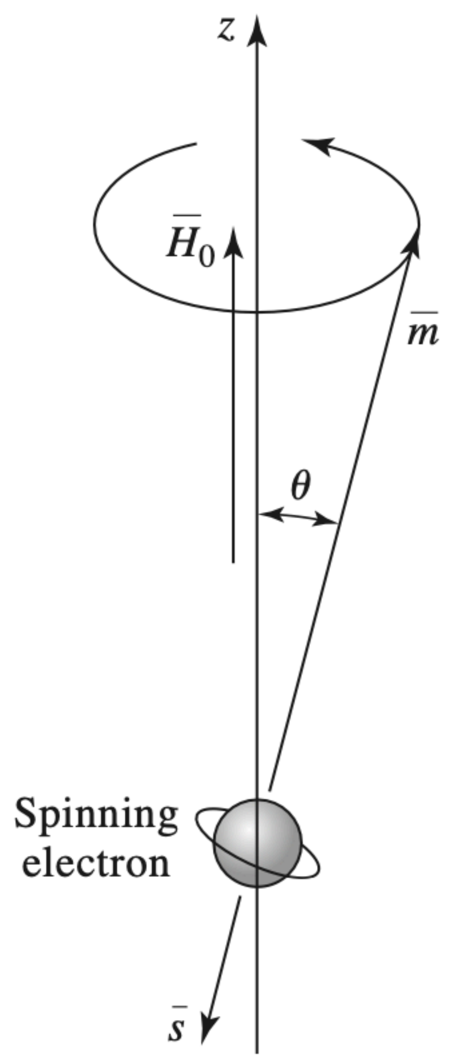




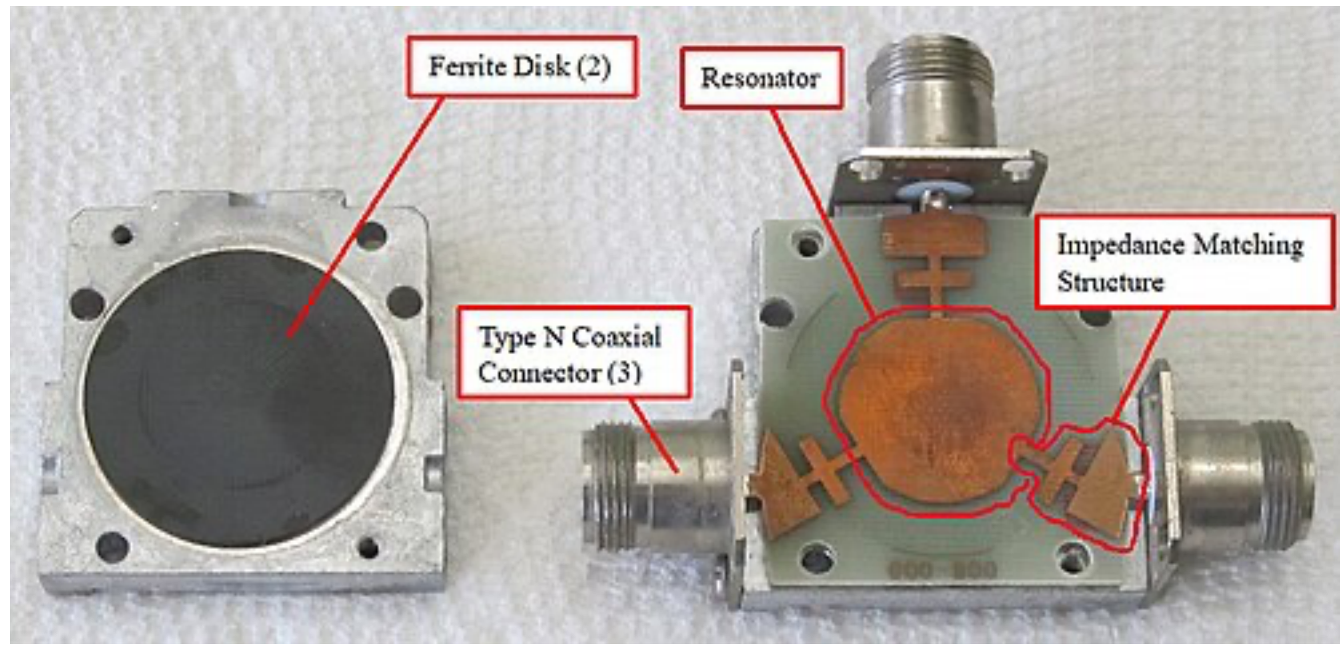
Microwave engineering in physics 2025:
what we did not cover

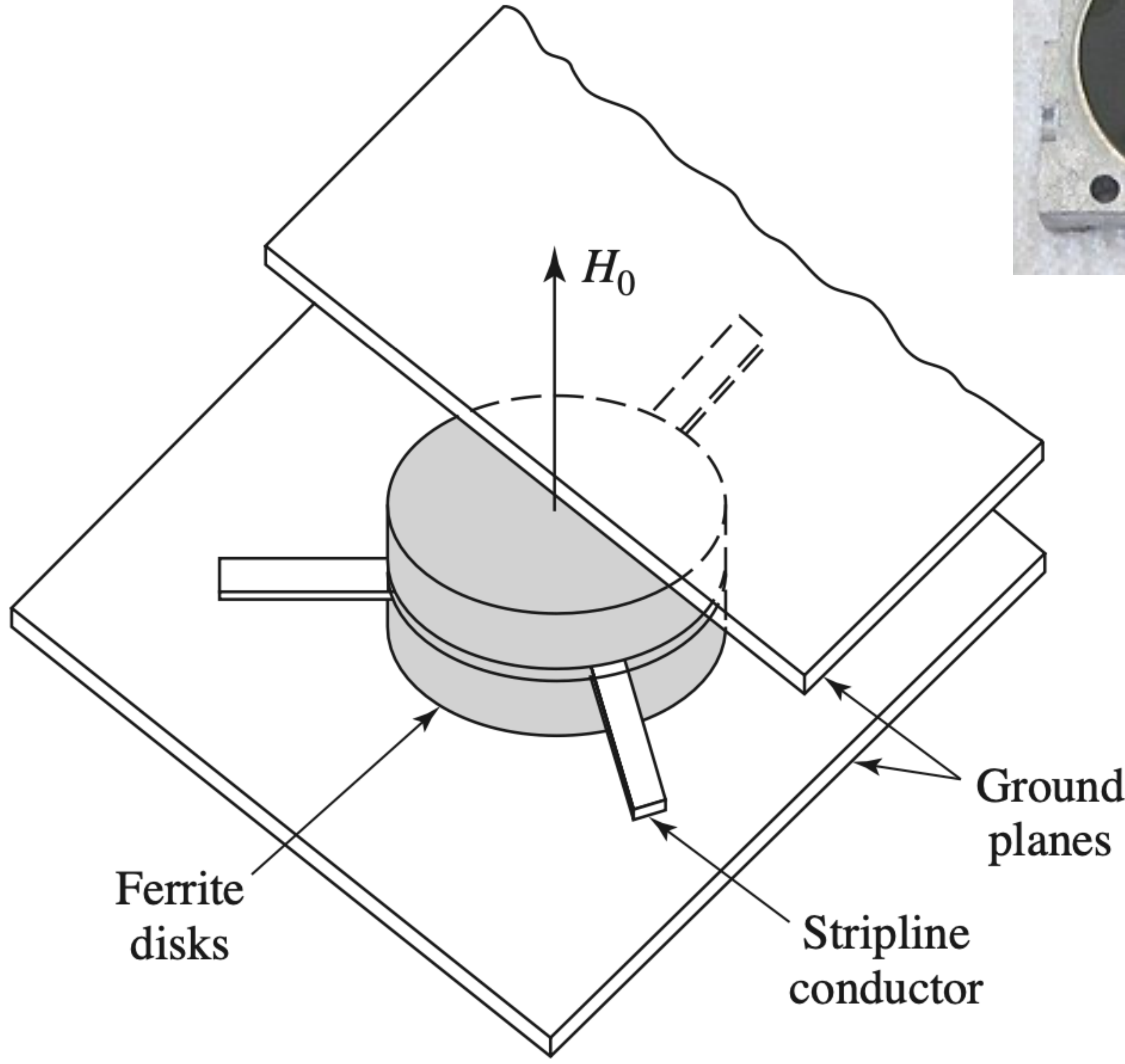
Non-reciprocal devices

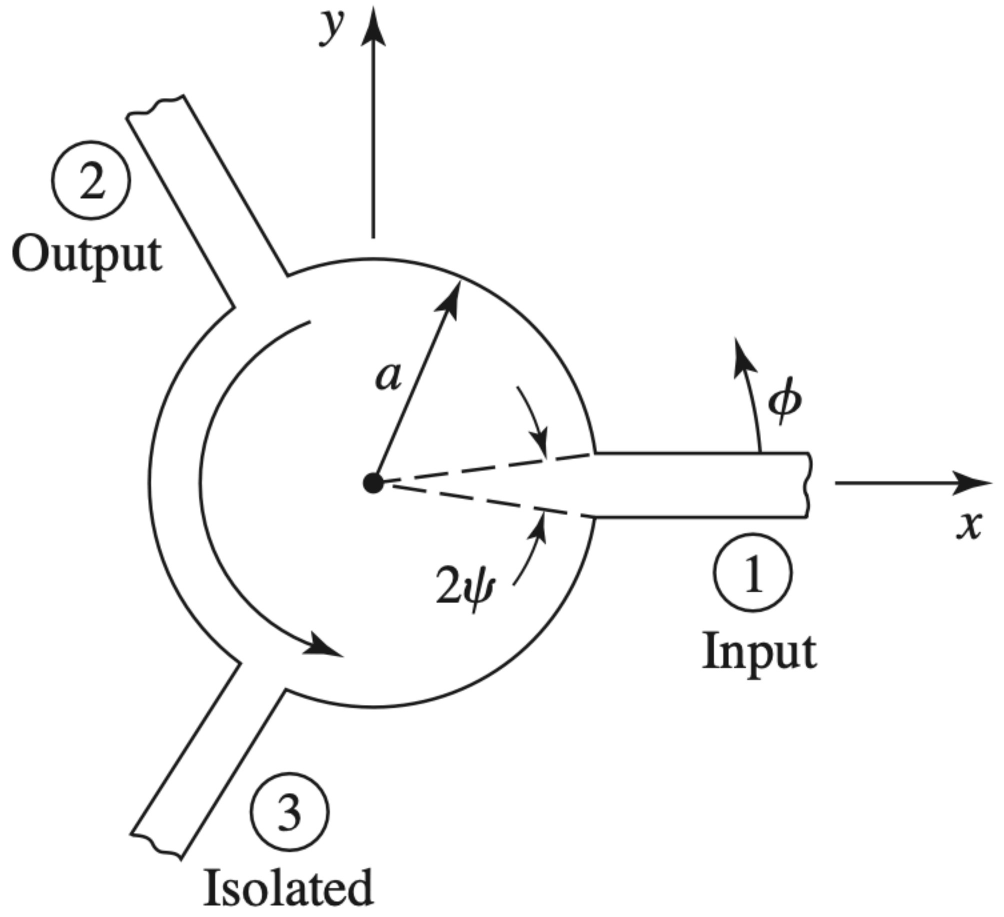
$$\begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix} = [\mu] \begin{bmatrix} H_\rho \\ H_\phi \\ H_z \end{bmatrix}$$



Larmor precession of magnetization







$$[\mu] = \mu_0([U] + [\chi]) = \begin{bmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & \mu_0 \end{bmatrix}$$

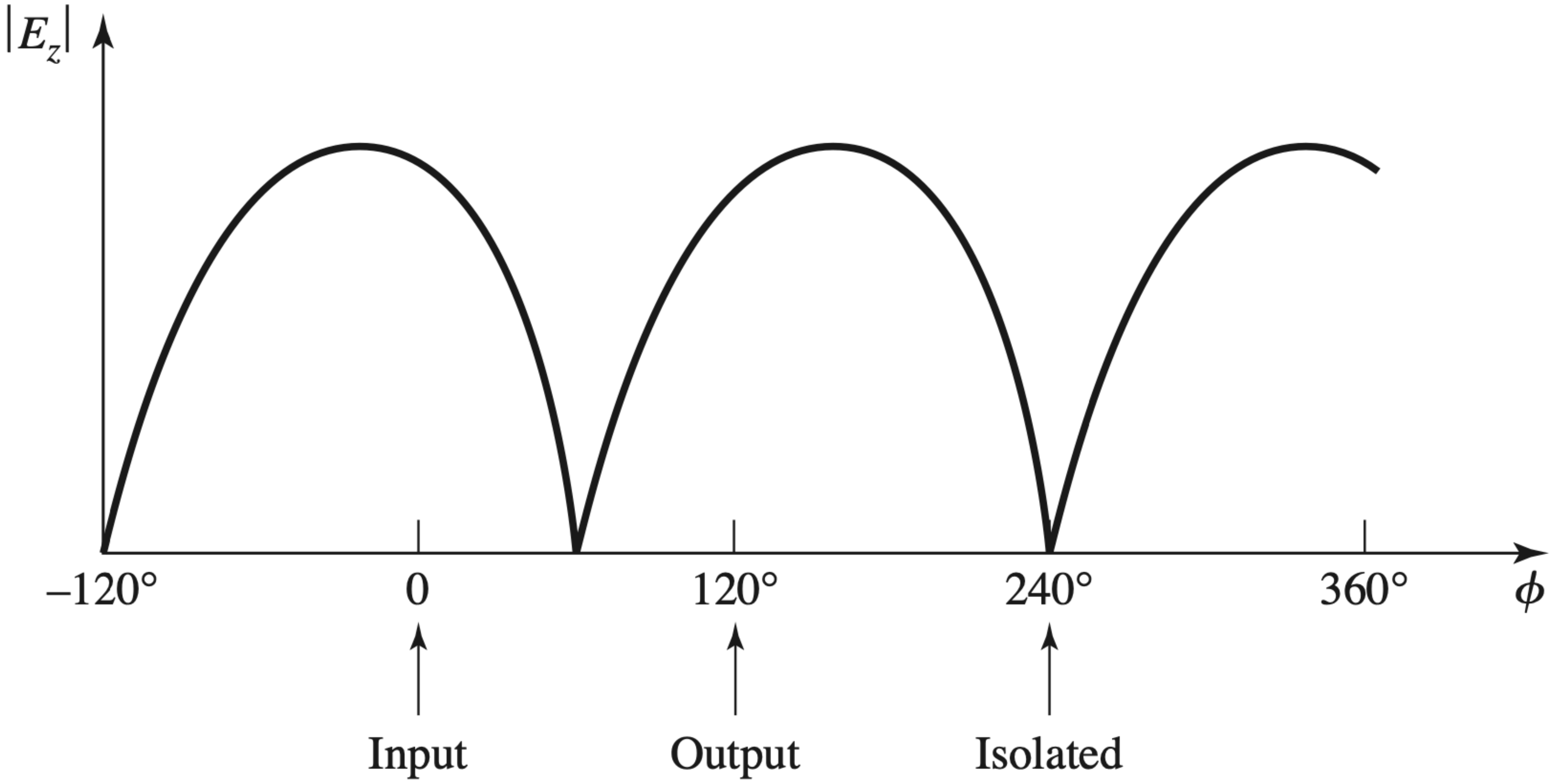
non-reciprocity

$$\mu = \mu_0(1 + \chi_{xx}) = \mu_0(1 + \chi_{yy}) = \mu_0 \left(1 + \frac{\omega_0 \omega_m}{\omega_0^2 - \omega^2}\right)$$

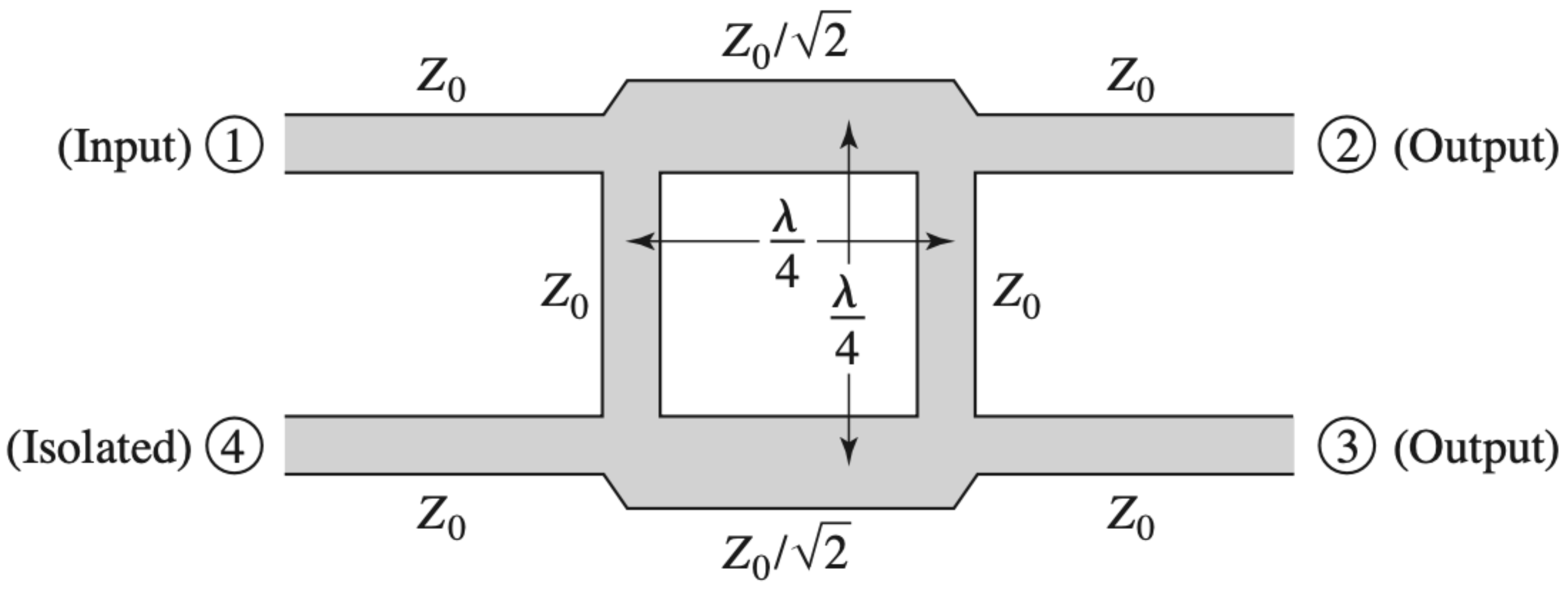
$$\kappa = -j\mu_0\chi_{xy} = j\mu_0\chi_{yx} = \mu_0 \frac{\omega\omega_m}{\omega_0^2 - \omega^2}.$$

Larmor frequency

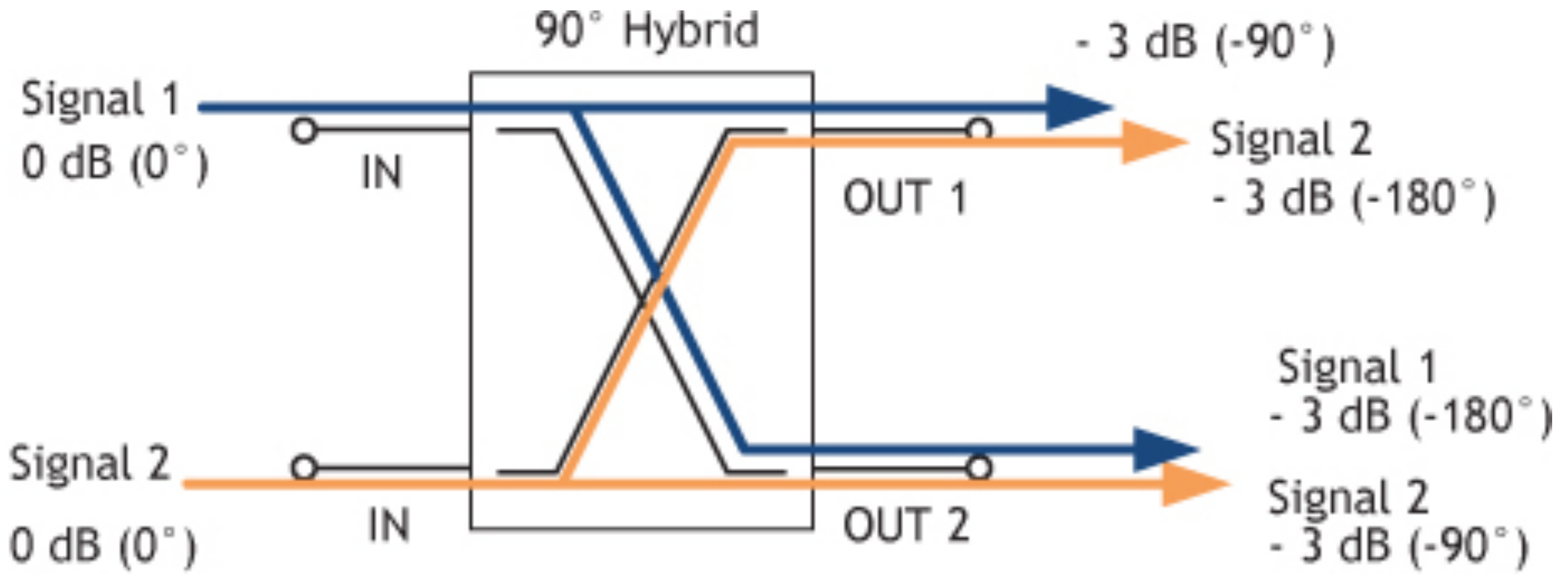
Sets the difference between clockwise and counterclockwise waves

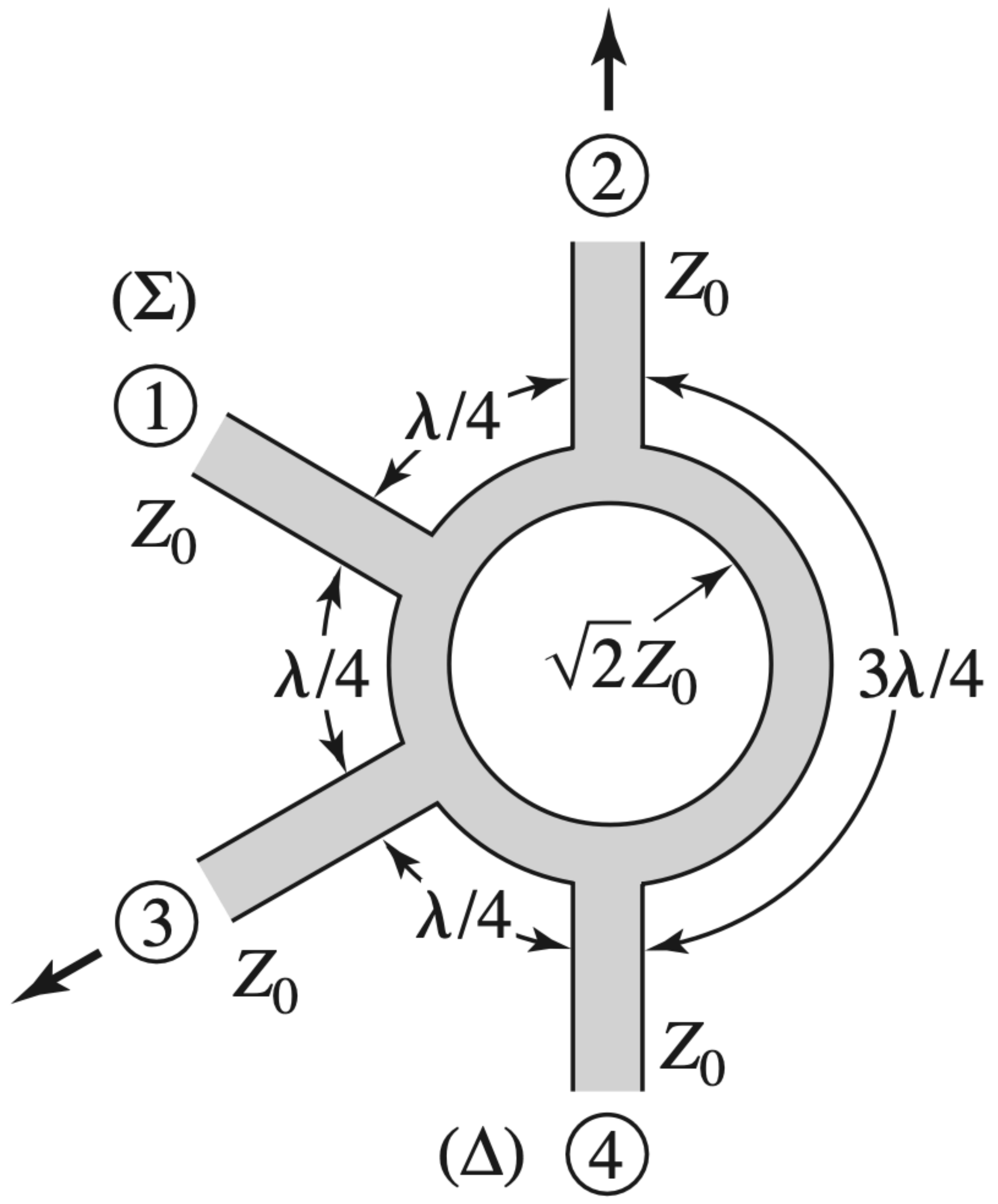


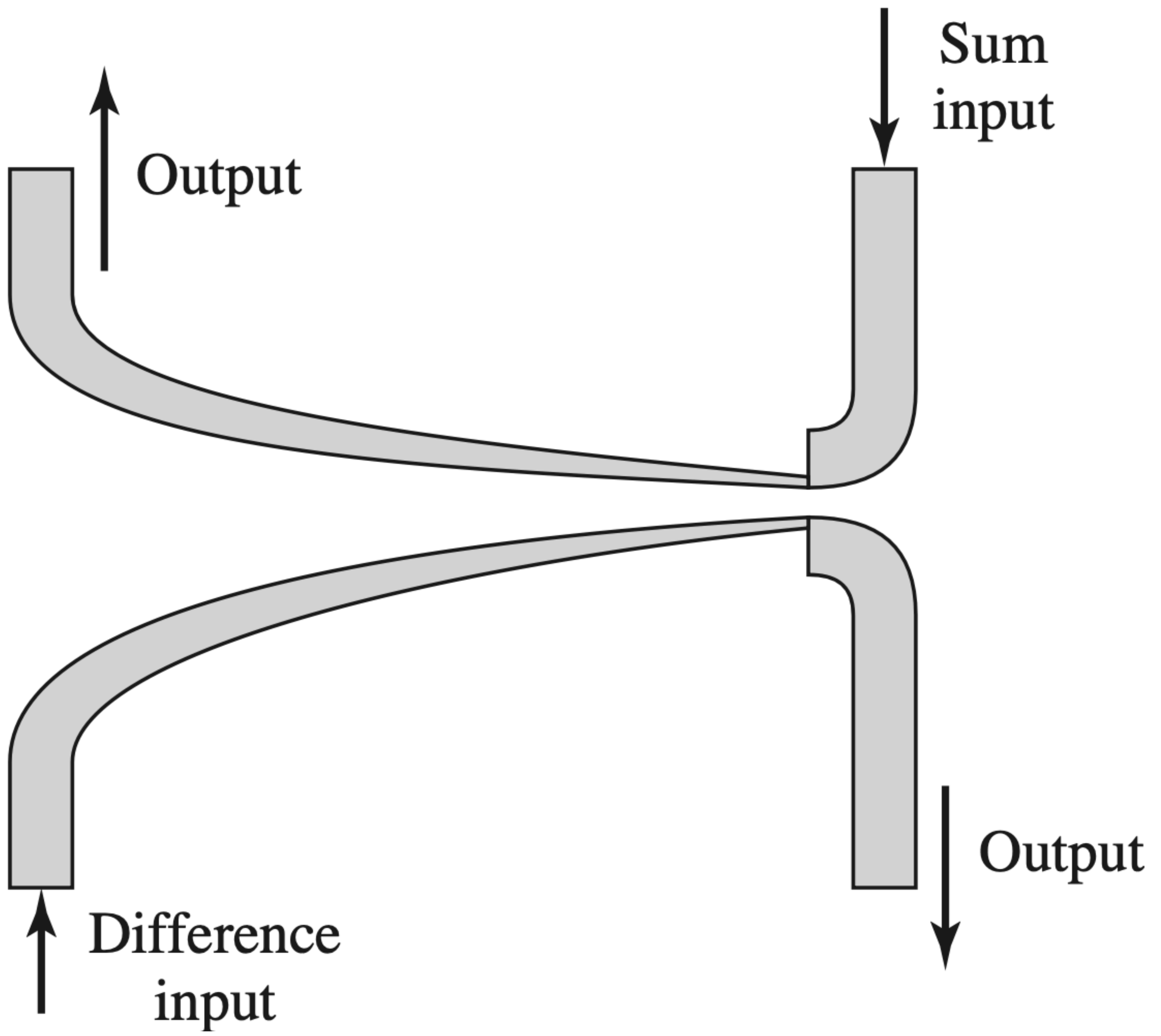
Hybrid couplers



Analog of optical beam-splitters



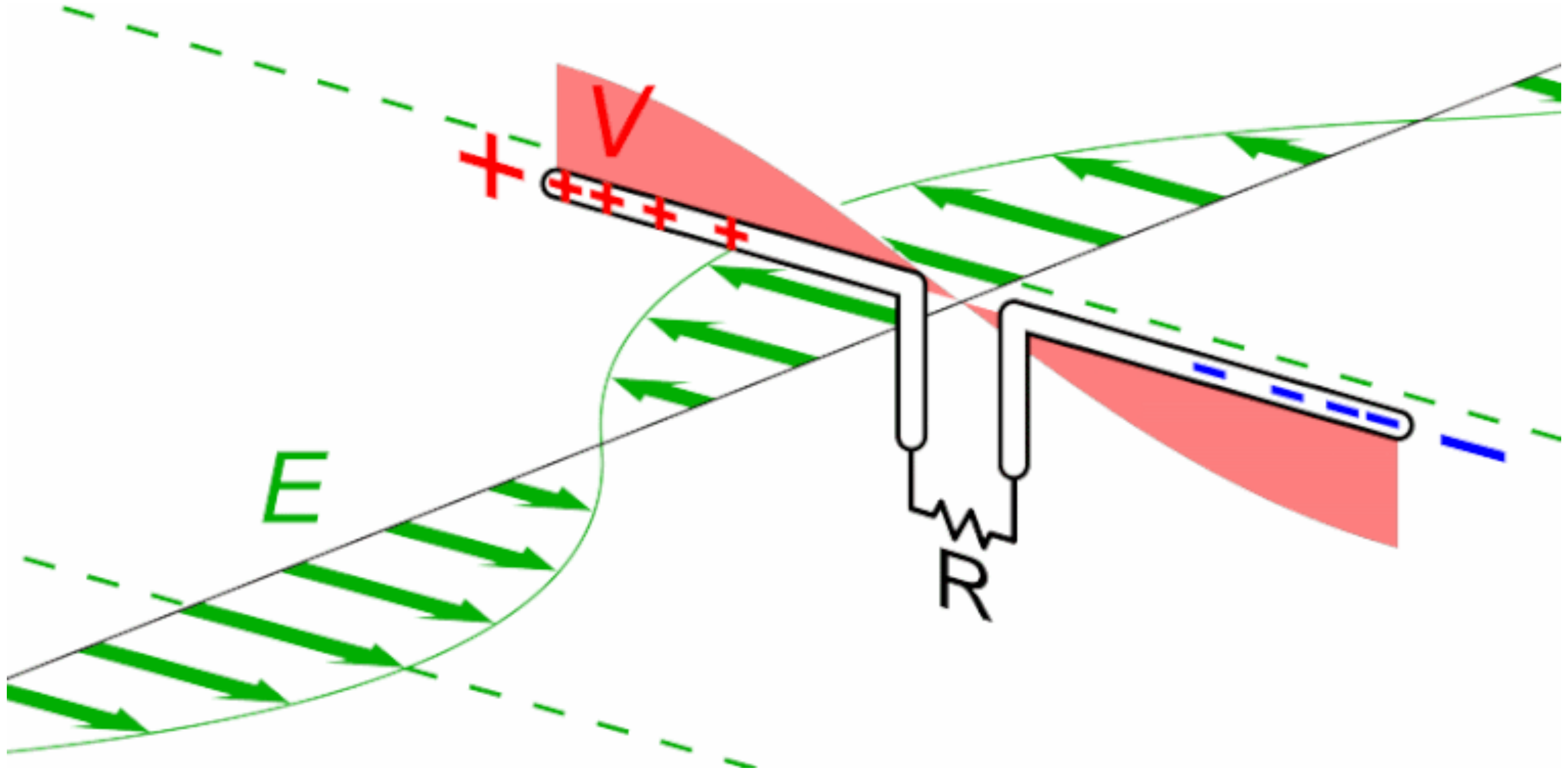


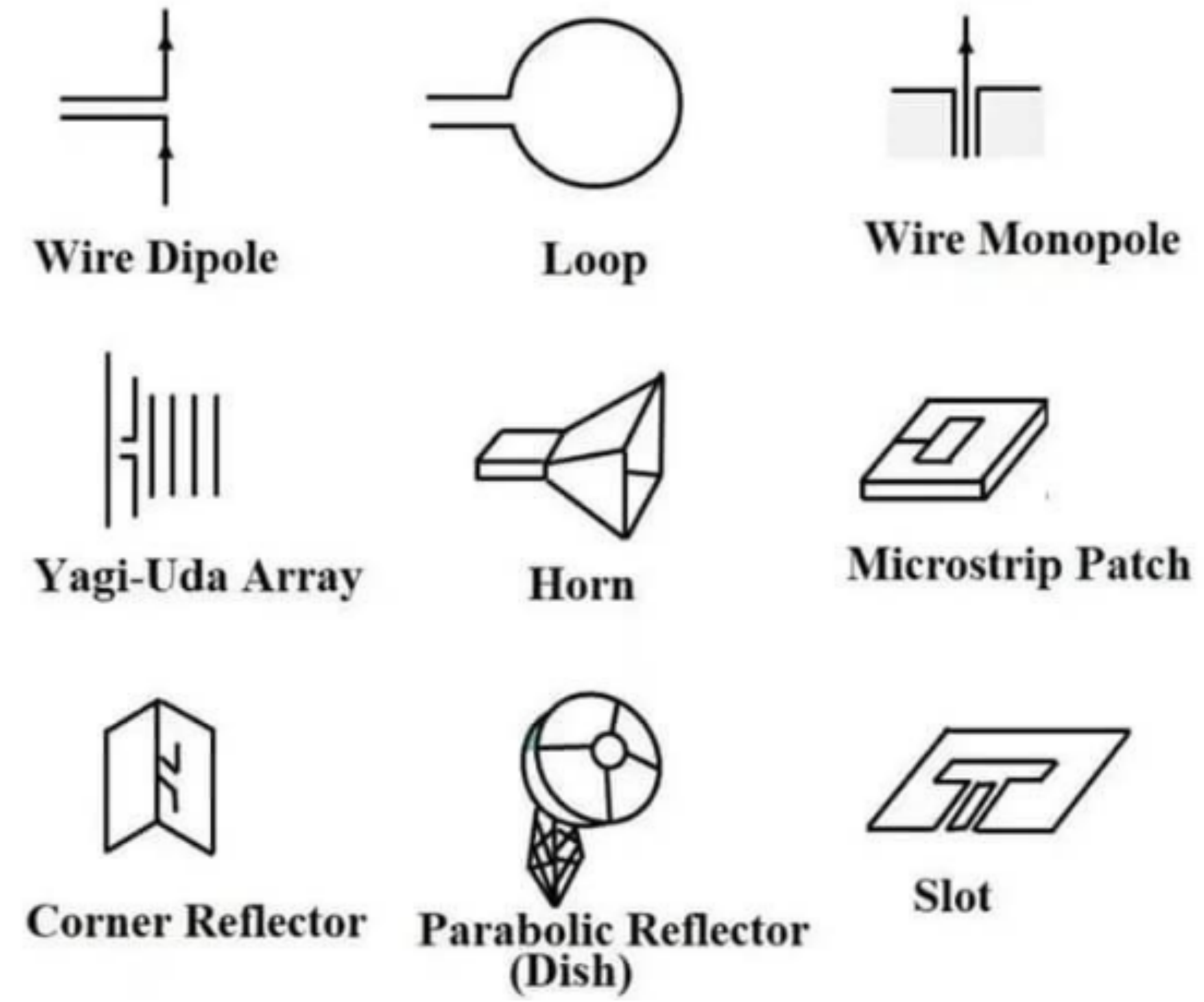


Antena designs

EM waves come from accelerating charges

$$P = \frac{2}{3}\frac{q^2}{4\pi\varepsilon_0 c}\left(\frac{\dot{v}}{c}\right)^2$$

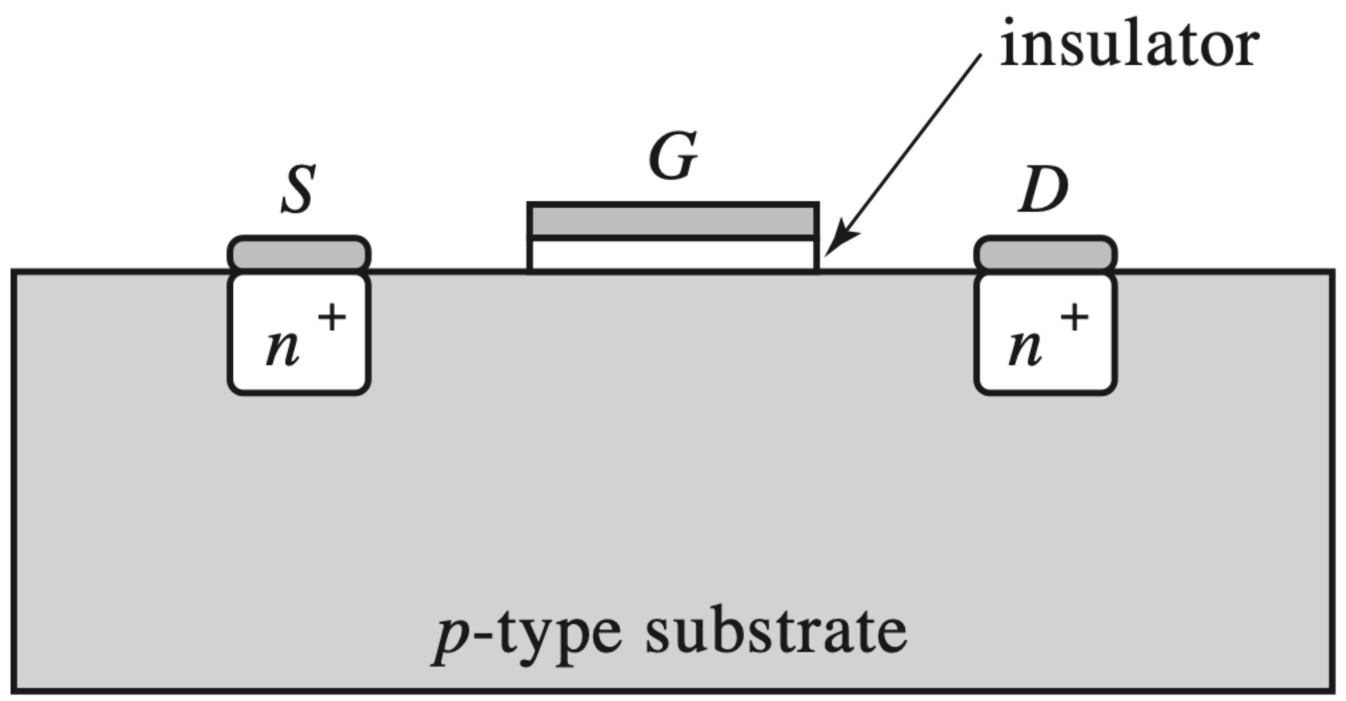




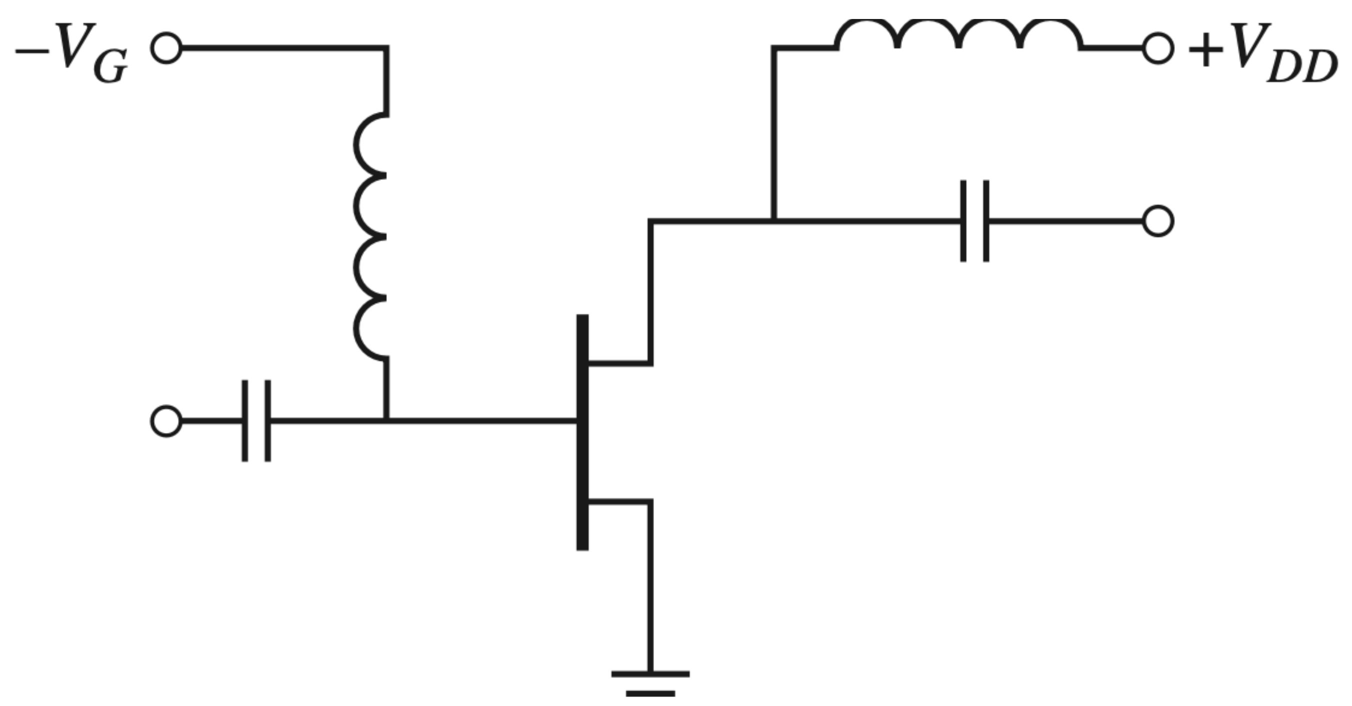
$$R_{\text{radiation}} = \frac{\pi}{6}\,\zeta_o\left(\frac{\ell}{\lambda}\right)^2 \approx \left(\frac{\ell}{\lambda}\right)^2 (197\ \Omega)$$

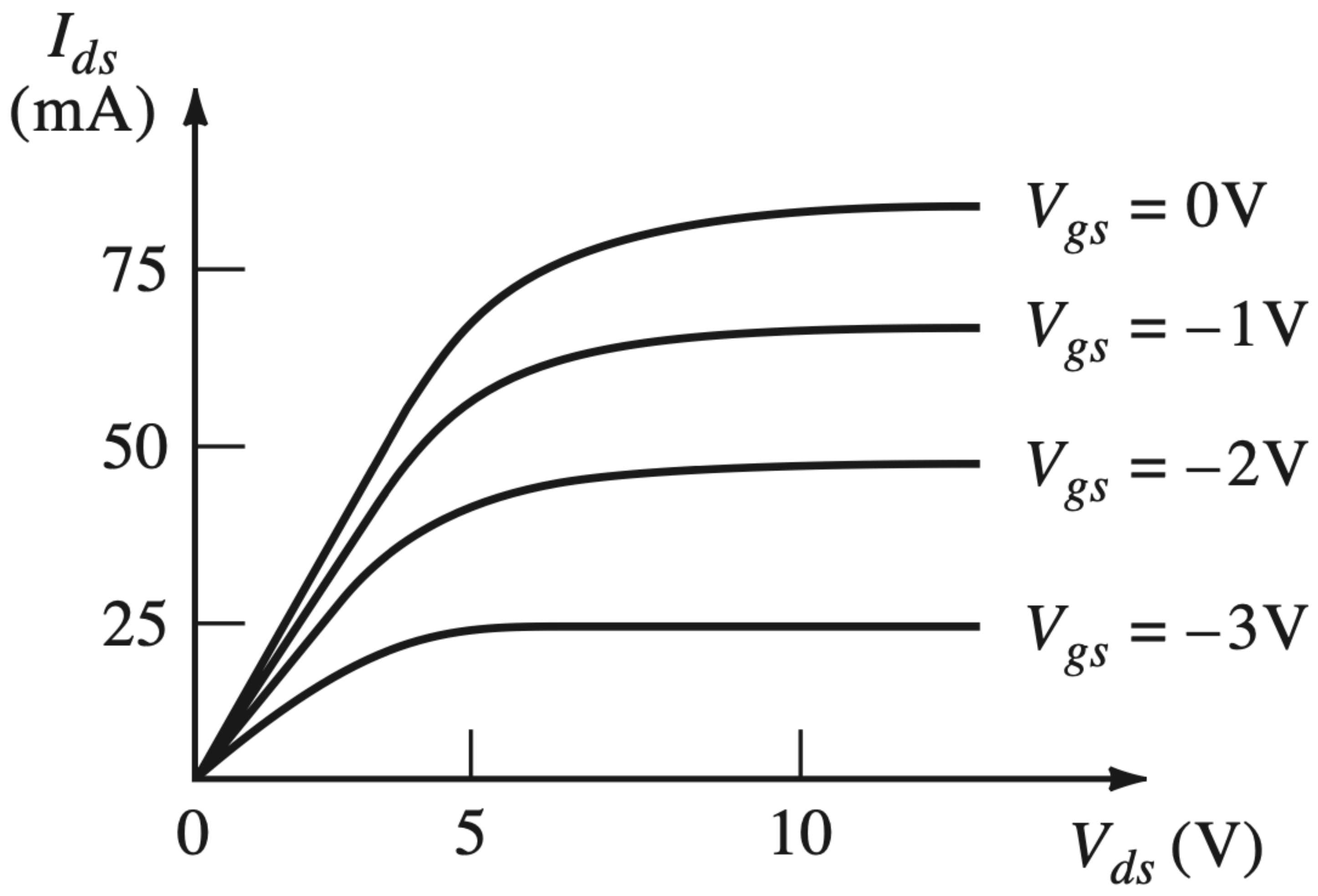
$$\sqrt{\mu_0/\epsilon_0} \approx 377\ \Omega$$

Semiconductor electronics

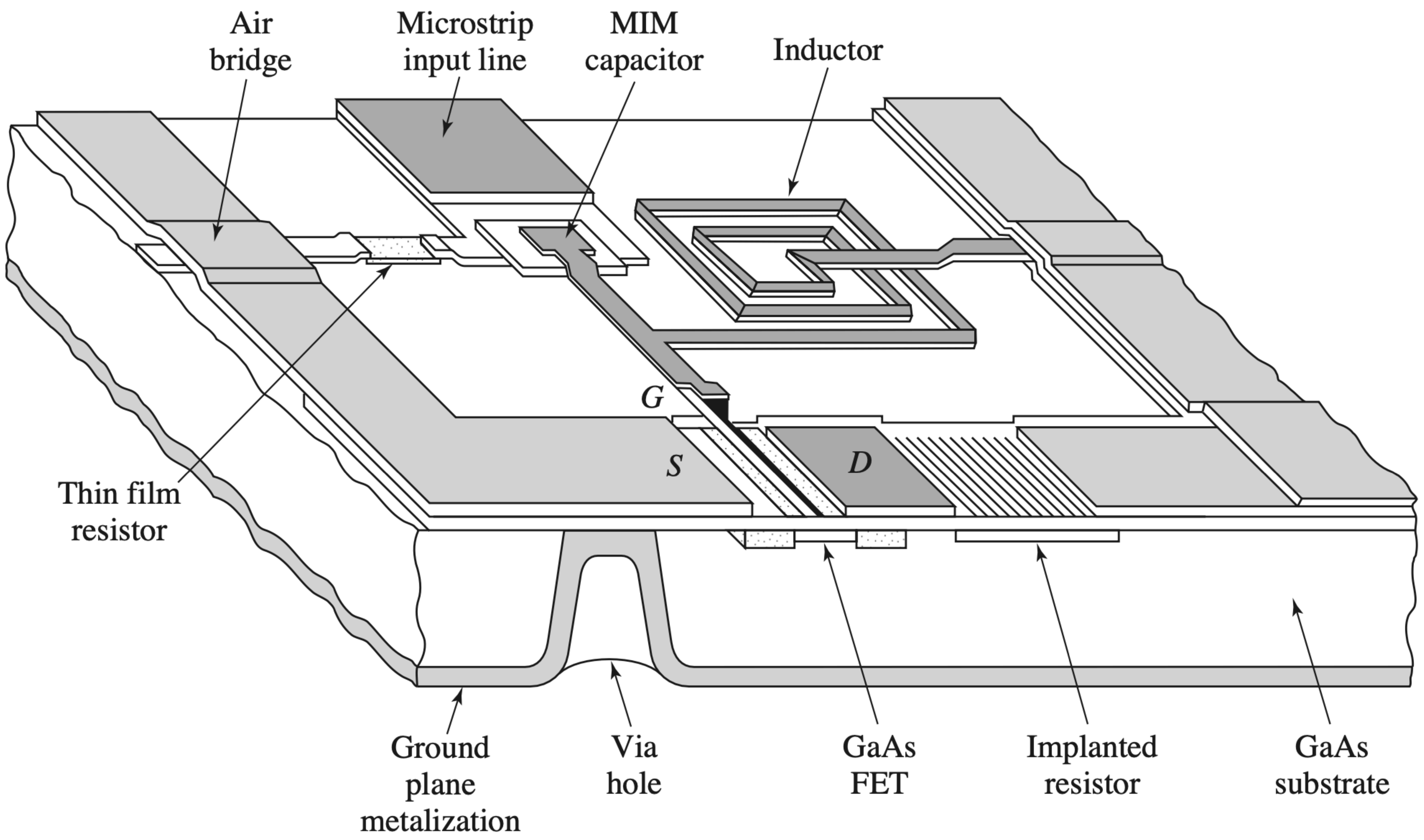


Cross section of an n-channel MOSFET.

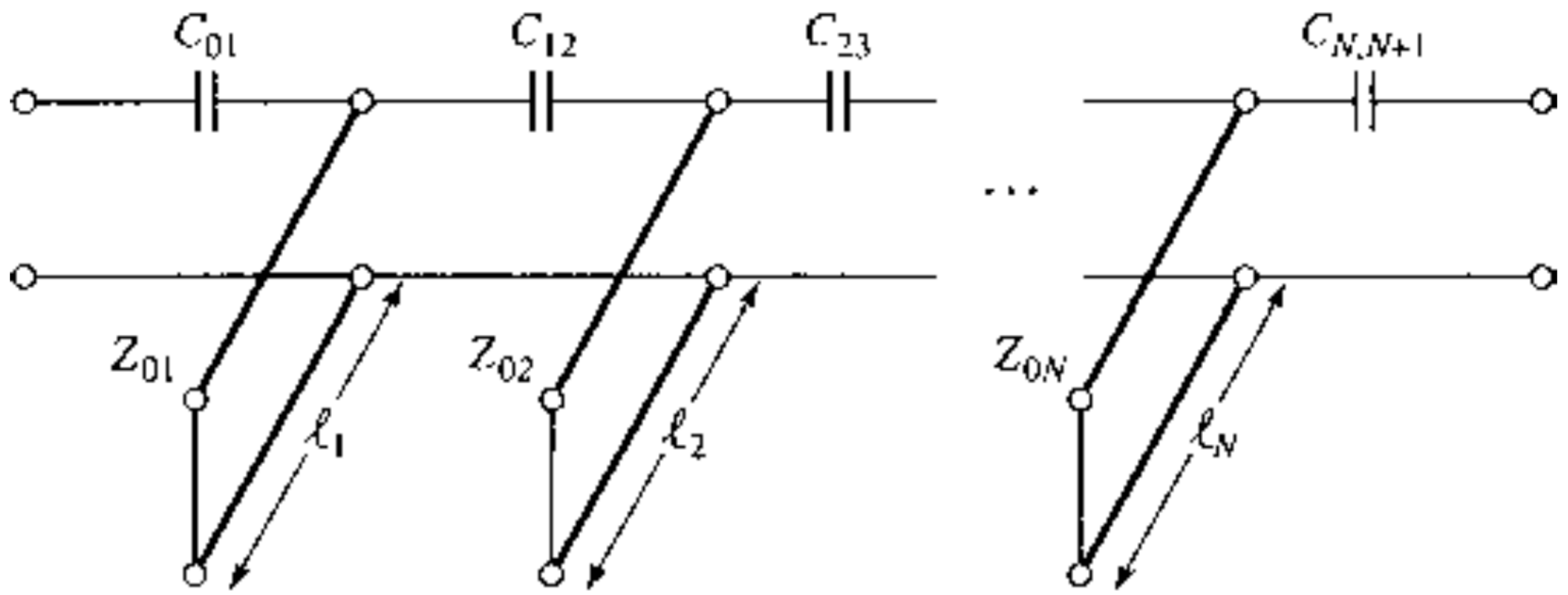


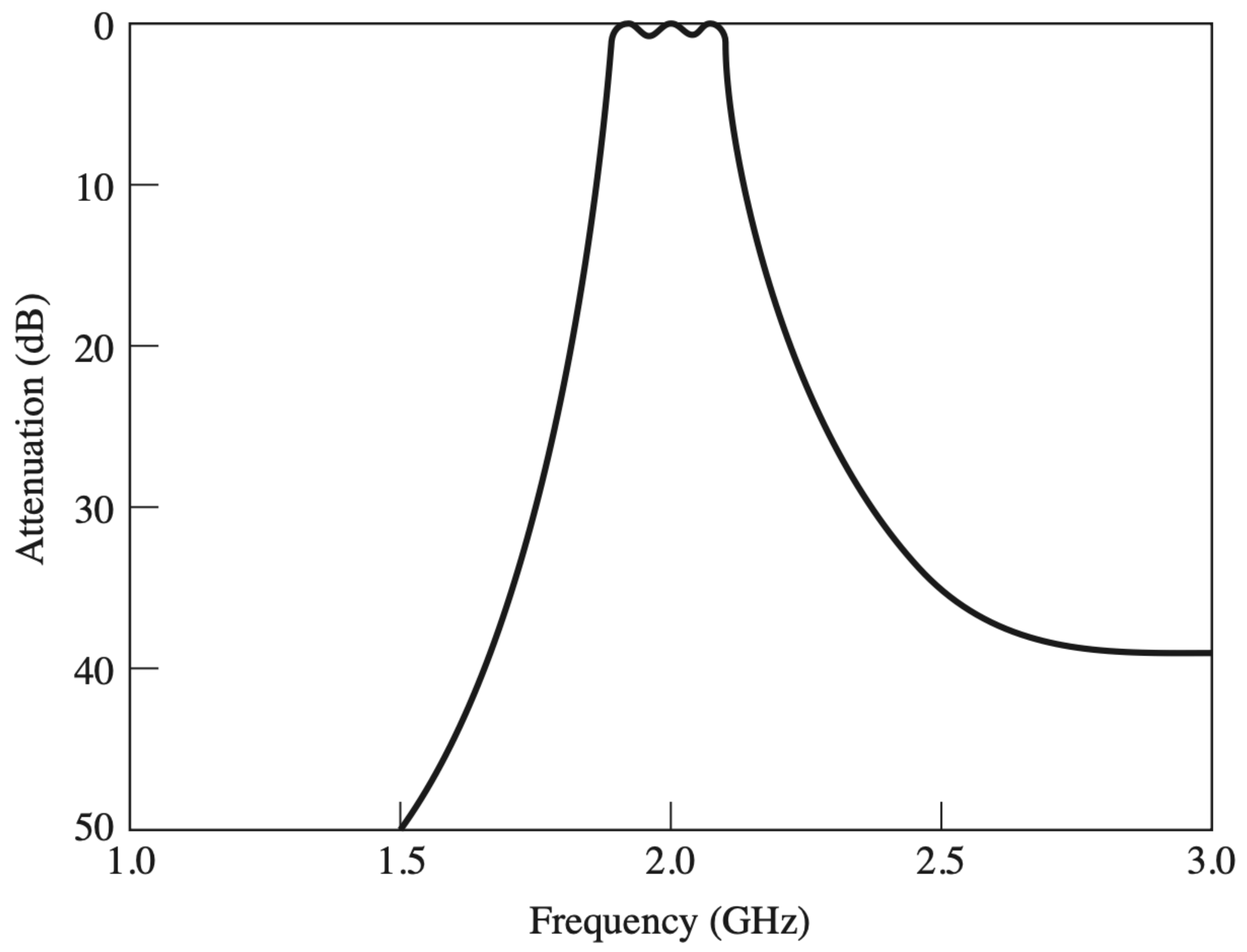


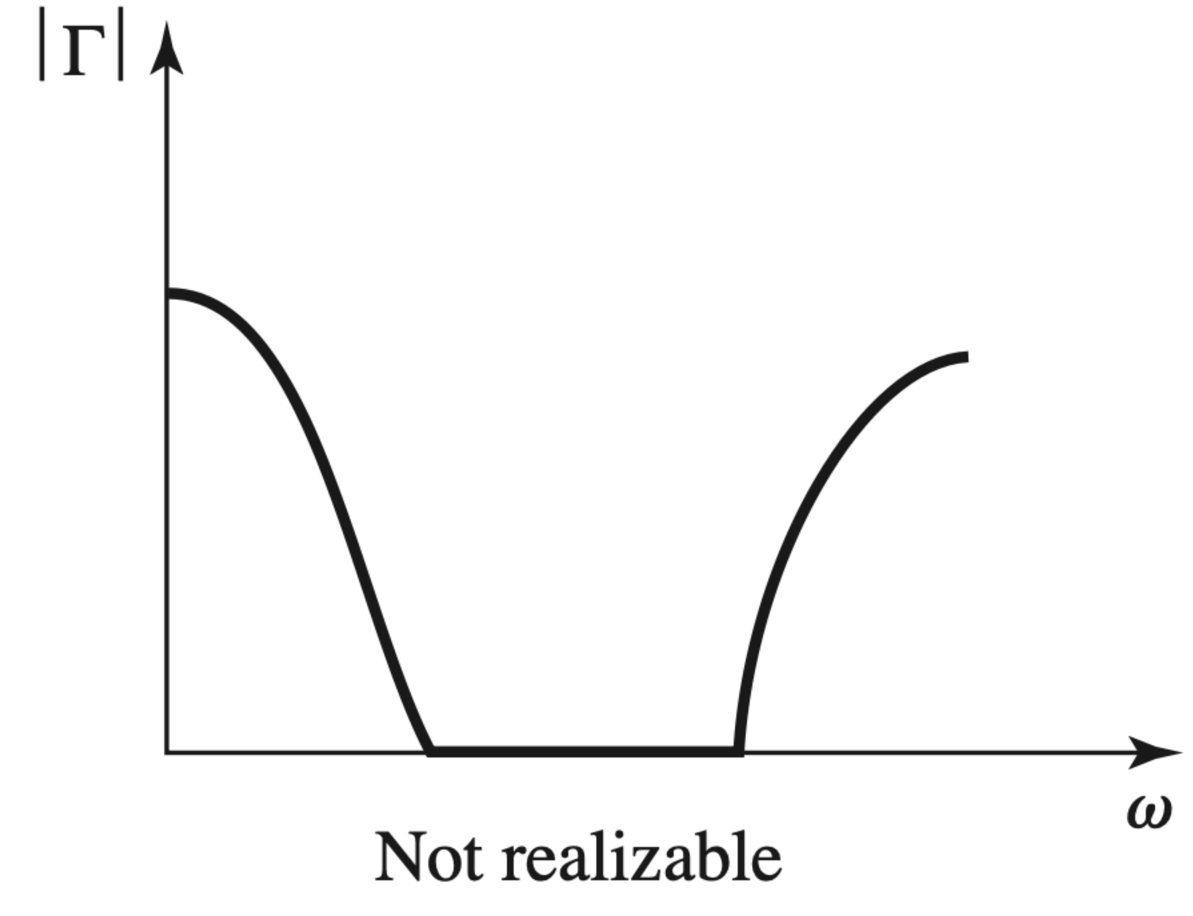
Microwave IC

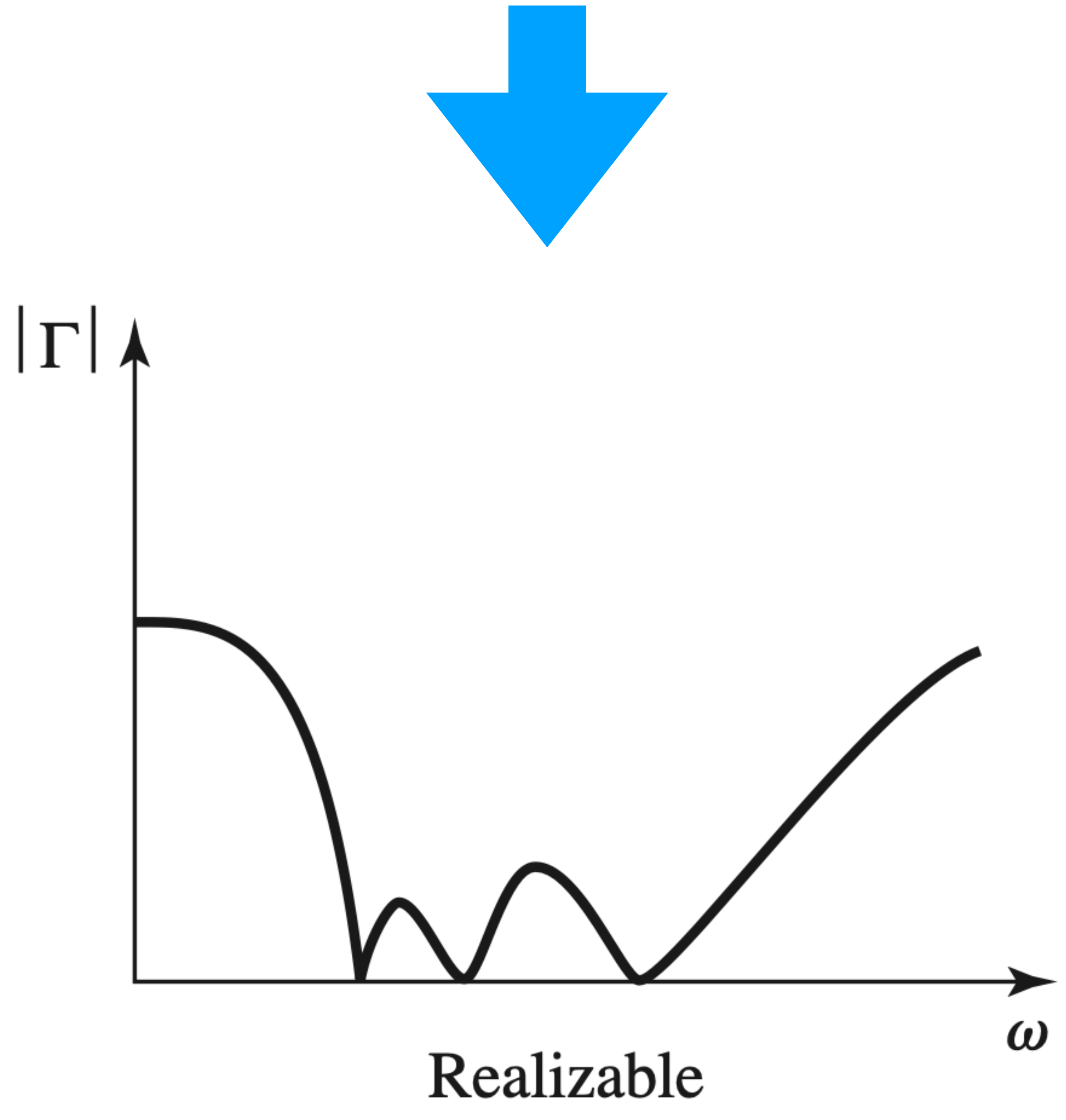


Filter design

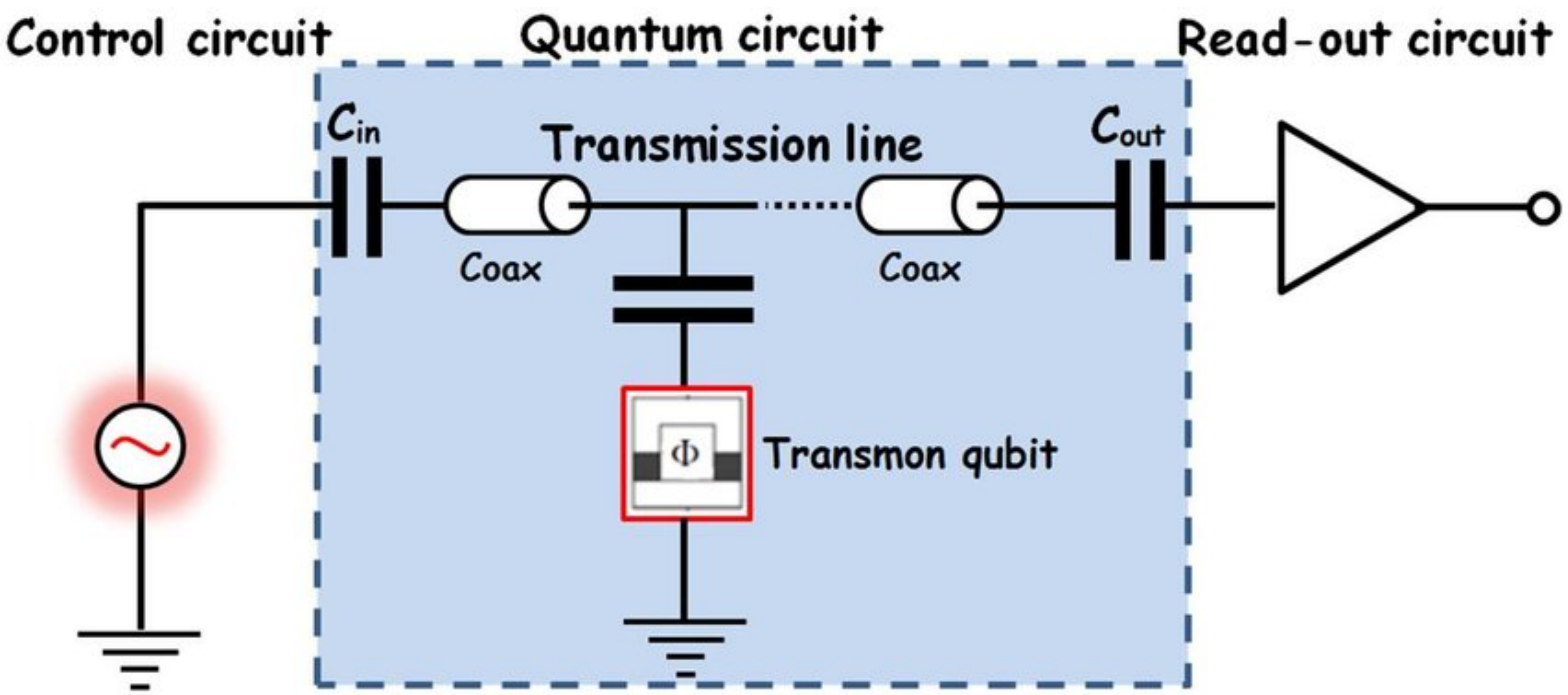








Qubit measurements



Come do a project in our labs!
(Brantut, Scarlino, Kippenberg, VM, ...)

From cavity QED to circuit QED