Introduction to Quantum Science and Technology

Final exam
Fall term 2023

Assignment date: February 01, 2024, 9h15 - 12h15
QUANT 400 – Exam – room GCC330

- There are 5 problems with equal weight. You have 3 hours.
- **For each problem, write your solutions in the indicated space.** Scrap paper will not be corrected.
- No electronic devices are allowed.
- Dont forget to clearly write your name below as well as on other problem sheets.
- Good luck!

Name: ____________________________

Section: ______________

Sciper No.: ______________

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Problem 1. *Zoë Holmes*

Student Name: ____________________________

Section: ______________

Sciper No.: ______________

1. Which of the following are quantum states:

   a) $|\psi\rangle = 0.5|0\rangle + 0.5|1\rangle$
   b) $|\psi\rangle = 0.7|0\rangle + 0.3|1\rangle$
   c) $|\psi\rangle = \cos(\theta)|0\rangle + \sin(\theta)|1\rangle$
   d) $|\psi\rangle = \cos(\theta)|0\rangle - i\sin(\theta)|1\rangle$
   e) $|\psi\rangle = \cos(\theta)^2|0\rangle - \sin(\theta)^2|1\rangle$
   f) $|\psi\rangle = 0.5|0\rangle + 0.5|1\rangle + 0.5|2\rangle + 0.5|3\rangle$

   2 marks

2. a) Write down the unitary that represents the upside-down CNOT gate in Fig. a)

   2 marks

   b) What is the state resulting from the circuit shown in Fig. b) to $|0\rangle^{\otimes 6}$?

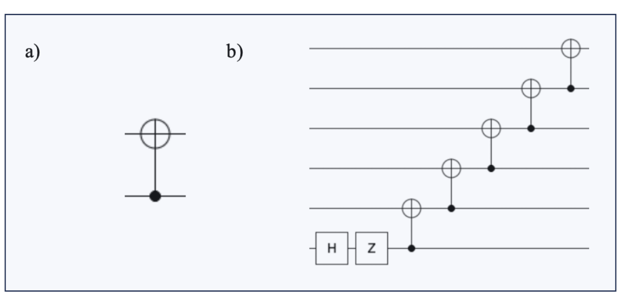
   3 marks

   c) What is the result of measuring the X operator on the first qubit of that state?

   2 marks

   d) How would you perform the $X^{\otimes n}$ measurement in practise? (i.e. assuming your quantum computer can only measure in the computational basis)

   3 marks (be explicit!)



3. a) What is meant by an entangled state? (An answer for pure states only is acceptable)

   1 mark

   b) Let $|\Psi\rangle = (p, q, r, s)$ be a 2 qubit state where p, q, r, s are arbitrary complex numbers. Show that $|\Psi\rangle$ is entangled if and only if $ps \neq qr$.

   (You may assume for simplicity that p, q, r and s are non-zero)

   6 marks

   c) Use the criterion in b) to show that the state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ is entangled.

   1 mark

Solution to Problem 1:

1. c,d,f

2. a) $U = \mathbb{I} \otimes |0\rangle\langle 0| + X \otimes |1\rangle\langle 1|$

   b) $|0\rangle^{\otimes 6} \rightarrow |-\rangle |0\rangle^{\otimes 5} \rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes 6} - |1\rangle^{\otimes 6}\right) := |\phi_6^{\mathrm{GHZ}-}\rangle$

   c) $1/2(\langle 000000| + \langle 111111|)(|100000\rangle + |011111\rangle) = 0$

   d) Apply Hadamard gate on the first qubit to rotate into X basis and then measure in the computational basis: $\langle X\rangle = \langle\psi|HZH|\psi\rangle := \langle\psi'|Z|\psi'\rangle = \frac{1}{2}(p_0 - p_1) = \frac{1}{2}(2p_0 - 1)$ where p_0 is estimated as N_0/N where N_0 is the number of times 0 is measured and N is the total no. of shots.

3. a) A composite state $|\Psi_{AB}\rangle$ such $|\Psi_{AB}\rangle \neq |\Psi_A\rangle \otimes |\Psi_B\rangle$

   b) Consider an unentangled state $(a, b) \otimes (c, d) = (ac, ad, bc, bd) \equiv (p, q, r, s)$. We have $ps - qr = acbd - adbc = 0$. So if the state is unentangled we have $ps = qr$.

   To prove the opposite direction, if $ps = qr$ then $s = qr/p$ so $(p, q, r, s) = (p, q, r, qr/p) = (p, q) \otimes (1, r/p)$. That is, if $ps = qr$ we have that the state is unentangled.

   This establishes that $|\Psi\rangle$ is unentangled iff $ps = qr$. Or equivalently $|\Psi\rangle$ is entangled if and only if $ps \neq qr$.

   c) $ps = 1/2$ and $qr = 0$ so $ps \neq qr$ and the state is entangled.

Problem 2. *Nicolas Macris*

Student Name: __________________________

Section: _____________

Sciper No.: _____________

We study two related 'Local Operation and Classical Communication' (LOCC) protocols. Alice and Bob are at two distant locations and are allowed to perform *local operations* (unitaries and measurements) and also communicate through *classical messages*.

I. First task (10 marks): Alice and Bob share a Bell pair $|\Phi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$. Their task is to transform it to $|\Psi_\theta\rangle = \cos\theta|00\rangle_{AB} + \sin\theta|11\rangle_{AB}$, $0 < \theta < 2\pi$. Alice also has an extra ancilla qubit. We guide you through the LOCC:

1. *(4 marks)* The extra ancilla qubit of Alice is in the state $|0\rangle_{A'}$ and she performs a local unitary on AA':

$$U = F_{0A} \otimes \mathbb{1}_{A'} + iF_{1A} \otimes X_{A'}$$

where

$$F_{0A} = \begin{pmatrix} \cos\theta & 0 \\ 0 & \sin\theta \end{pmatrix}, \quad F_{1A} = \begin{pmatrix} \sin\theta & 0 \\ 0 & \cos\theta \end{pmatrix}, \quad \mathbb{1}_{A'} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad X_{A'} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Compute the final *global state* of Alice and Bob (for the three qubits ABA').
Hint: the initial state can be written $|\Phi\rangle_{AB}\otimes|0\rangle_{A'} = \frac{1}{\sqrt{2}}(|0\rangle_A\otimes|0\rangle_{A'}\otimes|0\rangle_B+|1\rangle_A\otimes|0\rangle_{A'}\otimes|1\rangle_B)$.

2. *(4 marks)* Alice then does a measurement of her *ancilla* qubit in the computational basis $\{|0\rangle_{A'}, |1\rangle_{A'}\}$. Suppose the outcome for A' is $|i\rangle_{A'}$, $i \in \{0, 1\}$. What are the possible outcome *global states* for each $i = 0, 1$ and their respective probabilities p_0, p_1 ?

3. *(2 mark)* Finally Alice transmits to Bob one classical bit $i \in \{0, 1\}$ corresponding to her outcome. Propose simple *local unitaries* that Alice and Bob must perform in order to succeed with their task. What is the overall success probability for this protocol ?

II. Second task (10 marks): This is a reverse task. Alice and Bob now initially share the pair $|\Psi_\theta\rangle = \cos\theta|00\rangle_{AB} + \sin\theta|11\rangle_{AB}$, $0 < \theta < 2\pi$. They should transform it to a pure Bell state $|\Phi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$. Alice can use an ancilla qubit again. We guide you through the LOCC:

4. *(4 marks)* Alice performs the same local unitary U on her two qubits AA'. Her ancilla qubit is initially in the satte $|0\rangle_{A'}$ as before. Compute the final *global state* of the three qubits ABA'.

5. *(4 marks)* Alice then measures her *ancilla* qubit in the basis $\{|0\rangle_{A'}, |1\rangle_{A'}\}$. Explain what are the possible outputs for the *global state* (of ABA') and their corresponding probabilities.

6. *(2 mark)* Finally Alice sends a classical bit to Bob to inform him about the outcome of her measurement. And now, what can they both conclude about the success probability of this protocol ?

Solution to Problem 2:

1) *(4 marks)* We find

$$U|0\rangle_A \otimes |0\rangle_{A'} = \cos\theta|0\rangle_A \otimes |0\rangle_{A'} + i\sin\theta|0\rangle_A \otimes |1\rangle_{A'}$$

and

$$U|0\rangle_A \otimes |1\rangle_{A'} = \sin\theta|1\rangle_A \otimes |0\rangle_{A'} + i\cos\theta|1\rangle_A \otimes |1\rangle_{A'}$$

From which we deduce the final global state:

$$\frac{1}{\sqrt{2}}(\cos\theta|00\rangle_{AB} + \sin\theta|11\rangle_{AB}) \otimes |0\rangle_{A'} + \frac{1}{\sqrt{2}}(\sin\theta|00\rangle_{AB} + \cos\theta|11\rangle_{AB}) \otimes |1\rangle_{A'}$$

2) *(4 marks)* Alice does a measurement on A'. The possible resulting states are

$$\frac{1}{\sqrt{2}}(\cos\theta|00\rangle_{AB} + \sin\theta|11\rangle_{AB}), \qquad \frac{1}{\sqrt{2}}(\sin\theta|00\rangle_{AB} + \cos\theta|11\rangle_{AB}) \otimes |1\rangle_{A'}$$

with probabilities

$$p_0 = \frac{1}{2}(\cos^2\theta + \sin^2\theta) = \frac{1}{2}, \qquad p_1 = \frac{1}{2}(\cos^2\theta + \sin^2\theta) = \frac{1}{2}$$

3) *(2 mark)* When Bob reeives the classical bit 0 or 1, he knows what is the resulting global state. If 0 is the outcome then Alice and Bob aplly the local unitaries $\mathbb{1}_A \otimes \mathbb{1}_B$. If 1 is the outcome then Alice and Bob aplly the local unitaries $X_A \otimes X_B$.
The success probability of the protocol is always $1/2 + 1/2 = 1$.
Remark: this protocol is a simple example of so called 'entanglement concentration'.

4) *(4 marks)* We compute:

$$\begin{aligned}
&U|\Psi_\theta\rangle \otimes |0\rangle_{A'} \\
&= \cos^2\theta|0\rangle_A|0\rangle_{A'}|0\rangle_B + i\cos\theta\sin\theta|0\rangle_A|1\rangle_{A'}|0\rangle_B + \sin^2\theta|1\rangle_A|0\rangle_{A'}|1\rangle_B + i\sin\theta\cos\theta|1\rangle_A|1\rangle_{A'}|1\rangle_B \\
&= (\cos^2\theta|00\rangle_{AB} + \sin^2\theta|11\rangle_{AB}) \otimes |0\rangle_{A'} + i\sin\theta\cos\theta(|00\rangle_{AB} + |11\rangle_{AB}) \otimes |1\rangle_{A'}
\end{aligned}$$

5) *(4 marks)* Alice Measures her ancilla qubit. The possible outcome states are

$$\frac{(\cos^2\theta|00\rangle_{AB} + \sin^2\theta|11\rangle_{AB}) \otimes |0\rangle_{A'}}{\sqrt{\cos^4\theta + \sin^4\theta}} \otimes 0\rangle_{A'}, \qquad \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB}) \otimes |1\rangle_{A'}$$

with probabilities

$$p_0 = \cos^4\theta + \sin^4\theta, \qquad p_1 = 2(\sin\theta\cos\theta)^2$$

6) *(2 mark)* Alice send s a classical bit 0 or 1 to Bob according to her measurement result. Hence they both know what is the resulting state. The success probability of the protocol is thus

$$p_1 = 2(\sin\theta\cos\theta)^2$$

Remark: this protocol is an example of 'entanglement dilution'.
We see that entanglement concentration and dilution do not have symmetric success probabilities.

Problem 3. *Pasquale Scarlino*

Student Name: ____________________________

Section: ______________

Sciper No.: ______________

Electron spin resonance (ESR) in a gate-defined semiconducting quantum dot (10 marks)

A single electron (or hole) confined in a semiconducting gate-defined quantum dot (QD) can be used to implement the simplest example of a spin qubit: the Loss-DiVincenzo qubit. In the case of GaAs, the computational basis consists in spin up $|\uparrow_z\rangle = |0\rangle$ and spin down $|\downarrow_z\rangle = |1\rangle$, obtained by applying a static magnetic field $B_z e_z$ along the z-axis. An oscillating magnetic field $B_x \cos(\omega t + \phi) e_x$ is applied in the x-direction, perpendicular to the spin quantization axis. The Hamiltonian of a single electron in an external magnetic field can be written as: $H = g\mu_{\mathrm{B}}\mathbf{B}\cdot\boldsymbol{\sigma}/2$, where g is the g-factor, μ_{B} is the Bohr magneton and $\boldsymbol{\sigma}$ is a vector of Pauli matrices. In the frame rotating at the same frequency of the drive ω, the Hamiltonian reads as the typical driven two-level system one:

$$\hat{H} = \frac{\hbar\Delta}{2}\hat{\sigma_z} - \frac{\hbar\Omega}{2}(\hat{\sigma_x}\cos(\phi) + \hat{\sigma_y}\sin(\phi)) = -\frac{\hbar}{2}\begin{pmatrix} -\Delta & \Omega e^{-i\phi} \\ \Omega e^{i\phi} & \Delta \end{pmatrix}, \tag{1}$$

where $\Delta = \omega - \omega_0$ is the drive-to-qubit detuning and Ω is the Rabi frequency.

1. Find the expressions of the Larmor frequency ω_0 and the Rabi frequency Ω as a function of the magnetic field components, μ_{B} and g.

   2 marks

2. Now, we remind the identity:

   $$e^{i\frac{\theta}{2}(\sum n_j\sigma_j)} = \cos(\frac{\theta}{2})I + i\sin(\frac{\theta}{2})(\sum n_j\sigma_j), \tag{2}$$

   with $j = x, y, z$ Consider an initial state at the ground state of the system: $\psi(0) = |0\rangle$. We perform the following actions:

   - Turn on drive at resonance, phase $\phi = 0$ and a for a time $t = \frac{\pi}{2\Omega}$.
   - There is a delay between the pulses, resulting in free evolution for a time $t = \epsilon$
   - Turn on drive at resonance, phase $\phi = \pi$ and for a time $t = \frac{\pi}{2\Omega}$.

   What is the final state of the system?

   3 marks

3. If there was no delay, the pulse sequence would bring the state back to the ground state. What does the delay induce in the measurement and what can be done experimentally to counteract that?

   2 mark

4. Explain qualitatively and briefly how it is possible to define a double quantum dot (DQD) in a semiconducting material and how it is possible to confine electrons (or holes) in all the 3 dimensions. Why can DQDs be useful?

   3 marks

SQUIDed transmon (10 marks)

A transmon can be used as a two level system and it is composed of a Josephson junction, acting as a non-linear inductance, and a capacitance made with superconducting material. We now consider the following picture of a slightly modified transmon:

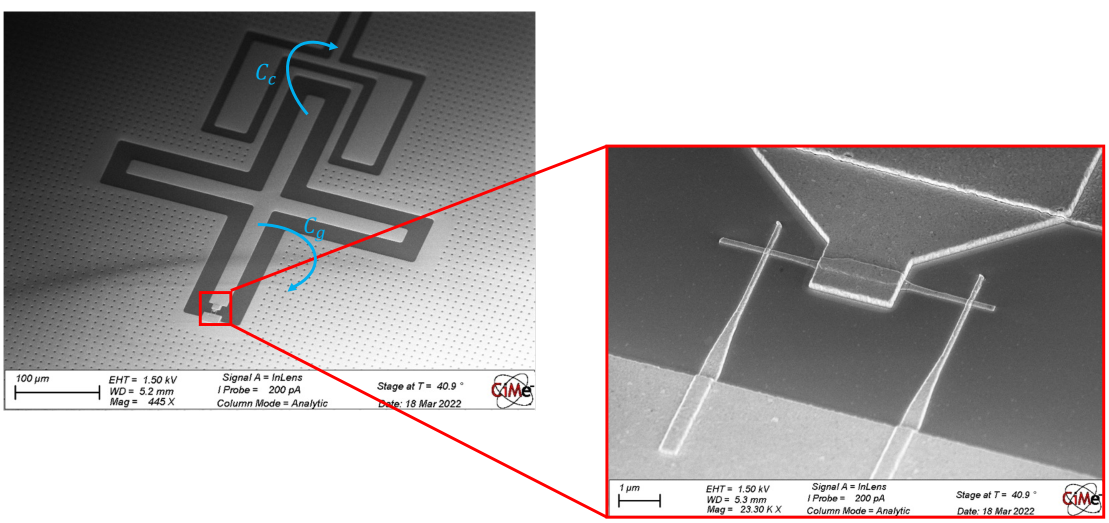


Figure 1: SEM pictures of a SQUIDed transmon (HQC) coupled to a readout resonator. The picture in the right is a zoom in of the picture on the left.

1. First, draw the equivalent electrical circuit scheme of Fig.1 taking care to write where the specified capacitances C_c and C_g are (other parameters are free).

   3 marks

2. Briefly detail what a SQUID is and what does it bring compared to a more classical transmon design having only one Josephson junction? Give one application on how this can be useful.

   2 mark

3. Write the Hamiltonian of the SQUIDed transmon in terms of the charge operator $\hat{n}$ and phase operators associated to each junction $\hat{\varphi}_1$ and $\hat{\varphi}_2$ (assume that they are identically fabricated with the same dimensions). Show that it can be modeled as a classical transmon with a tunable parameter.
   For the last point, use the fact that the phase difference in a SQUID is given by $\varphi_1 - \varphi_2 = 2\pi \frac{\phi}{\phi_0}$ where ϕ, ϕ_0 are the applied magnetic flux and flux quantum respectively, and use the trigonometric identity $\cos(x) + \cos(y) = 2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})$.

   3 marks

4. Finally, give the full Hamiltonian of the transmon AND readout system if the coupling factor between the two is given by κ and the readout resonant frequency is ω_r.

   2 marks

Solution to problem 3:

Electron spin resonance (ESR) in a gate-defined semiconducting quantum dot (10 marks)

1. We first develop the Hamiltonian for the single electron in magnetic field:

$$H = g\mu_{\mathrm{B}}\mathbf{B}\cdot\boldsymbol{\sigma}/2. \tag{3}$$

In our case, a B field in the z-direction is applied, for obtaining Zeeman splitting of the electron spin, but also a field in the x-direction, for state driving. Reporting this in the Hamiltonian, we obtain:

$$H = \frac{g\mu_{\mathrm{B}}}{2}\begin{pmatrix} B_x\cos(\omega t+\phi) \\ 0 \\ B_z \end{pmatrix}\cdot\begin{pmatrix}\sigma_x \\ \sigma_y \\ \sigma_z\end{pmatrix} \tag{4}$$

$$H = \frac{g\mu_{\mathrm{B}}}{2}(B_x\cos(\omega t+\phi)\sigma_x + B_z\sigma_z). \tag{5}$$

On the other hand, the Hamiltonian for a typical driven two level system is:

$$\hat{H} = \frac{\hbar\Delta}{2}\sigma_z - \frac{\hbar\Omega}{2}(\sigma_x\cos(\phi)+\sigma_y\sin(\phi)). \tag{6}$$

By comparing the terms of the two Hamiltonians, we conclude that:

$$\omega_0 = \frac{g\mu_{\mathrm{B}}}{\hbar}B_z \tag{7}$$

$$\Omega = \frac{1}{2}\frac{g\mu_{\mathrm{B}}}{\hbar}B_x. \tag{8}$$

2.
   - The time evolution of a given state with a time-independent Hamiltonian is given by:

$$|\psi(t)\rangle = U(t)\,|\psi(0)\rangle = e^{-i\frac{\hat{H}}{\hbar}t}\,|0\rangle\,. \tag{9}$$

In the case of the first pulse there is no detuning so the σ_z term is turned off. The drive phase turns off the σ_y term so only the σ_x term remains which results in a rotation on the x axis of the state vector in the Bloch sphere.

$$U(t) = e^{-i\frac{\hat{H}}{\hbar}t} = e^{i\frac{\pi}{4}\sigma_x}, \tag{10}$$

So that,

$$|\psi(t)\rangle = e^{i\frac{\pi}{4}\sigma_x}|0\rangle \tag{11}$$
$$|\psi(t)\rangle = (\cos(\frac{\pi}{4})I + i\sin(\frac{\pi}{4})\sigma_x)|0\rangle \tag{12}$$
$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle). \tag{13}$$

We get a 50/50 distribution of the ground and excited state with a global phase.

- In the case of free evolution, there is no drive so the second term is turned off. The detuning is however non zero and is equal to $-\omega_0$, the resonance frequency of the two level system. We have:

$$|\psi(t)\rangle = e^{-i\frac{\omega_0}{2}\sigma_z\epsilon}\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \tag{14}$$
$$|\psi(t)\rangle = (\cos(-\frac{\omega_0}{2}\epsilon)I + i\sin(-\frac{\omega_0}{2}\epsilon)\sigma_z)\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \tag{15}$$
$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}((\cos(-\frac{\omega_0}{2}\epsilon) + i\sin(-\frac{\omega_0}{2}\epsilon))|0\rangle + (\sin(-\frac{\omega_0}{2}\epsilon)) + i\cos(-\frac{\omega_0}{2}\epsilon)|1\rangle) \tag{16}$$
$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(e^{-i\frac{\omega_0}{2}\epsilon}|0\rangle + ie^{i\frac{\omega_0}{2}\epsilon}|1\rangle) \tag{17}$$

- We repeat what was done in the first step but with a π phase shift on the drive:

$$|\psi(t)\rangle = (\cos(-\frac{\pi}{4})I + i\sin(-\frac{\pi}{4})\sigma_x)\frac{1}{\sqrt{2}}(e^{-i\frac{\omega_0}{2}\epsilon}|0\rangle + ie^{i\frac{\omega_0}{2}\epsilon}|1\rangle) \tag{18}$$
$$|\psi(t)\rangle = \frac{1}{2}(I - i\sigma_x)(e^{-i\frac{\omega_0}{2}\epsilon}|0\rangle + ie^{i\frac{\omega_0}{2}\epsilon}|1\rangle) \tag{19}$$
$$|\psi(t)\rangle = (\frac{e^{i\frac{\omega_0}{2}\epsilon} + e^{-i\frac{\omega_0}{2}\epsilon}}{2})|0\rangle + i(\frac{e^{i\frac{\omega_0}{2}\epsilon} - e^{-i\frac{\omega_0}{2}\epsilon}}{2})|1\rangle \tag{20}$$
$$|\psi(t)\rangle = \cos(\frac{\omega_0}{2}\epsilon)|0\rangle + i\sin(\frac{\omega_0}{2}\epsilon)|1\rangle. \tag{21}$$

3. Looking at the final state when a delay is introduced, we can see that the $|1\rangle$ state has a certain probability to appear, which would result in an error in our basic pulse sequence. The delay is in principle very small so the cos term would be approaching 1 and the sin term approaching 0 so that the error has a very small chance of occurring. So to avoid such possibilities in practice, the measurement is conducted multiple times and a statistic is made on the results. In this case the errors with small probabilities of appearing will be averaged out.

4. In order to construct a working DQD, one needs to confine one or a few electrons/holes in 3 dimensional space. First of all, the confinement in the z-direction is done by stacking multiple layers of semiconducting materials and engineering a band configuration that creates a 2D-plane with free electrons (or holes), called a 2D-electron (hole) gas. Here the carriers are free to move in this 2-dimensional space. The confinement in the two other directions is achieved by applying negative/positive voltages to gates that are on top on the 2D-electron gas, to repel or attract electrons/holes.

   In order to create two distinct dots, the gates are designed in a certain configuration. The side barrier gates isolate the dots from the surrounding electron reservoirs and tune the tunneling rate, the central barrier gate controls the interdot tunneling coupling and the plunger gates the number of electrons (holes) inside the dots. A DQD is more versatile with respect to a single dot, as it can be used to encode a charge qubit (where the information is encoded into the position of one electron in the two dots, left or right in the position basis), a singlet-triplet qubit or, most importantly, to allow 2-qubit gate operations by turning on or off the exchange interaction between the two spins confined in the two dots. The exchange interaction depends, among other factors, on the interdot tunneling coupling, meaning that it can be tuned by the voltage applied on the interdot barrier gate.

SQUIDed transmon (10 marks)

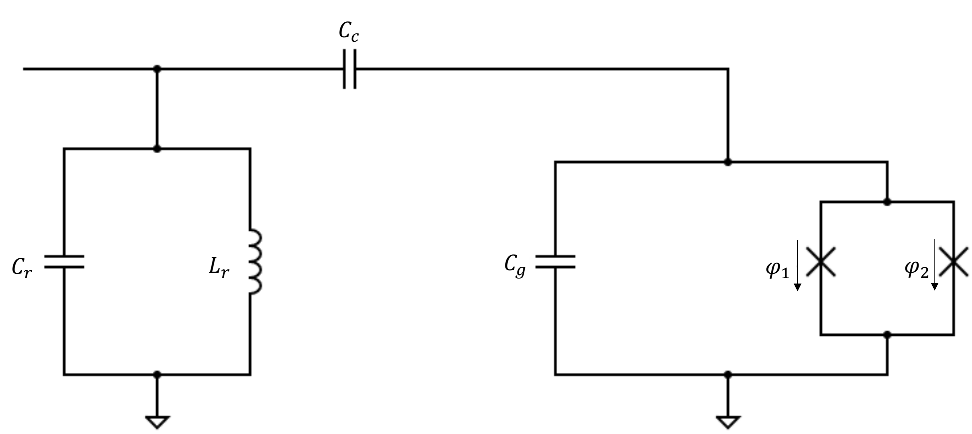


Figure 2: Circuit diagram of the SQUIDed transmon.

1.

2. The SQUID consists in two Josephson junctions forming a superconducting loop on the chip. By applying flux to the loop though a flux line or a coil, it is possible to address the loop and change the effective Josephson energy of the junctions, effectively tuning the resonance frequency of the transmon. This can be useful for coupling two qubits (taking them in and out of resonance), correcting fabrication errors or it can be useful for certain quantum algorithms.

3. To show that the SQUIDed transmon can be modeled as a classical transmon with tunable parameter, we must show that the two Hamiltonians are equivalent. The Hamiltonian of the SQUIDed transmon still shows the charge number operator present on the island and has two JJs instead of one, each having their own phase difference across themselves:

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_J\cos(\varphi_1) - E_J\cos(\varphi_2). \tag{22}$$

So to show that this device can be modelled by a frequency tunable transmon, we need to go back to the Hamiltonian of a classical transmon by using trigonometric and phase difference properties:

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_J(\cos(\varphi_1) + \cos(\varphi_2)) \tag{23}$$

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - 2E_J\cos(\frac{\varphi_1 + \varphi_2}{2})\cos(\frac{\varphi_1 - \varphi_2}{2}) \tag{24}$$

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - 2E_J\cos(\frac{\varphi_1 + \varphi_2}{2})\cos(\pi\frac{\phi}{\phi_0}) \tag{25}$$

$$\tag{26}$$

So we can define our new phase shift $\varphi = \frac{\varphi_1+\varphi_2}{2}$:

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - 2E_J\cos(\pi\frac{\phi}{\phi_0})\cos(\varphi) \tag{27}$$

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_{J\phi}\cos(\varphi). \tag{28}$$

Where $E_{J\phi}$ is the new effective Josephson energy of the system which value depends on the applied external flux. This term is associated to the non-linear inductance of the transmon. So with a magnetic flux, it is possible to change the effective inductance of the transmon, which directly impacts the resonance frequency as it depends on the inductance and capacitance of the system.

4. The Hamiltonian of the full system is composed of the sum of the Hamiltonian of the transmon (found above), the Hamiltonian of the RO resonator (defining its energy levels) and finally the interaction Hamiltonian between the two systems (for a RO resonator, the charge excitation of the transmon is transferred into photons in the resonator to be measured). So all and all:

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_{J\phi}\cos(\varphi) + \hbar\omega_r \hat{a}^\dagger \hat{a} + \kappa \hat{n}(\hat{a} + \hat{a}^\dagger). \tag{29}$$

Problem 4. *Adrian Ionescu*

Student Name: ___________________________

Section: _____________

Sciper No.: _____________

Please indicate the correct answers for each of the questions below by circling them. Multiple correct answers are possible.

- We will assign +1 point for each correct answer.
- We will Deduct -1 point for each incorrect answer.
- No selection will result in 0 points.

Comment: The overall score per question will be capped at zero to prevent additional penalties on the exam score, just in case there are too many negative points.

Question 1:

Single Electron Transistors (SET) are three-terminal devices that leverage discrete charge tunnelling based on a conductive nanodot as the central island, separated from the source and drain electrodes by tunnelling barriers. Select the correct statements from the list below that accurately reflect the features, operation, and technology implementation of SETs.

1. The SET device exhibits a threshold voltage in the drain voltage, corresponding to the Coulomb blockade (or gap) effect.

2. A parasitic background charge effect does not affect the periodicity of the SET transfer characteristics, specifically ID-VG, but only the level of the current at a certain fixed gate voltage.

3. Under the Coulomb blockade effect, the output characteristics (ID-VD) of a SET transistor are periodic, with a period equal to e/CG, where CG is the gate capacitance to the ground, and e is the elementary charge.

4. Using a metallic central island with a 5 nm radius surrounded by three metal electrodes (gate, source, and drain) with much larger radius sizes, the resulting SET is expected to show effective Coulomb blockade at T=20K, making it suitable for building a SET inverter.

5. In the current state of the art, the intrinsic frequency operation range (dictating the speed of the intrinsic device) for SETs is typically in the order of kHz to MHz in the best case due to their low on current (Ion).

6. The same SET device can have both positive and negative transconductance (gm=dId/dVg), depending only on the value of the applied gate voltage at the same drain voltage value. Therefore, one can build complementary logic using the same device for equivalent n- and p-type functionalities, such as constructing an inverter.

7. An inverter based on two SETs consumes only dynamic power during the transition between logic states.

8. Assuming a sub-1 nanometer central island diameter, the subthreshold slope of resulting SETs at room temperature is much more abrupt (smaller) than that of a MOSFET (60mV/dec).

9. SETMOS is a hybrid equivalent device made from a combination of SET and MOSFET, achieving high peaks of current (micro-Amps) with periodic Id-Vg transfer characteristics due to Coulomb blockade.

10. A silicon-based SET transistor can be used for both building logic circuit blocks (like inverters or others) and as a sensor of elementary charges for implementing an integrated qubit.

Question 2:

Select the correct statements about the semiconducting Tunnel FETs exploiting quantum mechanical band-to-band tunneling conduction mechanisms, from the list below:

1. Due to their low subthreshold slope at low voltages, Tunnel FETs can be employed to design more energy-efficient analog amplifiers at room temperature, operating at very low voltages and currents (sun-nanoAmps), especially in the current range where CMOS analog amplification is less effective.

2. The geometrical shape of the tunneling barrier in Tunnel FETs is identical to that in Single Electron Transistors; only the way they are technologically implemented differs.

3. Tunnel FETs inherit certain technology boosters (technological parameters that can enhance their switch performance) from MOSFETs. Among these, one can cite the use of high-k dielectrics, thinner semiconducting device bodies, and multi-gate control (such as FinFET and double gate architectures).

4. At high gate voltage, Tunnel FETs offer a higher on-current than MOSFETs due to their smaller subthreshold slope.

5. A Tunnel FET with an undoped silicon channel, a silicon-doped drain, and a doped InAs source is a homojunction tunneling device.

6. Trap-Assisted Tunneling (TAT) is significantly reduced as the temperature decreases, making Tunnel FETs an interesting candidate for implementing highly stable cryogenic electronic circuits.
7. Trap-Assisted Tunneling (TAT) is a phenomenon that does not depend on temperature.
8. The off current, Ioff, of homojunction Tunnel FETs is expected to decrease at cryogenic (sub-77K) temperatures compared to room temperature (T=300K).
9. In an ideal Tunnel FET, where Band-to-Band Tunneling is largely predominant over trap-assisted tunneling, the device's subthreshold slope is expected to be much smaller than 60mV/decade at room temperature (300K).
10. In a Tunnel FET without trap-assisted tunneling, the current has very little dependence on temperature because the semiconductor bandgap has a very low dependence on temperature.

Solution to problem 4:

Adrian: write your solutions here

Problem 5. *Edoardo Charbon*

Student Name: ___________________________

Section: _____________

Sciper No.: _____________

1. What is the purpose of a classical controller for qubits?
   2 marks

2. What are the advantages (if any) of operating the classical controller at cryogenic temperatures?
   3 marks

3. What are the best technologies to operate electronics at cryogenic temperatures? Why?
   2 marks

4. Draw a thermalization schematics from room temperature to 10mK. Indicate each intermediate temperature and a detail of the attenuator, including the input and output impedance.
   3 marks

5. What is the purpose of a low-noise amplifier in the context of a classical controller? Indicate its salient performance measures.
   3 marks

6. Compute the overall gain of a cascade of three 20-dB LNAs. How do you compute the input-referred noise of such a design?
   2 marks

7. Compute the quantization-noise power in a 12-bit (ENOB) ADC with 1V range and operating at twice the Nyquist frequency.
   3 marks

8. Can a perfect anti-aliasing filter suppress quantization noise in an ADC operating at twice the Nyquist frequency? Can you reduce quantization noise in an oversampled ADC? Why?
   2 marks

Solution to problem 5:

There are many possible solutions to these general questions.