$$A' = U^t A U =$$

$$\left(\begin{array}{c|c} P^tBP & P^tBT + P^tC \\ \hline T^tBP + C^tP & T^tBT + T^tC + C^tT + f \end{array}\right)$$

$$= \left(\begin{array}{c|c} B' & C' \\ \hline C'^t & f' \end{array}\right)$$

$$A = \left(\begin{array}{cc|c} a & b & d \\ b & c & e \\ \hline d & e & f \end{array}\right) = \left(\begin{array}{c|c} B & C \\ \hline C^t & f \end{array}\right)$$