

Calcul de primitives : exercices supplémentaires

Intégration par observation :

1. $\int x^5 dx$

2. $\int (x + \sqrt{x}) dx$

3. $\int \left(\frac{3}{\sqrt{x}} - \frac{x\sqrt{x}}{4} \right) dx$

4. $\int \frac{x^2}{\sqrt{x}} dx$

5. $\int \left(\frac{1}{x^2} + \frac{4}{x\sqrt{x}} + 2 \right) dx$

6. $\int \frac{1}{\sqrt[4]{x}} dx$

7. $\int \left(x^2 + \frac{1}{\sqrt[3]{x}} \right)^2 dx$

Intégration par observation et/ou changements de variables :

8. $\int e^{5x} dx$

9. $\int \cos(5x) dx$

10. $\int \sin(ax) dx$

11. $\int \frac{\ln(x)}{x} dx$

12. $\int \frac{1}{\sin^2(3x)} dx$

13. $\int \frac{1}{\cos^2(7x)} dx$

14. $\int \frac{1}{3x-7} dx$

15. $\int \frac{1}{1-x} dx$

16. $\int \frac{1}{5-2x} dx$

17. $\int \tan(2x) dx$

18. $\int \cot(5x-7) dx$

19. $\int \frac{1}{\cot(3x)} dx$

20. $\int \cot\left(\frac{x}{3}\right) dx$

21. $\int \tan(x) \frac{1}{\cos^2(x)} dx$

22. $\int (\cot(e^x))e^x dx$

23. $\int \left(\tan(4x) - \cot\left(\frac{x}{4}\right) \right) dx$

24. $\int \sin^2(x) \cos(x) dx$

25. $\int \cos^3(x) \sin(x) dx$

26. $\int x\sqrt{x^2+1} dx$

27. $\int \frac{x}{\sqrt{2x^2+3}} dx$

28. $\int \frac{x^2}{\sqrt{x^3+1}} dx$

29. $\int \frac{\cos(x)}{\sin^2(x)} dx$

$$30. \int \frac{\sin(x)}{\cos^3(x)} dx$$

$$31. \int \frac{\tan(x)}{\cos^2(x)} dx$$

$$32. \int \frac{\cot(x)}{\sin^2(x)} dx$$

$$33. \int \frac{1}{\cos^2(x)\sqrt{\tan(x)-1}} dx$$

$$34. \int \frac{\ln(x+1)}{x+1} dx$$

$$35. \int \frac{\cos(x)}{\sqrt{2\sin(x)+1}} dx$$

$$36. \int \frac{\sin(2x)}{(1+\cos(2x))^2} dx$$

$$37. \int \frac{\sin(2x)}{\sqrt{1+\sin^2(x)}} dx$$

$$38. \int \frac{\sqrt{\tan(x)+1}}{\cos^2(x)} dx$$

$$39. \int \frac{\cos(2x)}{(2+3\sin(2x))^3} dx$$

$$40. \int \frac{\sin(3x)}{\sqrt[3]{\cos^4(3x)}} dx$$

$$41. \int \frac{\ln^2(x)}{x} dx$$

$$42. \int \frac{\arcsin(x)}{\sqrt{1-x^2}} dx$$

$$43. \int \frac{\arctan(x)}{1+x^2} dx$$

$$44. \int \frac{\arccos^2(x)}{\sqrt{1-x^2}} dx$$

$$45. \int \frac{\operatorname{arccot}(x)}{1+x^2} dx$$

$$46. \int \frac{x}{x^2+1} dx$$

$$47. \int \frac{x+1}{x^2+2x+3} dx$$

$$48. \int \frac{\cos(x)}{2\sin(x)+3} dx$$

$$49. \int \frac{1}{x\ln(x)} dx$$

$$50. \int 2x(x^2+1)^4 dx$$

$$51. \int \tan^4(x) dx$$

$$52. \int \frac{1}{(1+x^2)\arctan(x)} dx$$

$$53. \int \frac{1}{\cos^2(x)(3\tan(x)+1)} dx$$

$$54. \int \frac{\tan^3(x)}{\cos^2(x)} dx$$

$$55. \int \frac{1}{\sqrt{1-x^2}\arcsin(x)} dx$$

$$56. \int \frac{\cos(2x)}{2+3\sin(2x)} dx$$

$$57. \int \frac{\cos(\ln(x))}{x} dx$$

$$58. \int \cos(a+bx) dx$$

$$59. \int e^{2x} dx$$

$$60. \int e^{\frac{x}{3}} dx$$

$$61. \int e^{\sin(x)} \cos(x) dx$$

$$62. \int a^{x^2} x dx$$

$$63. \int e^{\frac{x}{a}} dx$$

$$64. \int (e^{2x})^2 dx$$

$$65. \int 3^x e^x dx$$

$$66. \int e^{-3x} dx$$

67. $\int (e^{5x} + a^{5x}) dx$

68. $\int e^{x^2+4x+3}(x+2) dx$

69. $\int \frac{(a^x - b^x)^2}{a^x b^x} dx$

70. $\int \frac{e^x}{3 + 4e^x} dx$

71. $\int \frac{e^{2x}}{2 + e^{2x}} dx$

72. $\int \frac{1}{1 + 2x^2} dx$

73. $\int \frac{1}{\sqrt{1 - 3x^2}} dx$

74. $\int \frac{1}{\sqrt{16 - 9x^2}} dx$

75. $\int \frac{1}{\sqrt{9 - x^2}} dx$

76. $\int \frac{1}{4 + x^2} dx$

77. $\int \frac{1}{9x^2 + 4} dx$

78. $\int \frac{1}{4 - 9x^2} dx$

79. $\int \frac{1}{\sqrt{x^2 + 9}} dx$

80. $\int \frac{1}{\sqrt{b^2 x^2 - a^2}} dx$

81. $\int \frac{1}{\sqrt{b^2 + a^2 x^2}} dx$

82. $\int \frac{1}{a^2 x^2 - c^2} dx$

83. $\int \frac{x^2}{5 - x^6} dx$

84. $\int \frac{x}{\sqrt{1 - x^4}} dx$

85. $\int \frac{x}{x^4 + a^4} dx$

86. $\int \frac{e^x}{\sqrt{1 - e^{2x}}} dx$

87. $\int \frac{1}{\sqrt{3 - 5x^2}} dx$

88. $\int \frac{\cos(x)}{a^2 + \sin^2(x)} dx$

89. $\int \frac{1}{x\sqrt{1 - \ln^2(x)}} dx$

90. $\int \frac{\arccos(x) - x}{\sqrt{1 - x^2}} dx$

91. $\int \frac{x - \arctan(x)}{1 + x^2} dx$

92. $\int \frac{\sqrt{1 + \ln(x)}}{x} dx$

93. $\int \frac{\sqrt{1 + \sqrt{x}}}{\sqrt{x}} dx$

94. $\int \frac{1}{\sqrt{x}\sqrt{1 + \sqrt{x}}} dx$

95. $\int \frac{e^x}{1 + e^{2x}} dx$

96. $\int \frac{\cos(x)}{\sqrt[3]{\sin^2(x)}} dx$

97. $\int \sin(2x)\sqrt{1 + 3\cos^2(x)} dx$

98. $\int \frac{\sin(2x)}{\sqrt{1 + \cos^2(x)}} dx$

99. $\int \frac{\cos^3(x)}{\sin^4(x)} dx$

100. $\int \frac{\sqrt[3]{\tan^2(x)}}{\cos^2(x)} dx$

101. $\int \frac{1}{2\sin^2(x) + 3\cos^2(x)} dx$

Intégrales de fonctions du type $\int \frac{Ax + B}{ax^2 + bx + c} dx$:

$$102. \int \frac{1}{x^2 + 2x + 5} dx$$

$$109. \int \frac{3x - 2}{5x^2 - 3x + 2} dx$$

$$103. \int \frac{1}{3x^2 - 2x + 4} dx$$

$$110. \int \frac{3x - 1}{x^2 - x + 1} dx$$

$$104. \int \frac{1}{x^2 + 3x + 1} dx$$

$$111. \int \frac{7x + 1}{6x^2 + x - 1} dx$$

$$105. \int \frac{1}{x^2 - 6x + 5} dx$$

$$112. \int \frac{2x - 1}{5x^2 - x + 2} dx$$

$$106. \int \frac{1}{2x^2 - 2x + 1} dx$$

$$113. \int \frac{6x^4 - 5x^3 + 4x^2}{2x^2 - x + 1} dx$$

$$107. \int \frac{1}{3x^2 - 2x + 2} dx$$

$$114. \int \frac{1}{2 \cos^2(x) + \sin(x) \cos(x) + \sin^2(x)} dx$$

$$108. \int \frac{(6x - 7)}{3x^2 - 7x + 11} dx$$

Intégrales de fonctions du type $\int \frac{Ax + B}{\sqrt{ax^2 + bx + c}} dx$:

$$115. \int \frac{1}{\sqrt{2 - 3x - 4x^2}} dx$$

$$121. \int \frac{1}{\sqrt{5x^2 - x - 1}} dx$$

$$116. \int \frac{1}{\sqrt{1 + x + x^2}} dx$$

$$122. \int \frac{2ax + b}{\sqrt{ax^2 + bx + c}} dx$$

$$117. \int \frac{1}{\sqrt{2ax + x^2}} dx$$

$$123. \int \frac{x + 3}{\sqrt{4x^2 + 4x + 3}} dx$$

$$118. \int \frac{1}{\sqrt{5 - 7x - 3x^2}} dx$$

$$124. \int \frac{x - 3}{\sqrt{3 + 66x - 11x^2}} dx$$

$$119. \int \frac{1}{\sqrt{x(3x + 5)}} dx$$

$$125. \int \frac{x + 3}{\sqrt{3 + 4x - 4x^2}} dx$$

$$120. \int \frac{1}{\sqrt{2 - 3x - x^2}} dx$$

$$126. \int \frac{3x + 5}{\sqrt{x(2x - 1)}} dx$$

Intégration par partie :

127. $\int x e^x dx$

137. $\int \arctan(\sqrt{x}) dx$

128. $\int x \ln(x) dx$

138. $\int \frac{\arcsin(\sqrt{x})}{\sqrt{x}} dx$

129. $\int x \sin(x) dx$

139. $\int \arcsin\left(\sqrt{\frac{x}{x+1}}\right) dx$

130. $\int \ln(x) dx$

140. $\int x \cos^2(x) dx$

131. $\int \arcsin(x) dx$

141. $\int \frac{x \arcsin(x)}{\sqrt{1-x^2}} dx$

132. $\int \ln(1-x) dx$

142. $\int \frac{x \arctan(x)}{(x^2+1)^2} dx$

133. $\int x^n \ln(x) dx$

143. $\int x \arctan(\sqrt{x^2-1}) dx$

134. $\int x \arctan(x) dx$

144. $\int \frac{\arcsin(x)}{x^2} dx$

135. $\int x \arcsin(x) dx$

145. $\int \ln(x + \sqrt{1+x^2}) dx$

136. $\int \ln(x^2+1) dx$

146. $\int \arcsin(x) \frac{x}{\sqrt{(1-x^2)^3}} dx$

Introduire des variables trigonométriques :

147. $\int \frac{\sqrt{a^2-x^2}}{x^2} dx$

150. $\int \frac{\sqrt{x^2-a^2}}{x} dx$

148. $\int x^2 \sqrt{4-x^2} dx$

151. $\int \frac{1}{\sqrt{(a^2+x^2)^3}} dx$

149. $\int \frac{1}{x^2 \sqrt{1+x^2}} dx$

Intégration de fonctions rationnelles

152. $\int \frac{2x-1}{(x-1)(x-2)} dx$

154. $\int \frac{x^5+x^4-8}{x^3-4x} dx$

153. $\int \frac{x}{(x+1)(x+3)(x+5)} dx$

155. $\int \frac{x^4}{(x^2-1)(x+2)} dx$

$$156. \int \frac{1}{(x-1)^2(x-2)} dx$$

$$157. \int \frac{x-8}{x^3-4x^2+4x} dx$$

$$158. \int \frac{3x+2}{x(x+1)^3} dx$$

$$159. \int \frac{x^2}{(x+2)^2(x+4)^2} dx$$

$$160. \int \frac{1}{x(x^2+1)} dx$$

$$161. \int \frac{2x^2-3x-3}{(x-1)(x^2-2x+5)} dx$$

$$162. \int \frac{x^3-6}{x^4+6x^2+8} dx$$

$$163. \int \frac{1}{x^3+1} dx$$

$$164. \int \frac{3x-7}{x^3+x^2+4x+4} dx$$

$$165. \int \frac{4}{x^4+1} dx$$

$$166. \int \frac{x^5}{x^3-1} dx$$

Les trois intégrales ci-dessous sont hors du programme du CMS.

$$167. \int \frac{x^3+x-1}{(x^2+2)^2} dx$$

$$168. \int \frac{4x^2-8x}{(x-1)^2(x^2+1)^2} dx$$

$$169. \int \frac{1}{(x^2-x)(x^2-x+1)^2} dx$$

Intégration de fonctions irrationnelles

$$170. \int \frac{\sqrt{x}}{\sqrt[4]{x^3+1}} dx$$

$$171. \int \frac{\sqrt{x^3}-\sqrt[3]{x}}{6\sqrt[4]{x}} dx$$

$$172. \int \frac{\sqrt[6]{x}+1}{\sqrt[6]{x^7}+\sqrt[4]{x^5}} dx$$

$$173. \int \frac{2+\sqrt[3]{x}}{\sqrt[6]{x}+\sqrt[3]{x}+\sqrt{x}+1} dx$$

$$174. \int \frac{1}{x^2} \sqrt{\frac{1-x}{1+x}} dx$$

$$175. \int \frac{1}{x} \sqrt{\frac{1-x}{1+x}} dx$$

$$176. \int \frac{\sqrt[7]{x}+\sqrt{x}}{\sqrt[7]{x^8}+\sqrt[14]{x^{15}}} dx$$

$$177. \int \sqrt{\frac{2+3x}{x-3}} dx$$

Intégrales du type $\int R(x, \sqrt{ax^2+bx+c}) dx$:

$$178. \int \frac{1}{x\sqrt{x^2-x+3}} dx$$

$$182. \int \frac{1}{\sqrt{(2x-x^2)^3}} dx$$

$$179. \int \frac{1}{x\sqrt{2+x-x^2}} dx$$

$$183. \int \sqrt{2x-x^2} dx$$

$$180. \int \frac{1}{x\sqrt{x^2+4x-4}} dx$$

$$184. \int \frac{1}{x-\sqrt{x^2-1}} dx$$

$$181. \int \frac{\sqrt{x^2+2x}}{x} dx$$

$$185. \int \frac{1}{(1+x)\sqrt{1+x+x^2}} dx$$

$$186. \int \frac{x+1}{(2x+x^2)\sqrt{2x+x^2}} dx$$

$$188. \int \frac{\sqrt{x^2+4x}}{x^2} dx$$

$$187. \int \frac{1-\sqrt{1+x+x^2}}{x\sqrt{1+x+x^2}} dx$$

Intégration des fonctions trigonométriques :

$$189. \int \sin^3(x) dx$$

$$203. \int \frac{\cos(x)}{\sin^2(x)} dx$$

$$190. \int \sin^5(x) dx$$

$$204. \int \frac{\sin^3(x)}{\sqrt[3]{\cos^4(x)}} dx$$

$$191. \int \cos^4(x) \sin^3(x) dx$$

$$205. \int \sin(x) \sin(3x) dx$$

$$192. \int \frac{\cos^3(x)}{\sin^4(x)} dx$$

$$206. \int \cos(4x) \cos(7x) dx$$

$$193. \int \cos^2(x) dx$$

$$207. \int \cos(2x) \sin(4x) dx$$

$$194. \int \sin^4(x) dx$$

$$208. \int \sin\left(\frac{x}{4}\right) \cos\left(\frac{3x}{4}\right) dx$$

$$195. \int \cos^6(x) dx$$

$$209. \int \frac{1}{4-5\sin(x)} dx$$

$$196. \int \sin^4(x) \cos^4(x) dx$$

$$210. \int \frac{1}{5-3\cos(x)} dx$$

$$197. \int \tan^3(x) dx$$

$$211. \int \frac{\sin(x)}{1+\sin(x)} dx$$

$$198. \int \cot^5(x) dx$$

$$212. \int \frac{\cos(x)}{1+\cos(x)} dx$$

$$199. \int \cot^3(x) dx$$

$$213. \int \frac{\sin(2x)}{\cos^4(x) + \sin^4(x)} dx$$

$$200. \int \frac{1}{\cos^8(x)} dx$$

$$214. \int \frac{1}{(1+\cos(x))^2} dx$$

$$201. \int \frac{\tan^4(x)}{\cos^4(x)} dx$$

$$215. \int \frac{1}{\sin^2(x) + \tan^2(x)} dx$$

$$202. \int \frac{1}{\cos^4(x)} dx$$

$$216. \int \frac{\sin^2(x)}{1+\cos^2(x)} dx$$
