Série 19

Calculer les primitives des fonctions suivantes :

1. $a(x) \;=\; 12\,(x+1)\,(x-2)\,(x-1)\,.$

2. $b(x) \;=\; \dfrac{x^3 - \sqrt[3]{x}}{\sqrt{x}}\,.$

3. $c(x) \;=\; \sin(2x)\cdot\cos^2(x)\,.$

4. $d(x) \;=\; \sin(4x)\cdot\cos(x)\,.$

5. $e(x) \;=\; \dfrac{x}{e^{(x^2+1)}}\,.$

6. $f(x) \;=\; \tan^2(x)\,.$

7. $g(x) \;=\; \dfrac{1}{\sin(x)\cdot\cos(x)}\,.$

8. $h(x) \;=\; \dfrac{1}{1-e^x}\,.$

9. $i(x) \;=\; \dfrac{1}{\cosh(x)}\,.$

10. $j(x) \;=\; \dfrac{\ln x}{x}\,.$

11. $k(x) \;=\; \dfrac{1}{x\cdot\ln^2 x}\,.$

12. $l(x) \;=\; \dfrac{\cos\left(\sqrt{x}\,\right)}{\sqrt{x}}\,.$

13. $m(x) \;=\; \sin^2(a\,x)\,, \quad a\in\mathbb{R}^*.$

14. Calculer les primitives des fonctions suivantes :

a) $a(x) = x\cdot e^{2x}\,,$

b) $b(x) = \ln(x)\,,$

c) $c(x) = \arctan(x)\,,$

d) $d(x) = e^{ax}\,\cos(bx)\,.$

Réponses de la série 19

1. $\displaystyle\int 12\,(x+1)\,(x-2)\,(x-1)\,dx \quad = \quad 3x^4 - 8x^3 - 6x^2 + 24x + C\,.$

2. $\displaystyle\int \frac{x^3 - \sqrt[3]{x}}{\sqrt{x}}\;dx \quad = \quad \frac{2}{7}\,x^{7/2} - \frac{6}{5}\,x^{5/6} + C\,.$

3. $\displaystyle\int \sin(2x)\cdot\cos^2(x)\;dx \quad = \quad -\frac{1}{2}\,\cos^4(x) + C\,.$

4. $\displaystyle\int \sin(4x)\cdot\cos(x)\;dx \quad = \quad -\frac{1}{10}\,\cos(5x) - \frac{1}{6}\,\cos(3x) + C\,, \qquad \text{ou}$

$$\int \sin(4x)\cdot\cos(x)\;dx \quad = \quad -\frac{8}{5}\,\cos^5(x) + \frac{4}{3}\,\cos^3(x) + C'\,.$$

5. $\displaystyle\int \frac{x}{e^{(x^2+1)}}\;dx \quad = \quad -\frac{1}{2}\,e^{-(x^2+1)} + C\,.$

6. $\displaystyle\int \tan^2(x)\;dx \quad = \quad \tan(x) - x + C\,.$

7. $\displaystyle\int \frac{1}{\sin(x)\cdot\cos(x)}\;dx \quad = \quad \ln|\tan(x)| + C\,.$

8. $\displaystyle\int \frac{1}{1-e^x}\;dx \quad = \quad -\ln|e^{-x} - 1| + C\,.$

9. $\displaystyle\int \frac{1}{\cosh(x)}\;dx \;=\; -2\,\arctan(e^{-x}) + C \;=\; 2\,\arctan(e^x) + C' \;=\; \arctan[\sinh(x)] + C''\,.$

10. $\displaystyle\int \frac{\ln x}{x}\;dx = \frac{1}{2}\,\ln^2(x) + C$

11. $\displaystyle\int \frac{1}{x\cdot\ln^2 x}\;dx = -\frac{1}{\ln x} + C$

12. $\displaystyle\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\;dx = 2\,\sin\left(\sqrt{x}\right) + C$

13. $\displaystyle\int \sin^2(a\,x)\;dx = \frac{x}{2} - \frac{\sin(2a\,x)}{4a} + C$

14. a) $\displaystyle \int a(x)\ dx = \frac{1}{4}\,(2x-1)\cdot e^{2x} + C$

b) $\displaystyle \int b(x)\ dx = x\cdot[\,\ln(x)-1\,] + C$

c) $\displaystyle \int c(x)\ dx = x\cdot \arctan(x) - \frac{1}{2}\ \ln(1+x^2) + C$

d) $\displaystyle \int d(x)\ dx = \frac{e^{ax}}{a^2+b^2}\ [\,a\ \cos(bx) + b\ \sin(bx)\,] + C$
