ARTIFACTS MRSI – POINT SPREAD FUNCTION (PSF)

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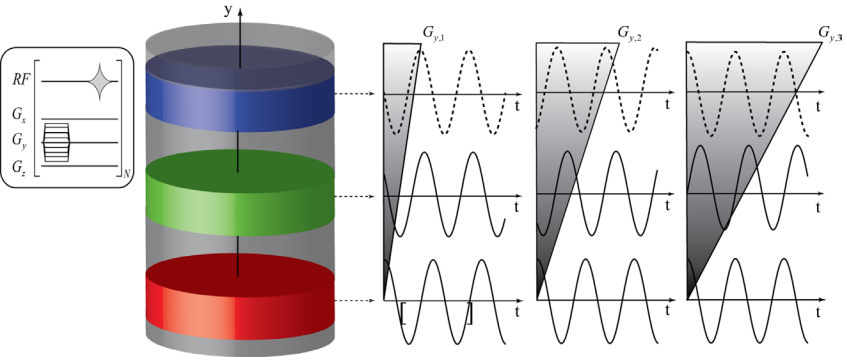
CIBM MRI EPFL AIT

PHYS-770

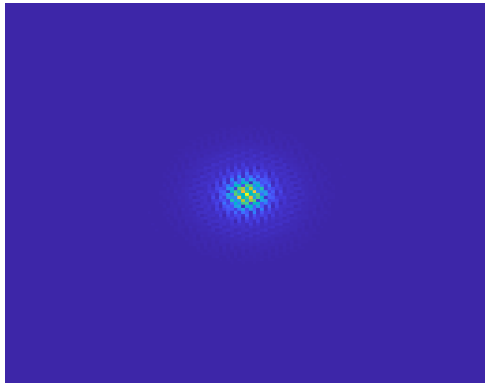
27th of March 2025

MRSI : REMINDER

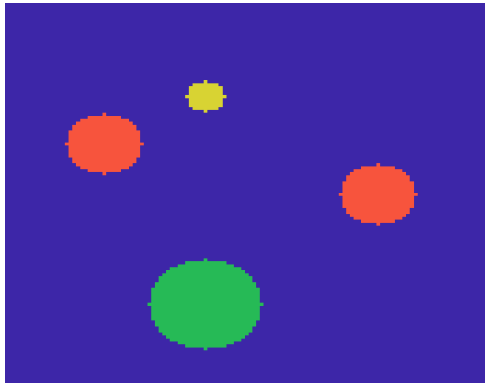
Acquisition & Gradients



2D *k*-space

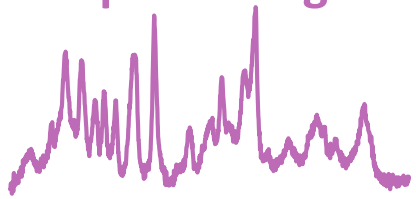


2D image-space



Temporal Signal

Spectral Signal



$$S(\vec{G},t) = \int_{\Omega} s(\vec{r},t)\, e^{-i\gamma\vec{r}\cdot\int_0^{\tau pe}\vec{G}(\tau')d\tau'}\, d\vec{r}$$

$$S(\vec{k},t) = \int_{\Omega} s(\vec{r},t)\, e^{-i\vec{r}\cdot\vec{k}}\, d\vec{r}$$

$$\vec{k} = \gamma \int_0^{\tau pe} \vec{G}\,(\tau')d\tau'$$

$$s(\vec{r},t) = \mathcal{F}_{\vec{k}}[S(\vec{k},t)]$$

$$f(\vec{r},\omega) = \mathcal{F}_t[s(\vec{r},t)]$$

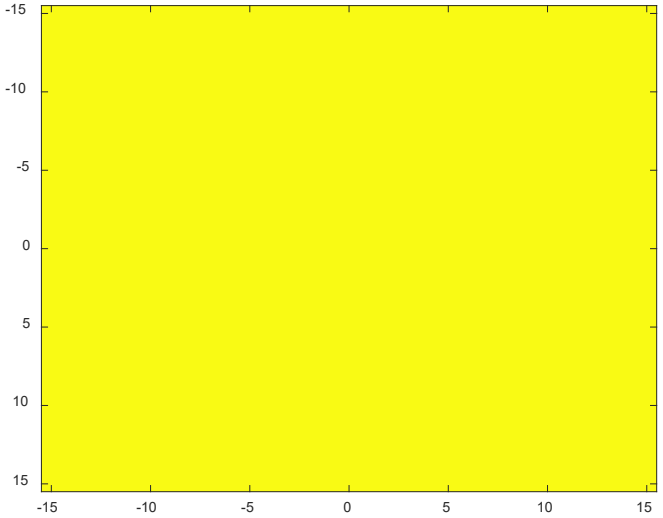
MRSI Spectrum : $f(\vec{r},\omega) = \int_{-\infty}^{+\infty} dt \int_{\Omega} d\vec{k}\, S(\vec{k},t)\, e^{i\vec{r}\cdot\vec{k}}\, e^{-i\omega t}$

THIS IS THEORETICAL !!!!

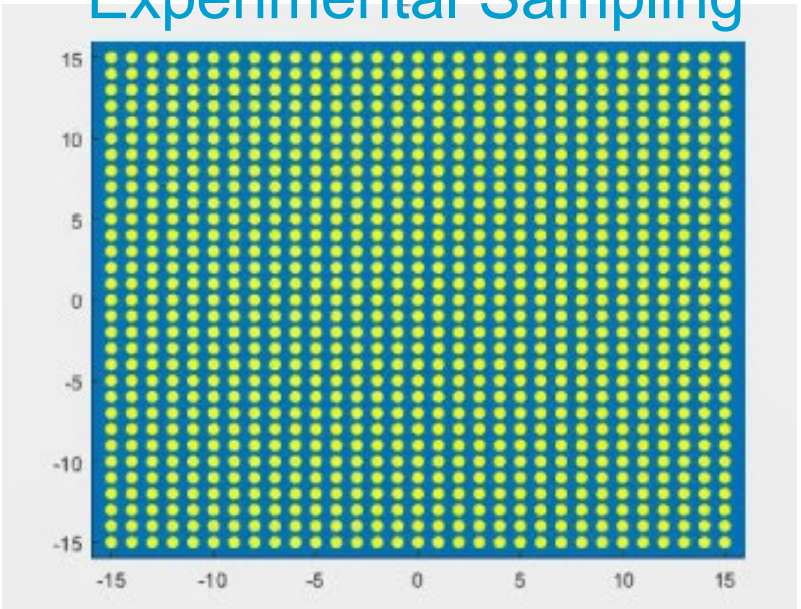
SAMPLING FUNCTION

$$f_{\text{eff}}(\vec{r},\omega) = \int_{-\infty}^{+\infty} dt \int_{\Omega} d\vec{k}\, S_{\text{eff}}(\vec{k},t) e^{i\vec{r}\cdot\vec{k}} e^{-i\omega t}$$

Theoretical Sampling



Experimental Sampling



Nothing really changes…

$$S_{\text{eff}}(\vec{k},t) = S(\vec{k},t)$$

Discreet sampling !!

$$S_{\text{eff}}(\vec{k},t) = F_S(\vec{k}) S(\vec{k},t)$$

SAMPLING FUNCTION

$$f_{\text{eff}}(\vec{r},\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t}dt \int_{\Omega} d\vec{k}\boldsymbol{F_S}(\vec{\boldsymbol{k}})\boldsymbol{S}\,(\vec{\boldsymbol{k}},\boldsymbol{t})e^{i\vec{r}\cdot\vec{k}}$$

$$\Rightarrow F_S(\vec{k})S\,(\vec{k},t) = \mathcal{F}_r[F_S(\vec{r})]\mathcal{F}_r[s(\vec{r},t)] = \mathcal{F}_r[\int_{-\infty}^{+\infty} F_S(\vec{r}')s(\vec{r}-\vec{r}',t)d\vec{r}']$$

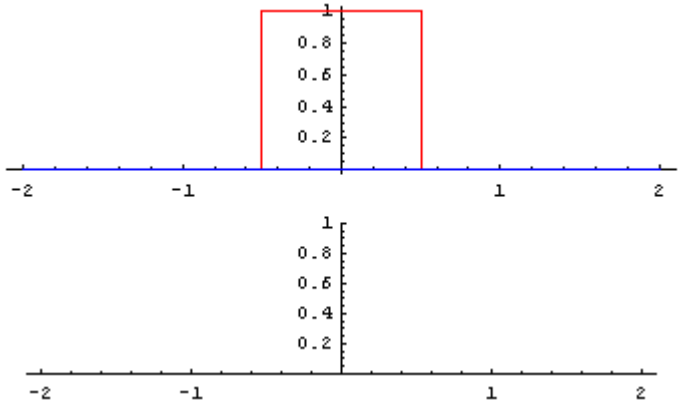
Convolution Theorem :

$$\mathcal{F}_r[f(\vec{r})]\mathcal{F}_r[g(\vec{r})] = \mathcal{F}_r[f(\vec{r}) * g(\vec{r})]$$

Where: $f(\vec{r}) * g(\vec{r}) = \int_{-\infty}^{+\infty} f(\vec{r}-\vec{r}')g(\vec{r}')d\vec{r}'$

After computation :

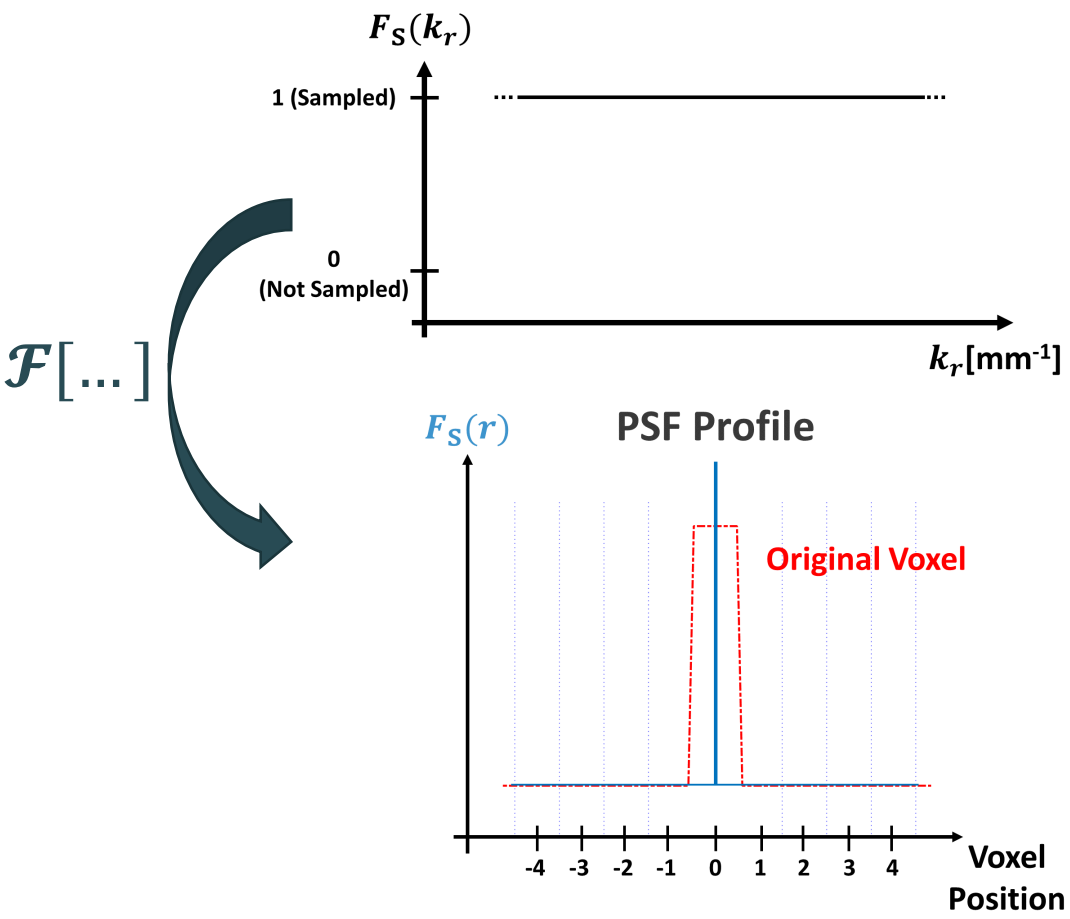
$$\boldsymbol{f_{\text{eff}}(\vec{r},\omega)} = \int_{-\infty}^{+\infty} \boldsymbol{F_S(\vec{r}')f(\vec{r}-\vec{r}',\omega)d\vec{r}'} = \underbrace{\boldsymbol{F_S(\vec{r})}}_{\textbf{Point Spread Function (PSF)}} * \boldsymbol{f(\vec{r},\omega)}$$



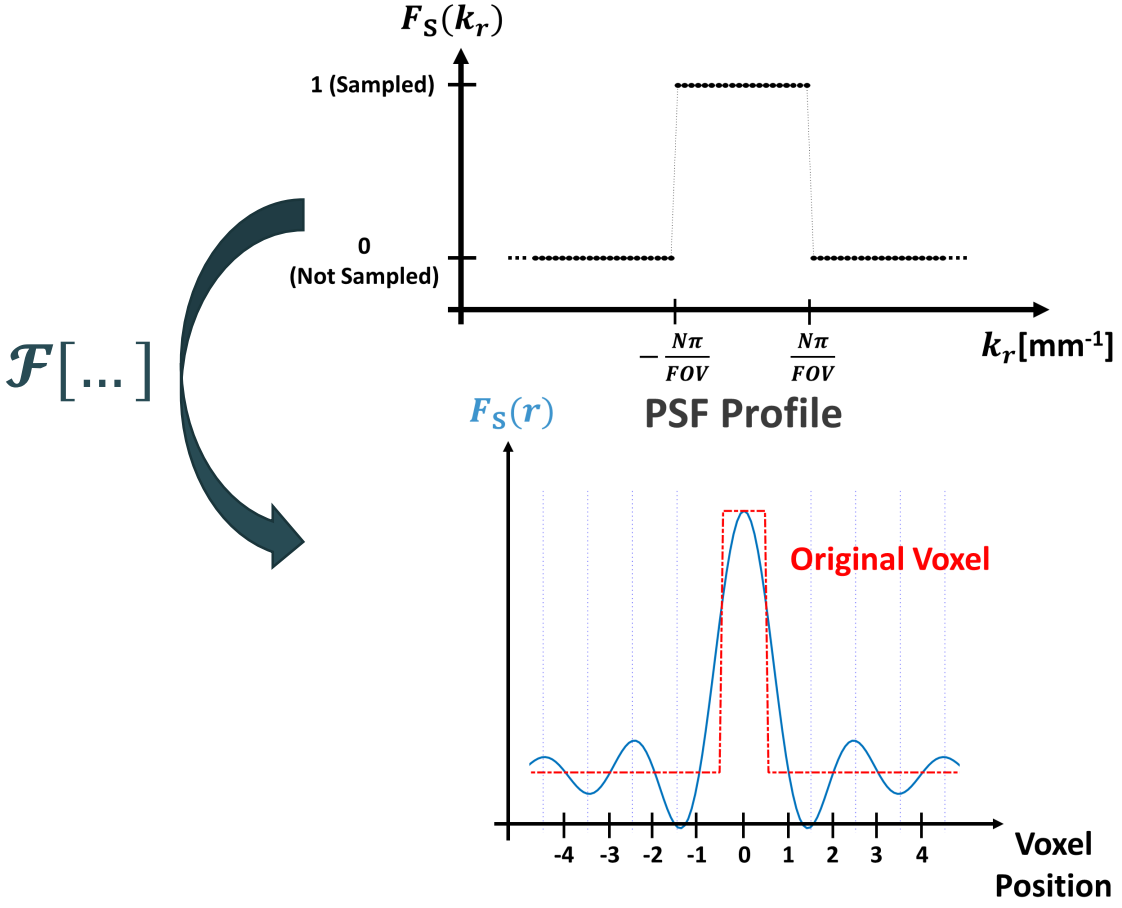
Courtesy of : https://fr.wikipedia.org/wiki/Produit_de_convolution#/media/Fichier:Convolucion_Funcion_Pi.gif

SAMPLING FUNCTION (1D EXAMPLE)

Theoretical Sampling



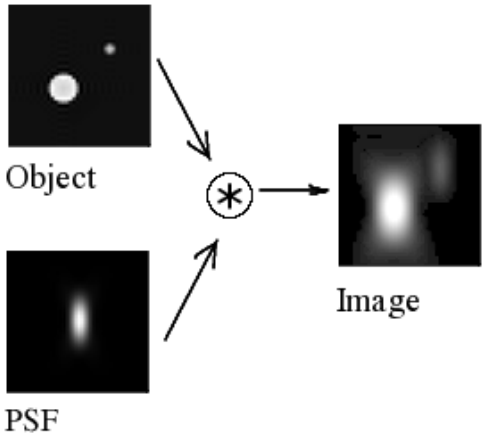
Experimental Sampling



POINT SPREAD FUNCTION - DESCRIPTION

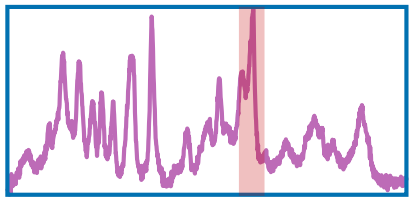
The [Point Spread function] describes what a single point in the object looks like in the image.
(https://svi.nl/Point-Spread-Function-(PSF))

$$I(\vec{r}) = \iiint O(\vec{r}')\mathrm{PSF}(\vec{r} - \vec{r}')d\vec{r}'$$

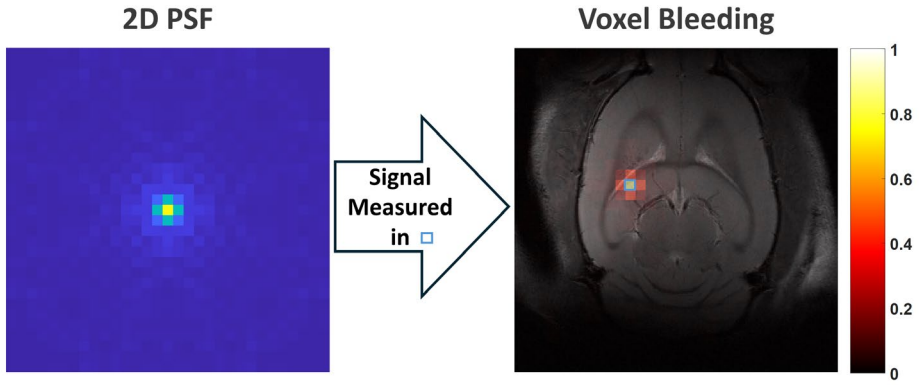


In MRSI…

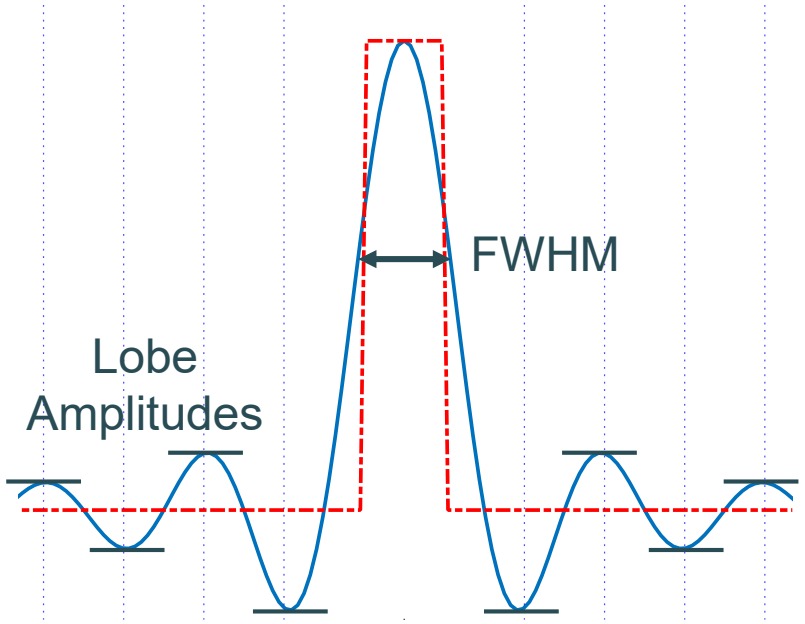
$$O(\vec{r}) \equiv f(\vec{r}, \omega)$$



$*$



Important Metrics

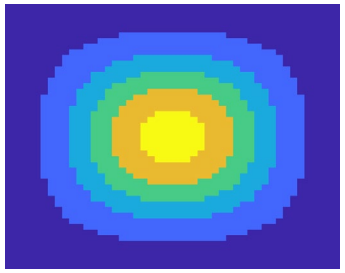


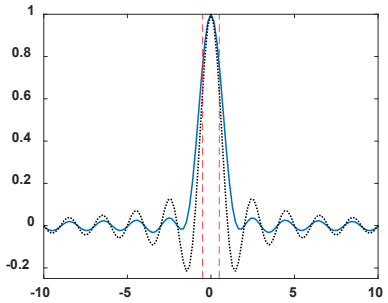
Effective Voxel size = $\Delta V \cdot \mathrm{FWHM}$
Lobe Amplitudes → «Far» contamination (Lipids)

POINT SPREAD FUNCTION - ALTERATION

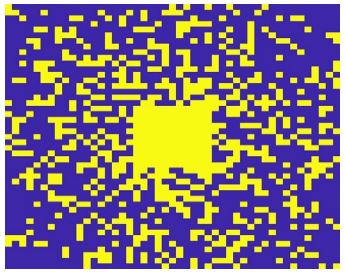
Acquisition Level

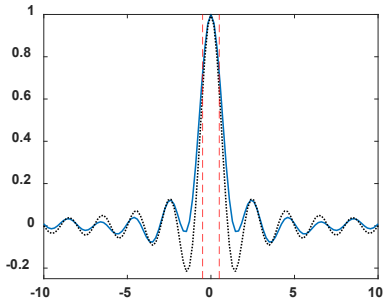
Weighting with averages



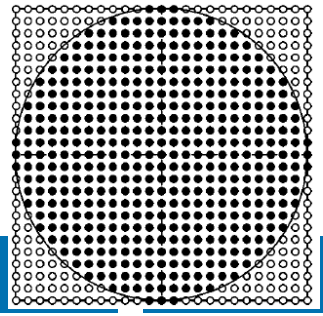


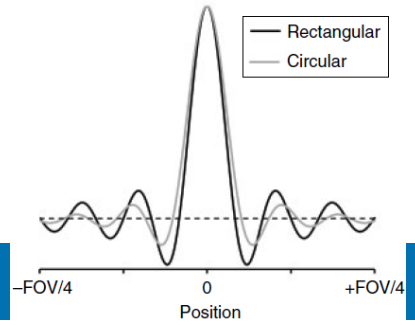
Compressed Sensing





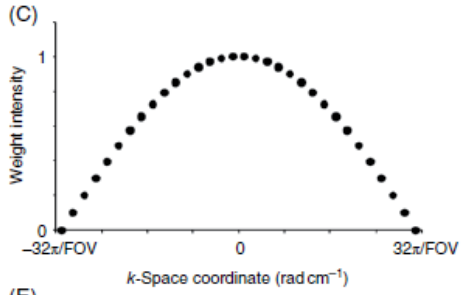
Non-Cartesian Sampling

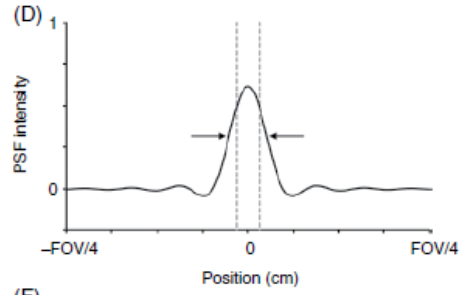




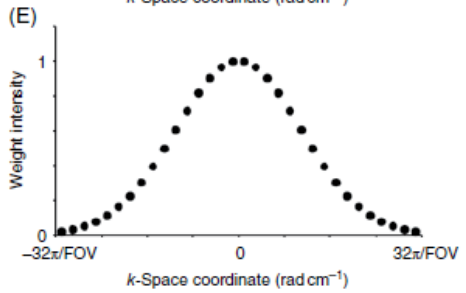
Processing Level (Filters)

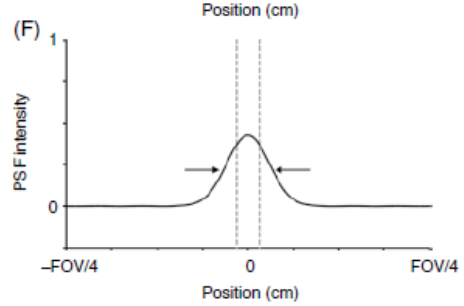
Cosine Filter



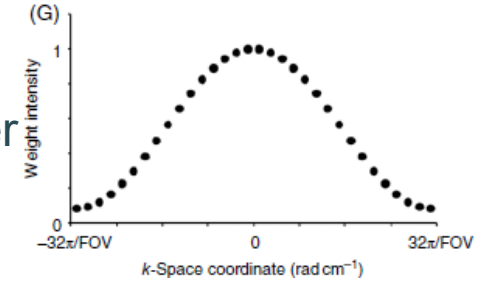


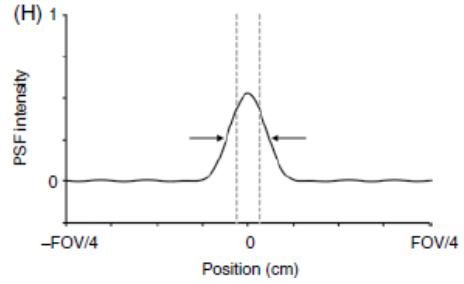
Gaussian Filter





Hamming Filter





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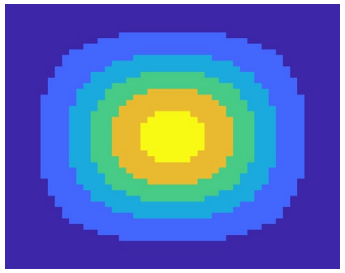
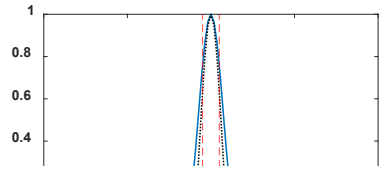
POINT SPREAD FUNCTION - ALTERATION

Acquisition Level

Weighting with averages





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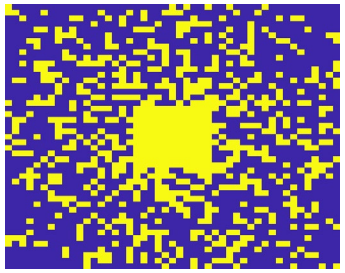
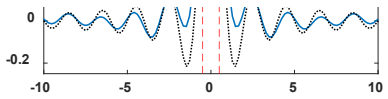
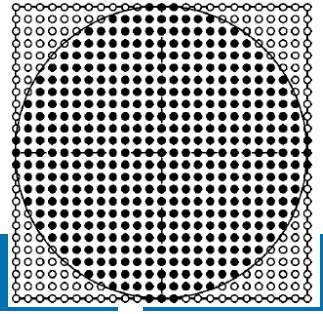
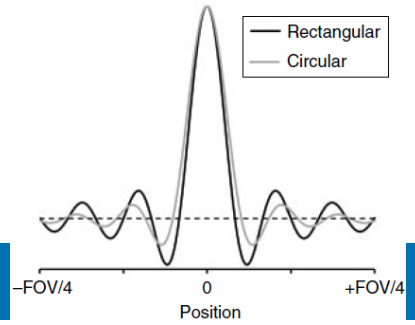




Non-Cartesian Sampling

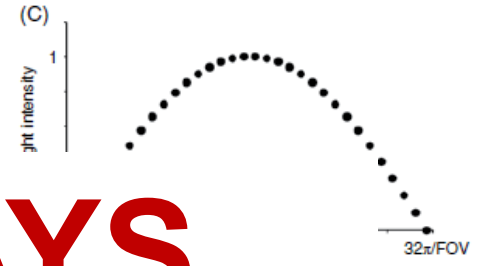


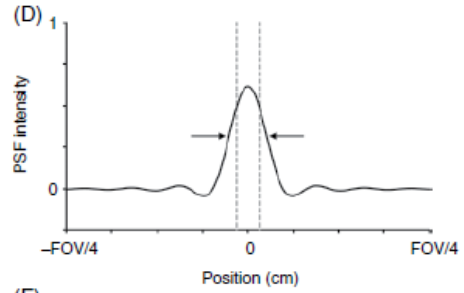


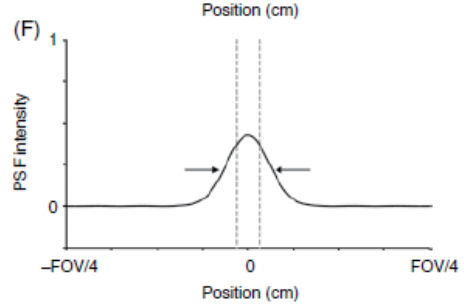
YOU ALWAYS PAY A PRICE !!!

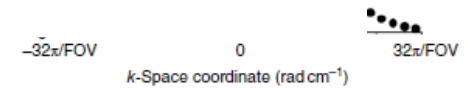
Processing Level (Filters)

Cosine Filter

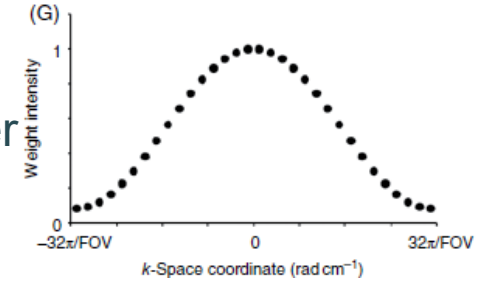


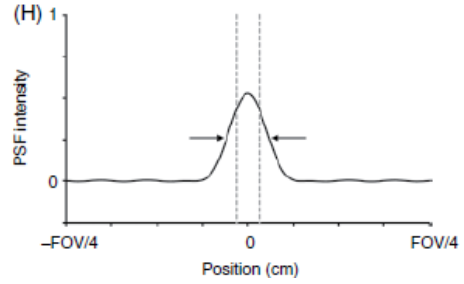






Hamming Filter





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CONCLUSION

- **MRSI spectrum is a combination of multiple spectrum**
- **Weighting is governed by the PSF (Fourier Transform of sampling)**
- **Contamination via voxel bleeding**
  - Neighbor : FWHM and effective voxel volume
  - “Far” : Lobes
- **There are ways to play with the PSF (Filters, sampling strategies…)**

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Center for Biomedical Imaging

THANK YOU FOR YOUR ATTENTION

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FONDATION LEENAARDS