

# Principles and applications of diffusion MRI

Ileana Jelescu, PhD

Assistant Professor, Dept of Radiology, Lausanne University Hospital

Microstructure Mapping Lab



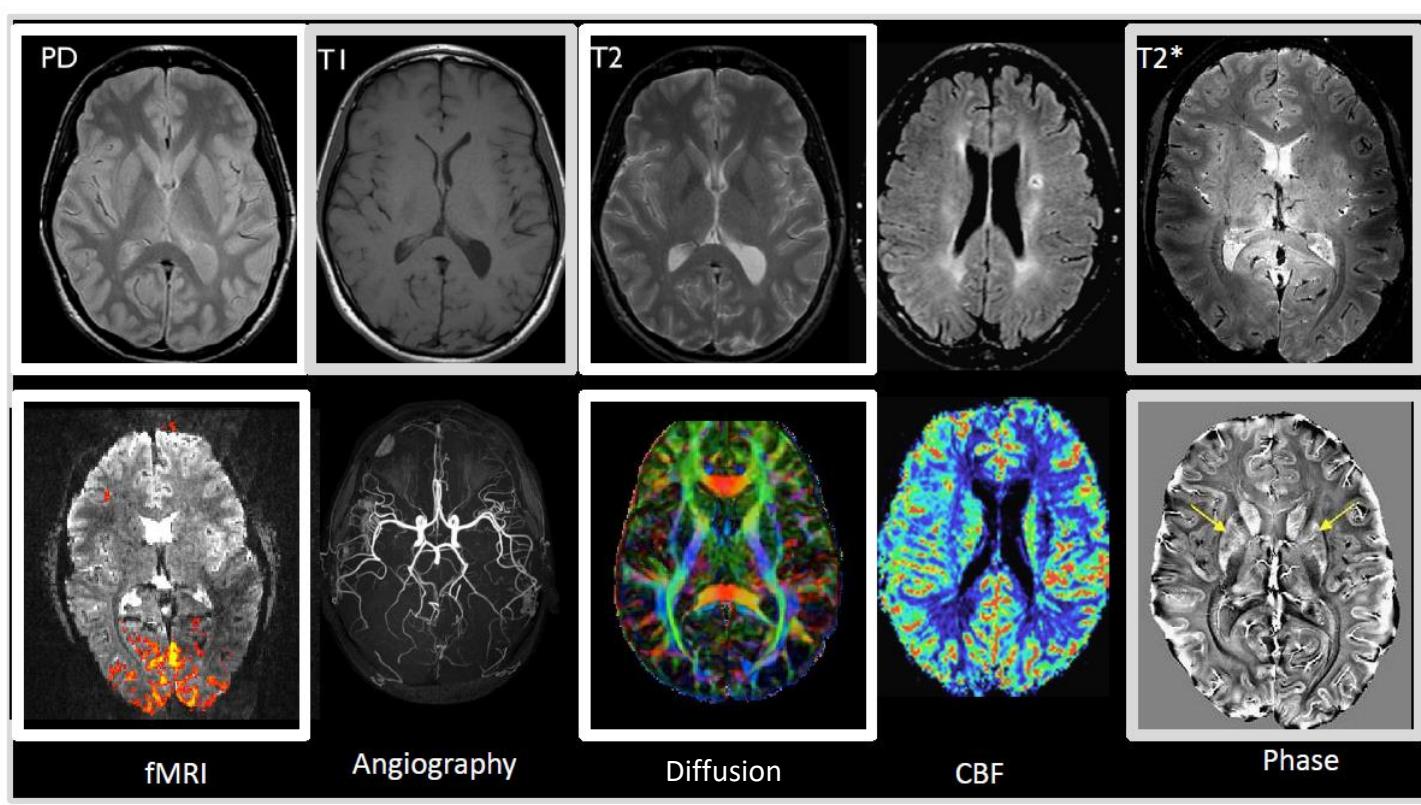
@ijelescu.bsky.social

ijelescu@chuv.ch

# MRI – imaging water

Image contrast can be manipulated to be sensitized to a variety of features

- Depending on: chemical & magnetic properties of immediate environment, mobility of molecules...



Diffusion MRI: sensitizing the signal to the Brownian motion (random walk) of water molecules

# Outline

## I. Diffusion-weighting

## II. Gaussian approximation regime: ADC and DTI

## III. Beyond Gaussian:

### i. Signal representations: DKI

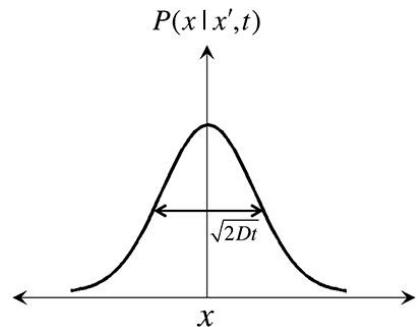
### ii. White matter models & applications

### iii. Gray matter models & applications

### iv. Thinking outside the brain

# Diffusion MRI gives access to the mesoscale

## ➤ Unrestricted homogeneous medium:

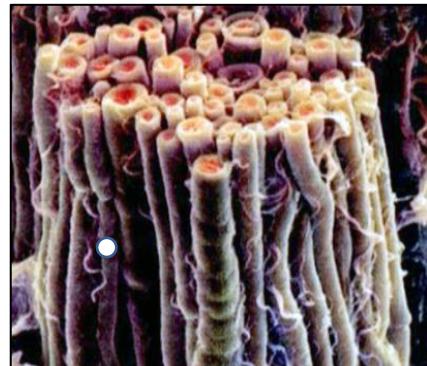


$$\langle x^2 \rangle = 2nDt$$

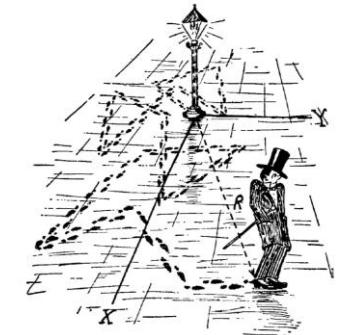
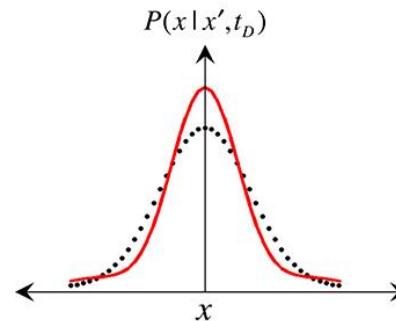
MRI voxel size:  
1 – 2 mm



Typical length scale for  
microstructure: 1 – 10 μm



## ➤ Biological tissue:

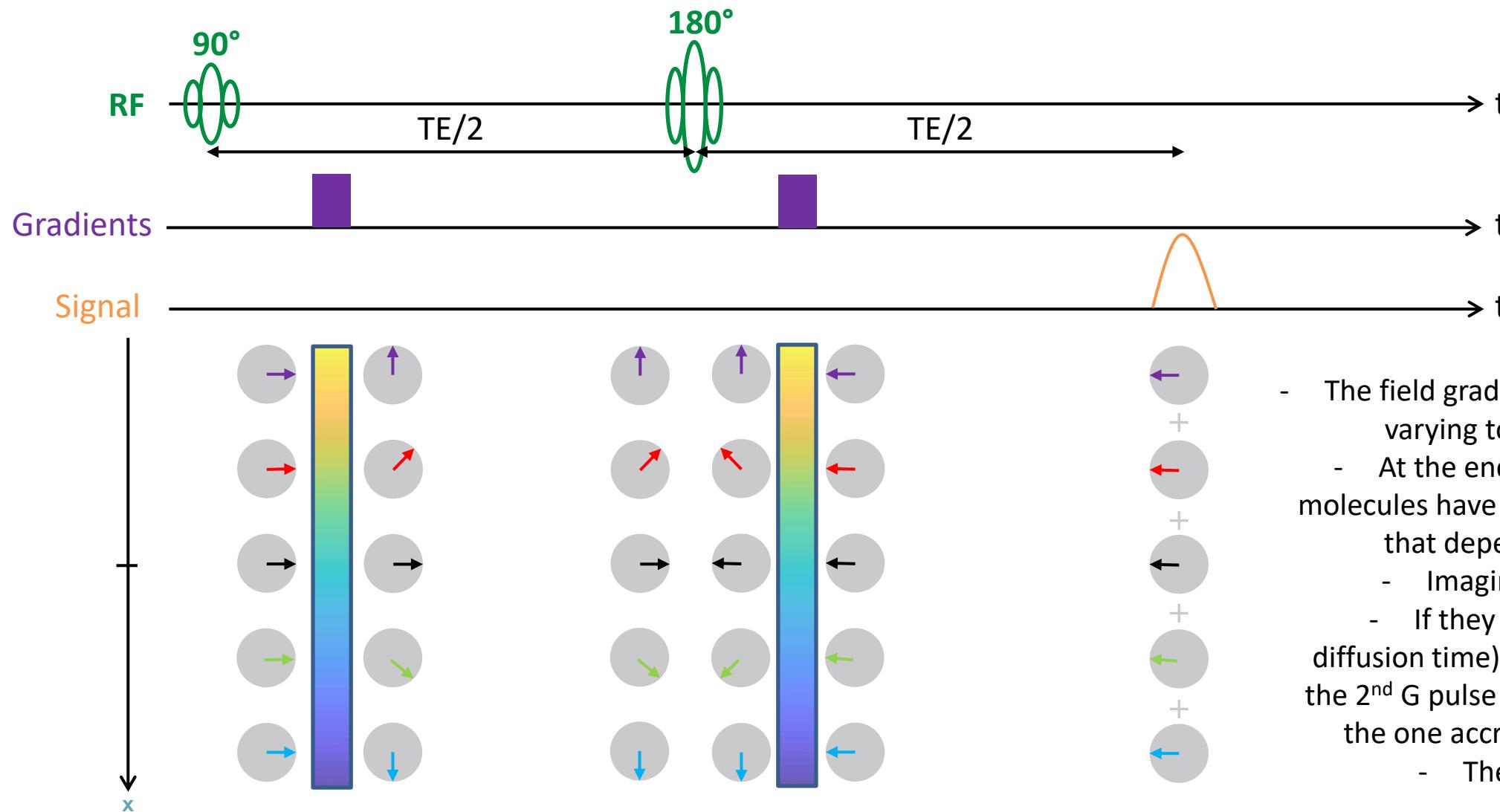


**Mean displacement of water molecules :**

- $t = 2 - 500 \text{ ms} \longrightarrow \sqrt{\langle x^2 \rangle} \approx 2 - 50 \mu\text{m}$

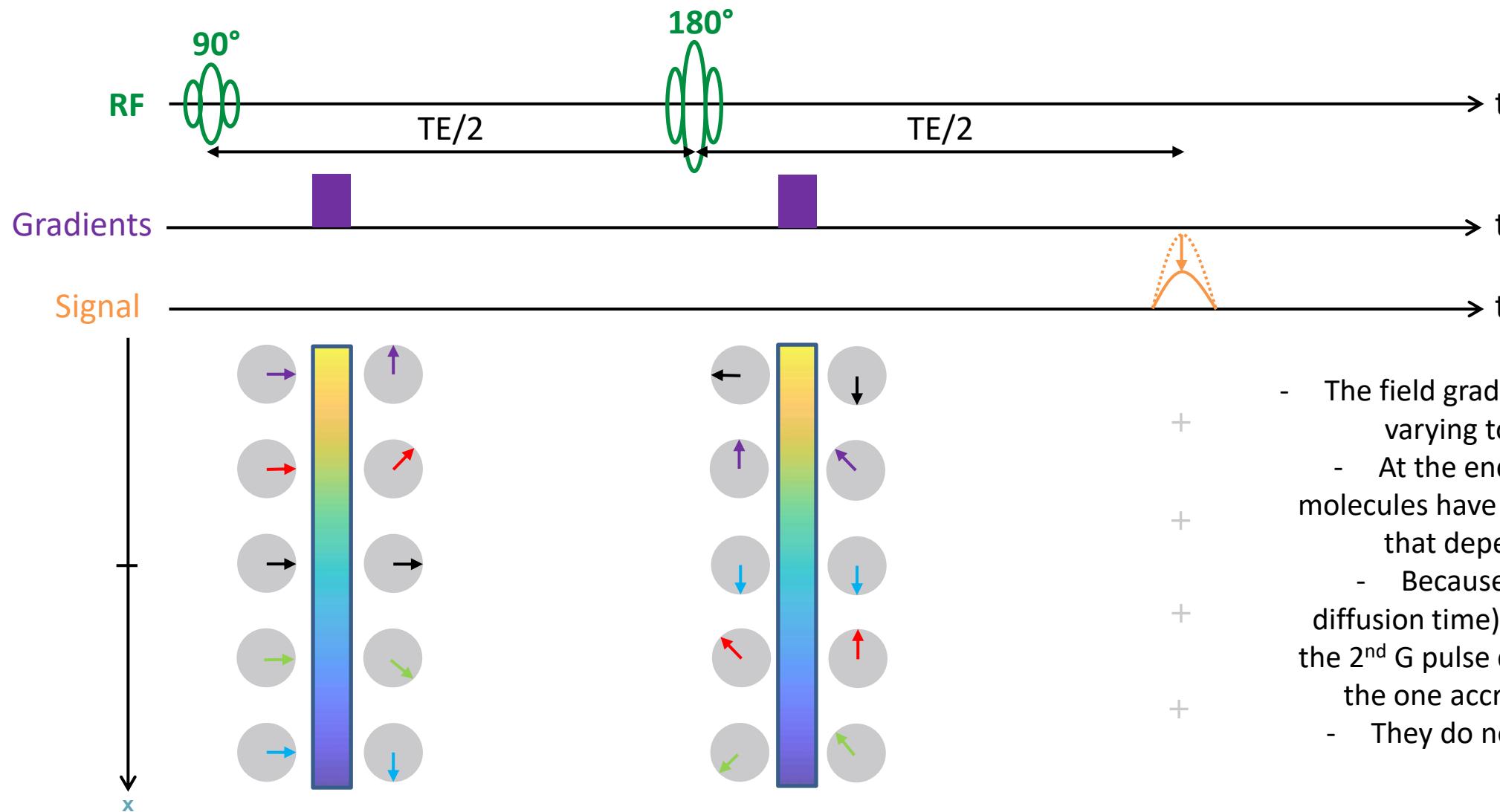
## ➤ Excellent probe of tissue microstructure

# Diffusion weighting in MRI – pulsed gradient spin-echo (PGSE)



- The field gradient creates a linearly varying total field:  $B(x)=B_0+G.x$
- At the end of the (short) pulse, molecules have accumulated a phase that depends on their position
- Imagine: no diffusion ( $D=0$ )
- If they don't mix (during the diffusion time) the phase accrued in the 2<sup>nd</sup> G pulse will fully compensate the one accrued in the 1<sup>st</sup> G pulse
- They add up coherently!

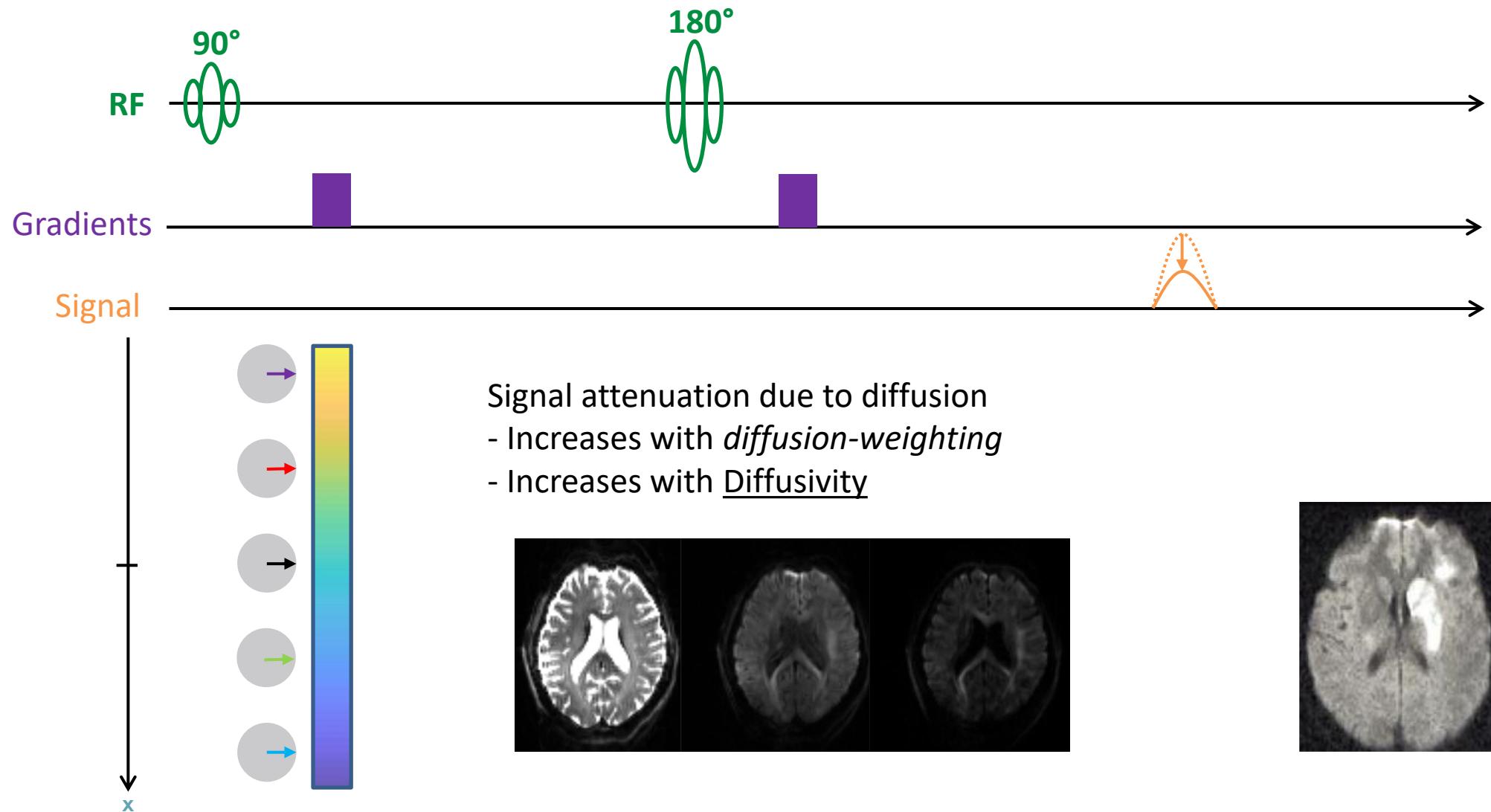
# Diffusion weighting in MRI – pulsed gradient spin-echo (PGSE)



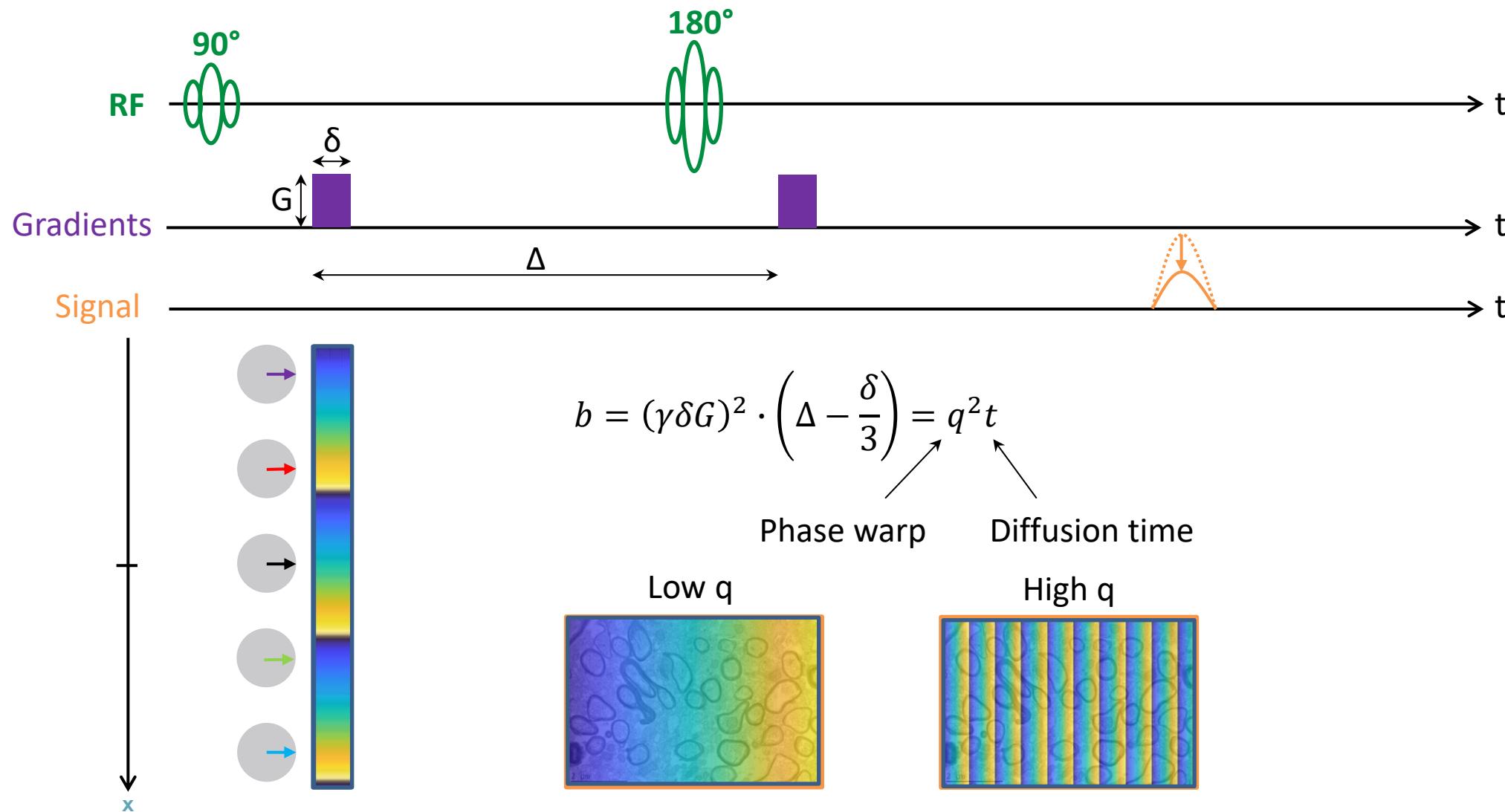
- The field gradient creates a linearly varying total field:  $B(x)=B_0+G.x$
- At the end of the (short) pulse, molecules have accumulated a phase that depends on their position
- Because they mix (during the diffusion time) the phase accrued in the 2<sup>nd</sup> G pulse does not compensate the one accrued in the 1<sup>st</sup> G pulse
- They do not add up coherently!

Signal attenuation

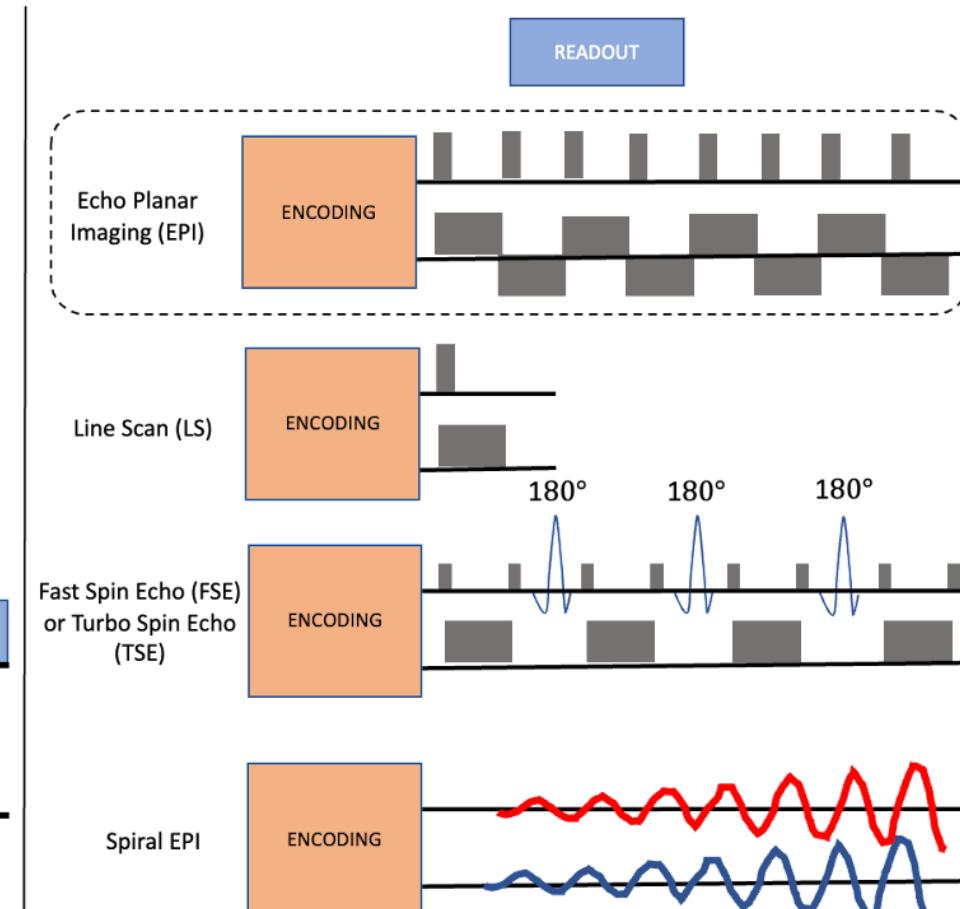
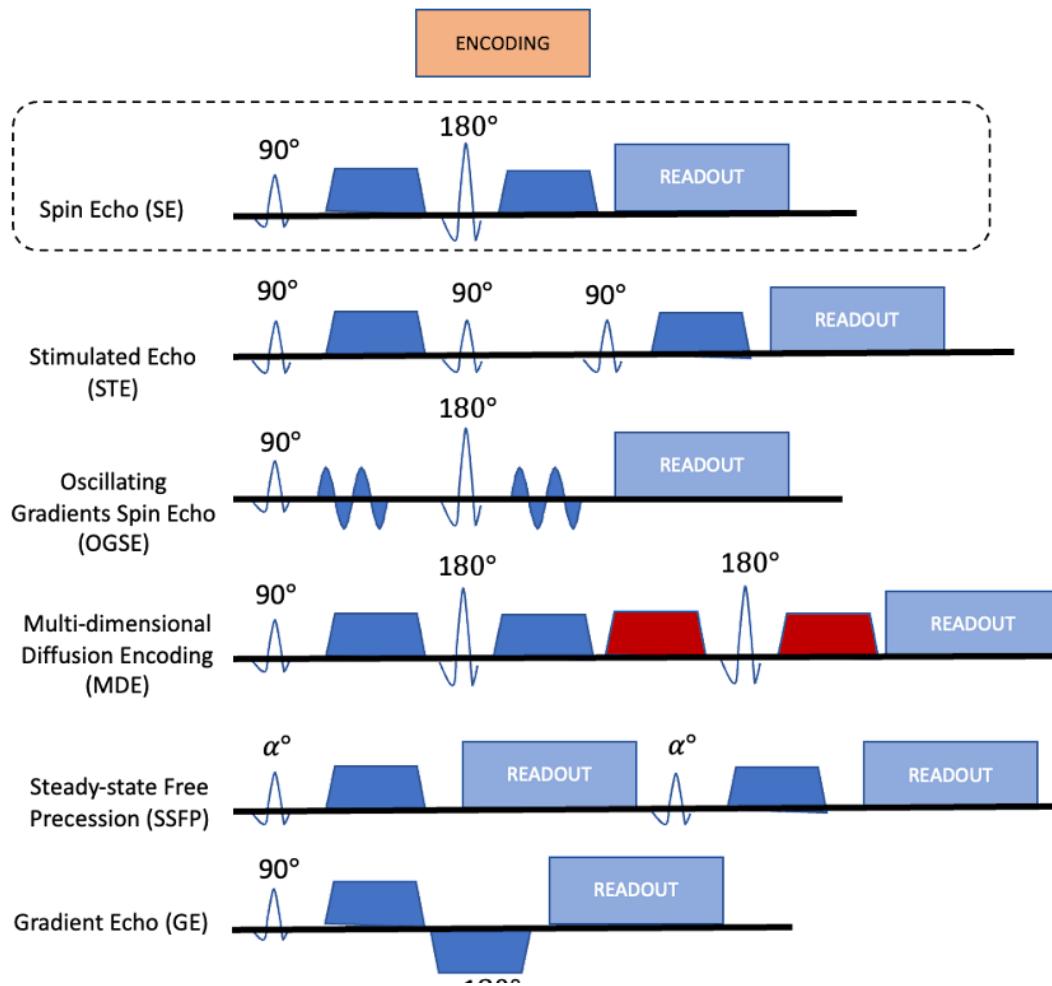
# Diffusion weighting: signal attenuation & contrast



# Diffusion weighting in MRI – q, t and the b-value



# Diffusion encoding schemes and image read-out schemes



# Summary so far

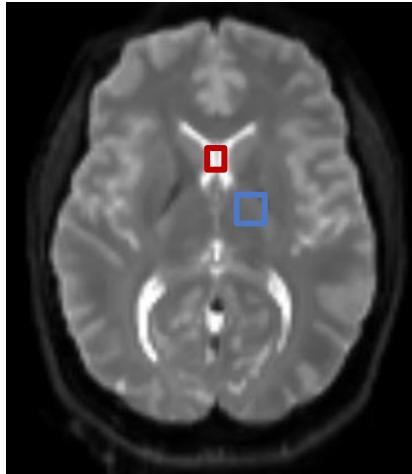
- Diffusion of water molecules in biological tissue (e.g. brain):
  - Is a stochastic process due to thermal agitation, it is always present
  - Is not free: it is affected by the cellular structures (restriction & hindrance)
  - Can provide information about tissue microstructure below the MR image spatial resolution
- In Diffusion MRI, we sensitize the signal amplitude to the diffusion of water molecules using strong magnetic field gradients  $G$
- The MRI signal attenuation depends on:
  - The diffusion properties of the medium (that we want to measure)
  - The diffusion weighting imparted:  $b=q^2.t$  (that we set as acquisition parameter)

# Outline

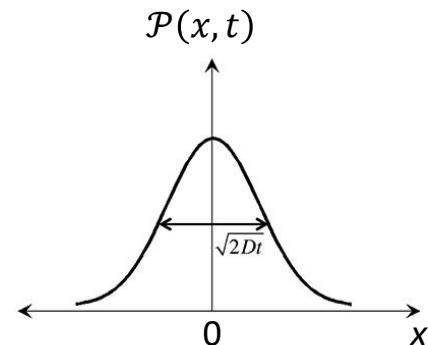
## Agenda

- I. Diffusion-weighting
- II. Gaussian approximation regime: ADC and DTI
- III. Beyond Gaussian:
  - i. Signal representations: DKI
  - ii. White matter models & applications
  - iii. Gray matter models & applications
  - iv. Thinking outside the brain

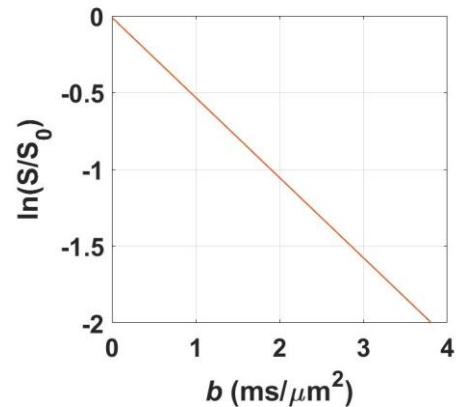
# From a free medium to biological tissue (and from D to ADC)



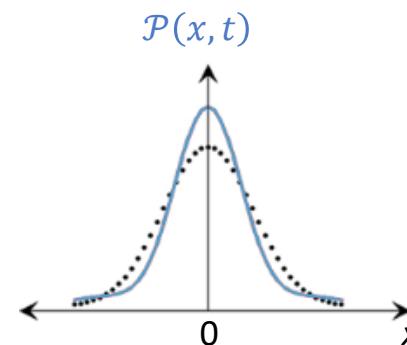
Free homogeneous medium  
Gaussian diffusion



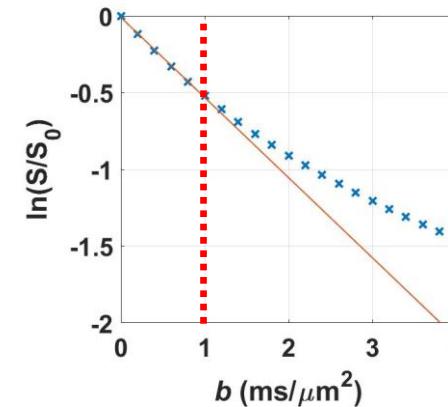
$$\ln\left(\frac{S}{S_0}\right) = -bD$$



Biological tissue  
Non-Gaussian diffusion



$$\ln\left(\frac{S}{S_0}\right) = F(b)$$



Low b-value regime:  
Gaussian approximation  
“DTI” regime

$$\ln\left(\frac{S}{S_0}\right) \approx -b \cdot ADC + \mathcal{O}(b^2)$$



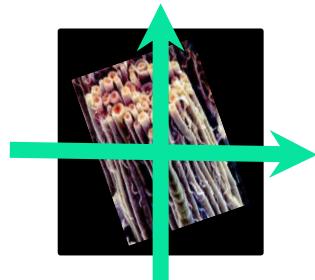
Apparent Diffusion Coefficient

Le Bihan, Radiology 1986

# From the ADC to the diffusion tensor (DTI)

## Apparent Diffusion Coefficient

...  
along a direction  $\vec{g}$ ,  
given by  $\vec{G} = G \cdot \vec{g}$



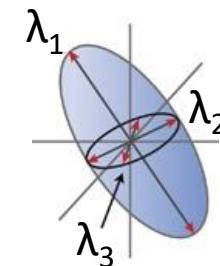
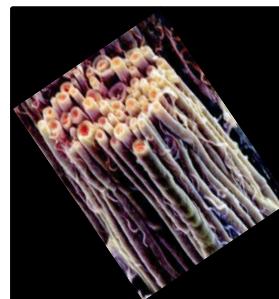
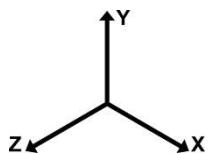
$$\widehat{D}_{\{\vec{x}, \vec{y}, \vec{z}\}} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{pmatrix}$$

$$\ln \frac{S(b, \vec{g})}{S_0} = -b \cdot \text{ADC}_{\vec{g}}$$

$$\ln \frac{S(b, \vec{g})}{S_0} = -b \sum_{i,j=1}^3 g_i g_j D_{ij} = -b \vec{g}^T \widehat{D} \vec{g}$$

$$\vec{g} = [1 \ 0 \ 0]$$

$$\vec{g} = [0 \ 1 \ 0]$$



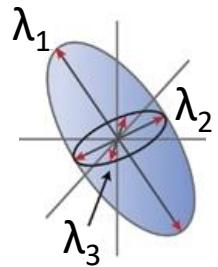
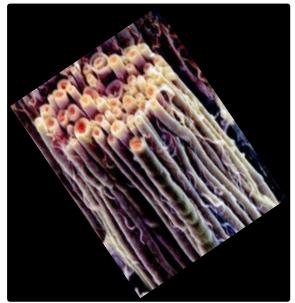
$$\widehat{D}_{\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

- Symmetric  $\rightarrow$  6 coefficients to estimate
- Invertible with all eigenvalues real and positive

Minimal data:

- **1  $b = 0$**
- **6 non collinear directions on 1 shell (e.g.  $b = 1 \text{ ms}/\mu\text{m}^2$ )**

# DTI scalars

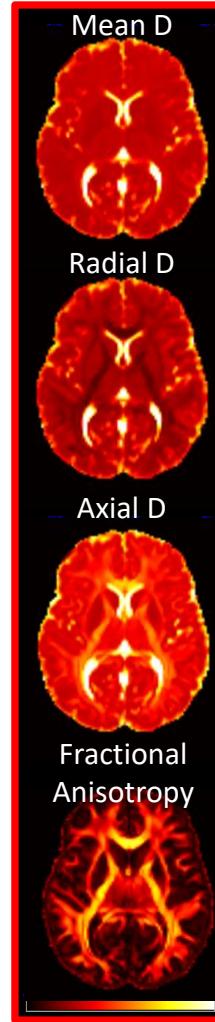


$$\widehat{D}_{\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

In white matter in particular:

$\vec{e}_1$ : direction of faster diffusion in the voxel  
(along the axon bundle)

→ Useful in early days of tractography for tracking fiber orientations from voxel to voxel



Mean diffusivity:

$$MD = (\lambda_1 + \lambda_2 + \lambda_3)/3$$
$$MD = \text{Tr}(\widehat{D})/3$$

$MD \sim 1 \frac{\mu\text{m}^2}{\text{ms}}$  in brain, little GM/WM contrast

Radial diffusivity:

$$RD = (\lambda_2 + \lambda_3)/2$$

Good GM/WM contrast, lowest in WM

Axial diffusivity:

$$AD = \lambda_1$$

Some GM/WM contrast, highest in WM

Fractional anisotropy:

$$FA = \sqrt{\frac{1}{2} \frac{\sqrt{(\lambda_1 - \lambda_2)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_3 - \lambda_1)^2}}{\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}}$$

FA = 0 : isotropic diffusion, FA = 1 : fully anisotropic (1D)

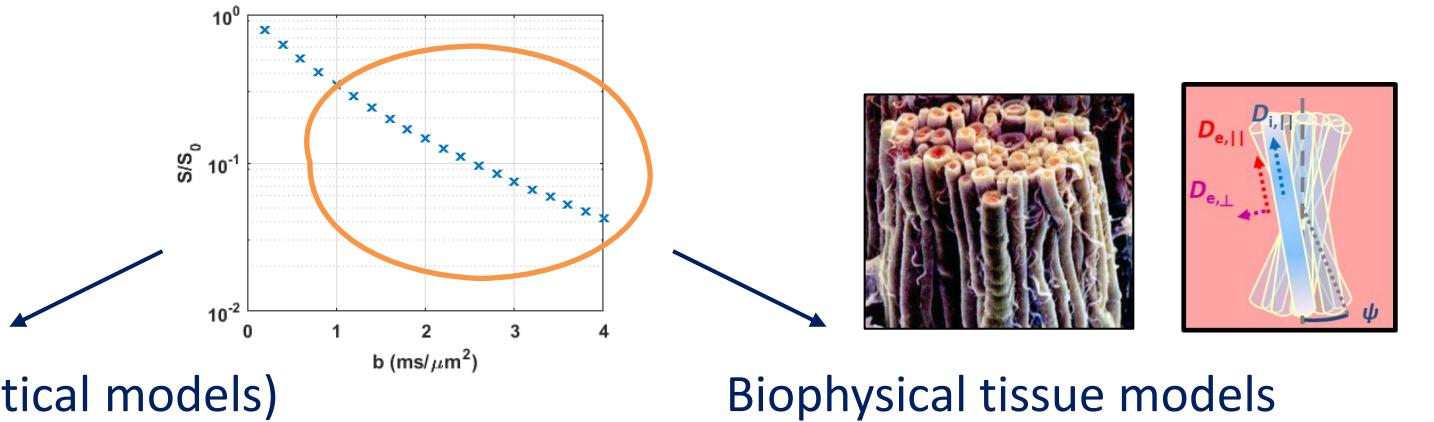
Excellent GM/WM contrast, highest in WM

# Outline

## Agenda

- I. Diffusion-weighting
- II. Gaussian approximation regime: ADC and DTI
- III. Beyond Gaussian:
  - i. Signal representations: DKI
  - ii. White matter models & applications
  - iii. Gray matter models & applications
  - iv. Thinking outside the brain

# Signal representations vs biophysical models



## Signal representations (statistical models)

- “Formulas” to describe signal decay
- ☺ Applicable to any type of tissue
- ☺ Provide sensitive biomarkers
- ☹ Lack specificity: difficult to make inferences about microstructural changes

## Biophysical tissue models

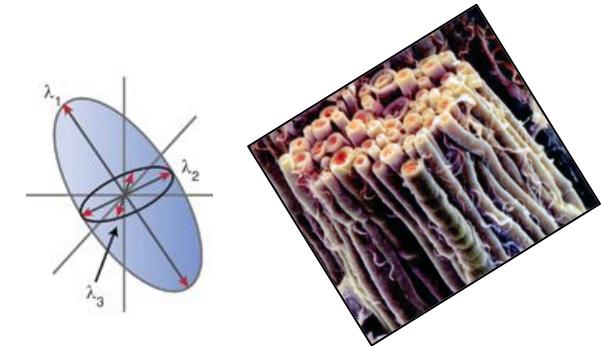
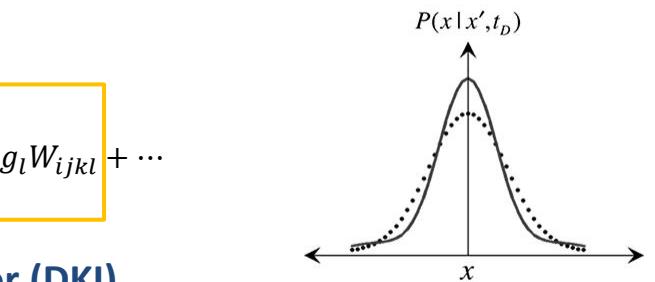
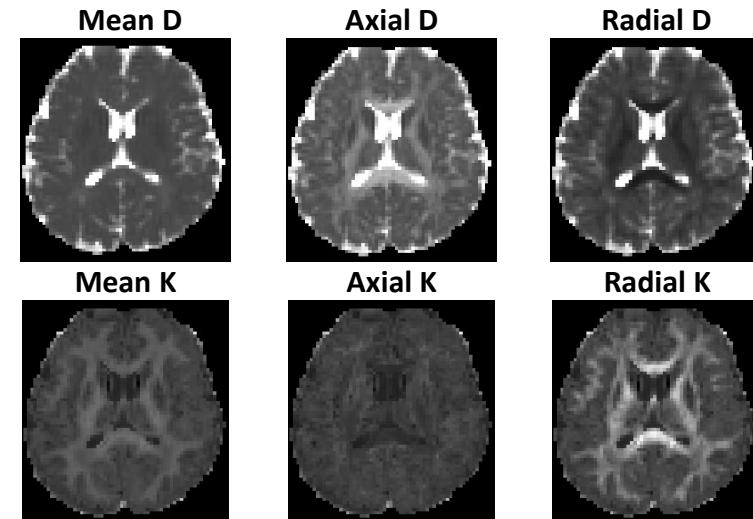
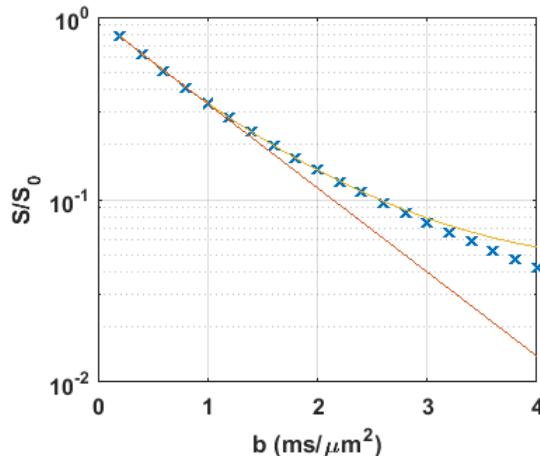
- Simplified “pictures” of medium in which water molecules are diffusing
- ☺ Tissue-specific
- ☺ Specific biomarkers that characterize microstructure
- ☹ Assumption validity?
- ☹ Fit stability?

# Representations: Diffusion Tensor Imaging (DTI) and Kurtosis (DKI)

$$\ln \frac{S}{S_0} = -bD + \frac{1}{6}b^2D^2K + \dots$$

$$\ln \frac{S}{S_0} = -b \sum_{i,j=1}^3 g_i g_j D_{ij} + \frac{1}{6}b^2 \bar{D}^2 \sum_{i,j,k,l=1}^3 g_i g_j g_k g_l W_{ijkl} + \dots$$

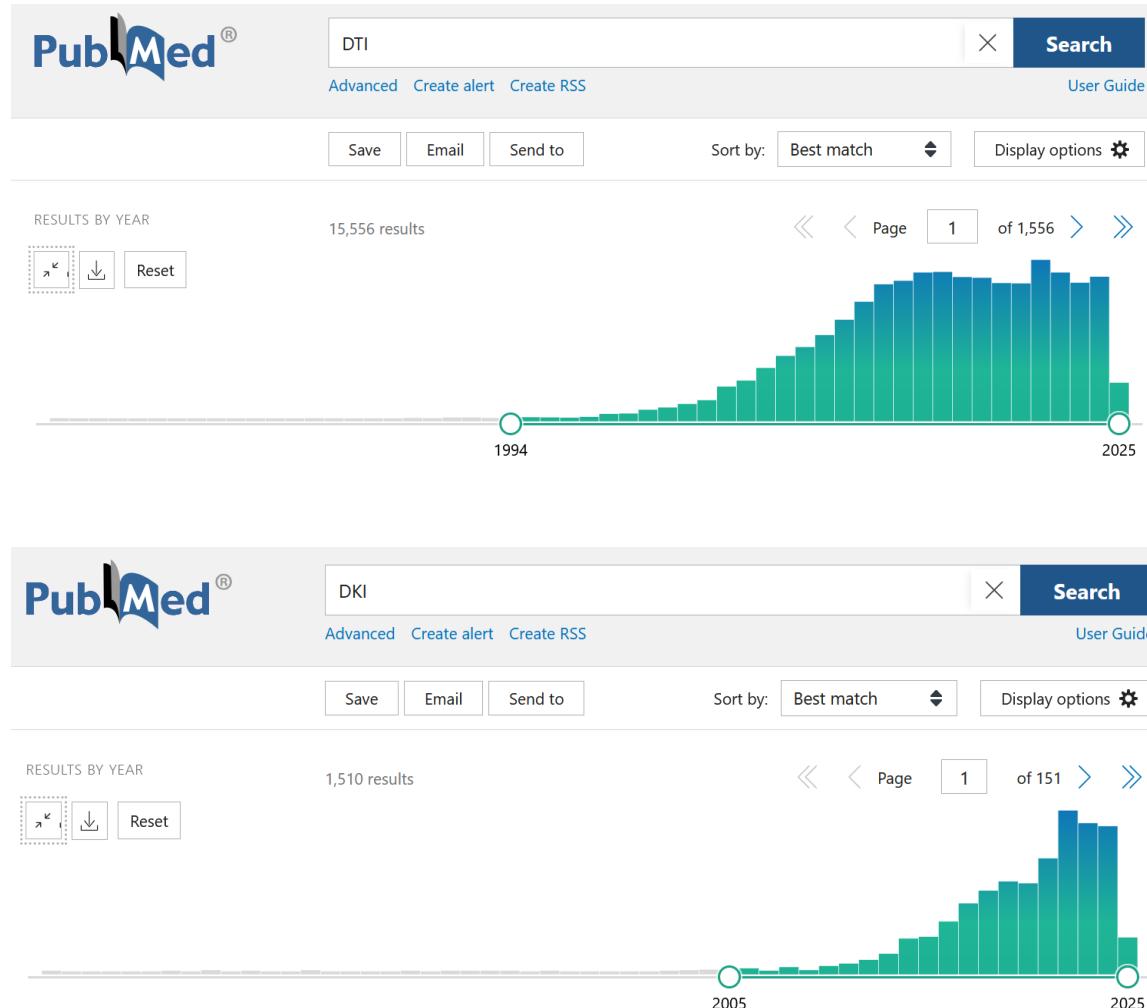
diffusion tensor (DTI) & kurtosis tensor (DKI)



Minimal data:

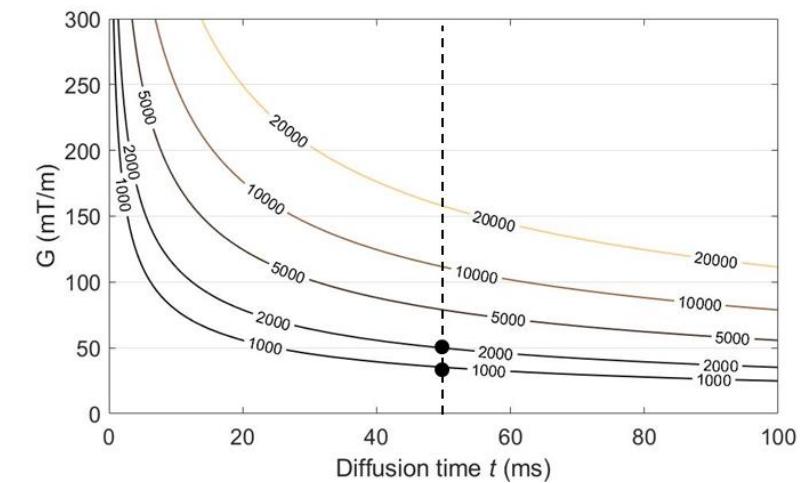
- 1  $b = 0$
- 6 directions on 1 shell (e.g.  $b = 1 \text{ ms}/\mu\text{m}^2$ )
- 15 directions on a 2<sup>nd</sup> shell (e.g.  $b = 2/2.5$ )

# DKI: Valuable



## Practicality

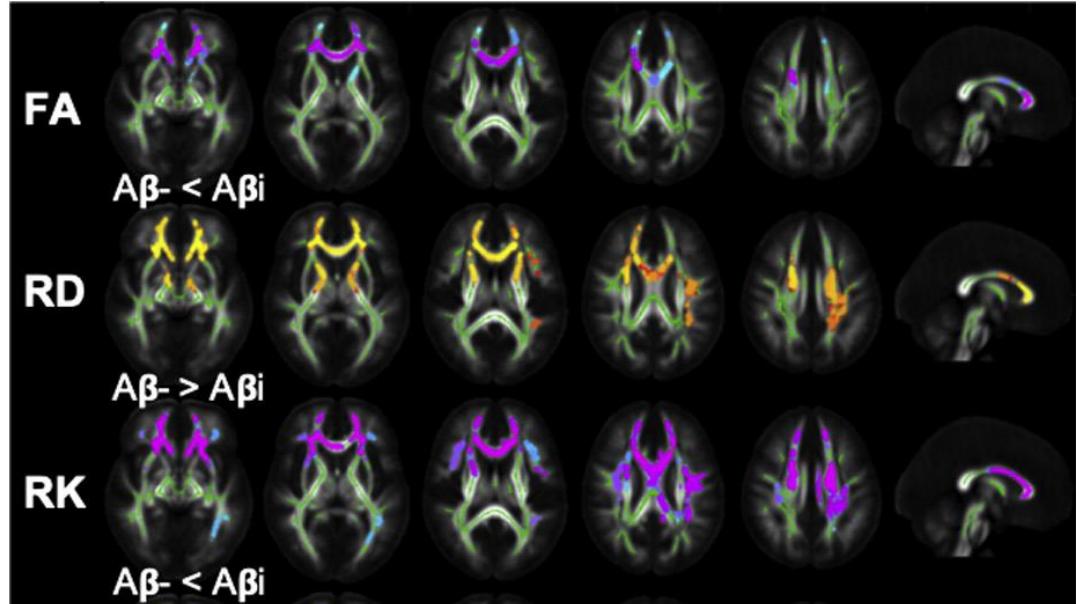
- “Standard” multi-shell acquisition:
  - $1 b = 0, 30 b_1, 60 b_2$
  - ~7-8 min using acceleration (GRAPPA, multiband...)



# DKI: Valuable

- Sensitive to tissue complexity
- Complementary information to DTI:
  - Brain development / aging
  - Dementia
  - Stroke
  - Traumatic Brain Injury
  - Glioma
  - Prostate cancer
  - Other body cancers

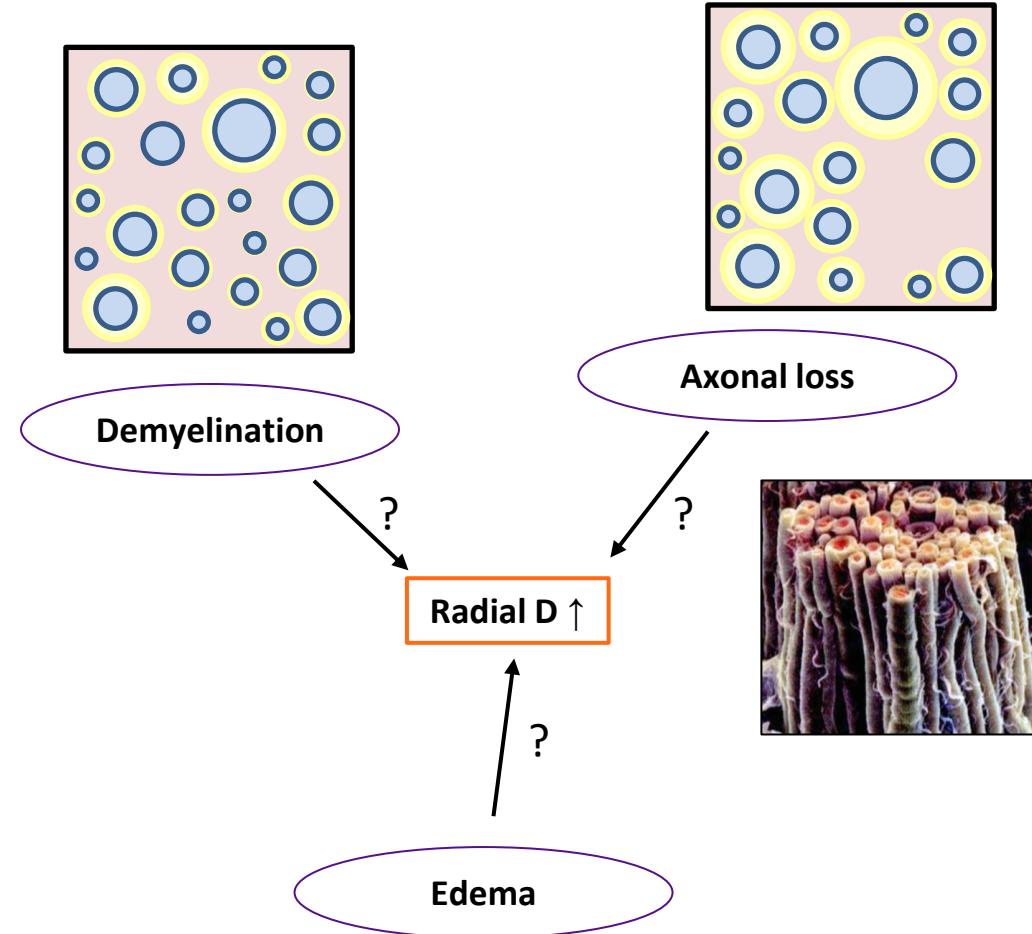
Jensen and Helpern, NMR Biomed 2010  
Hui, Stroke 2012  
Paydar, AJNR 2014  
Rosenkrantz, JMRI 2015  
McKenna, Cortex 2019



Relationship between altered WM microstructure & amyloid load in MCI patients

Dong, Neurobiol Aging 2020

# Representations: Valuable... but still lack specificity

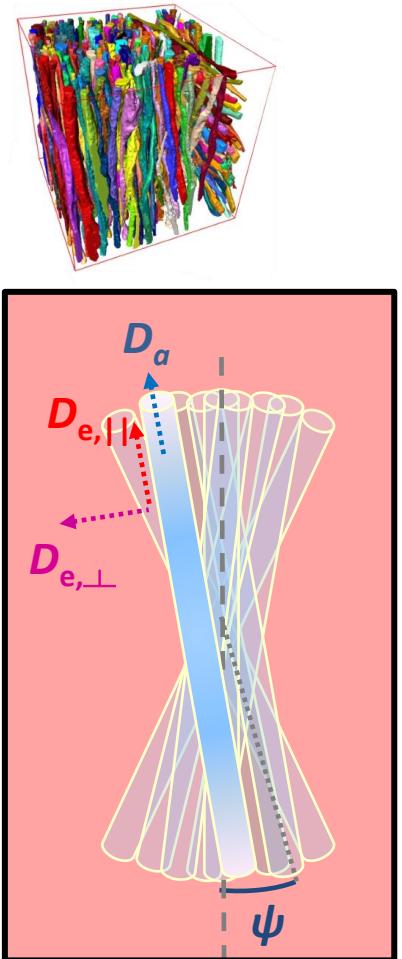


# Outline

## Table of contents

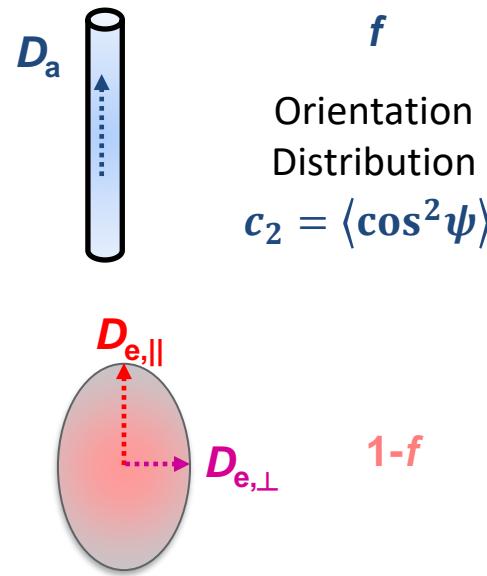
- I. Diffusion-weighting
- II. Gaussian approximation regime: ADC and DTI
- III. Beyond Gaussian:
  - i. Signal representations: DKI
  - ii. White matter models & applications
  - iii. Gray matter models & applications
  - iv. Thinking outside the brain

# White Matter «Standard Model»



Intra-axonal compartment:  
Collection of sticks

Extra-axonal compartment



“Standard Model” of diffusion in white matter:

$$\mathbf{p} = [f, c_2, D_a, D_{e,||}, D_{e,\perp}]$$

Markers of:

$f$ : axonal loss

$c_2$ : pruning

$D_a$ : axonal injury

$D_{e,\perp}$ : demyelination

$D_{e,||}$ : inflammation

# Moving away from spurious assumptions

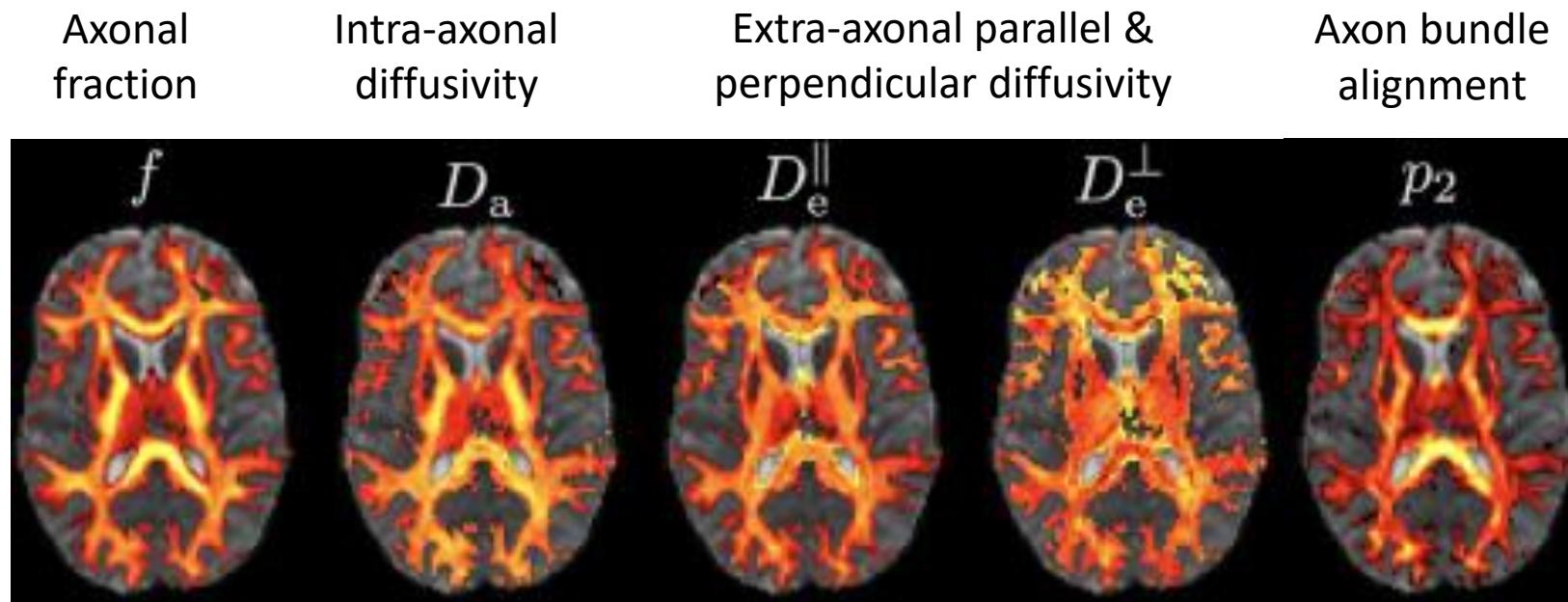
- Favor Standard Model frameworks that introduce minimum assumptions

- WMTI-Watson (from DKI)
- SMI (from Rotational Invariants)

[https://github.com/Mic-map/WMTI-Watson\\_DL](https://github.com/Mic-map/WMTI-Watson_DL)

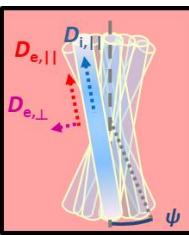
<https://github.com/NYU-DiffusionMRI/SMI>

Jespersen, NeuroImage 2018; Novikov, NeuroImage 2018



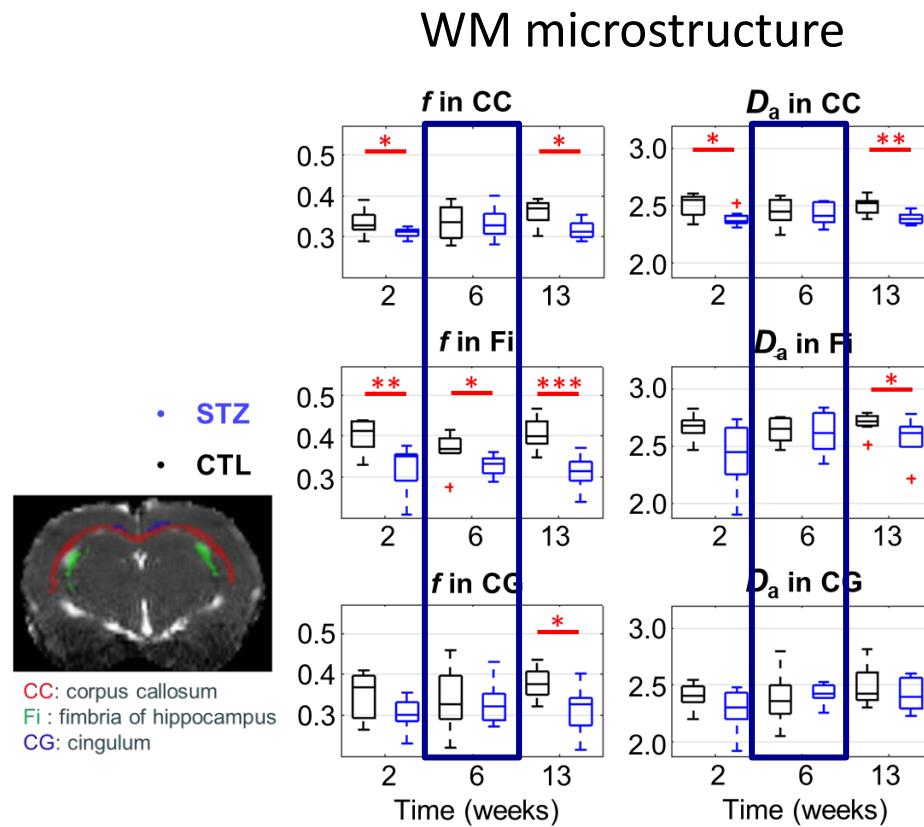
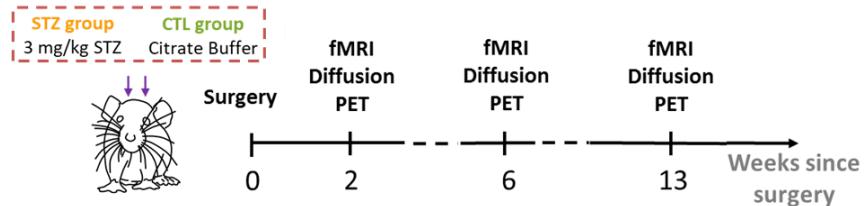
Coelho, NeuroImage 2022

# Applications: Rat model of Alzheimer's ("brain diabetes")



## Injection of streptozotocin (STZ) in lateral ventricles

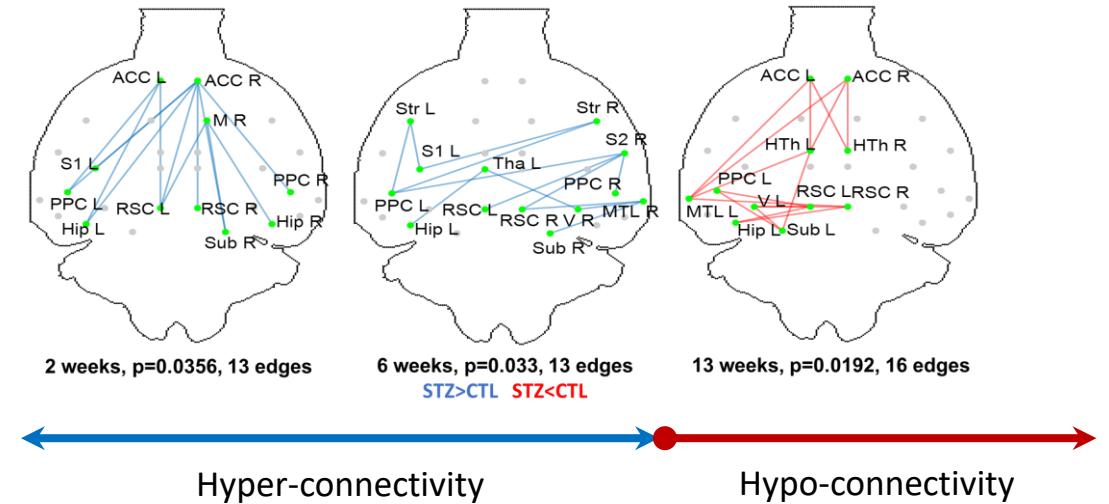
- Insulin-resistant brain state – but no systemic diabetes



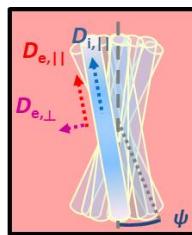
$$[f, c_2, D_a, D_{e,||}, D_{e,\perp}]$$

- Axonal microstructural alterations
- Inflection point: ~6 weeks (temporary recovery?)

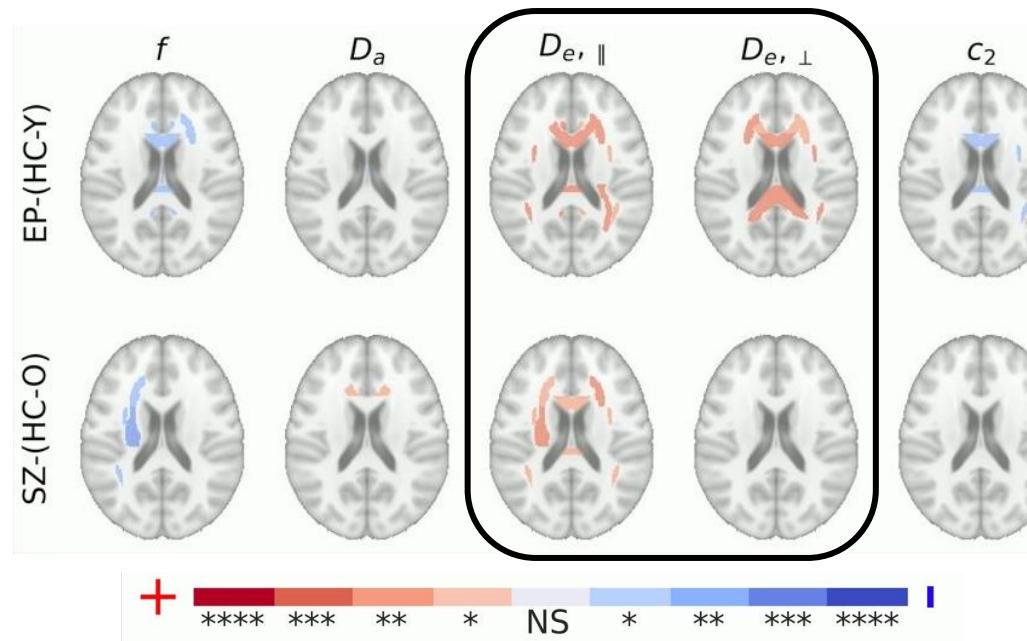
## Functional connectivity



# Applications of WM Standard Model to patient populations



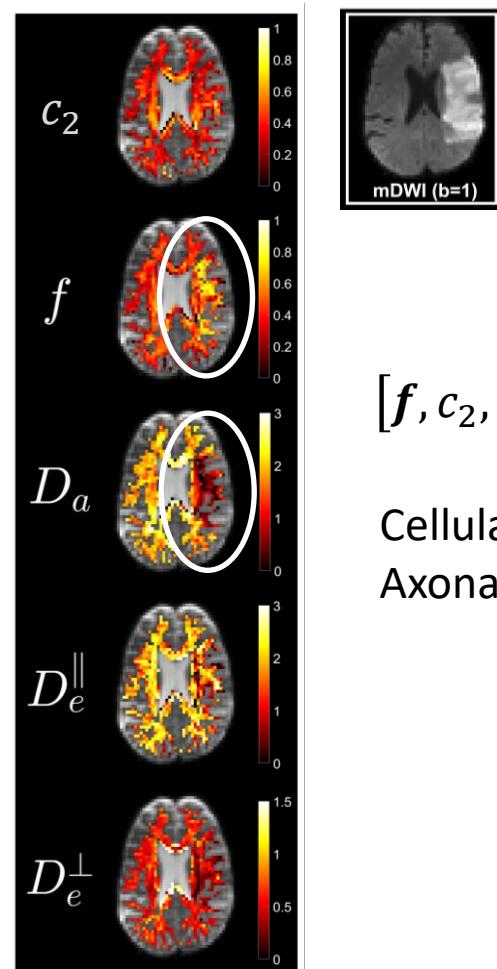
## Early psychosis & schizophrenia



$[f, c_2, D_a, D_{e,\parallel}, D_{e,\perp}]$

Extra-axonal changes  
Myelin defects

## Stroke



$[f, c_2, D_a, D_{e,\parallel}, D_{e,\perp}]$

Cellular swelling  
Axonal beading

# Outline

I. Diffusion-weighting

II. Gaussian approximation regime: ADC and DTI

III. Beyond Gaussian:

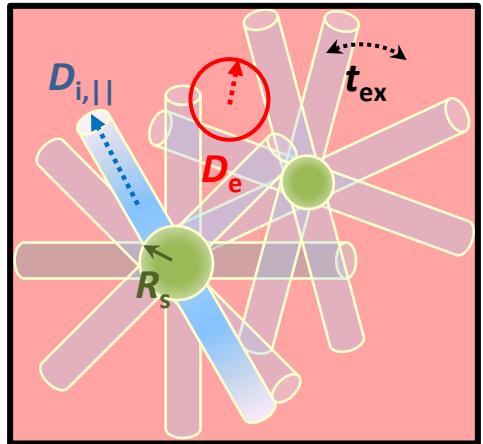
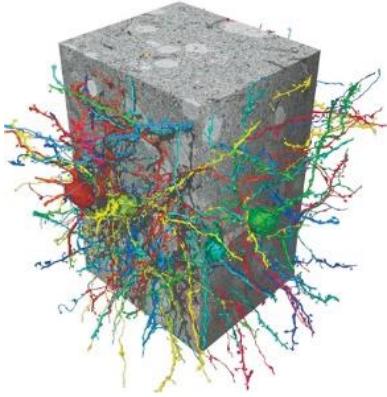
i. Signal representations: DKI

ii. White matter models & applications

iii. Gray matter models & applications

iv. Thinking outside the brain

# What is a relevant model for gray matter?



$$\mathbf{p} = [f_{in}, D_{i,||}, D_e, f_{is}, R_s, t_{ex}]$$

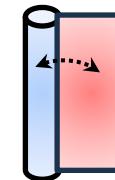
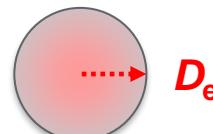
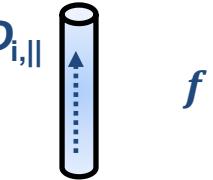
- Cell bodies: a third compartment?

Palombo, NeuroImage 2020

- Little myelin: water exchange between compartments ( $t_{ex}$ )?

Jelescu, NeuroImage 2022; Olesen NeuroImage 2022

$$\begin{aligned} \frac{\text{blue square}}{\text{blue square} + \text{red square} + \text{green square}} &= f_{in} \\ \frac{\text{green square}}{\text{blue square} + \text{red square} + \text{green square}} &= f_{is} \end{aligned}$$



Markers of:

$f$  : loss of cell processes (neurites, ...)

$D_{i,||}$  : intra-neurite injury

$D_e$  : inflammation

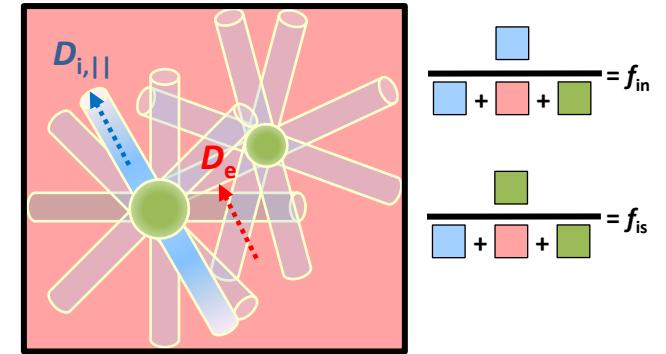
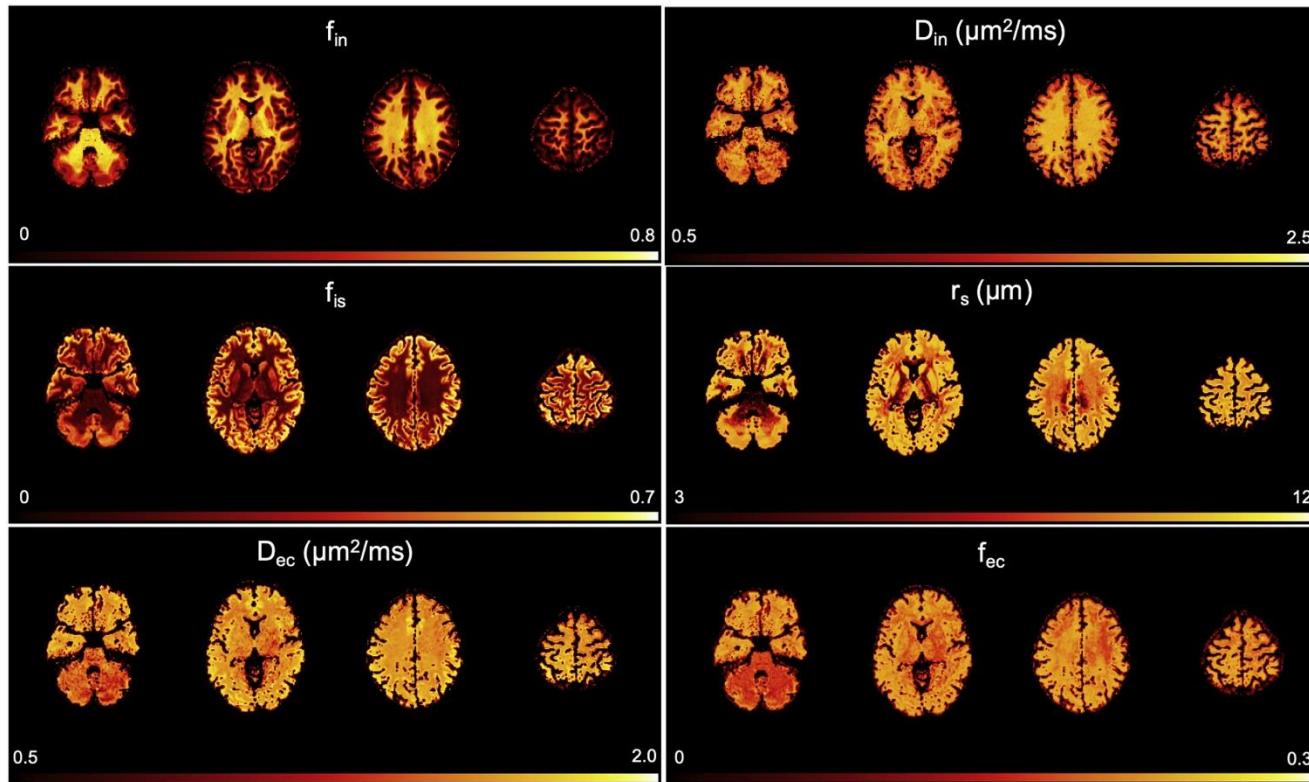
$t_{ex}$ : membrane permeability:  
myelination, membrane dysfunction

$R_s$ : soma size, reactivity

$f_s$ : cell proliferation or loss

# Modeling soma: SANDI – Soma and Neurite Density Imaging

- Assumption: non-exchanging compartments
  - $t_d \leq 20 \text{ ms}$



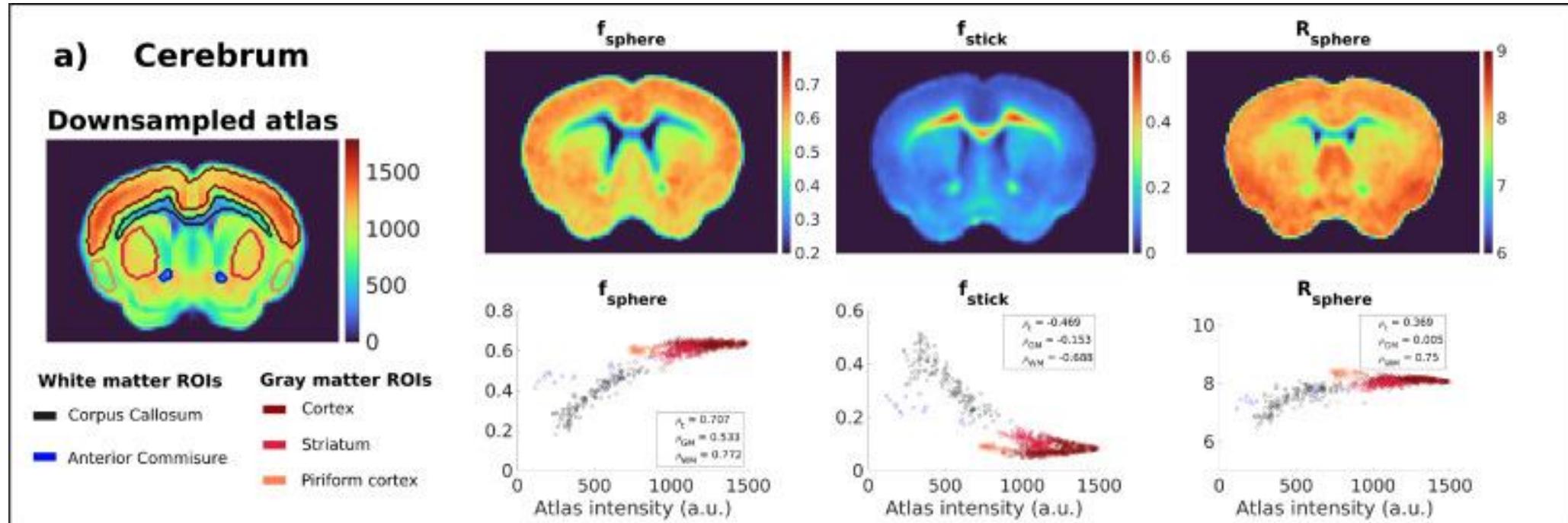
$$\mathbf{p} = [f_{in}, D_{i,||}, D_e, f_{is}, R_s ]$$

- ☺ Reasonable estimate of soma density & size
- ☹ Not feasible on a typical MRI clinical scanner

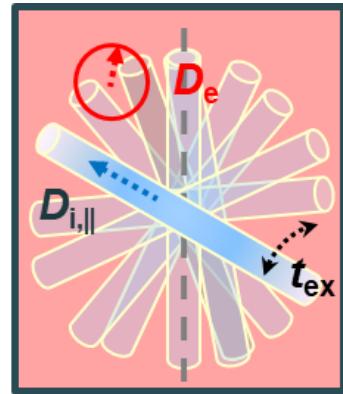
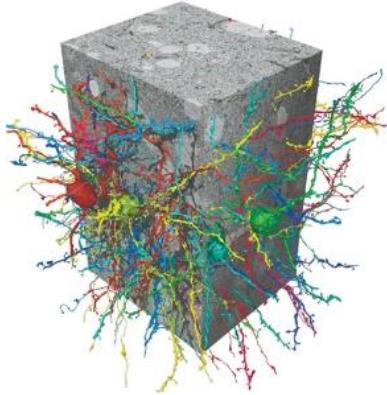
$$b = (\gamma \delta G)^2 \cdot \left( \Delta - \frac{\delta}{3} \right)$$

If  $\left( \Delta - \frac{\delta}{3} \right)$  is to be kept short,  
Need very high G to achieve high b-values ( $b > 5 \text{ ms}/\mu\text{m}^2$ )

# SANDI vs cell density (Allen Brain atlas)

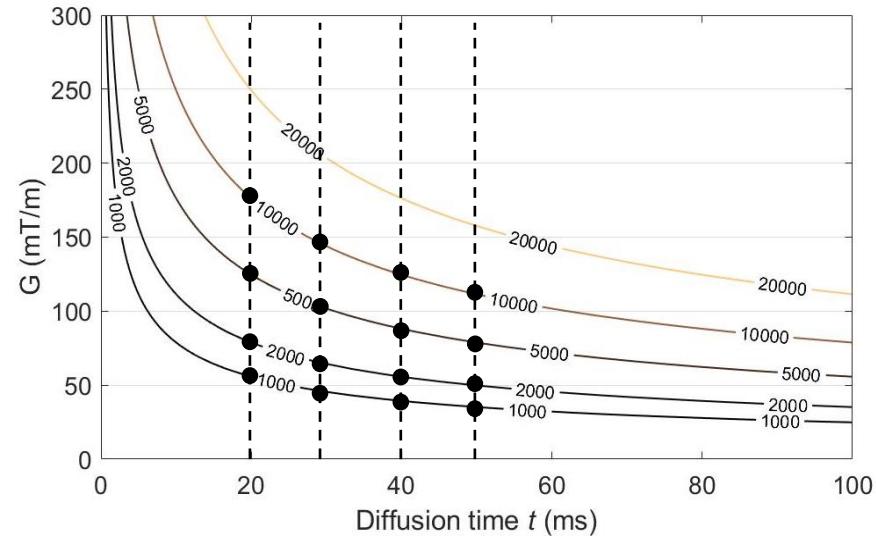


# Modeling exchange: NEXI (Neurite EXchange Imaging)

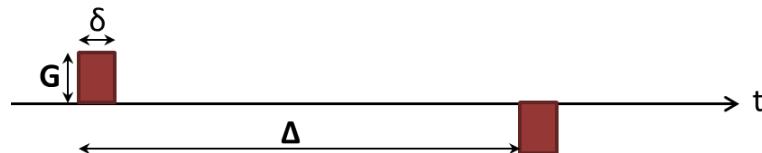


$$\mathbf{p} = [f, D_{i,||}, D_e, t_{ex}]$$

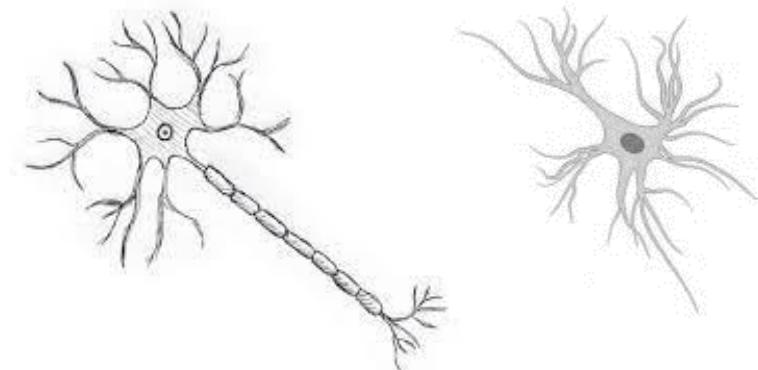
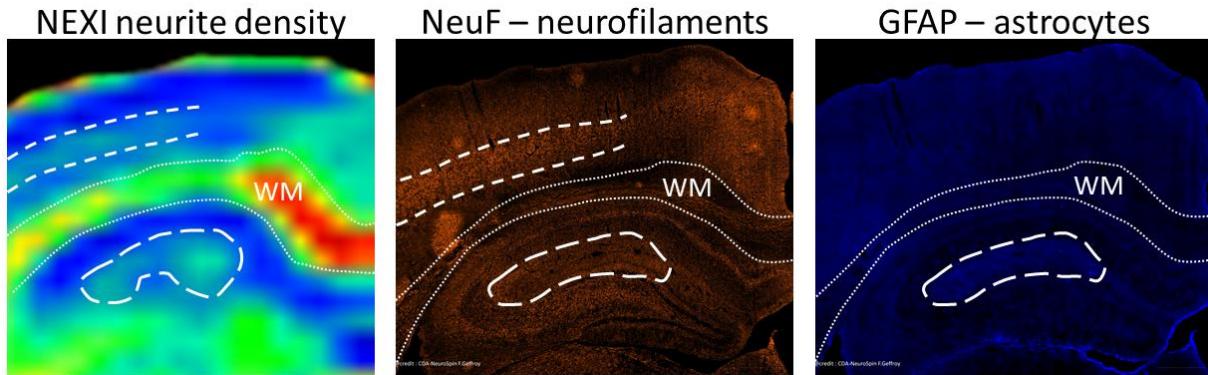
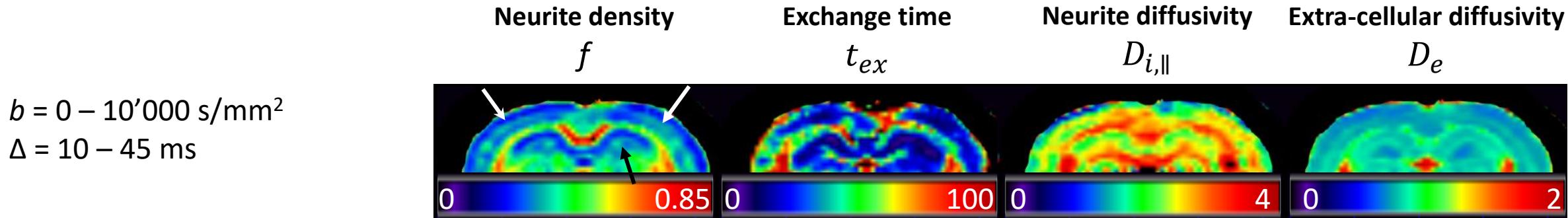
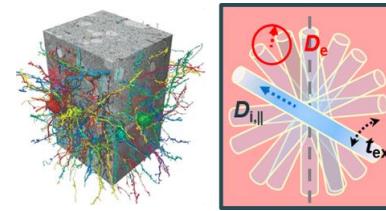
$$\frac{\square}{\square + \square} = f$$



→ Vary G and  $\Delta$

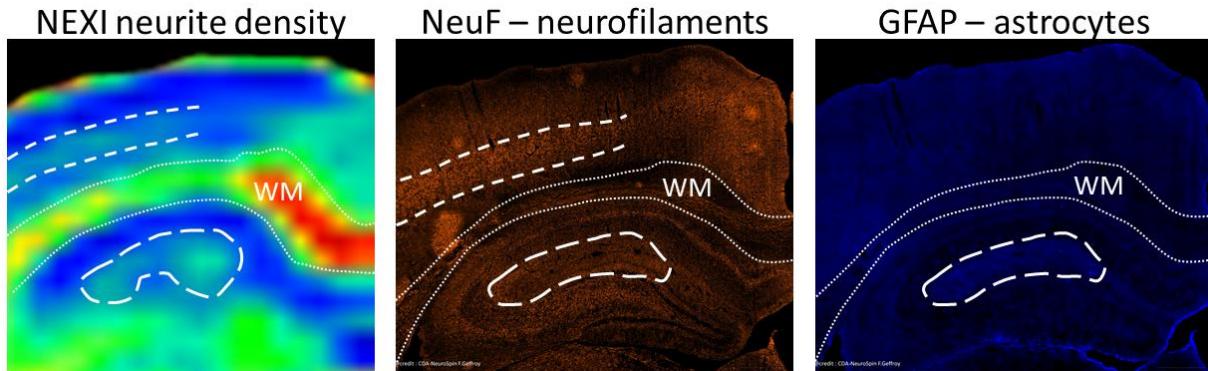
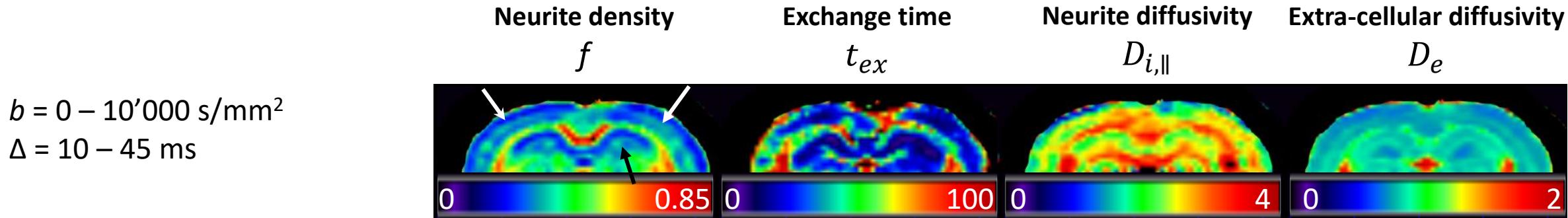
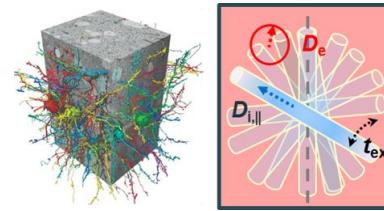


# NEXI – rat brain, in vivo



“Neurite density” = “Cell process density”

# NEXI – rat brain, in vivo



“Neurite density” = “Cell process density”

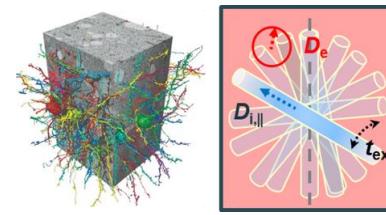
$$t_{ex} \sim 15 - 35 \text{ ms} \cong \frac{d}{4P}$$

$$\text{Permeability} \sim [2 - 33] \times 10^{-3} \mu\text{m/ms}$$

Vs literature:  
 Permeability  $\sim [6 - 30] \times 10^{-3} \mu\text{m/ms}$

Harkins, 2009; Finkelstein, 1987; Latour, 1994; Stanisz, 1997

# NEXI – human *in vivo*



## 3T Connectom scanner

$G_{\max} = 300 \text{ mT/m}$

$b = 1000 - 7500 \text{ s/mm}^2$

$\Delta = 20 - 40 \text{ ms}$

Scan time = 40 min

Uhl, Imaging Neuroscience 2024

## 3T Clinical scanner

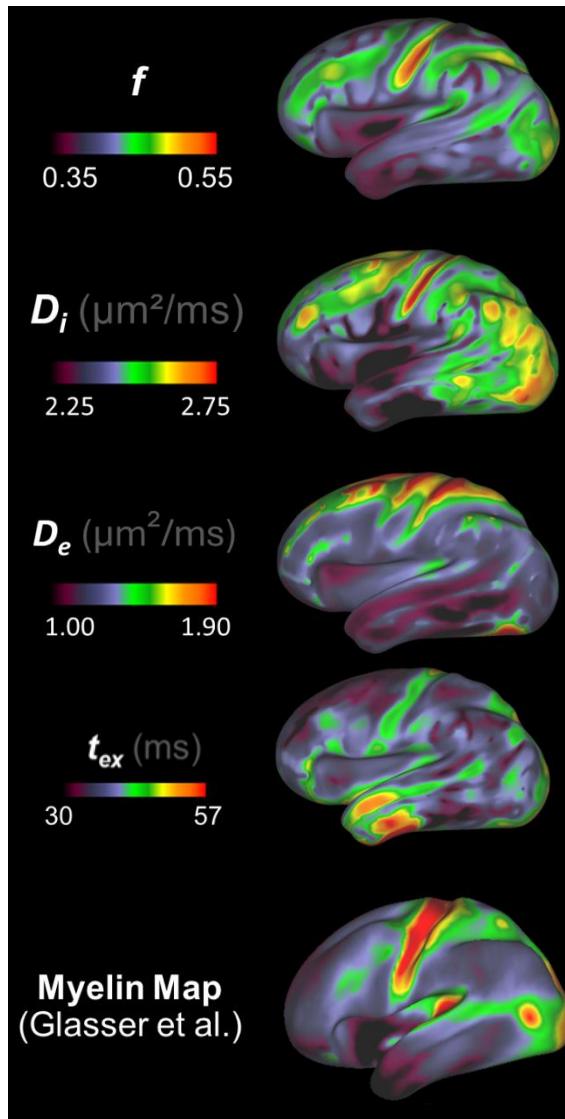
$G_{\max} = 80 \text{ mT/m}$

$b = 1000 - 5000 \text{ s/mm}^2$

$\Delta = 28 - 65 \text{ ms}$

Scan time = 30 min

Uhl, bioRxiv 2024



[https://github.com/Mic-map/graymatter\\_swissknife](https://github.com/Mic-map/graymatter_swissknife)

### Next steps:

- More with less: reduce scan time
- Model exchange + soma

Olesen, NeuroImage 2022

### Applications:

- Multiple sclerosis
- Traumatic Brain Injury
- ...

# Outline

I. Diffusion-weighting

II. Gaussian approximation regime: ADC and DTI

III. Beyond Gaussian:

i. Signal representations: DKI

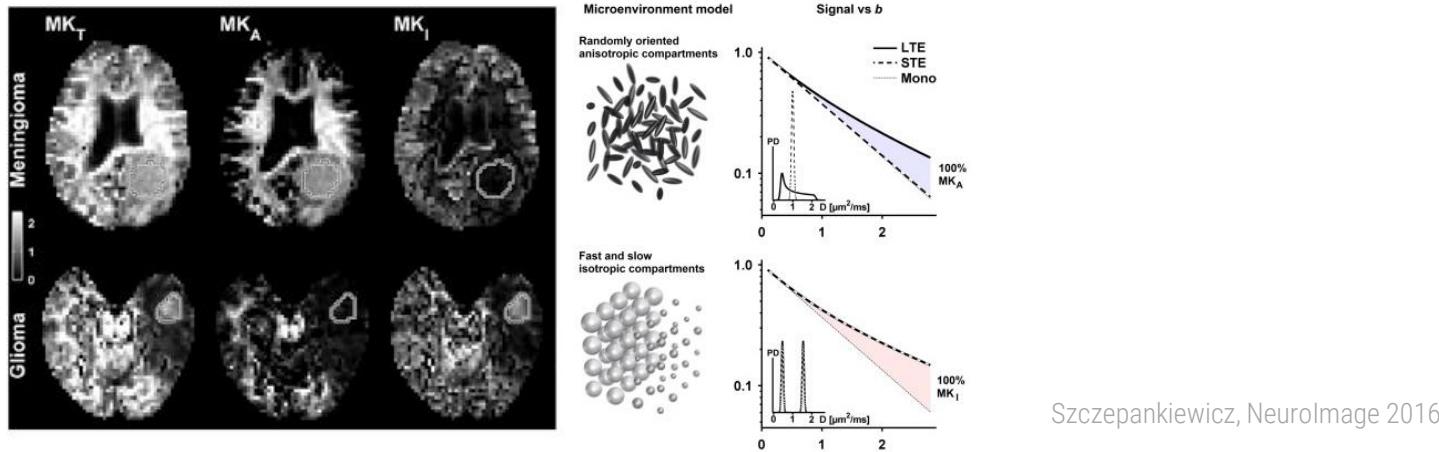
ii. White matter models & applications

iii. Gray matter models & applications

iv. Thinking outside the brain

# Microstructure models (almost) outside the brain

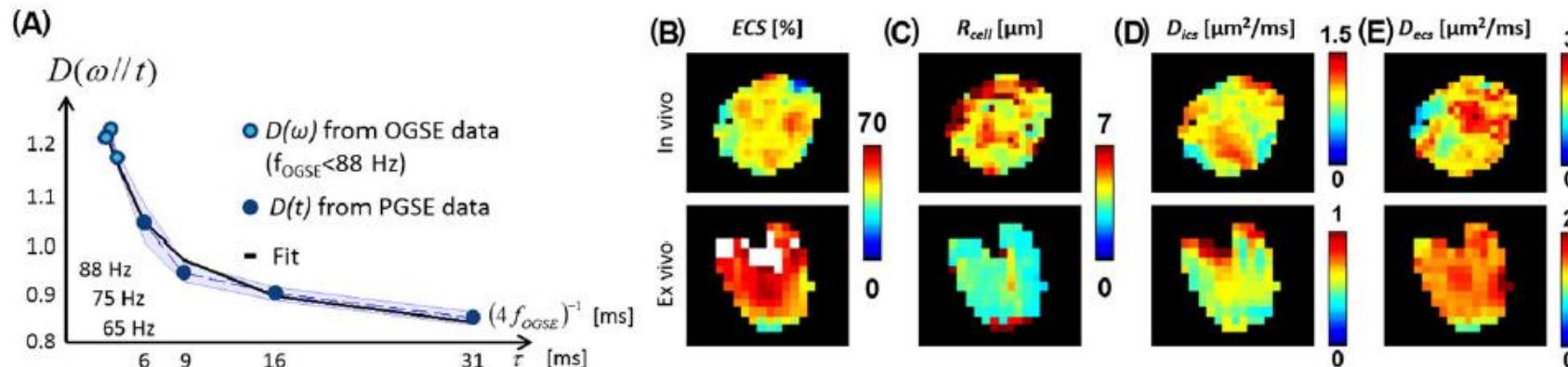
## A. Diffusional variance decomposition



Szczerpaniewicz, NeuroImage 2016

## Glioma

Quantification of cell density, radius, and compartment diffusivities



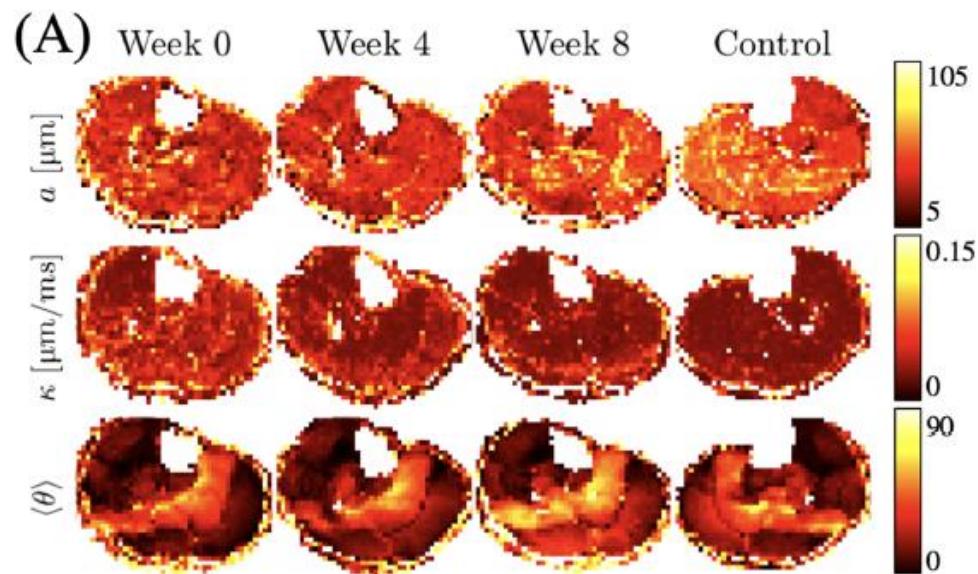
Reynaud, NMR in Biomed 2016

# Microstructure models outside the brain

## Random Permeable Barrier Model

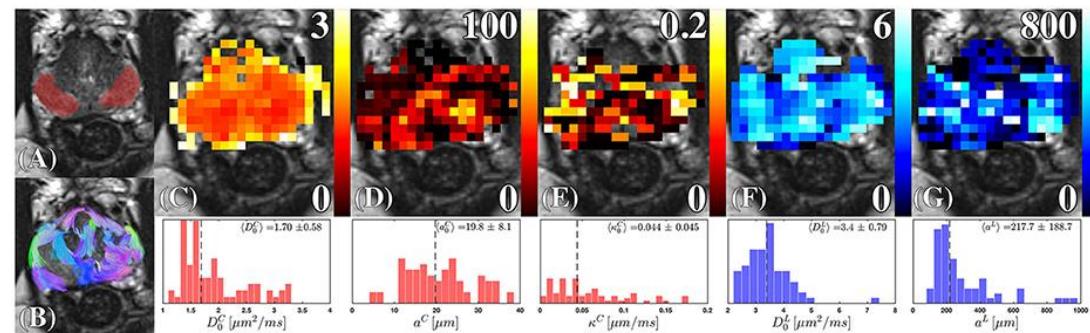
### Muscle

Quantification of myofiber size, permeability and orientation



Lemberskiy, NMR in Biomed 2021

### Prostate



Cellular diffusivity, fiber diameter, and membrane permeability

Luminal diffusivity and diameter

Lemberskiy, Front Phys 2018

# Summary

- **Biophysical tissue models:**
  - In theory: specific metrics of microstructure
  - In practice: dependent on validity of model assumptions & fitting procedure
    - Interpret output with caution!
- **Substantial progress in white matter modeling**
  - Estimation of main Standard Model parameters comprehensively
  - Accessible applications and translation to preclinical & clinical studies of disease
- **Current focus: gray matter modeling**
  - Accounting for inter-compartment exchange appears critical
  - Parameter estimation may require (q-t) coverage
    - Challenge for retrospective studies & prospective clinical studies
  - Eventually: propose a more comprehensive model (exchange + soma + ...)
- **Other organs / tissue types**
  - High interest

Reviews:

Jelescu and Budde, Front Phys 2017  
Novikov, MRM 2018  
Nilsson, NeuroImage 2018  
Alexander, NMR in Biomed 2019  
Novikov, NMR in Biomed 2019  
Jelescu, J Neurosci Meth 2020

Recommendations Papers:

Jelescu, [...], Schilling, MRM 2025  
Schilling, [...], Jelescu, MRM 2025

# Questions?

