

Principles and applications of diffusion MRI

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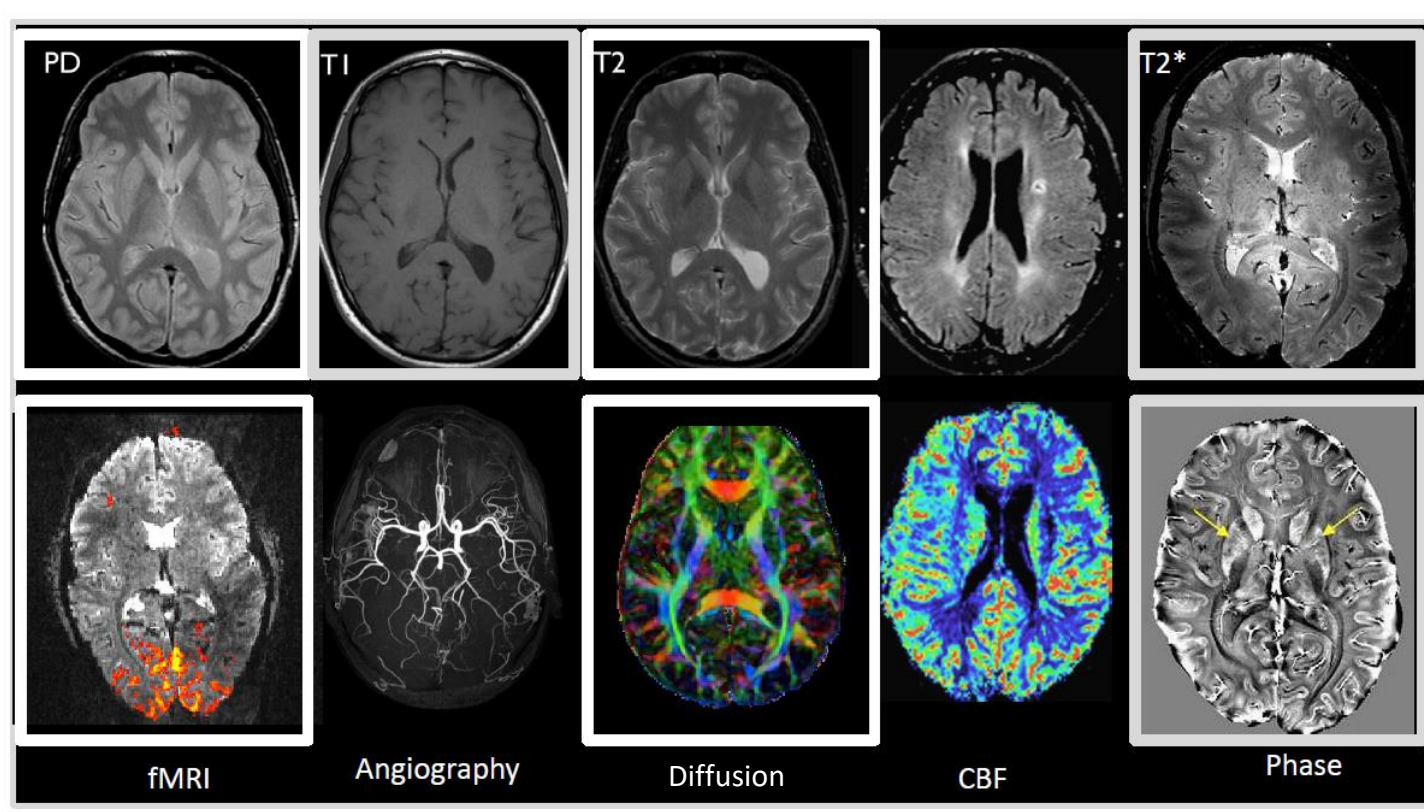
Faculty of Biology and Medicine



MRI – imaging water

Image contrast can be manipulated to be sensitized to a variety of features

- Depending on: chemical & magnetic properties of immediate environment, mobility of molecules...



Diffusion MRI: sensitizing the signal to the Brownian motion (random walk) of water molecules

Outline

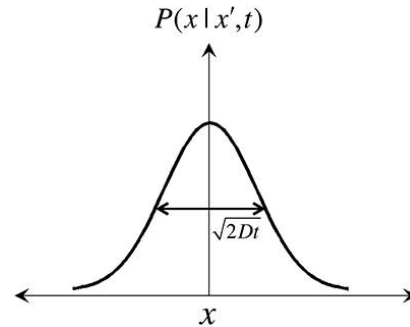
- I. Diffusion-weighting
- II. Gaussian approximation regime: ADC and DTI
- III. Beyond Gaussian:
 - i. Signal representations: DKI
 - ii. White matter models & applications
 - iii. Gray matter models & applications
 - iv. Thinking outside the brain

Diffusion MRI gives access to the mesoscale

➤ Unrestricted homogeneous medium:



MRI voxel size:
1 – 2 mm

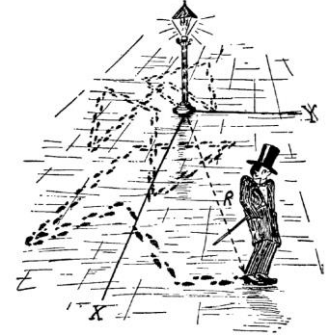
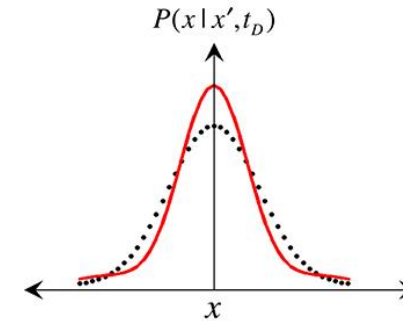


$$\langle x^2 \rangle = 2nDt$$

Typical length scale for
microstructure: 1 – 10 μm



➤ Biological tissue:

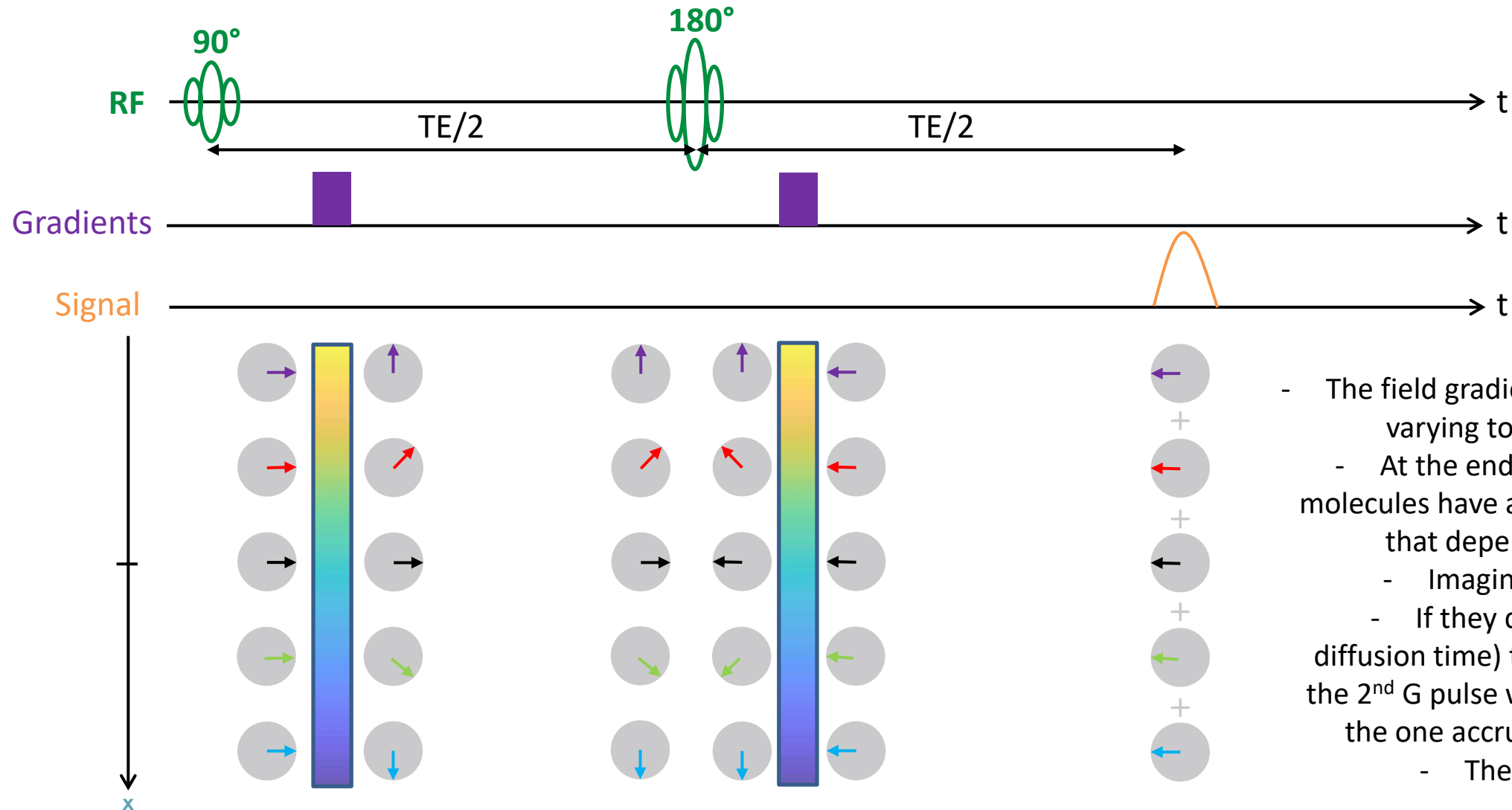


Mean displacement of water molecules :

- $t = 2 - 500 \text{ ms} \longrightarrow \sqrt{\langle x^2 \rangle} \approx 2 - 50 \mu\text{m}$

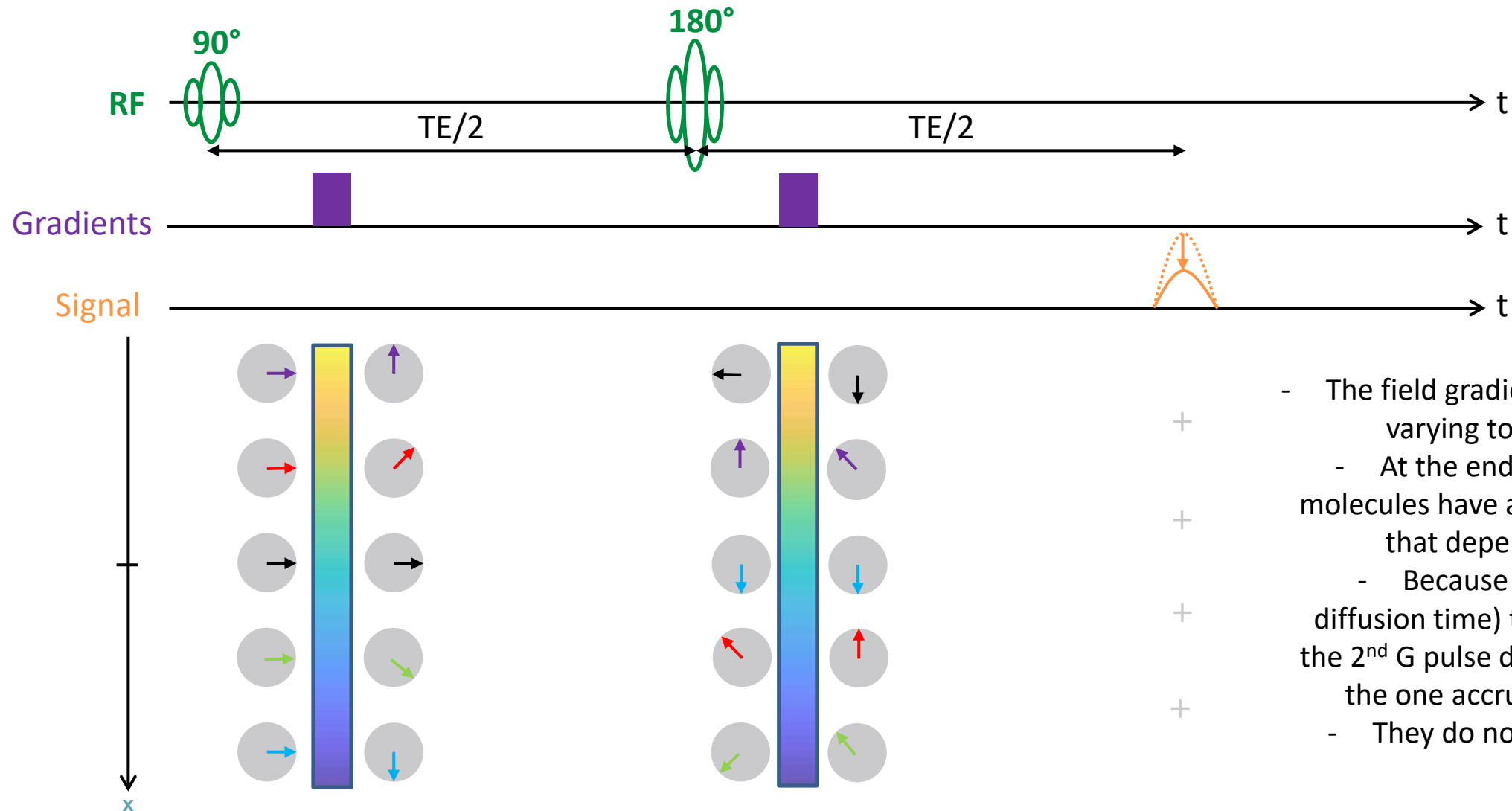
➤ Excellent probe of tissue microstructure

Diffusion weighting in MRI – pulsed gradient spin-echo (PGSE)



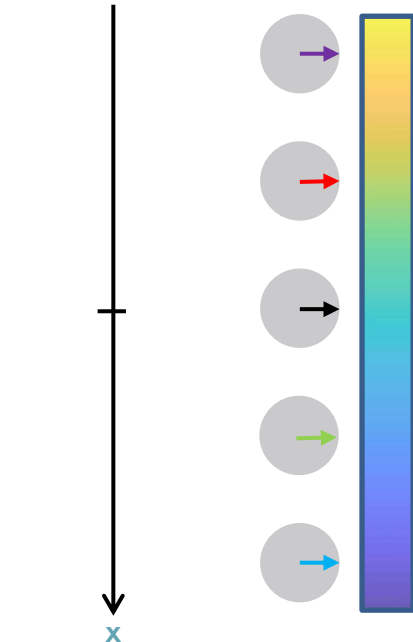
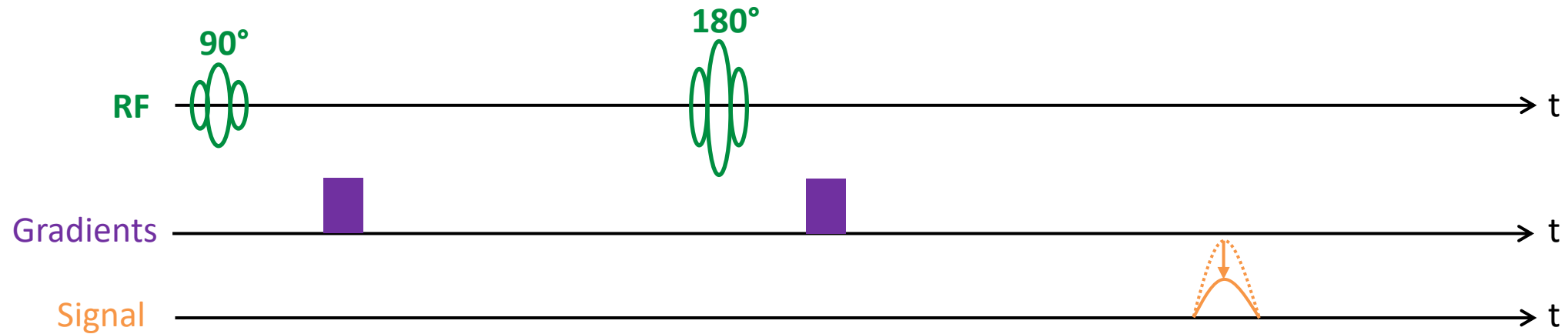
- The field gradient creates a linearly varying total field: $B(x) = B_0 + G \cdot x$
- At the end of the (short) pulse, molecules have accumulated a phase that depends on their position
 - Imagine: no diffusion ($D=0$)
 - If they don't mix (during the diffusion time) the phase accrued in the 2nd G pulse will fully compensate the one accrued in the 1st G pulse
 - They add up coherently!

Diffusion weighting in MRI – pulsed gradient spin-echo (PGSE)



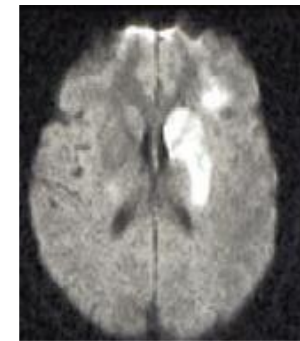
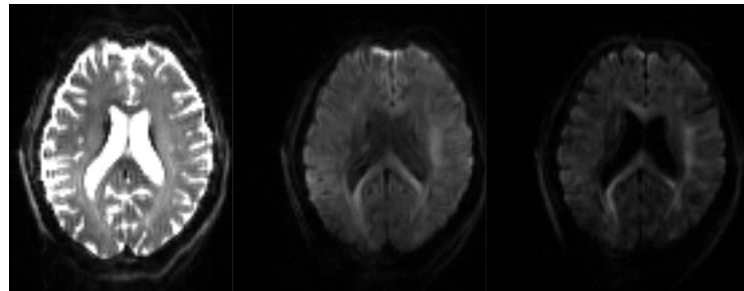
- The field gradient creates a linearly varying total field: $B(x) = B_0 + G \cdot x$
- At the end of the (short) pulse, molecules have accumulated a phase that depends on their position
- Because they mix (during the diffusion time) the phase accrued in the 2nd G pulse does not compensate the one accrued in the 1st G pulse
- They do not add up coherently!
Signal attenuation

Diffusion weighting: signal attenuation & contrast



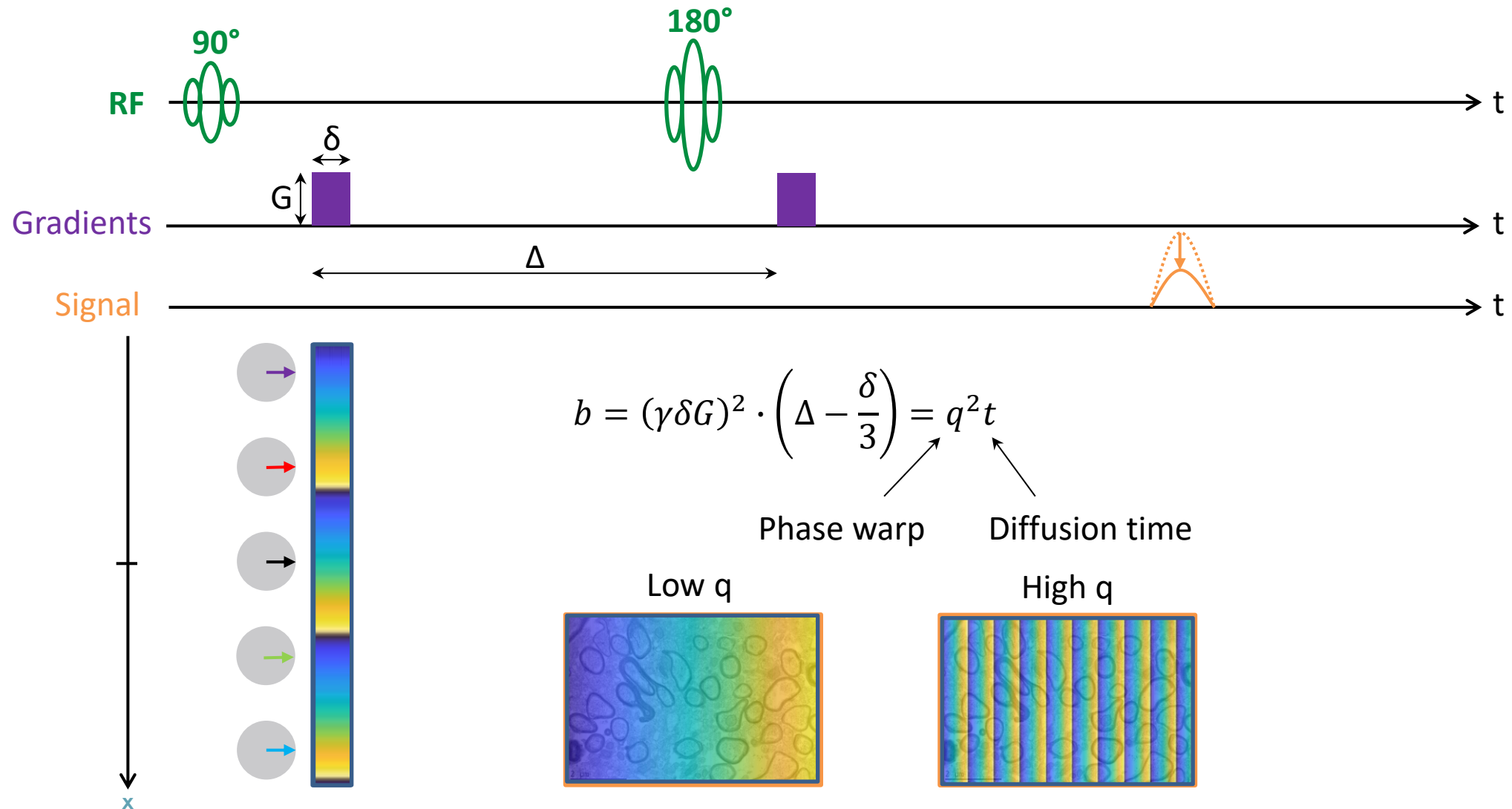
Signal attenuation due to diffusion

- Increases with *diffusion-weighting*
- Increases with Diffusivity

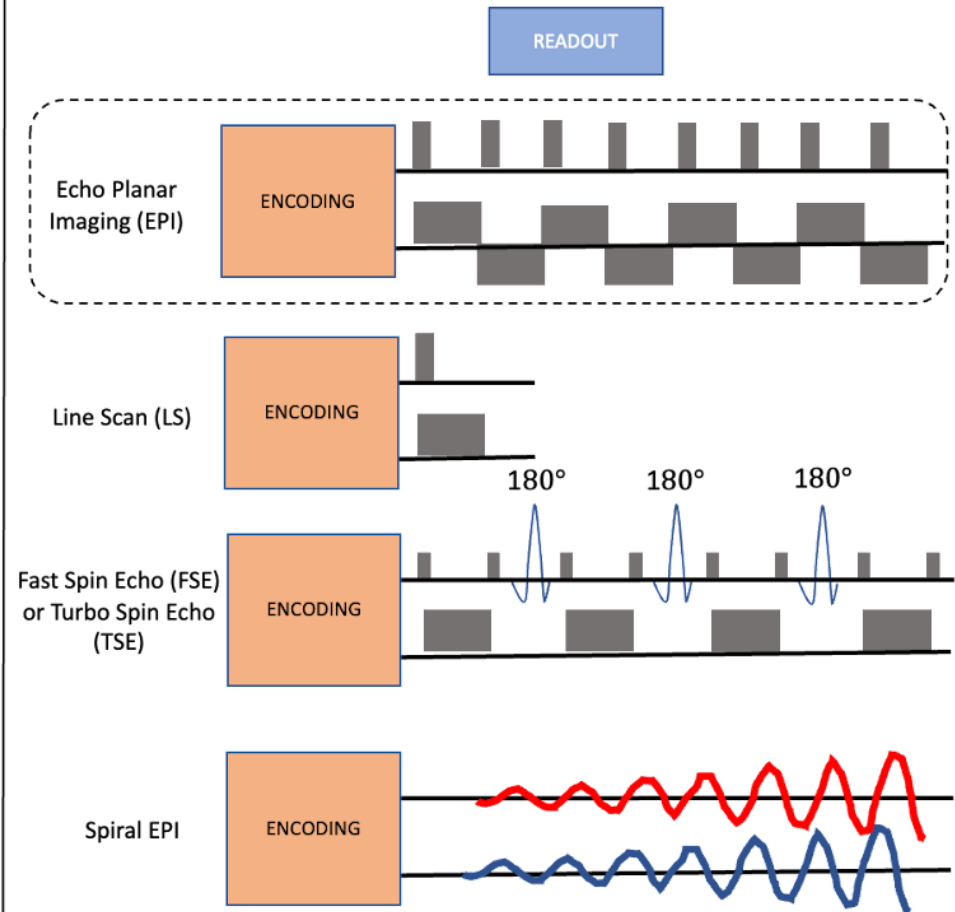
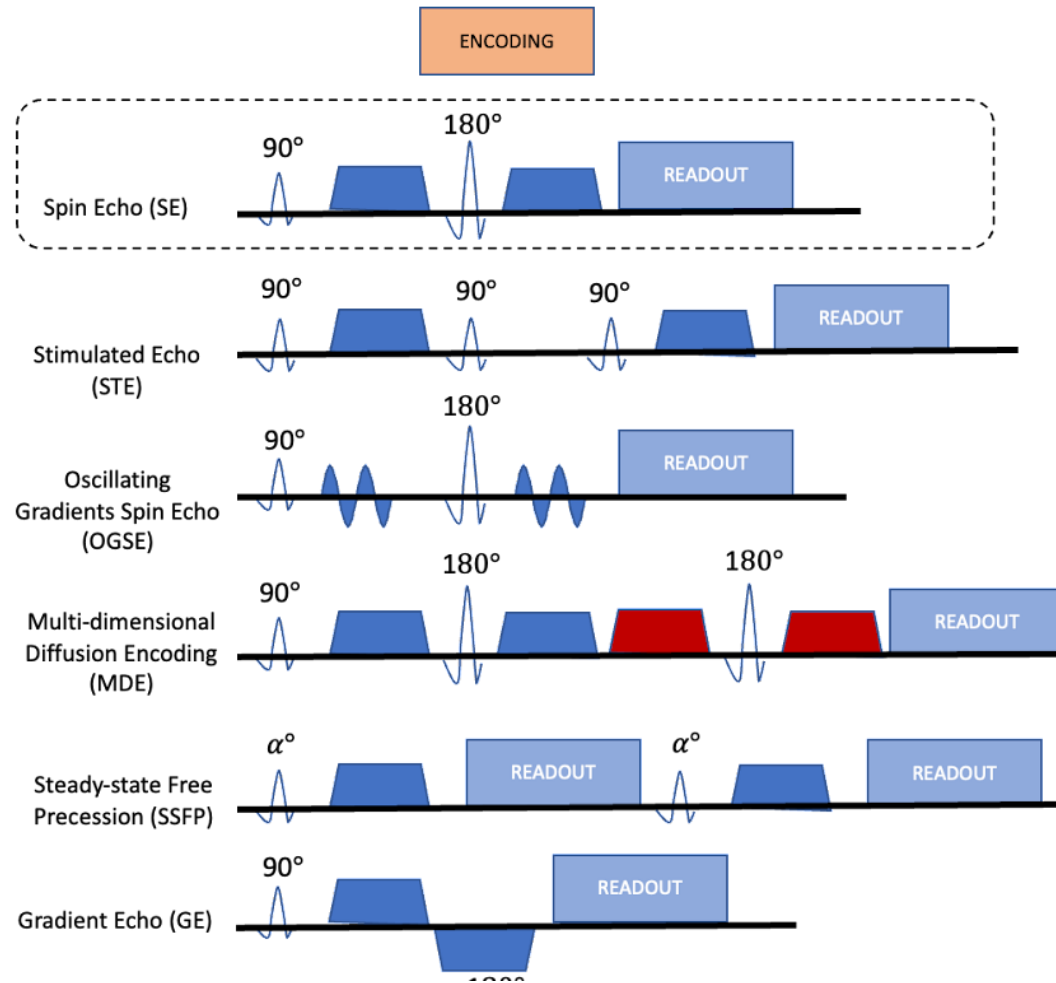


Stroke

Diffusion weighting in MRI – q, t and the b-value



Diffusion encoding schemes and image read-out schemes



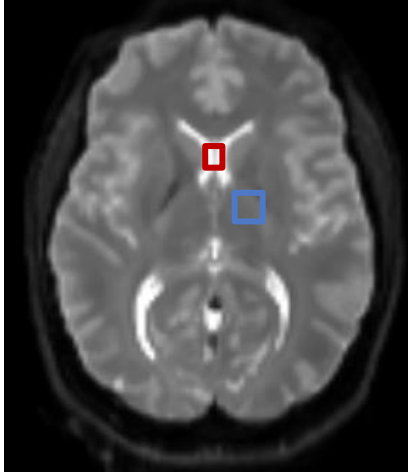
Summary so far

- Diffusion of water molecules in biological tissue (e.g. brain):
 - Is a stochastic process due to thermal agitation, it is always present
 - Is not free: it is affected by the cellular structures (restriction & hindrance)
 - Can provide information about tissue microstructure below the MR image spatial resolution
- In Diffusion MRI, we sensitize the signal amplitude to the diffusion of water molecules using strong magnetic field gradients G
- The MRI signal attenuation depends on:
 - The diffusion properties of the medium (that we want to measure)
 - The diffusion weighting imparted: $b=q^2 \cdot t$ (that we set as acquisition parameter)

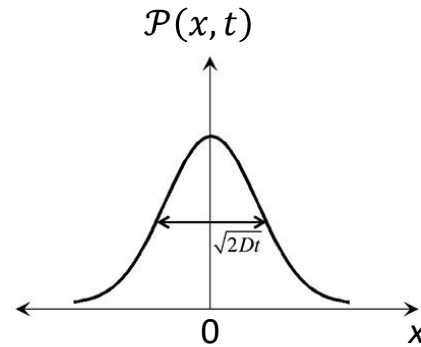
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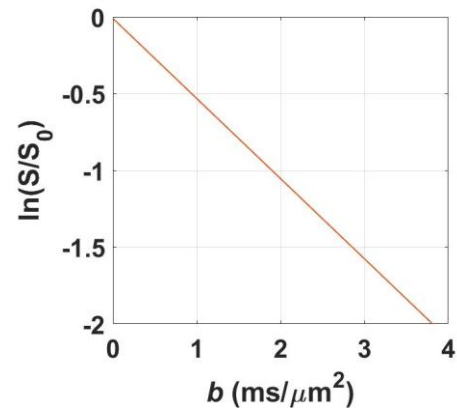
From a free medium to biological tissue (and from D to ADC)



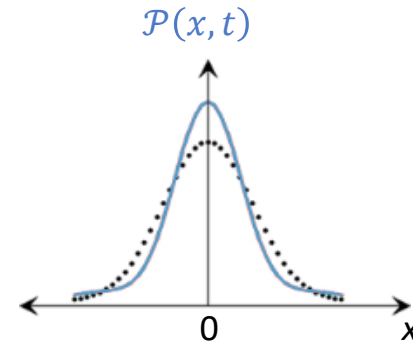
Free homogeneous medium
Gaussian diffusion



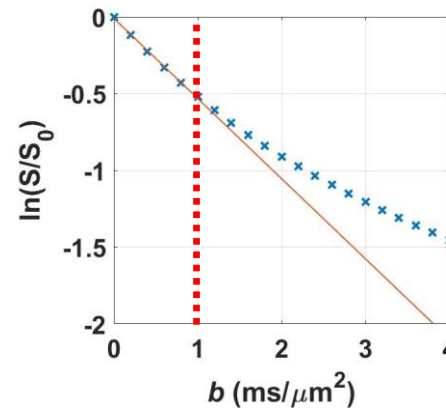
$$\ln\left(\frac{S}{S_0}\right) = -bD$$



Biological tissue
Non-Gaussian diffusion



$$\ln\left(\frac{S}{S_0}\right) = F(b)$$



Low b-value regime:
Gaussian approximation
“DTI” regime

$$\ln\left(\frac{S}{S_0}\right) \approx -b \cdot ADC + \mathcal{O}(b^2)$$

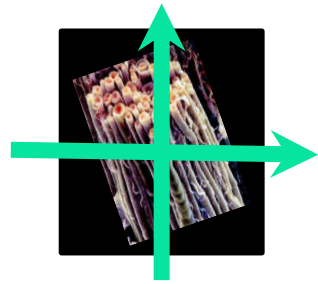
Apparent Diffusion Coefficient

Le Bihan, Radiology 1986

From the ADC to the diffusion tensor (DTI)

Apparent Diffusion Coefficient

...
along a direction \vec{g} ,
given by $\vec{G} = G \cdot \vec{g}$



$$\ln \frac{S(b, \vec{g})}{S_0} = -b \cdot ADC_{\vec{g}}$$

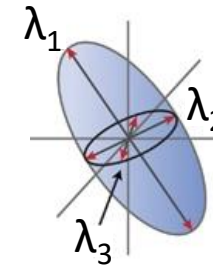
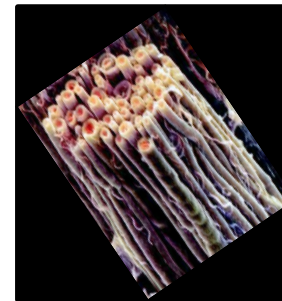
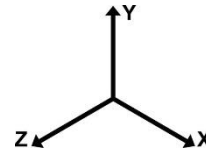
$$\vec{g} = [g_1 \ g_2 \ g_3]$$

$$\ln \frac{S(b, \vec{g})}{S_0} = -b \sum_{i,j=1}^3 g_i g_j D_{ij} = -b \vec{g}^T \hat{D} \vec{g}$$

$$\vec{g} = [1 \ 0 \ 0]$$

$$\vec{g} = [0 \ 1 \ 0]$$

$$\hat{D}_{\{\vec{x}, \vec{y}, \vec{z}\}} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{pmatrix}$$



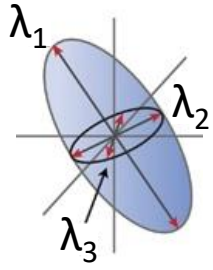
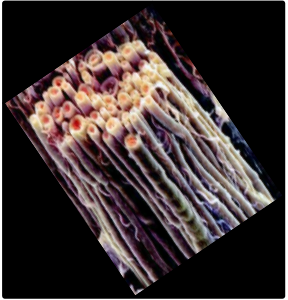
$$\hat{D}_{\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

- Symmetric \rightarrow 6 coefficients to estimate
- Invertible with all eigenvalues real and positive

Minimal data:

- $b = 0$
- 6 non collinear directions on 1 shell (e.g. $b = 1 \text{ ms}/\mu\text{m}^2$)

DTI scalars

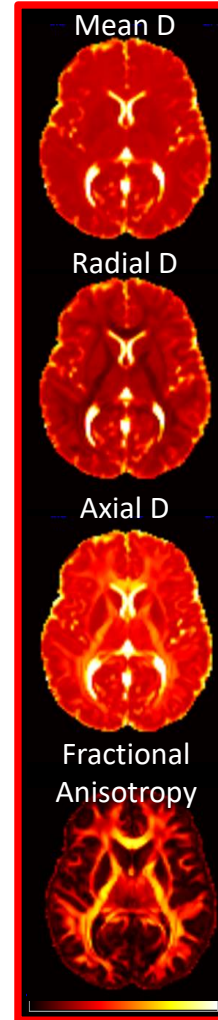


$$\hat{D}_{\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

In white matter in particular:

\vec{e}_1 : direction of faster diffusion in the voxel
(along the axon bundle)

→ Useful in early days of tractography for tracking fiber orientations from voxel to voxel



Mean diffusivity:

$$MD = (\lambda_1 + \lambda_2 + \lambda_3)/3$$

$$MD = \text{Tr}(\hat{D})/3$$

$MD \sim 1 \frac{\mu m^2}{ms}$ in brain, little GM/WM contrast

Radial diffusivity:

$$RD = (\lambda_2 + \lambda_3)/2$$

Good GM/WM contrast, lowest in WM

Axial diffusivity:

$$AD = \lambda_1$$

Some GM/WM contrast, highest in WM

Fractional anisotropy:

$$FA = \frac{\sqrt{\frac{1}{2}[(\lambda_1 - \lambda_2)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_3 - \lambda_1)^2]}}{\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}$$

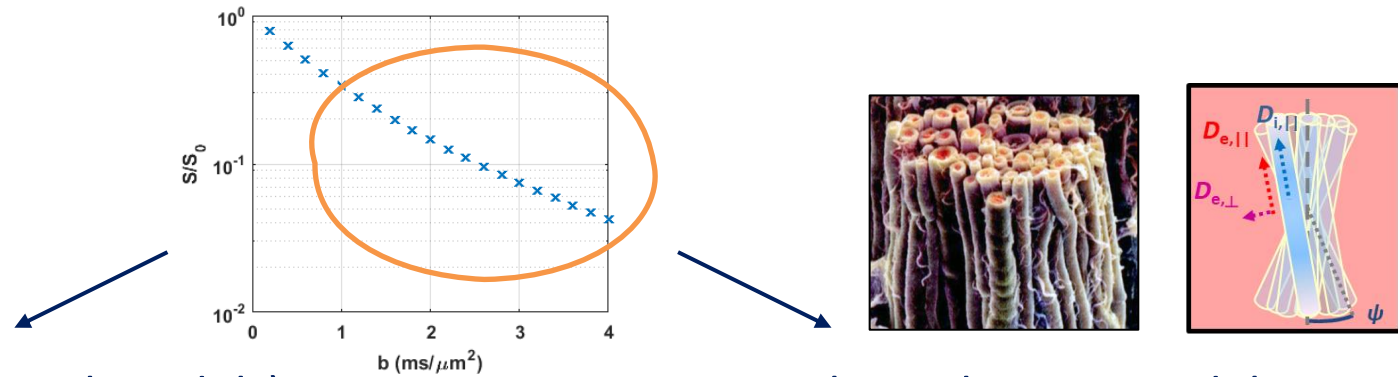
FA = 0 : isotropic diffusion, FA = 1 : fully anisotropic (1D)

Excellent GM/WM contrast, highest in WM

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Signal representations vs biophysical models



Signal representations (statistical models)

- “Formulas” to describe signal decay
- ☺ Applicable to any type of tissue
- ☺ Provide sensitive biomarkers
- ☹ Lack specificity: difficult to make inferences about microstructural changes

Biophysical tissue models

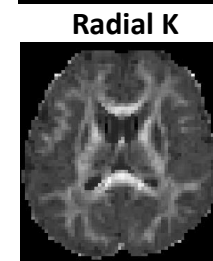
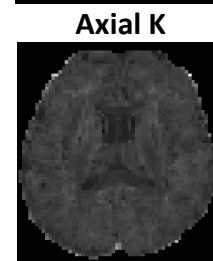
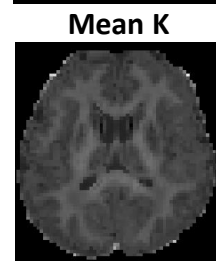
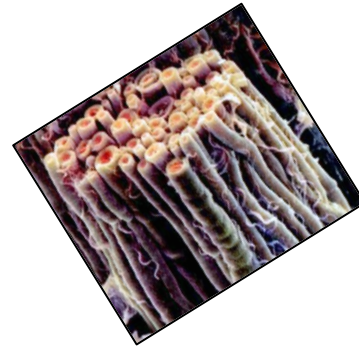
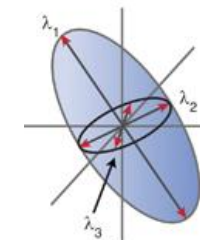
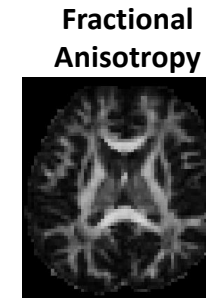
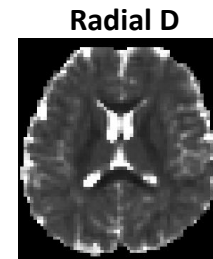
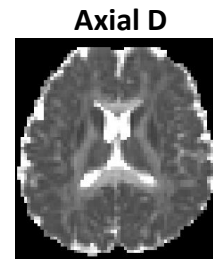
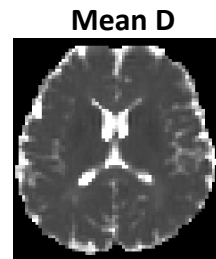
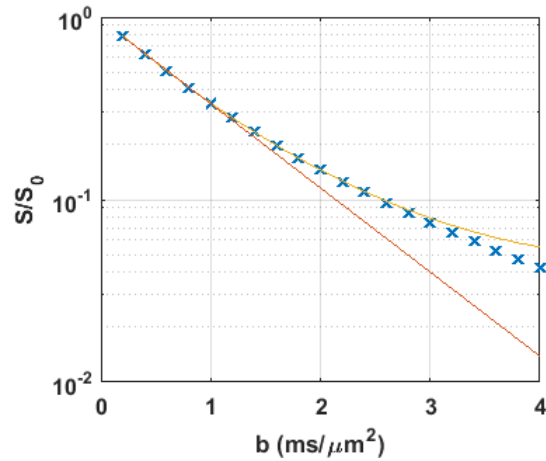
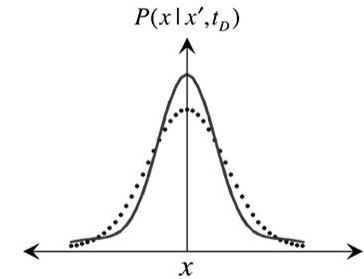
- Simplified “pictures” of medium in which water molecules are diffusing
- ☹ Tissue-specific
- ☺ Specific biomarkers that characterize microstructure
- ☹ Assumption validity?
- ☹ Fit stability?

Representations: Diffusion Tensor Imaging (DTI) and Kurtosis (DKI)

$$\ln \frac{S}{S_0} = -bD + \frac{1}{6} b^2 D^2 K + \dots$$

$$\ln \frac{S}{S_0} = -b \sum_{i,j=1}^3 g_i g_j D_{ij} + \frac{1}{6} b^2 \bar{D}^2 \sum_{i,j,k,l=1}^3 g_i g_j g_k g_l W_{ijkl} + \dots$$

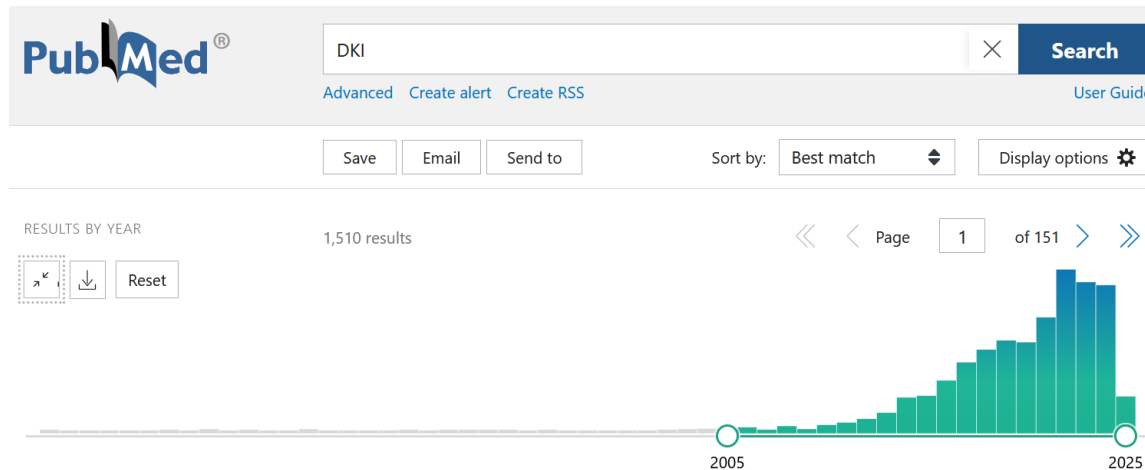
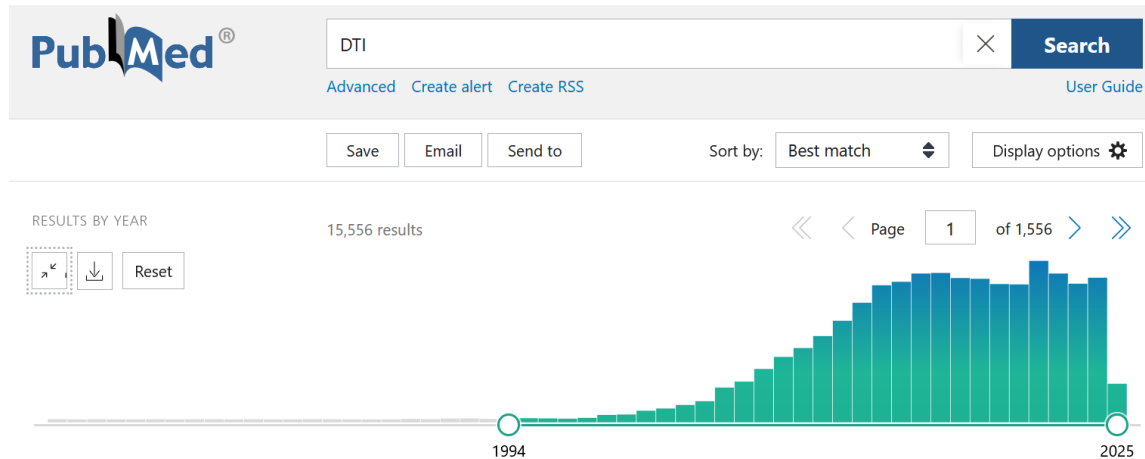
diffusion tensor (DTI) & kurtosis tensor (DKI)



Minimal data:

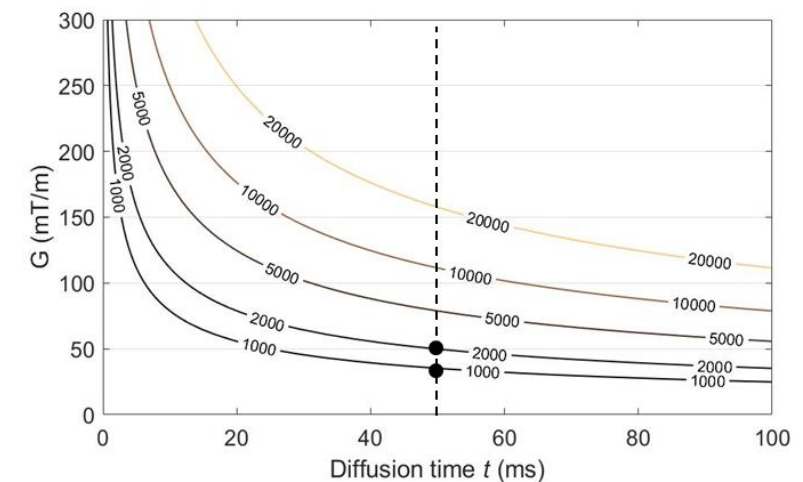
- **1 $b = 0$**
- **6 directions on 1 shell (e.g. $b = 1 \text{ ms}/\mu\text{m}^2$)**
- **15 directions on a 2nd shell (e.g. $b = 2/2.5$)**

DKI: Valuable



Practicality

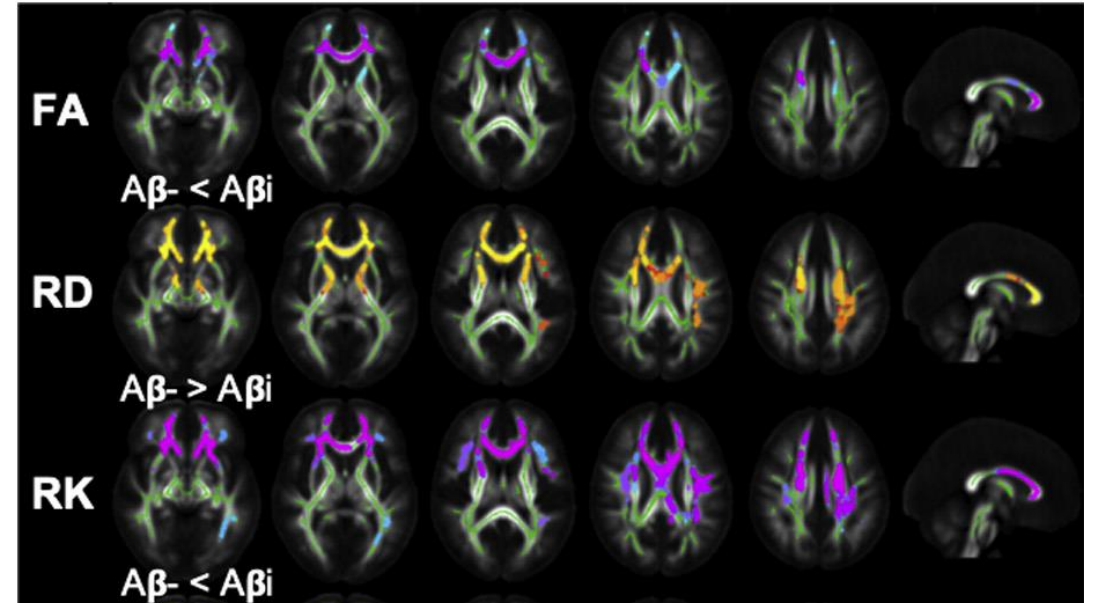
- “Standard” multi-shell acquisition:
 - $1\ b = 0, 30\ b_1, 60\ b_2$
 - ~7-8 min using acceleration (GRAPPA, multiband...)



DKI: Valuable

- Sensitive to tissue complexity
- Complementary information to DTI:
 - Brain development / aging
 - Dementia
 - Stroke
 - Traumatic Brain Injury
 - Glioma
 - Prostate cancer
 - Other body cancers

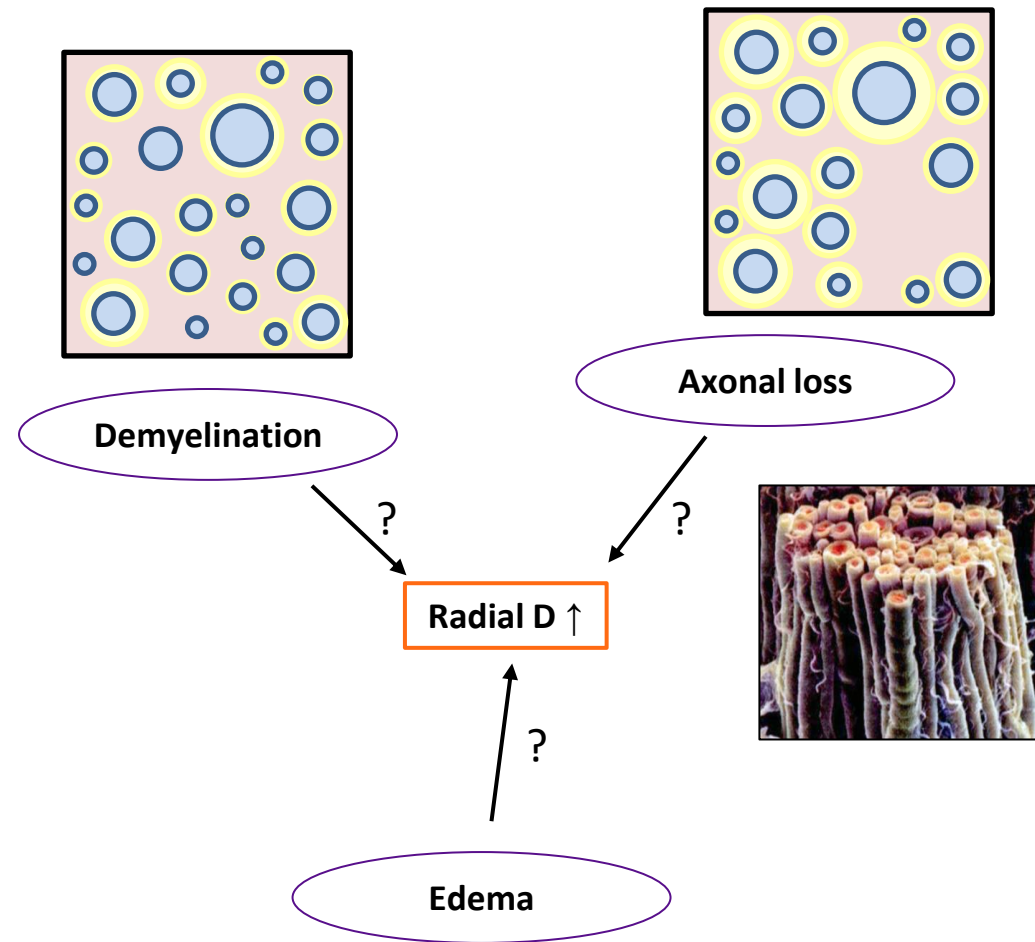
Jensen and Helpert, NMR Biomed 2010
Hui, Stroke 2012
Paydar, AJNR 2014
Rosenkrantz, JMIR 2015
McKenna, Cortex 2019



Relationship between altered WM microstructure & amyloid load in MCI patients

Dong, Neurobiol Aging 2020

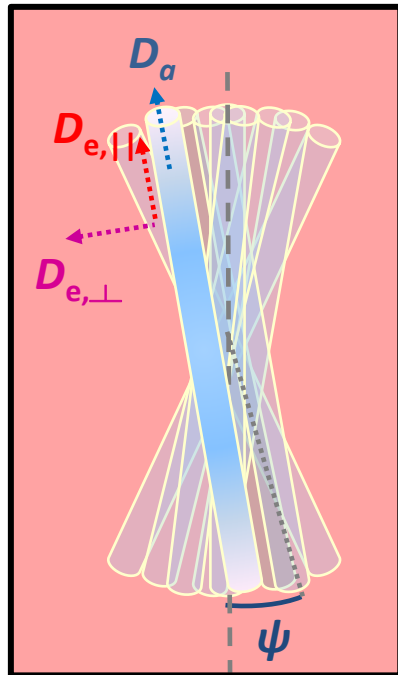
Representations: Valuable... but still lack specificity



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White Matter «Standard Model»



$$\frac{\text{blue box}}{\text{blue box} + \text{red box}} = f$$

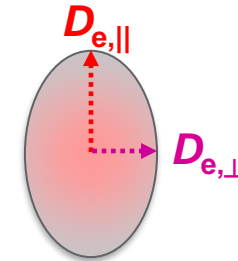
■ Intra-axonal space
■ Extra-axonal space

Intra-axonal compartment:
Collection of sticks

Extra-axonal compartment



f
 Orientation
 Distribution
 $c_2 = \langle \cos^2 \psi \rangle$



$1-f$

Markers of:

f : axonal loss

c_2 : pruning

D_a : axonal injury

$D_{e,\perp}$: demyelination

$D_{e,||}$: inflammation

“Standard Model” of diffusion in white matter:

$$\mathbf{p} = [f, c_2, D_a, D_{e,||}, D_{e,\perp}]$$

Callaghan, 1979
 Jespersen, 2007
 Fieremans, 2011
 Zhang, 2012
 Reisert, 2017
 Novikov, 2018

Moving away from spurious assumptions

➤ Favor Standard Model frameworks that introduce minimum assumptions

- WMTI-Watson (from DKI)
- SMI (from Rotational Invariants)

https://github.com/Mic-map/WMTI-Watson_DL

<https://github.com/NYU-DiffusionMRI/SMI>

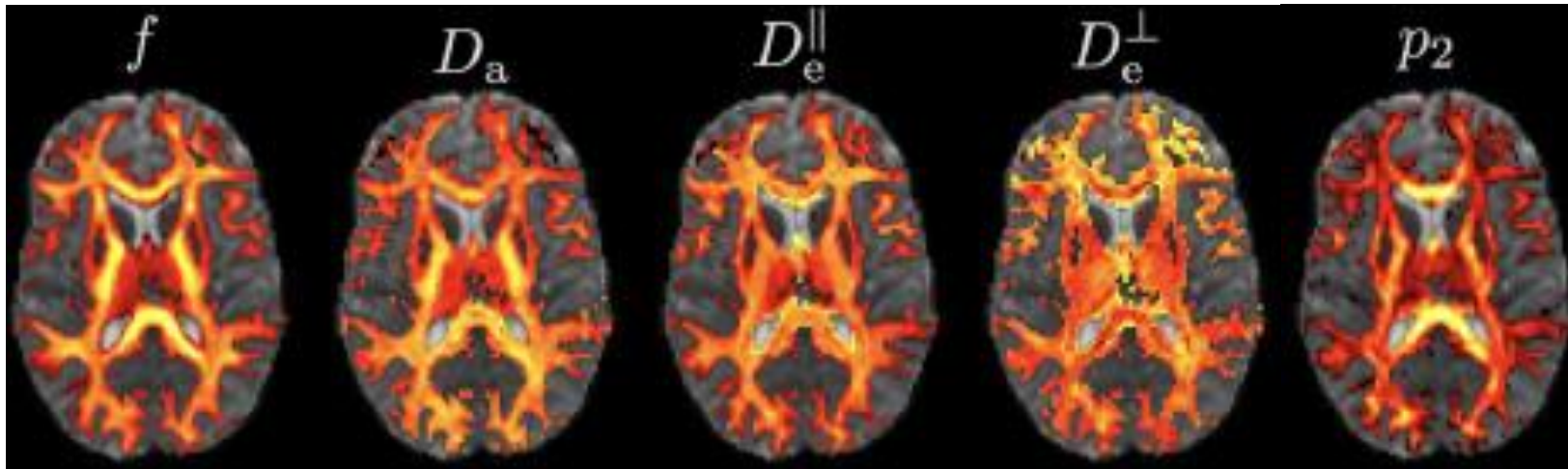
Jespersen, NeuroImage 2018; Novikov, NeuroImage 2018

Axonal
fraction

Intra-axonal
diffusivity

Extra-axonal parallel &
perpendicular diffusivity

Axon bundle
alignment

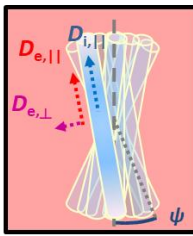
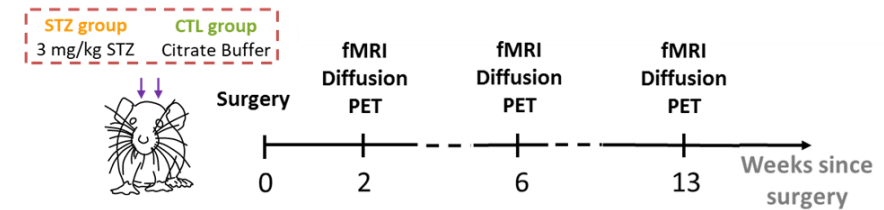


Coelho, NeuroImage 2022

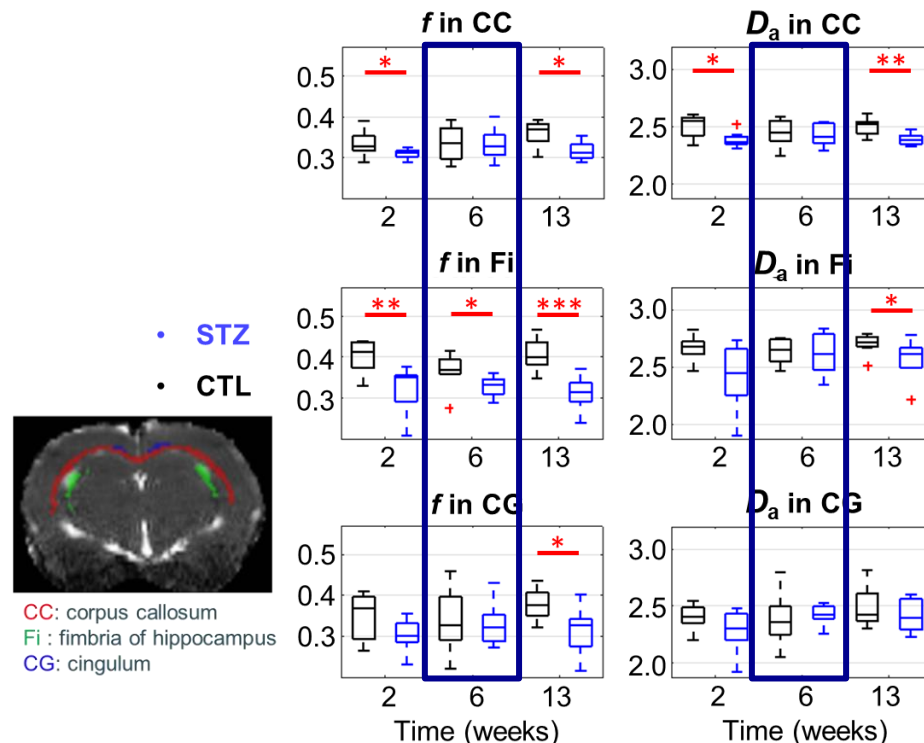
Applications: Rat model of Alzheimer's (“brain diabetes”)

Injection of streptozotocin (STZ) in lateral ventricles

- Insulin-resistant brain state – but no systemic diabetes



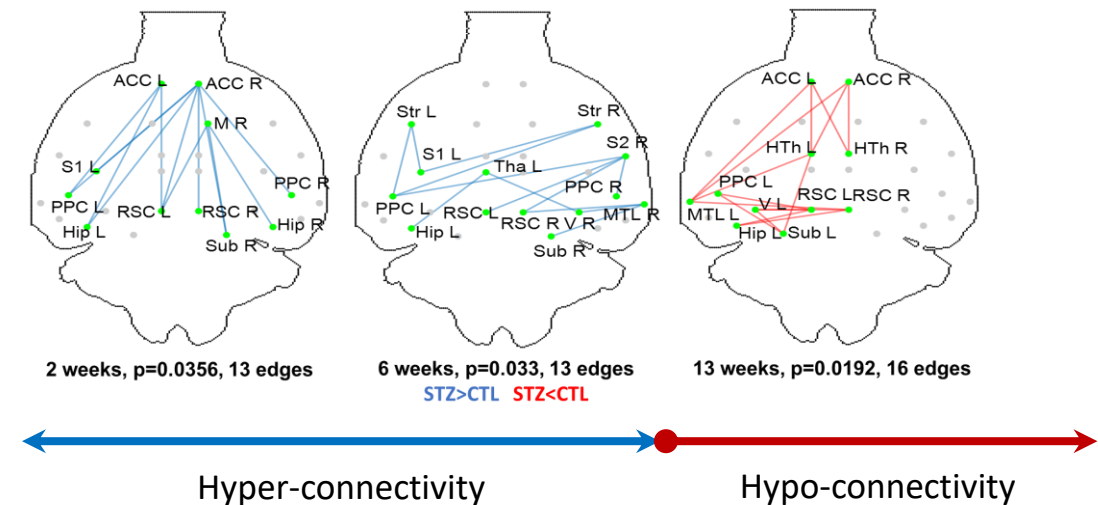
WM microstructure



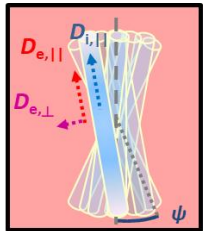
$$[f, c_2, D_a, D_{e,||}, D_{e,\perp}]$$

- Axonal microstructural alterations
- Inflection point: ~6 weeks (temporary recovery?)

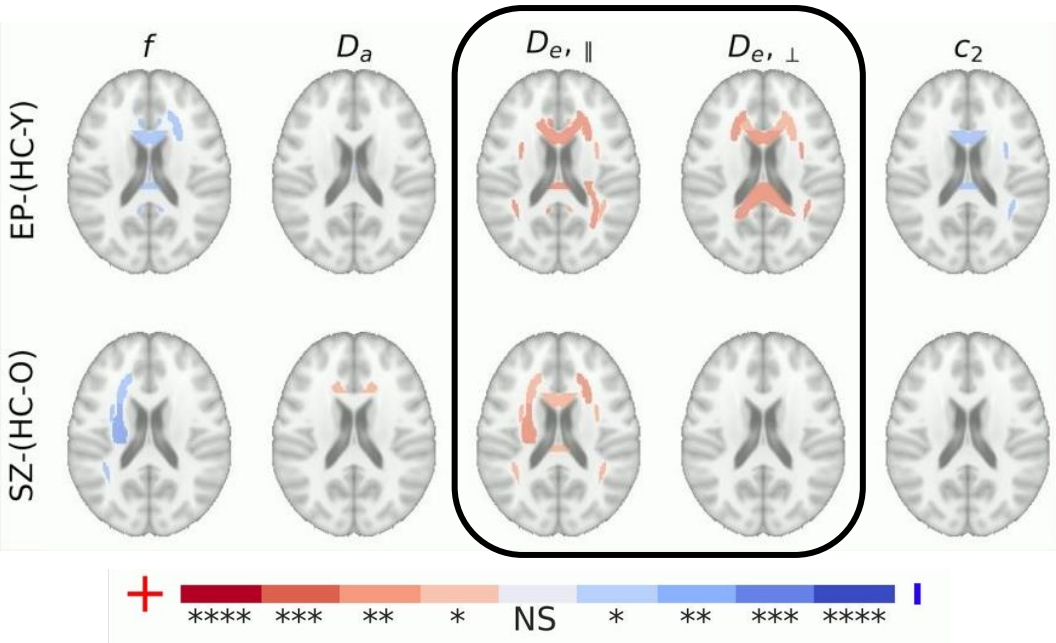
Functional connectivity



Applications of WM Standard Model to patient populations



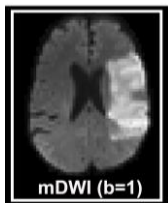
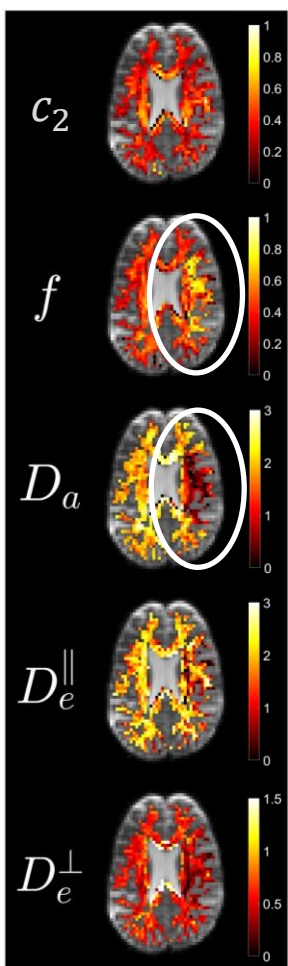
Early psychosis & schizophrenia



$$[f, c_2, D_a, D_{e,||}, D_{e,\perp}]$$

Extra-axonal changes
Myelin defects

Stroke



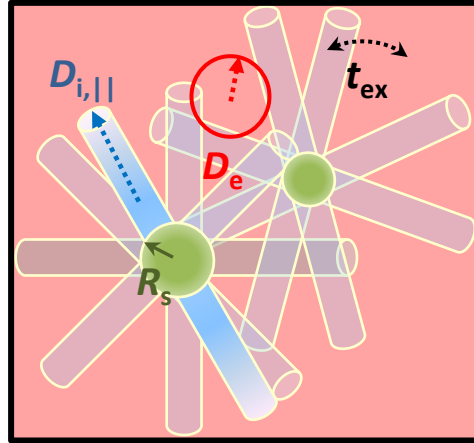
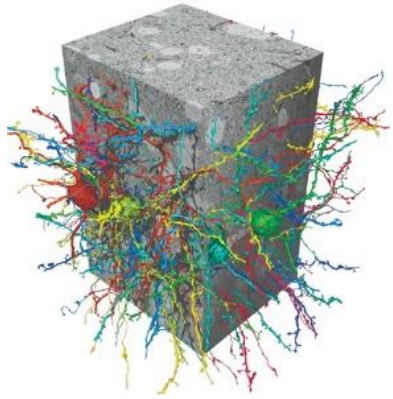
$$[f, c_2, D_a, D_{e,||}, D_{e,\perp}]$$

Cellular swelling
Axonal beading

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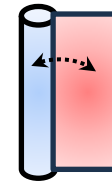
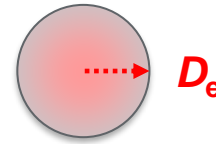
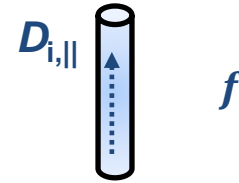
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What is a relevant model for gray matter?



$$\frac{\text{blue}}{\text{blue} + \text{red} + \text{green}} = f_{in}$$

$$\frac{\text{green}}{\text{blue} + \text{red} + \text{green}} = f_{is}$$



Markers of:

f : loss of cell processes (neurites, ...)

$D_{i,||}$: intra-neurite injury

D_e : inflammation

t_{ex} : membrane permeability:
myelination, membrane dysfunction

R_s : soma size, reactivity

f_s : cell proliferation or loss

$$\mathbf{p} = [f_{in}, D_{i,||}, D_e, f_{is}, R_s, t_{ex}]$$

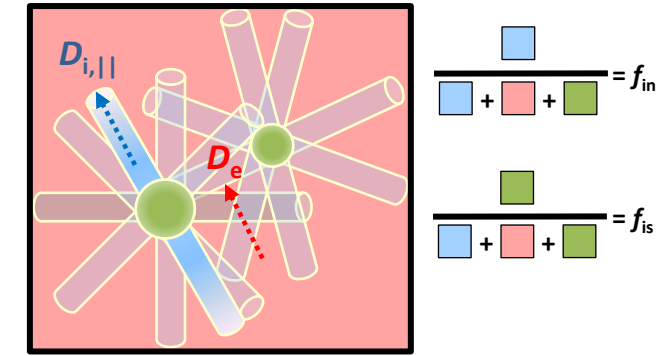
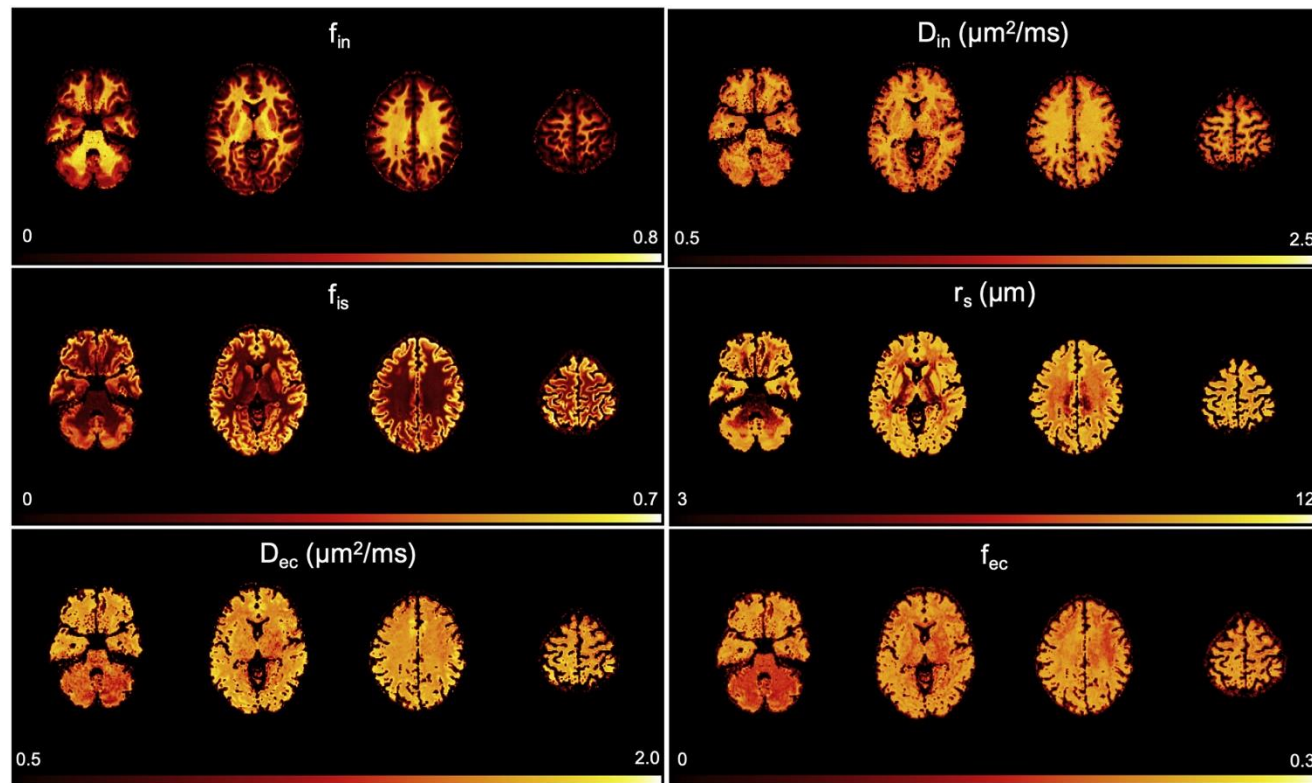
- Cell bodies: a third compartment?

Palombo, NeurolImage 2020

- Little myelin: water exchange between compartments (t_{ex})?

Modeling soma: SANDI – Soma and Neurite Density Imaging

- Assumption: non-exchanging compartments
 - $t_d \leq 20$ ms



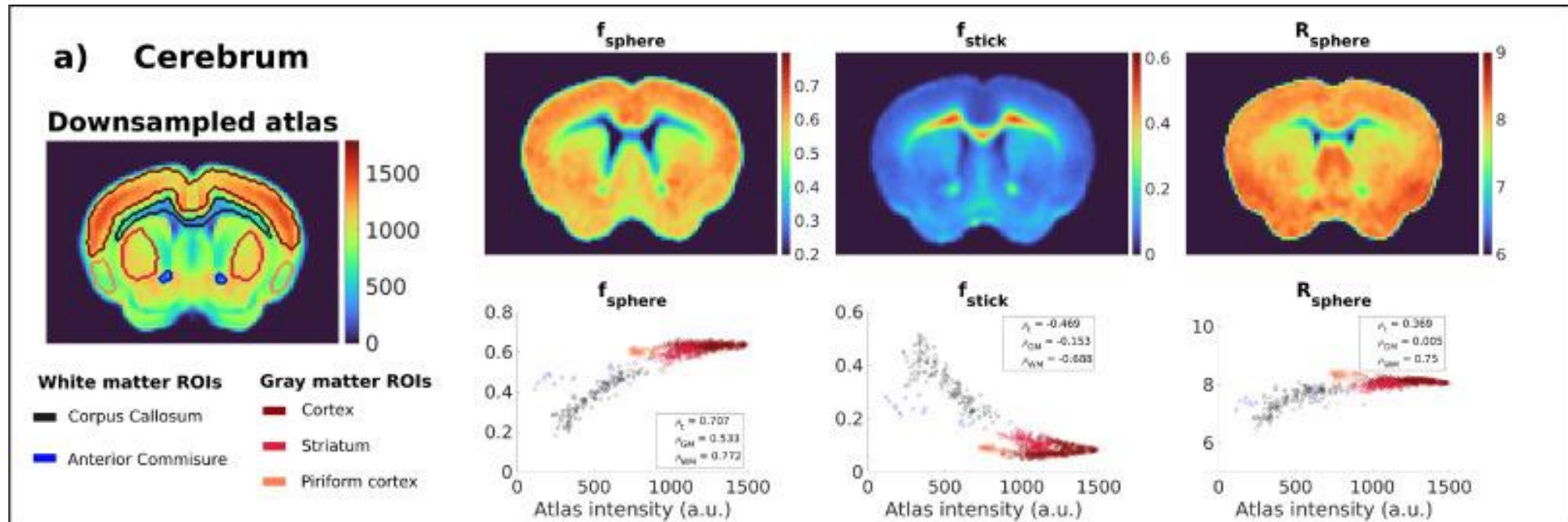
$$\mathbf{p} = [f_{in}, D_{i,\parallel}, D_e, f_{is}, R_s]$$

- ☺ Reasonable estimate of soma density & size
- ☹ Not feasible on a typical MRI clinical scanner

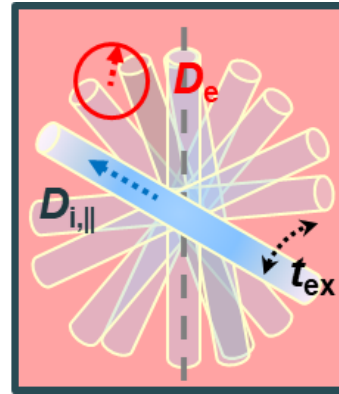
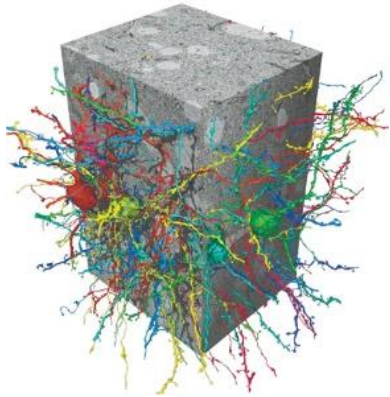
$$b = (\gamma \delta G)^2 \cdot \left(\Delta - \frac{\delta}{3} \right)$$

If $\left(\Delta - \frac{\delta}{3} \right)$ is to be kept short,
Need very high G to achieve high b -values ($b > 5$ ms/ μm^2)

SANDI vs cell density (Allen Brain atlas)

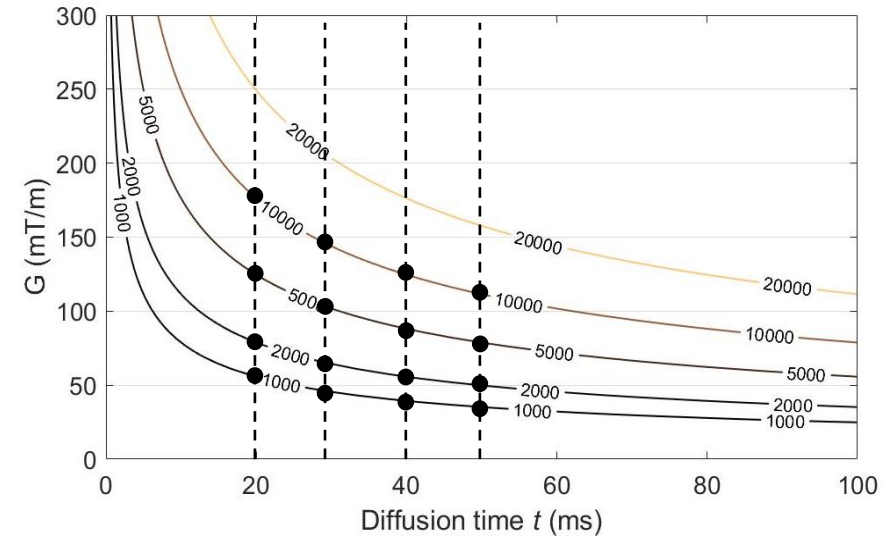


Modeling exchange: NEXI (Neurite EXchange Imaging)

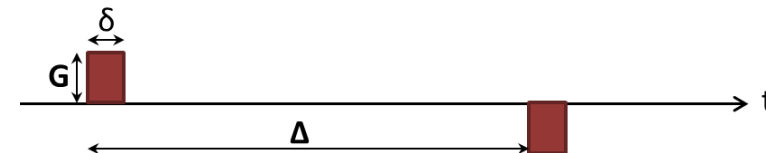


$$\frac{\text{blue box}}{\text{blue box} + \text{pink box}} = f$$

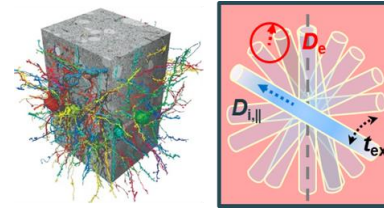
$$\mathbf{p} = [f, D_{i,\parallel}, D_e, t_{ex}]$$



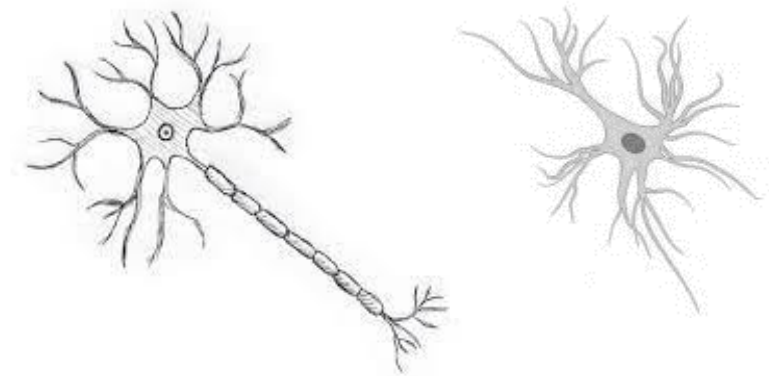
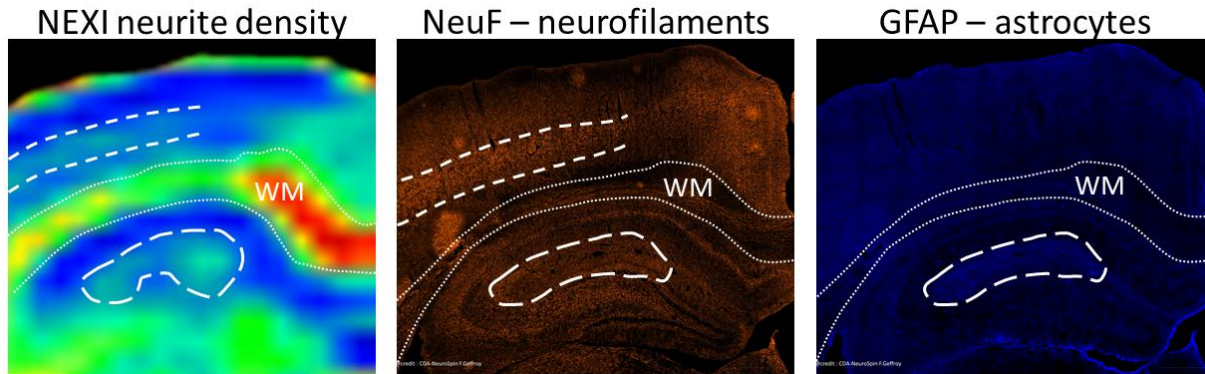
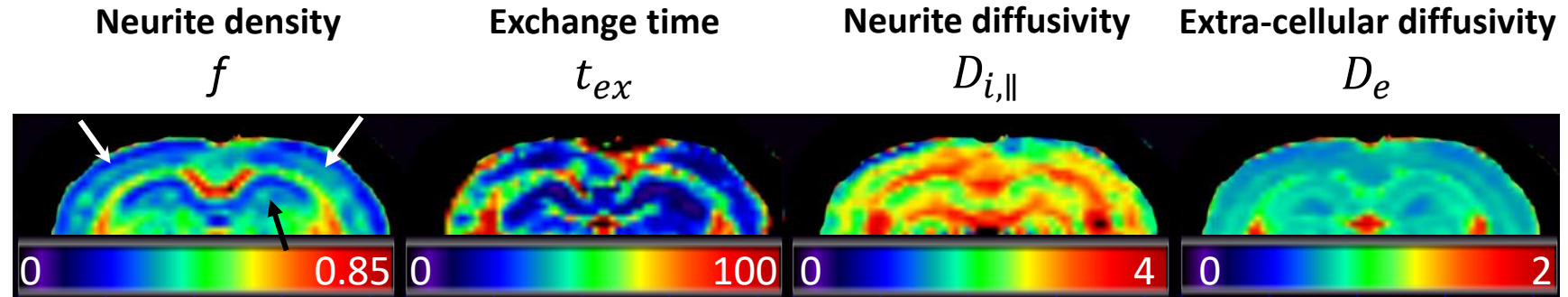
→ Vary G and Δ



NEXI – rat brain, in vivo

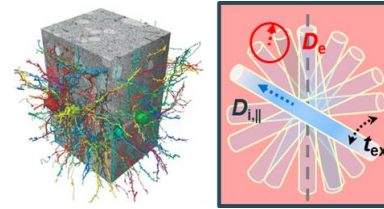


$b = 0 - 10'000 \text{ s/mm}^2$
 $\Delta = 10 - 45 \text{ ms}$

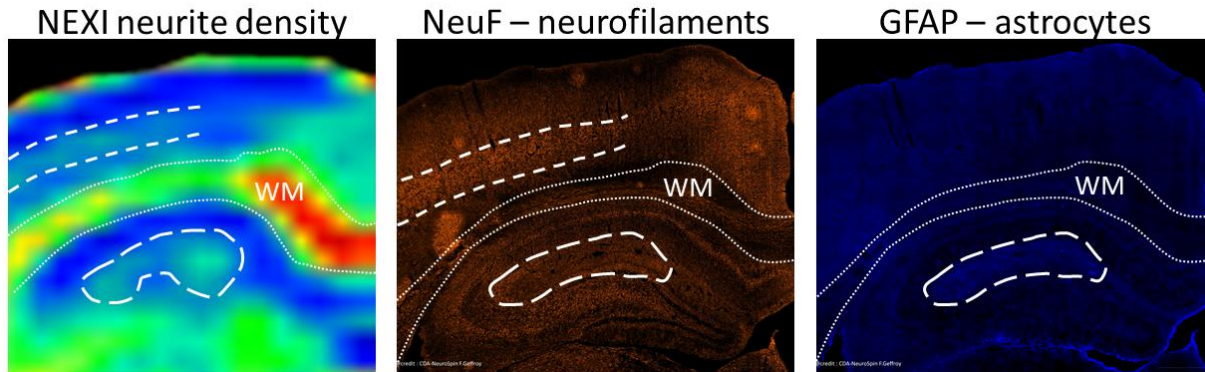
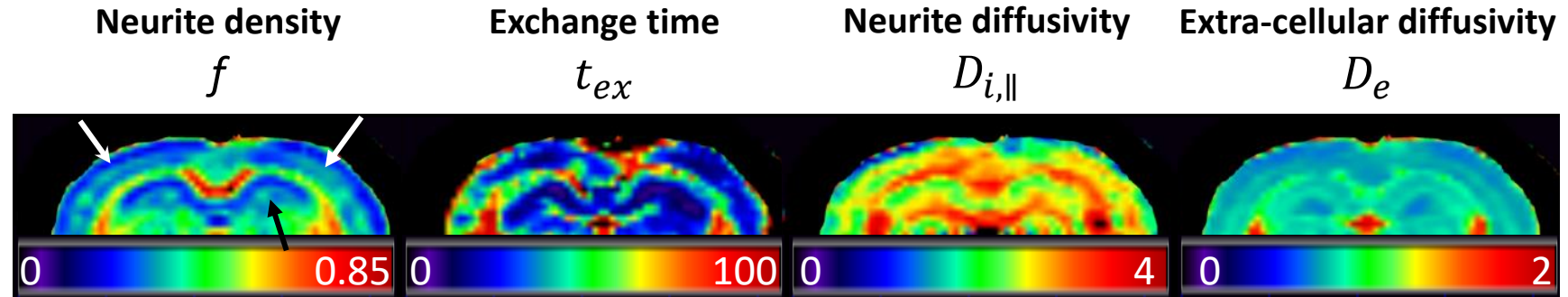


“Neurite density” = “Cell process density”

NEXI – rat brain, in vivo



$b = 0 - 10'000 \text{ s/mm}^2$
 $\Delta = 10 - 45 \text{ ms}$



$$t_{ex} \sim 15 - 35 \text{ ms} \cong \frac{d}{4P}$$

Permeability $\sim [2 - 33] \times 10^{-3} \mu\text{m/ms}$

Vs literature:
 Permeability $\sim [6 - 30] \times 10^{-3} \mu\text{m/ms}$

“Neurite density” = “Cell process density”

Harkins, 2009; Finkelstein, 1987; Latour, 1994; Staniszc, 1997

NEXI – human *in vivo*

3T Connectom scanner

$$G_{\max} = 300 \text{ mT/m}$$

$$b = 1000 - 7500 \text{ s/mm}^2$$

$$\Delta = 20 - 40 \text{ ms}$$

Scan time = 40 min

Uhl, Imaging Neuroscience 2024

3T Clinical scanner

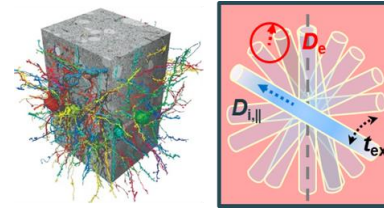
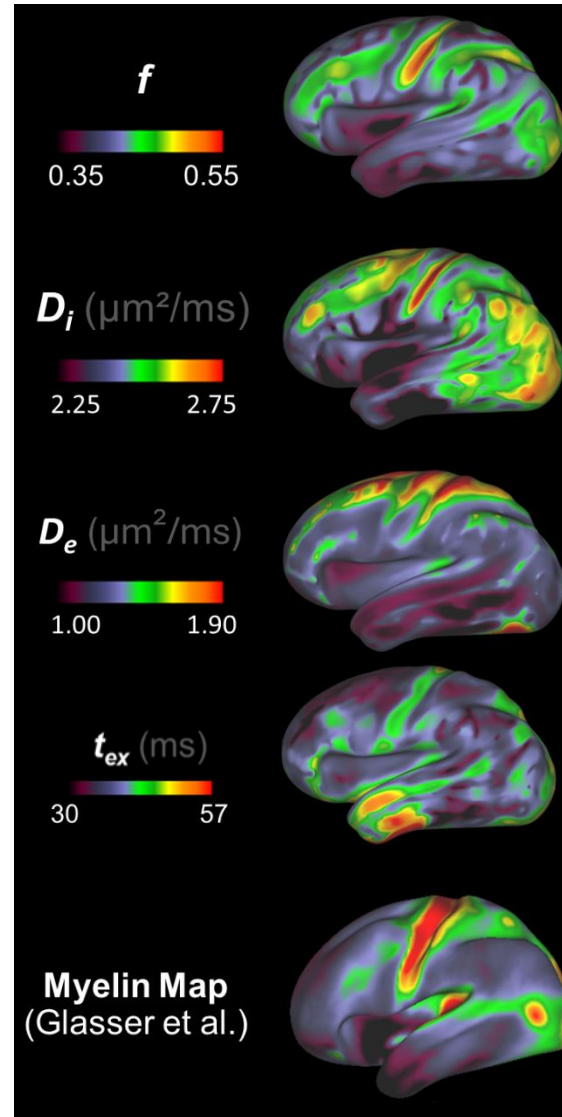
$$G_{\max} = 80 \text{ mT/m}$$

$$b = 1000 - 5000 \text{ s/mm}^2$$

$$\Delta = 28 - 65 \text{ ms}$$

Scan time = 30 min

Uhl, bioRxiv 2024



https://github.com/Mic-map/graymatter_swissknife

Next steps:

- More with less: reduce scan time
- Model exchange + soma

Olesen, NeuroImage 2022

Applications:

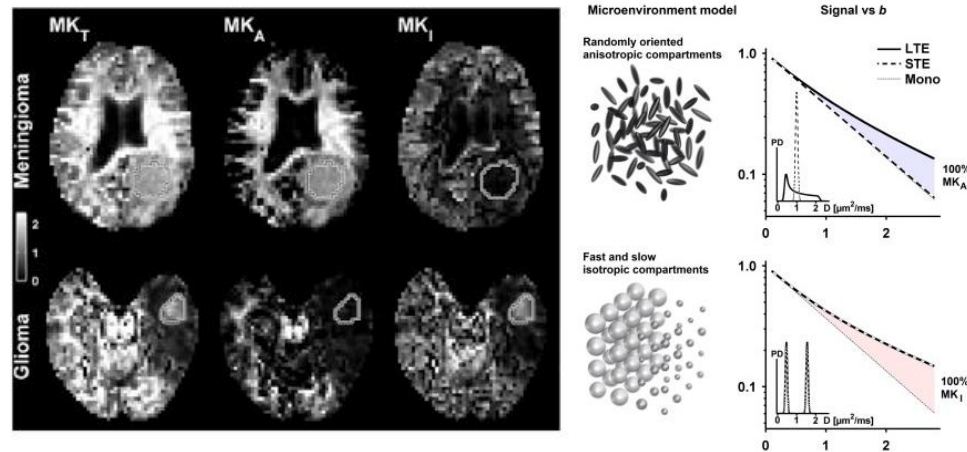
- Multiple sclerosis
- Traumatic Brain Injury
- ...

Outline

- I. Diffusion-weighting
- II. Gaussian approximation regime: ADC and DTI
- III. Beyond Gaussian:
 - i. Signal representations: DKI
 - ii. White matter models & applications
 - iii. Gray matter models & applications
 - iv. Thinking outside the brain

Microstructure models (almost) outside the brain

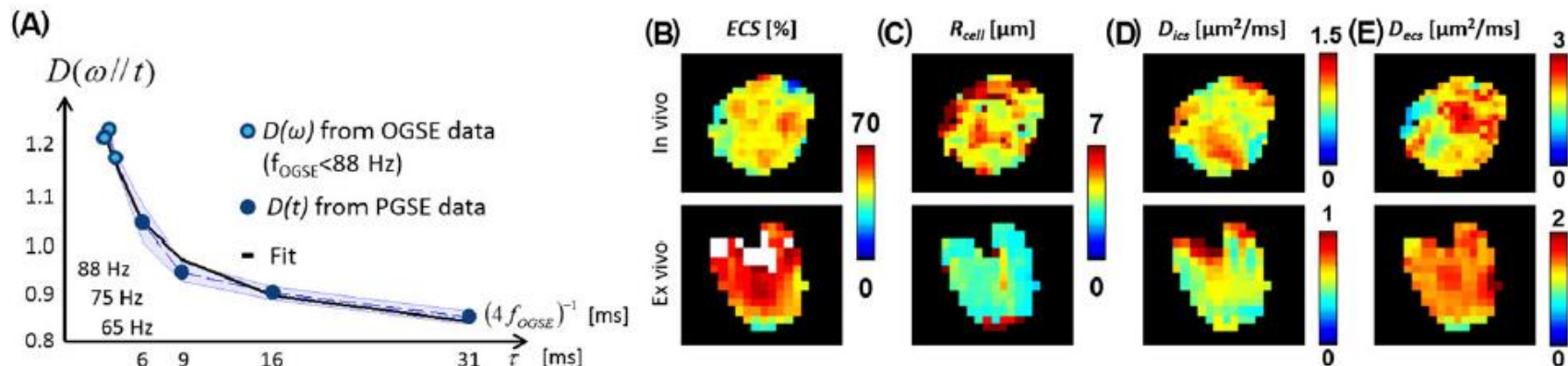
A. Diffusional variance decomposition



Szczepankiewicz, NeuroImage 2016

Glioma

Quantification of cell density, radius, and compartment diffusivities



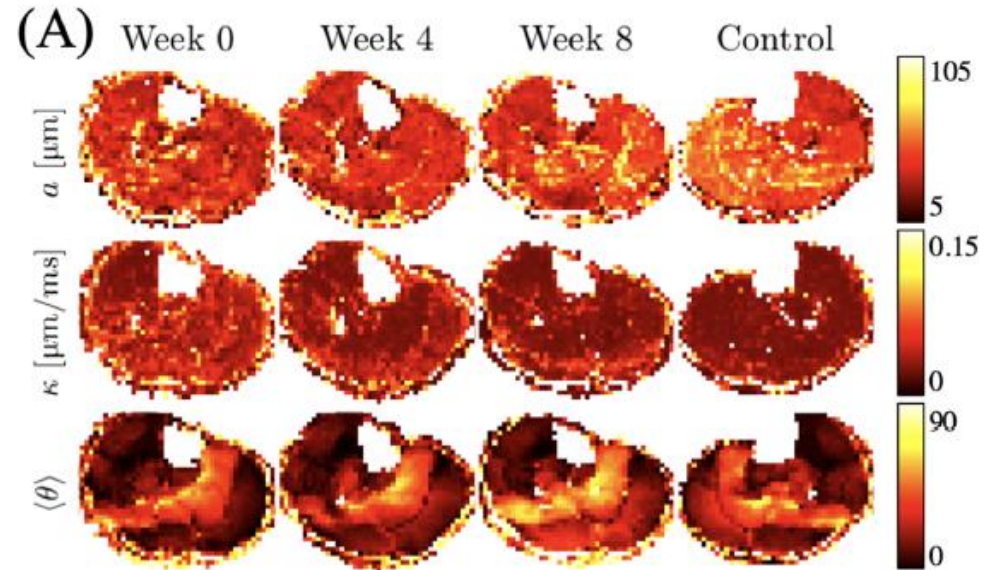
Reynaud, NMR in Biomed 2016

Microstructure models outside the brain

Random Permeable Barrier Model

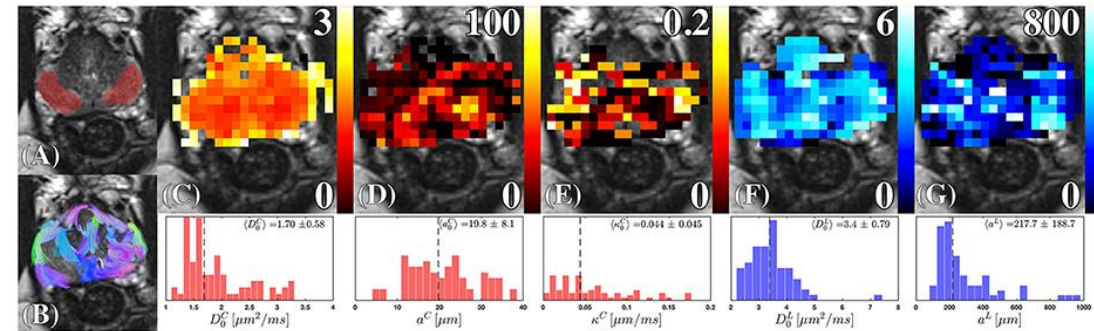
Muscle

Quantification of myofiber size, permeability and orientation



Lemberskiy, NMR in Biomed 2021

Prostate



Cellular diffusivity, fiber diameter, and membrane permeability

Luminal diffusivity and diameter

Lemberskiy, Front Phys 2018

Summary

- Biophysical tissue models:
 - In theory: specific metrics of microstructure
 - In practice: dependent on validity of model assumptions & fitting procedure
 - Interpret output with caution!
- Substantial progress in white matter modeling
 - Estimation of main Standard Model parameters comprehensively
 - Accessible applications and translation to preclinical & clinical studies of disease
- Current focus: gray matter modeling
 - Accounting for inter-compartment exchange appears critical
 - Parameter estimation may require (q-t) coverage
 - Challenge for retrospective studies & prospective clinical studies
 - Eventually: propose a more comprehensive model (exchange + soma + ...)
- Other organs / tissue types
 - High interest

Reviews:

Jelescu and Budde, Front Phys 2017

Novikov, MRM 2018

Nilsson, NeuroImage 2018

Alexander, NMR in Biomed 2019

Novikov, NMR in Biomed 2019

Jelescu, J Neurosci Meth 2020

Recommendations Papers:

Jelescu, [...], Schilling, MRM 2025

Schilling, [...], Jelescu, MRM 2025

Questions?

