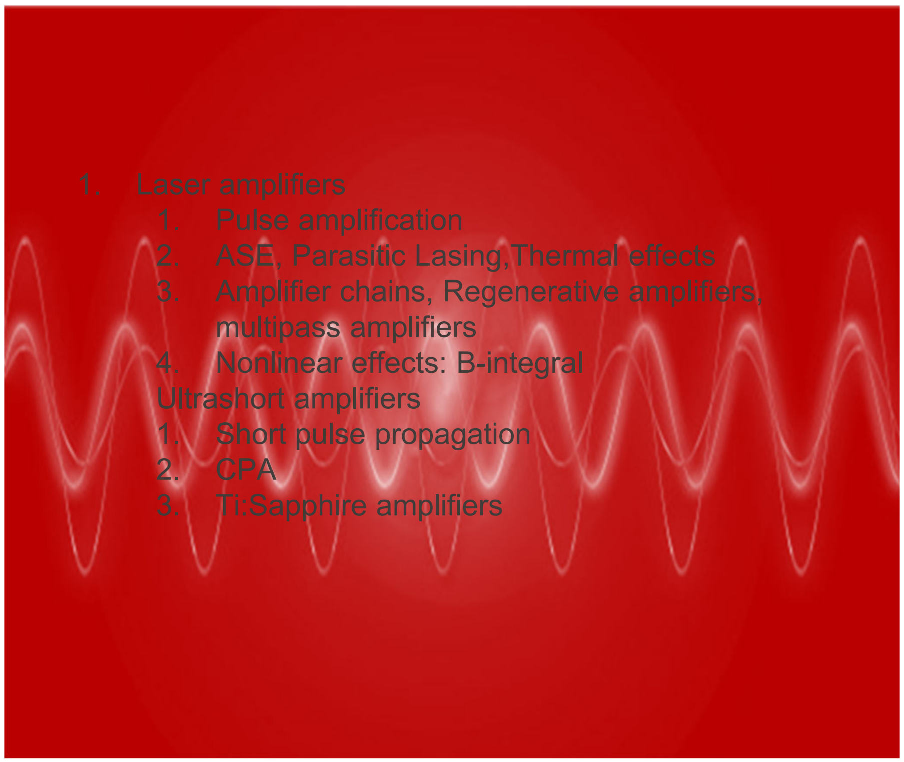
EPFL

Summary



1. Laser amplifiers
   1. Pulse amplification
   2. ASE, Parasitic Lasing,Thermal effects
   3. Amplifier chains, Regenerative amplifiers, multipass amplifiers
   4. Nonlinear effects: B-integral

Ultrashort amplifiers
   1. Short pulse propagation
   2. CPA
   3. Ti:Sapphire amplifiers

Advanced Radiation Sources - PHSY761

Lecture 05

8 October 2024

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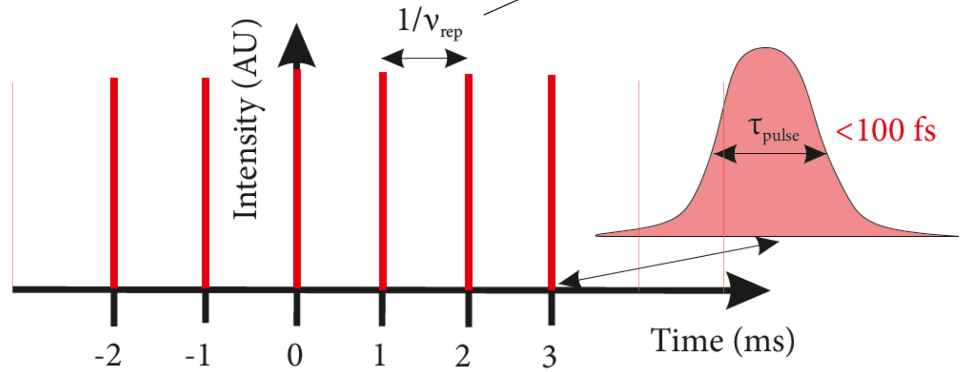
Laser pulse trains

Measured with an oscilloscope



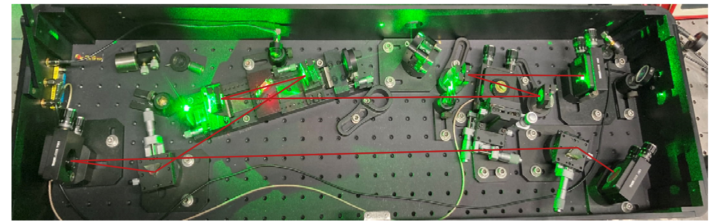
Train of equally-spaced pulses: The «repetition rate» is the inverse of the temporal pulse separation («duty cycle»)

Pulse duration: we define it as the FWHM of the intensity → Oscilloscope (ns) Special methods are needed for shorter pulse

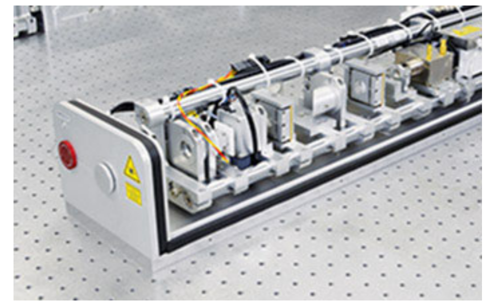
Average power: energy averaged over a time T>> $1/\nu_{rep}$ → Measure of the energy carried in time: measured with a slow power meter

Pulse power: pulse energy / pulse duration → Measure of the **peak** power of the pulse (even very moderate average powers can have huge pulse power)

Pulsed laser sources we have seen so far:



Typical specifications of laser oscillators E (ν_{rep}) :



Mode-locked fs Ti:Sapphire lasers (≈ 10 fs) -> ≈10 nJ (100 MHz)

Q-switched Nd laser (≈ 10 ns)-> ≈ 100 mJ (Hz - kHz)

These correspond to MW pulses -> Different strategies are required for higher peak intensities

MW -> GW -> TW -> PW

Perturbative Nonlinear optics

Strong field effects

Relativistic nonlinear optics

PEAK INTENSITY

10^{13} W/cm^2

10^{18} W/cm^2

↳ LINKED TO MODE QUALITY ⟶ DIFFRACTION LIMITED SOURCES

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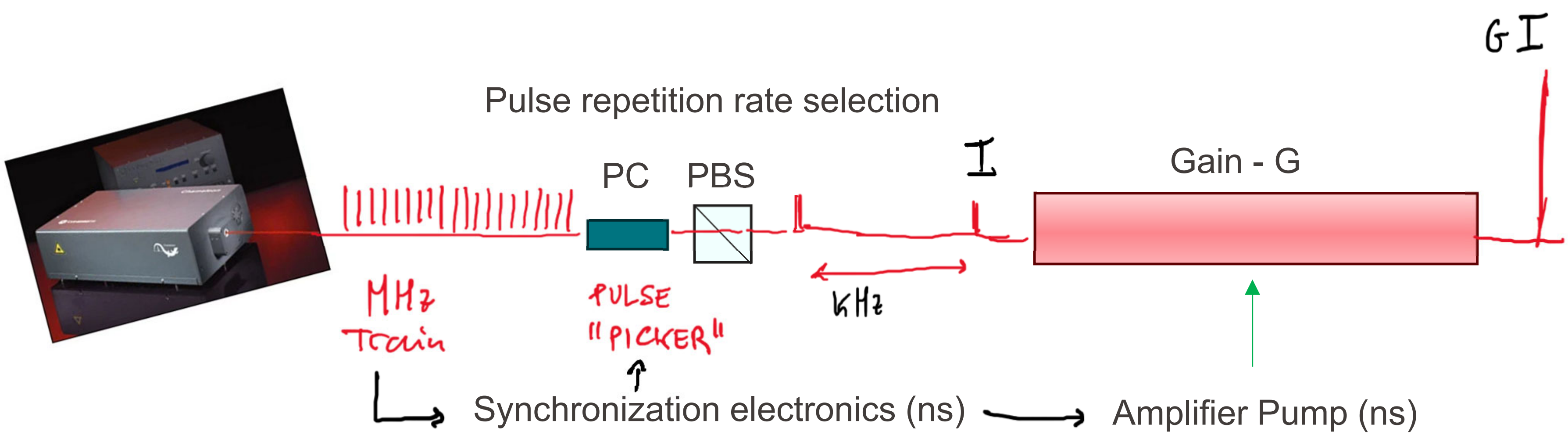
Why so hard to increase the power of a femtosecond oscillator?

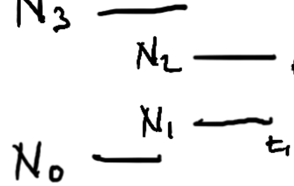
- Lack of suitable pump lasers
- High intracavity power and intensity: thermal and nonlinear effects.

Master Oscillator / Power Amplifier (MOPA)



- 4-LEVEL SYSTEM
- τ_p pulse duration

$t_1 \ll \tau_p \ll t_2$



$N = N_2 - N_1$

t_2 UPPER STATE LIFETIME

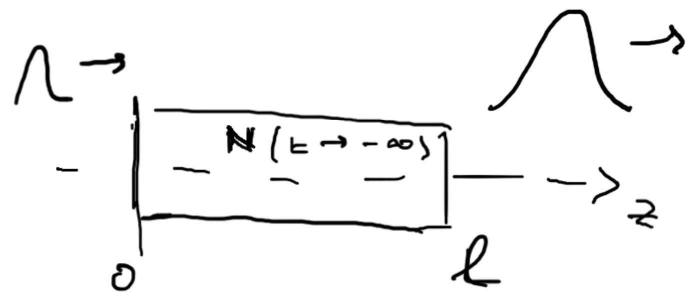
t_1 LOWER STATE LIFETIME

σ X SECTION

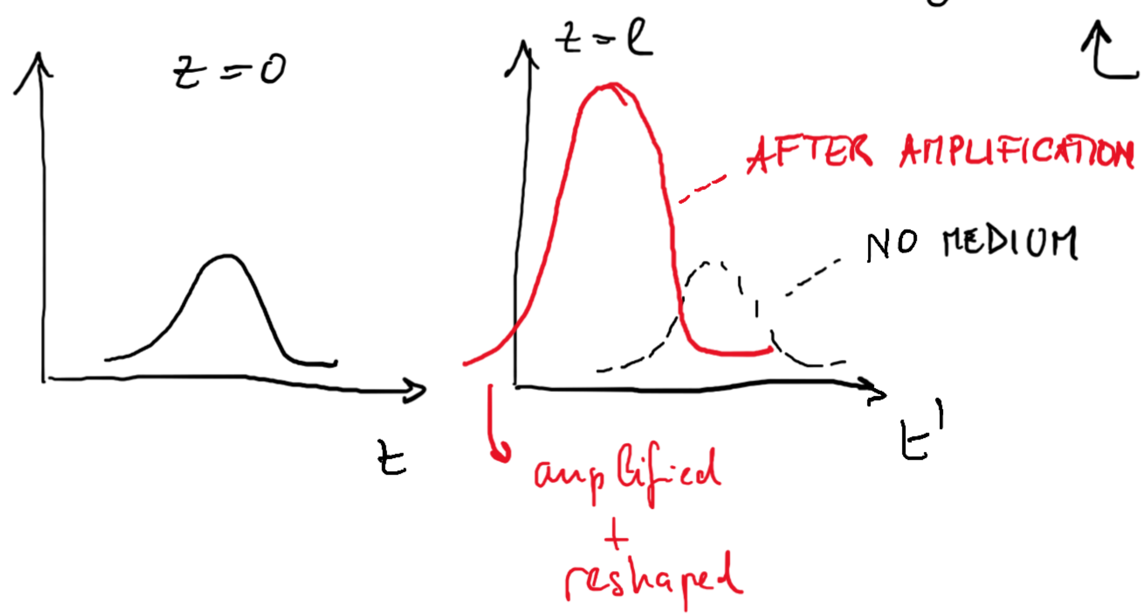
1d MODEL → WE WANT TO KNOW

$I(z,t)$; $N(z,t)$

INTENSITY — POP. INVERSION



AMPLIFYING MEDIUM OF LENGHT L



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$t_1 \ll \tau_p \ll \tau$

NEGLECT OTHER TRANSITIONS MODIFYING INVERSION

Transition rate

RATE EQ. FOR POP. INVERSION $\longrightarrow$

$$\frac{\partial N(z,t)}{\partial t} = \overbrace{W}^{\text{Transition rate}} N(z,t) + \ldots$$

$$= \phi \sigma N = \frac{I}{h\nu} \sigma N$$

$$= \frac{I(z,t)\, N(z,t)}{\Gamma_s}$$

$$\Gamma_s = \frac{h\nu}{\sigma}$$

SATURATION EN. DENSITY $\left[J/cm^2 \right]$

$\Gamma_s \simeq 1 \; J/cm^2$

Ti : Sapphire

$\Gamma_s = 1 - 2 \; mJ/cm^2$

Dye Laser

E.M. energy density

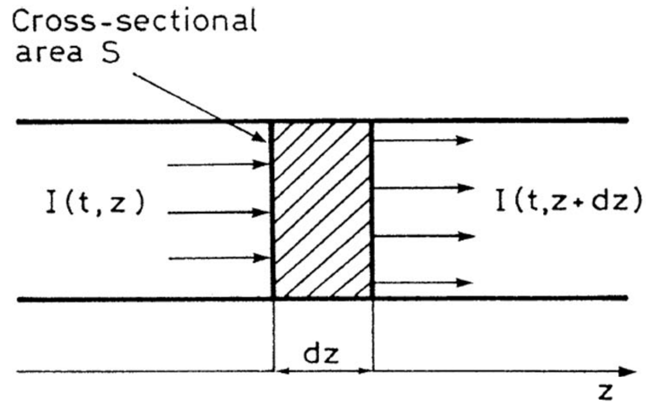
$$\frac{\partial \rho}{\partial t} = \left.\frac{\partial \rho}{\partial t}\right|_1 + \left.\frac{\partial \rho}{\partial t}\right|_2 + \left.\frac{\partial \rho}{\partial t}\right|_3$$

STIMULATED PROCESSES

LOSSES (e.g. defects..)

photon flux in/out

$dV = S\,dz$



$$\left(\frac{\partial \rho}{\partial t}\right)_1 = W N h\nu = \sigma I N$$

$$\left(\frac{\partial \rho}{\partial t}\right)_3 S\,dz = S\left(I(t,z+dz) - I(t,z)\right)$$

$$\left(\frac{\partial \rho}{\partial t}\right)_2 = -\alpha_L I$$

due to ampl. losses..

$$\frac{\partial \rho}{\partial t} = \frac{1}{c}\frac{\partial I}{\partial t} = \sigma I N - \alpha_L I + \frac{\partial I}{\partial z}$$

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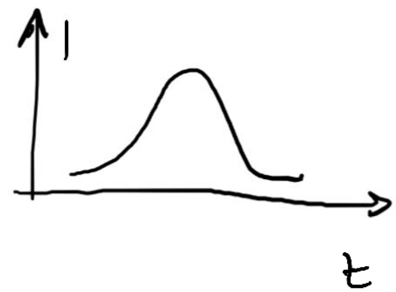
$$\text{①} \quad \frac{1}{c}\frac{\partial I}{\partial t} + \frac{\partial I}{\partial z} = \sigma N I - \alpha I$$

$$\text{②} \quad \frac{\partial N}{\partial t} = -\frac{N I}{\Gamma_s}$$

CAN BE NUMERICALLY INTEGRATED..

EXACT SOLUTION DEPENDS ON THE INPUT SHAPE



$I(t \to \pm\infty) = 0$

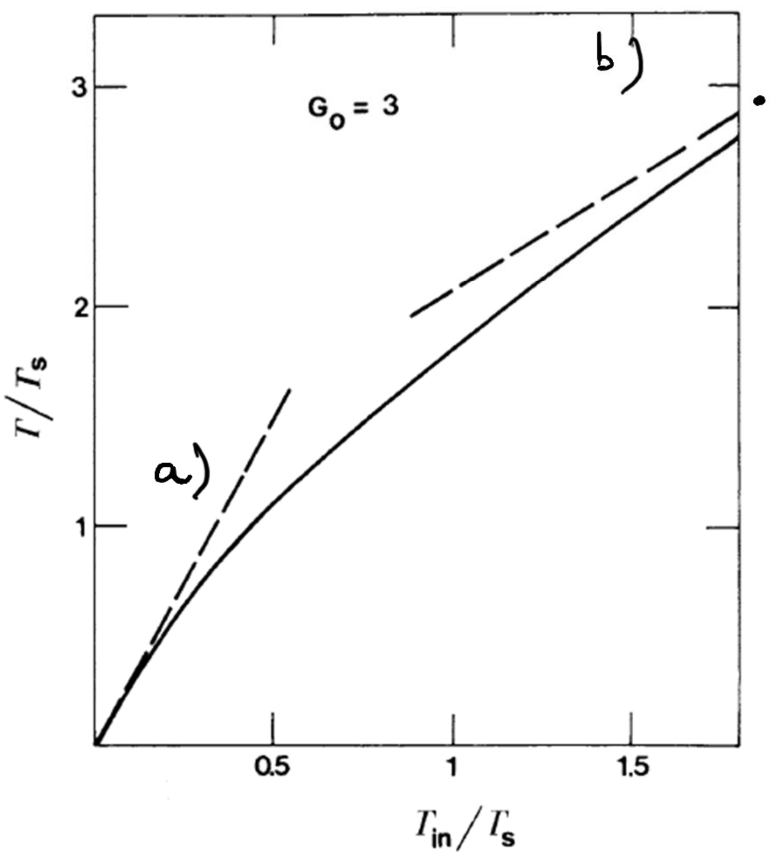
PULSE ENERGY density $\quad \Gamma = \int_{-\infty}^{+\infty} I \, dt$ ← SOME GENERAL CONSIDERATION CAN BE DRAWN FOR THE ENERGY FLUENCE Γ

$$\text{②} \rightarrow N(t \to +\infty) = N(t \to -\infty)\, e^{-\Gamma/\Gamma_s} = N_0 e^{-\Gamma/\Gamma_s}$$ (INTEGRATE ② $\int_{-\infty}^{+\infty} dt$)

① $\int_{-\infty}^{+\infty} dt$ BOTH SIDES ; INSERT ② ; $I(\infty) = 0$

$$\frac{d\Gamma}{dz} = g\,\Gamma_s \left[1 - \exp\left(-\Gamma/\Gamma_s \right) \right] - \alpha \Gamma$$

$g = N_0 \sigma$ UNSATURATED GAIN COEFFICIENT



ASSUMING $\alpha_L \simeq 0$ HAS ANALYTIC SOL.

$$\Gamma(\ell) = \Gamma_s \ln\left[1 + G_0\left[\exp\left(\frac{\Gamma_{in}}{\Gamma_s}\right) - 1\right]\right]$$

$G_0 = e^{-g\ell}$ UNSATURATED GAIN (SMALL-SIGNAL GAIN) $\Gamma_{in} = \Gamma(0)$

Two extreme cases.

linearize.. $\ln(1+x) \sim x$

a) $\frac{\Gamma_{in}}{\Gamma_s} \ll 1 \Rightarrow \Gamma(\ell) = G_0 \Gamma_{in}$

LINEAR REGIME

b) $\frac{\Gamma_{in}}{\Gamma_s} \gg 1 \Rightarrow \Gamma(\ell) = \Gamma_{in} + g\ell\Gamma_s$

DEEP SATURATION REGIME

MAXIMUM ENERGY EXTRACTION

$g\Gamma_s \ell = \underbrace{N_0 h\nu}\, \ell$ (~ALL THE ATOMS CONTRIBUTE)

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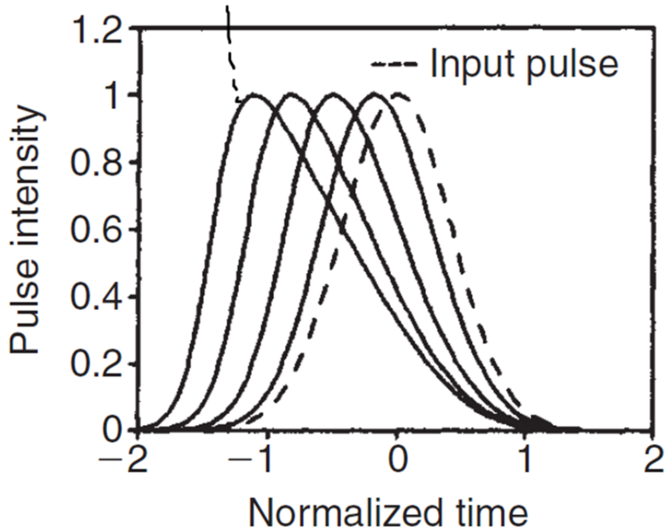
Transverse mode and temporal reshaping during amplification

$\Gamma/\Gamma_s \sim 1$

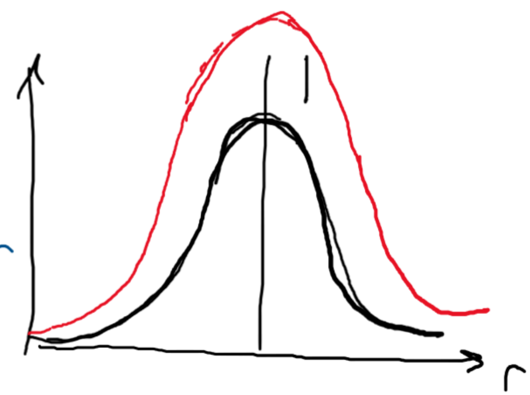


NUMERICAL RESULTS FOR :

GAUSSIAN INPUT

$10^{-4} < \Gamma/\Gamma_s < 1$

$G_0 = 10^4$



TRANSVERSE MODE

SATURATION AMPLIFY THE WINGS MORE THAN THE PEAK

IN GENERAL → SPATIAL PROFILE WORSEN

What limits the maximum fluence which can be extracted by an amplifier?

- Medium losses $\longrightarrow \Gamma_{max} \rightarrow \dfrac{g\Gamma_s}{\alpha_L} \quad (\alpha \ll g)$
- Optical damage $\longrightarrow \Gamma_d \cong 10\ J/cm^2 \quad \Gamma \cong g l \Gamma_s < \Gamma_d$

Effects arising due to a high gain G_0

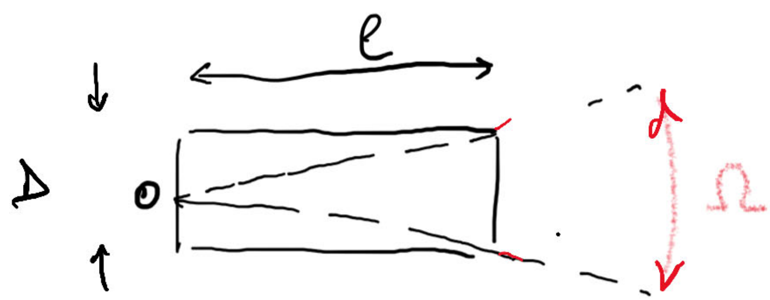
- Amplified spontaneous emission (ASE)
- Parasitic lasing

Limitation due to high average power and high intensity

- Thermal effects
- Nonlinear effects

$\longrightarrow$ Mode distortion
Temporal distortion

- Amplified spontaneous emission (ASE)



SOLID ANGLE

CYLINDRICAL GAIN MEDIUM ("ROD")

$G = \exp[\sigma (N_2 - N_1) \ell]$ gain (assume to be large e.g. 10^4)

$\Longrightarrow$ STRONG AMPLIFICATION OF THE FLUORESCENCE FROM O

$D \ll \ell \qquad \Omega \sim \frac{\pi D^2}{4\ell^2}$

"MIRRORLESS LASER"

SUPERLUMINESCENCE

- NARROWER THAN SPONT. EMISSION (SPECTRALLY) MORE "COHERENT"
  ↳ gain narrowing
- DIRECTIONAL (Ω)
- SOFT "THRESHOLD"

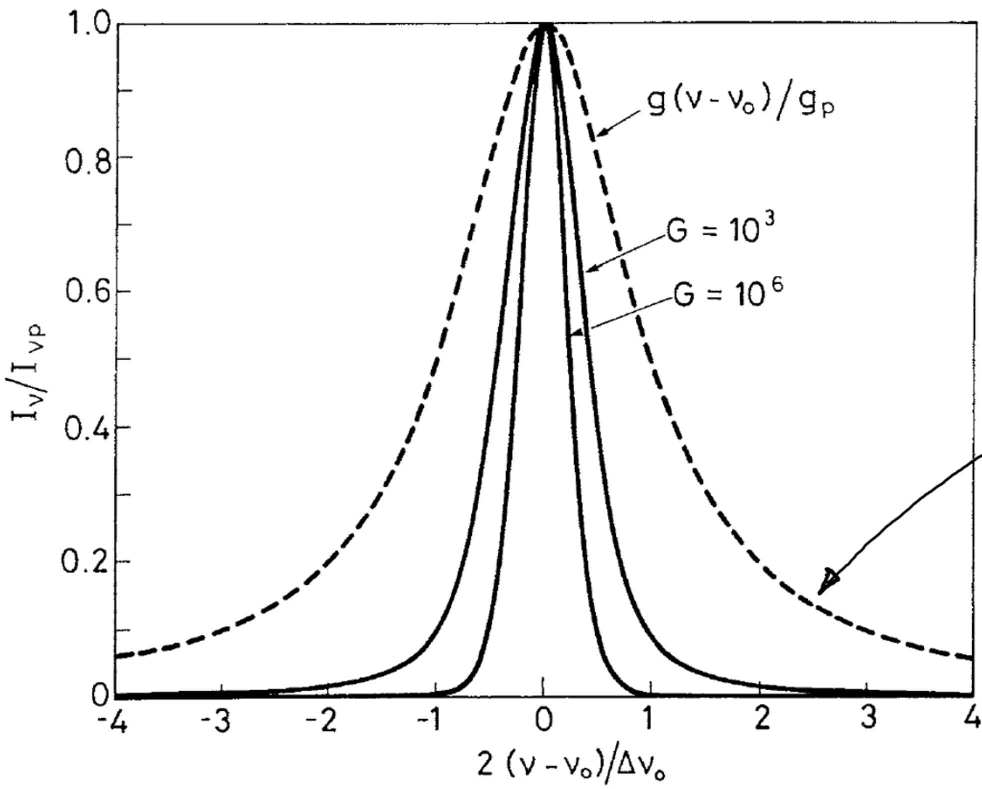
Gain Narrowing

SPECTRAL NARROWING OF THE PULSE ⟶ TRANSITION LINE SHAPE

$\Downarrow$

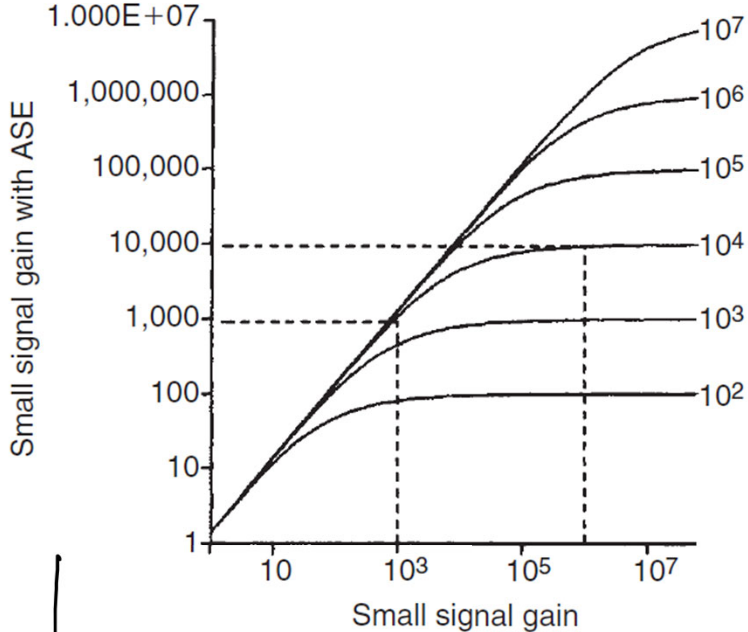
$G_0 = \exp(g\ell)$

$\underline{\sigma(\omega)}\, N_0\, \ell$



$$G(\omega) = \exp[\sigma(\omega)\, N_0\, \ell]$$

Mitigation of ASE:

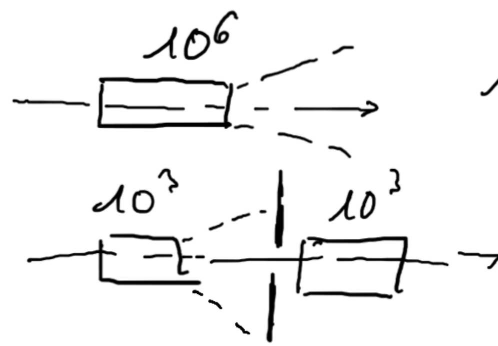


Actual gain considering ASE

Theoretical gain w/o ASE

$$G_0 = e^{\sigma N_0 \ell}$$

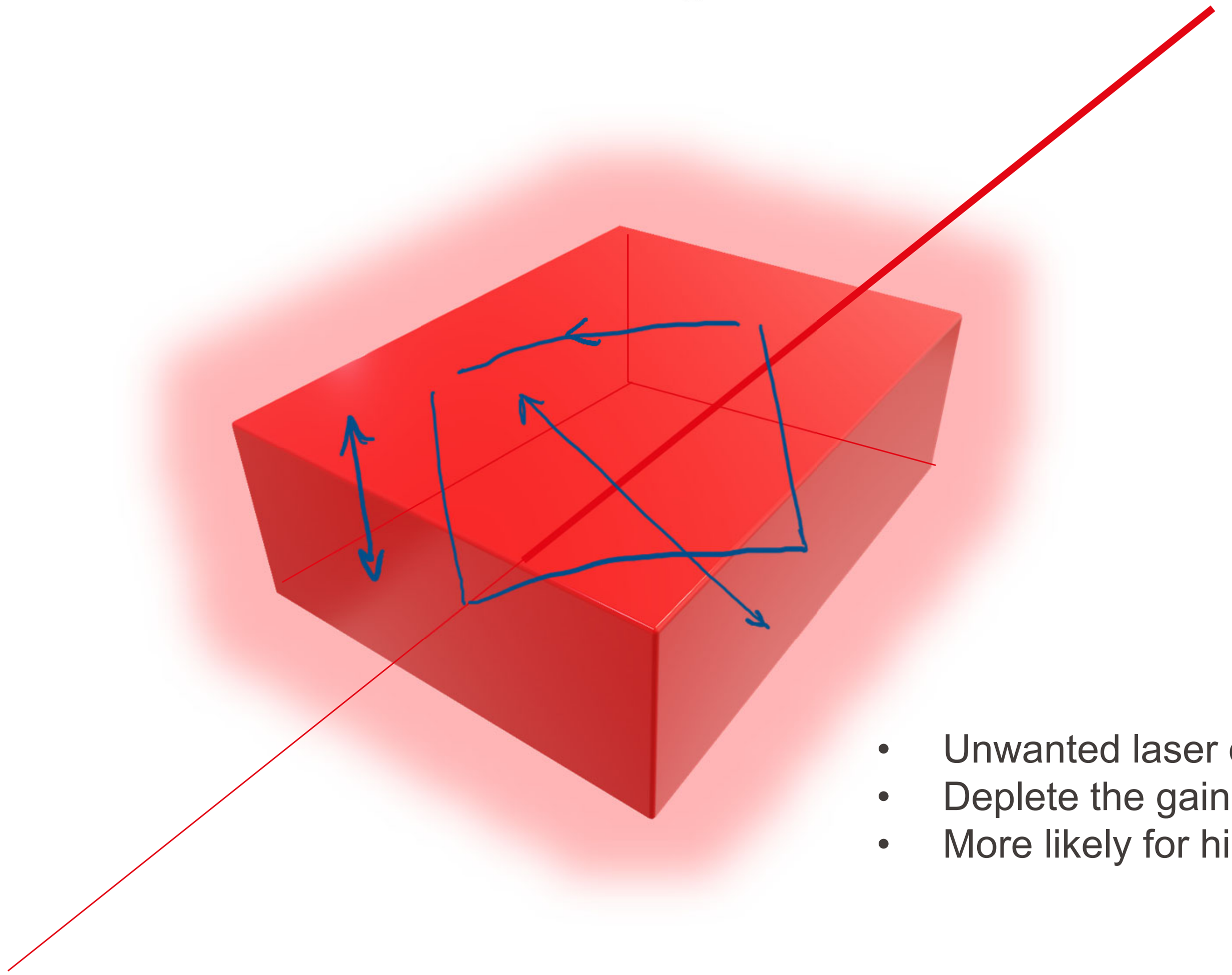
IDEA: 2X GAIN SECTIONS



10^4 gain

10^6 gain

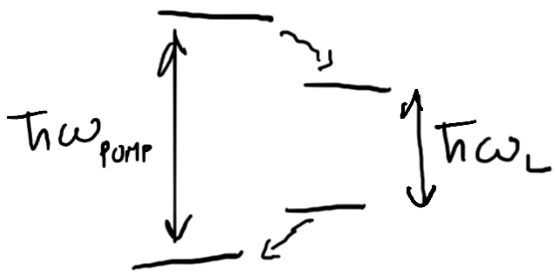
Parasitic lasing



- Unwanted laser oscillations
- Deplete the gain and limit the maximum
- More likely for high G

Thermal effects

Quantum defect in a lasing transition



$$\hbar(\omega_p - \omega_L) = \underbrace{\delta > 0}_{\text{HAS TO BE DISSIPATED AS HEAT}}$$

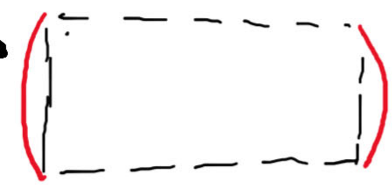
⇝ NON RADIATIVE SCATTERING WITH THE LATTICE

Thermo-optical effects → $\frac{\partial n}{\partial T}$

Thermo-elastic effects → STRAIN →

NON UNIFORM PUMP!

Thermal "lensing"

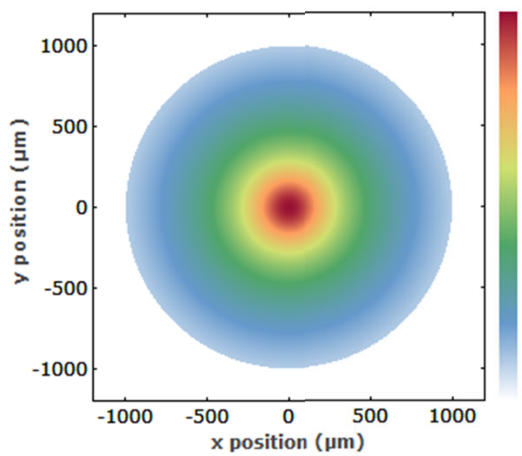


BULGING OF SURFACES

$$f_{TH}^{-1} = \frac{\partial n}{\partial T} \underbrace{\frac{1}{2KA}}_{\text{Thermal conductivity}} P_{heat}$$

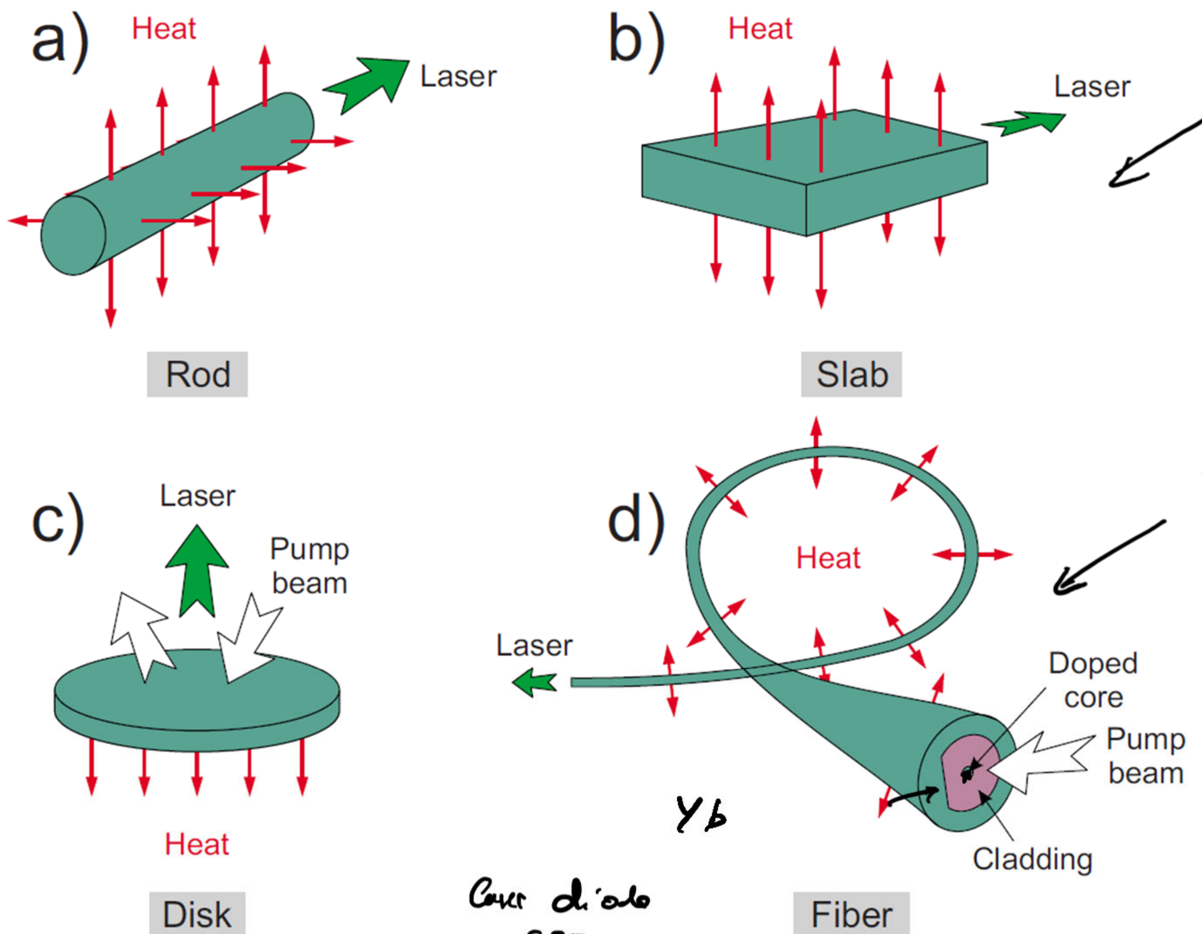
P_{heat}: dissipated power

CAN BE ACCOUNTED FOR IN THE DESIGN



T DISTRIBUTION ON END-PUMPED ROD

Shape of the amplifying medium:



∘ BETTER COOLING

∘ 1d Flow
cylindrical
Thermal
lens

MOSTLY
IMMUNE
TO THERMAL
EFFECTS

∘ KW
CW
LASERS

∘ SMALL CORE
(nonlinear effects
large)

∘ 1d flow
∘ very efficient
cooling
− low gain

Yb

Laser diode
975 nm
HIGH

1030 nm

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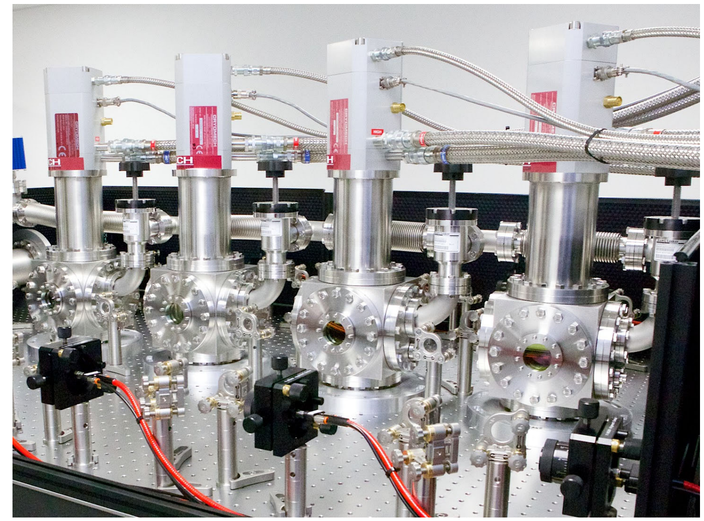
Cryogenic cooling:

TABLE I
SAPPHIRE PROPERTIES AT 300 AND 77 K[a]

Property	At 300 K	At 77 K
Thermal Conductivity, κ	0.33 W cm^{-1} K^{-1}	10 W cm^{-1} K^{-1}
Dependence of Refractive Index on Temperature, dn/dT	1.28×10^{-5} K^{-1}	0.19×10^{-5} K^{-1}
Thermal Expansion Coefficient, α	5.0×10^{-6} K^{-1}	0.34×10^{-6} K^{-1}
Volumetric Heat Capacity, $c\rho$	3.1 J cm^{-3} K^{-1}	0.25 J cm$-^{3}$ K^{-1}

IEEE JOURNAL OF QUANTUM ELECTRONICS, VOL. 21, NO. **4**, APRIL 1991



Thor-300 Cryogenic Yb:YAG Laser Amplifier System

- Improved thermal parameters
- Closed loop cryostats

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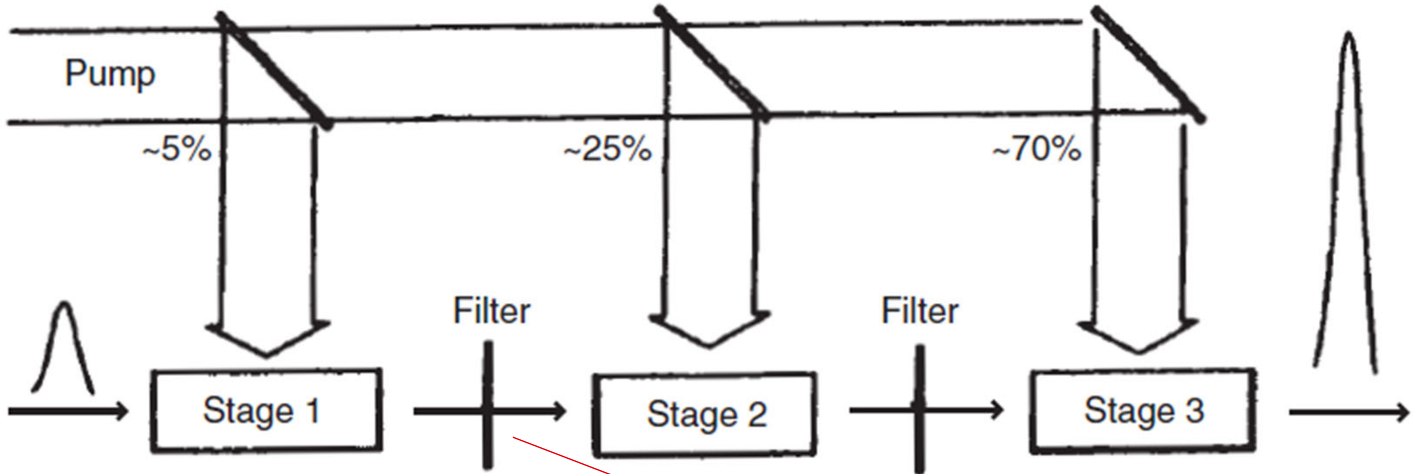
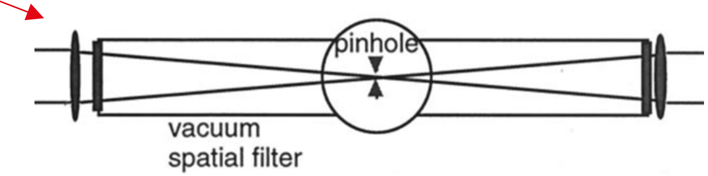
Amplifier chains: Given a target total amplification, better to separate the amplification in several stages

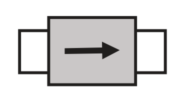


- High G pre-amplifiers
- Low G power amplifiers
- Spatial filters



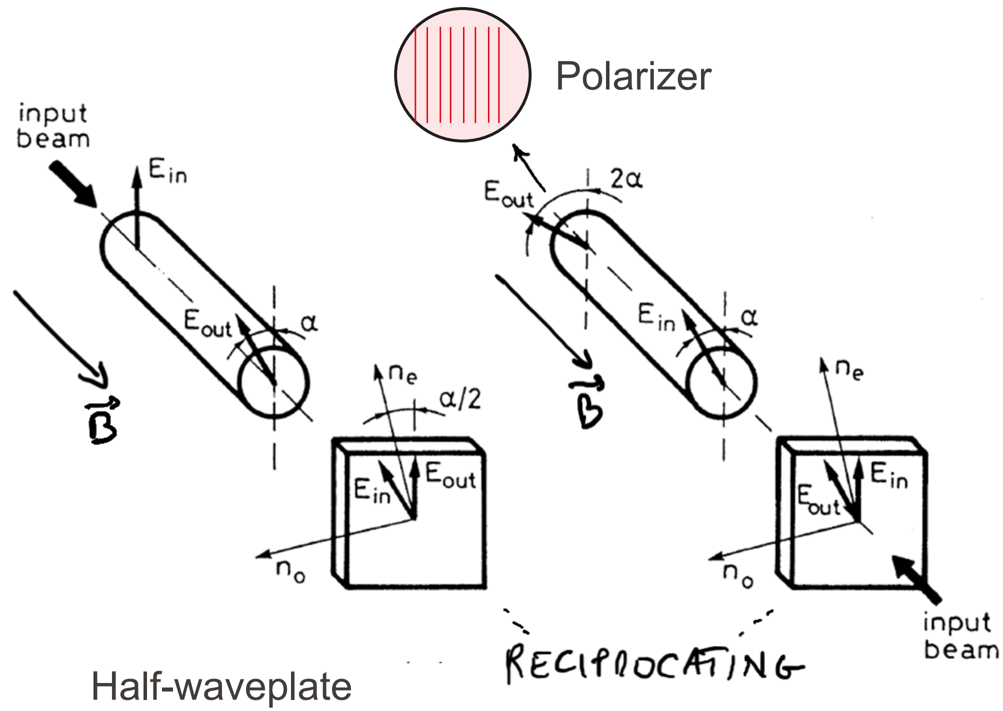
- How to prevent back-amplification?

Optical isolator



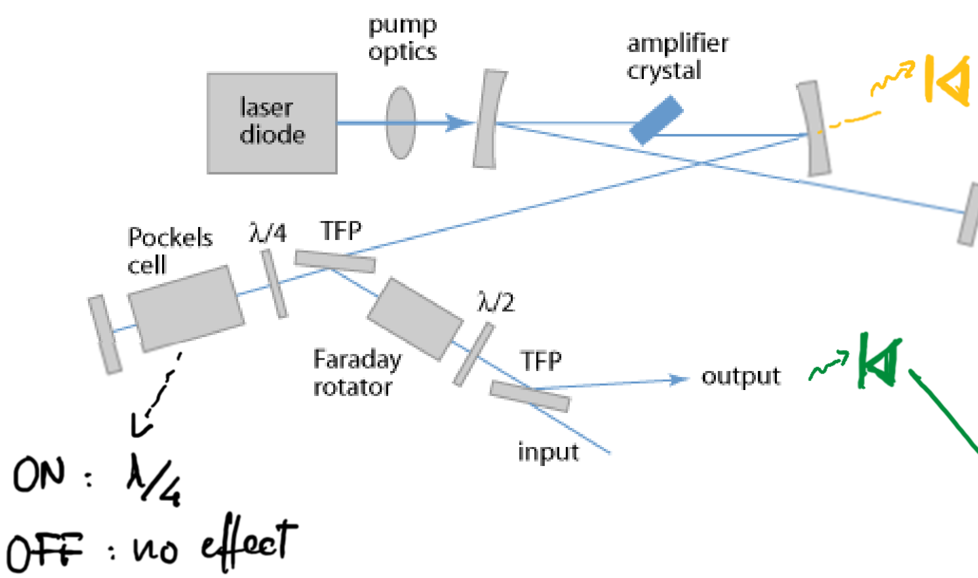
Optical isolators

- Faraday effect
- Non-reciprocating optical element



Optical isolator is realized for α=45° by inserting a polarizer at the input

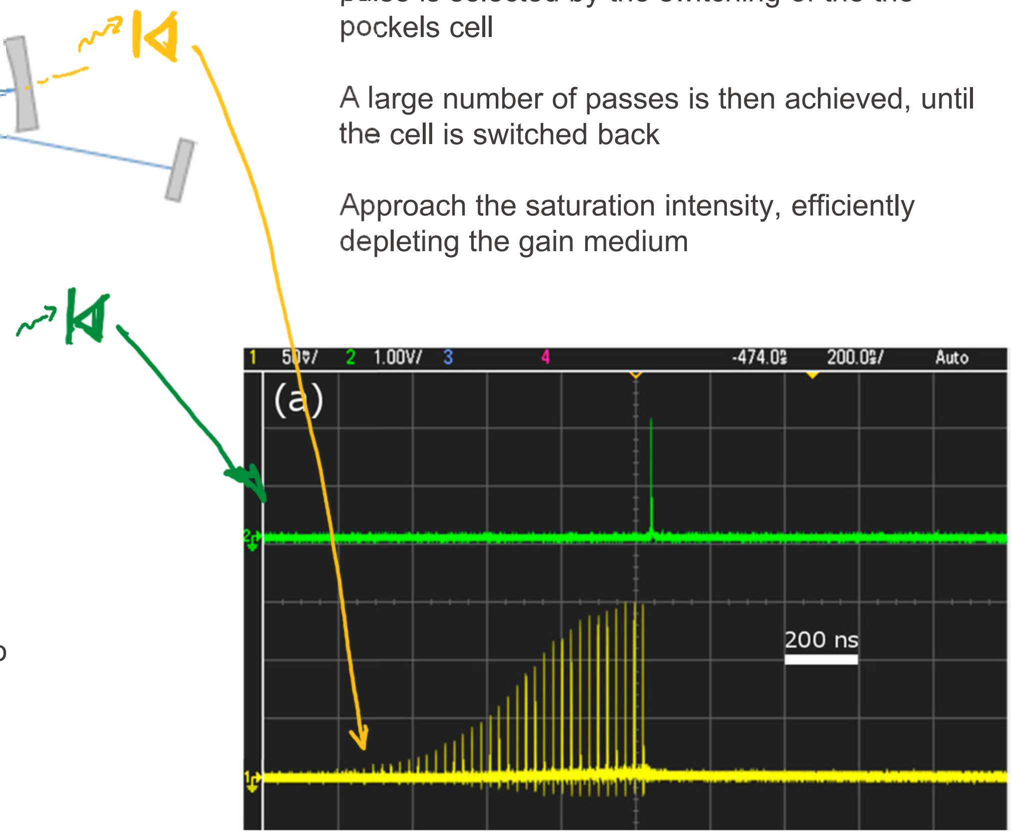
Regenerative amplifier



The pulse train is incoupled in a cavity: only one pulse is selected by the switching of the the pockels cell

A large number of passes is then achieved, until the cell is switched back

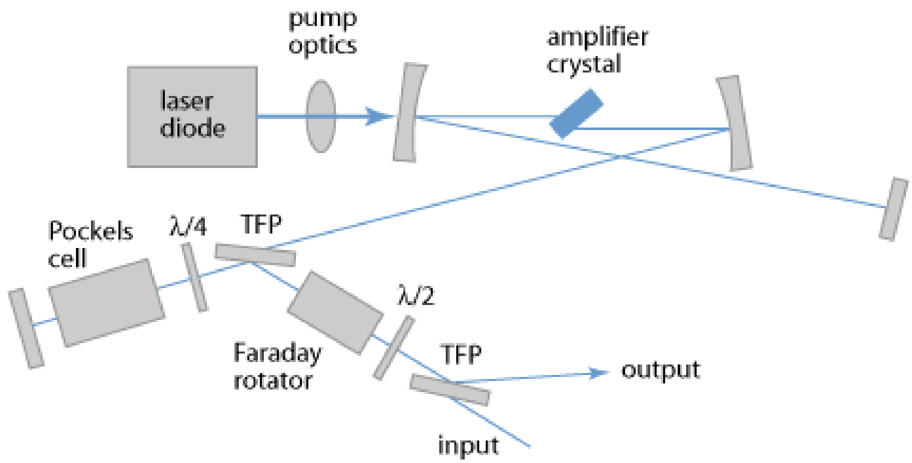
Approach the saturation intensity, efficiently depleting the gain medium



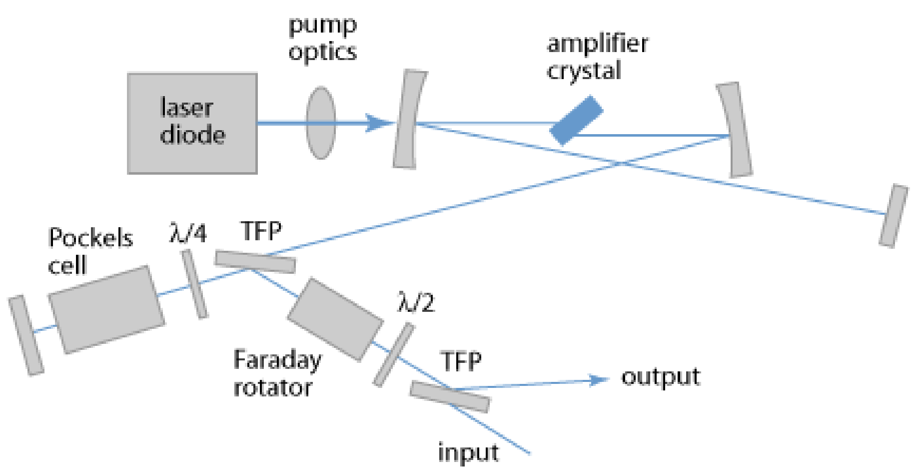
Good efficiency, good mode quality.
kHz, mJ pulses at <100 fs (> 10 GW)
The Q-switch of the pump and the cell have to be well synchronized (<ns).

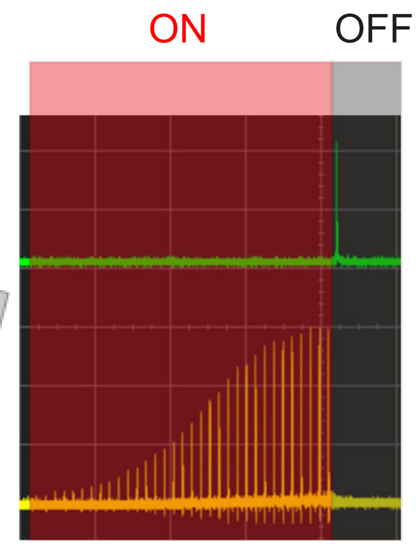
OFF
No polarization rotation

Light does 1 x roundtrip

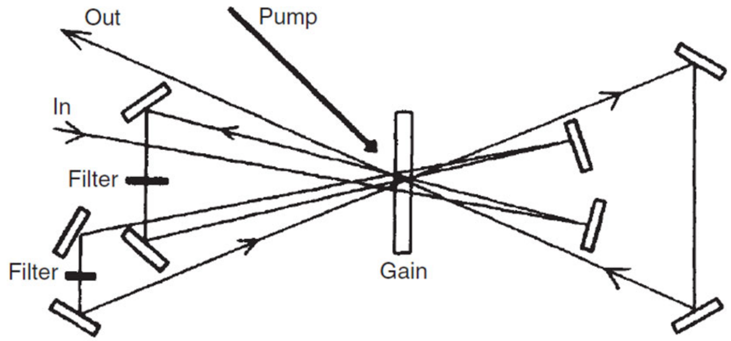


ON
Voltage $V=V_{\lambda/4}$
Equivalent to a $\lambda/4$ plate

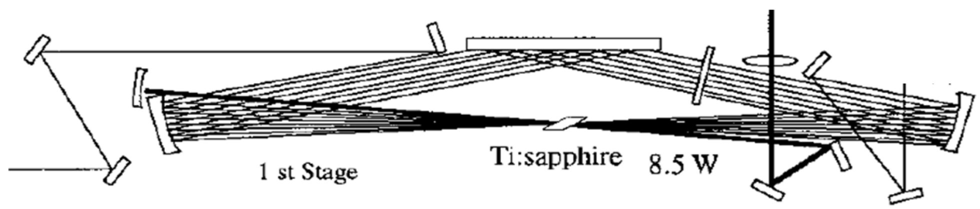




Multi-pass amplifiers

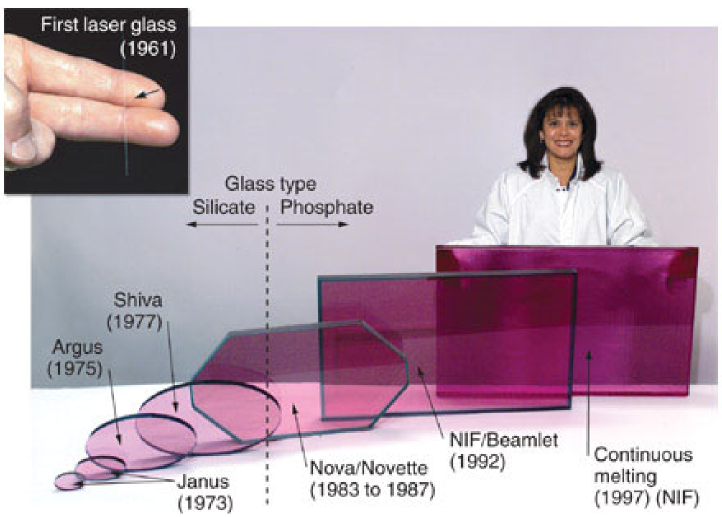


- Complex optical alignment
- Often used as “booster” stage of a reg. amplifier
- Flexibility between successive passes (change the mode size/apply filters)



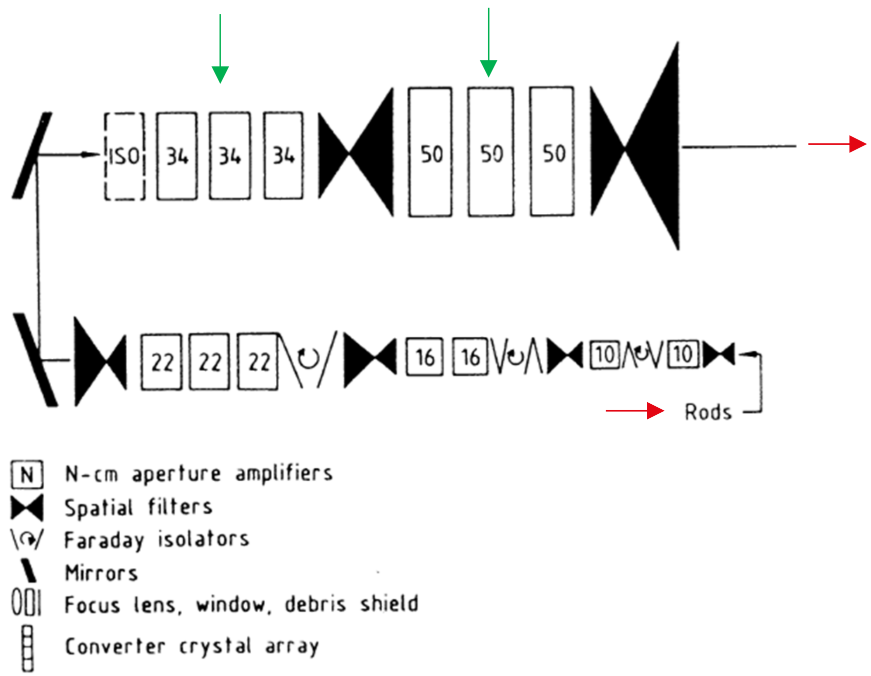
- Complex optical alignment

- In very high power system (>TW) regen. and multipass amplifiers are followed by amplification in low-gain, high aperture amplification stages



Nd-doped glasses

Flash-lamp pumped, very low rep. rate



Example: 1 amplification arm of the NOVA laser (LLNL)

10s kJ, ns pulses

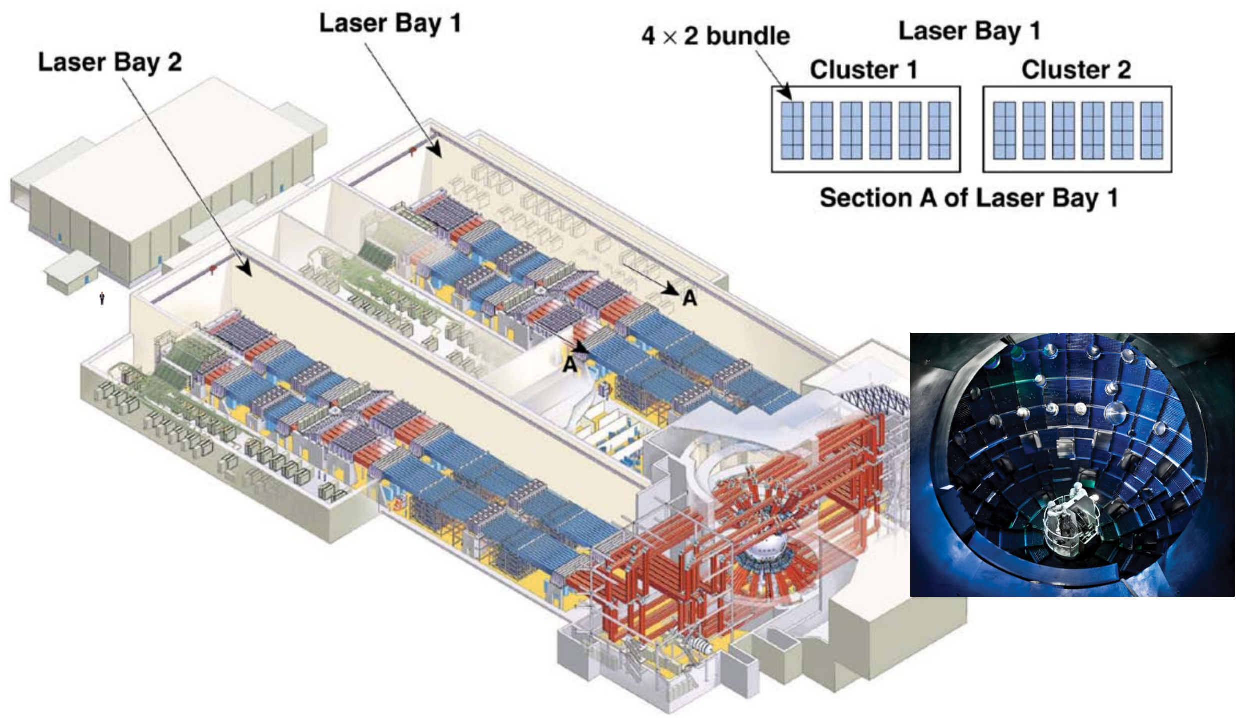
The national ignition facility

Lasers for fusion research /plasma physics /high energy physics

1 x oscillator
192 lasers Nd:glass amplifiers (20 kJ) in parallel

National Ignition Facility (USA)
Laser Megajoule (France)

Pulse duration **20 ns**, 4.2 MJ (1 µm wavelength); (1.8 MJ @ 0.34 µm)



Hard to further scale the pulse power!
A decrease of the pulse duration is used to reach the **Petawatt level (10^{15} W)**:

Required the introduction of CPA – Chirped pulse amplification
VULCAN laser UK 1 PW, ≈ 0.5 ps
LFEX Japan : 2 PW, >1 ps
ELI-NP (EU, planned) Extreme light infrastructure
2 x 10 PW (with <50 fs; Ti:Sapphire amplifiers)

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Part 2 Amplifiers of short pulses

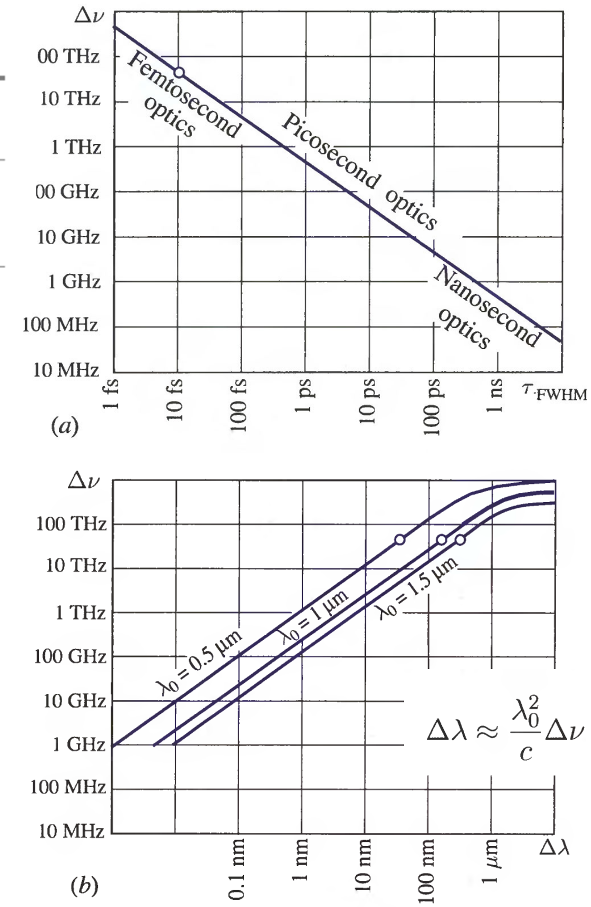
Time-bandwidth product

$$\Delta\nu \cdot \tau_p \geq k_{TBP}$$

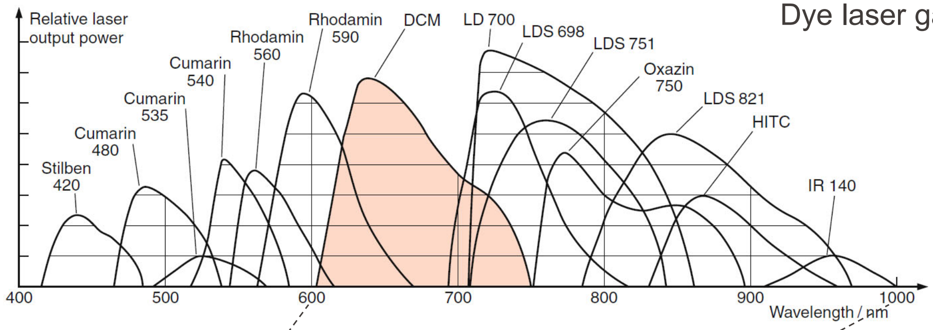
~ 1

	$I(t)\ (x \equiv t/\tau)$	τ_p/τ	$\Delta\nu_p \times \tau_p$
Gaussian	$I(t) = e^{-x^2}$	$2\sqrt{\ln 2}$	0.4413
Hyperbolic secant (soliton pulse)	$I(t) = \mathrm{sech}^2 x$	1.7627	0.3148

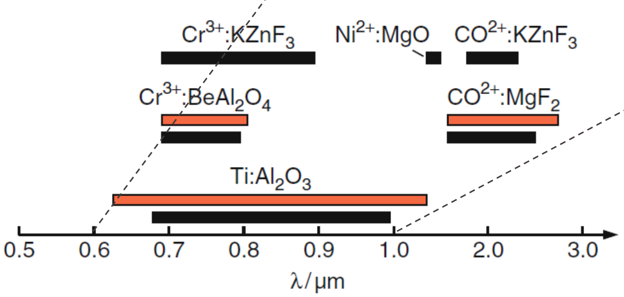
Freq. Bandwith vs time duration



Dye laser gain



Vibronic lasers gain



Below 1 ps, we speak of ultrashort pulses: the spectral bandwidth is necessarily high!
Ti:Sapphire solid state material with the largest known bandwidth (230 nm)

$\chi^{(3)}$ effects: the nonlinear index of refraction

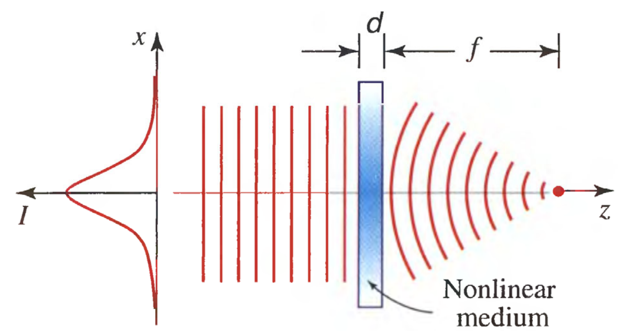
$$\tilde{P}^{(3)}(t) = \chi^{(3)}\tilde{E}(t)^3$$

$$n(\omega, I) = n_0(\omega) + n_2(\omega)I.$$

Note: this is one of the many terms of four-wave mixing, where three photon at frequency ω mix to generate the same frequency (ω+ω-ω->ω)

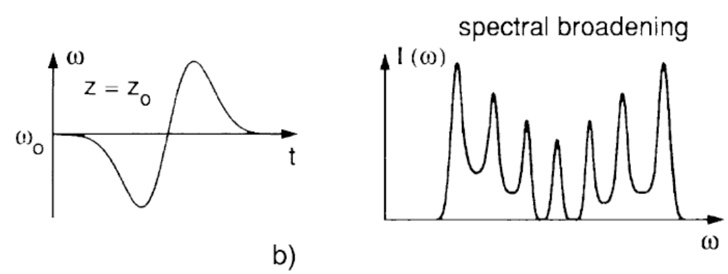
In space: Self-focusing

The radial variation of intensity translate into a radially varying optical path: this acts as a lens!



In time: Self-phase modulation and spectral broadening

$$\omega = \frac{\mathrm{d}\phi}{\mathrm{d}t} = \omega_0 - A\frac{\mathrm{d}I}{\mathrm{d}t}$$



B integral

- The peak intensity during amplification increases as the pulse energy grows
- How to quantify the impact of n_2 ?

$$B = \frac{2\pi}{\lambda} \int_0^L \underbrace{n_2}_{\text{NONLINEAR INDEX OF REFRACTION}} I(z)\, dz$$

Remember, $\chi^{(3)}$ in time = self-phase modulation.

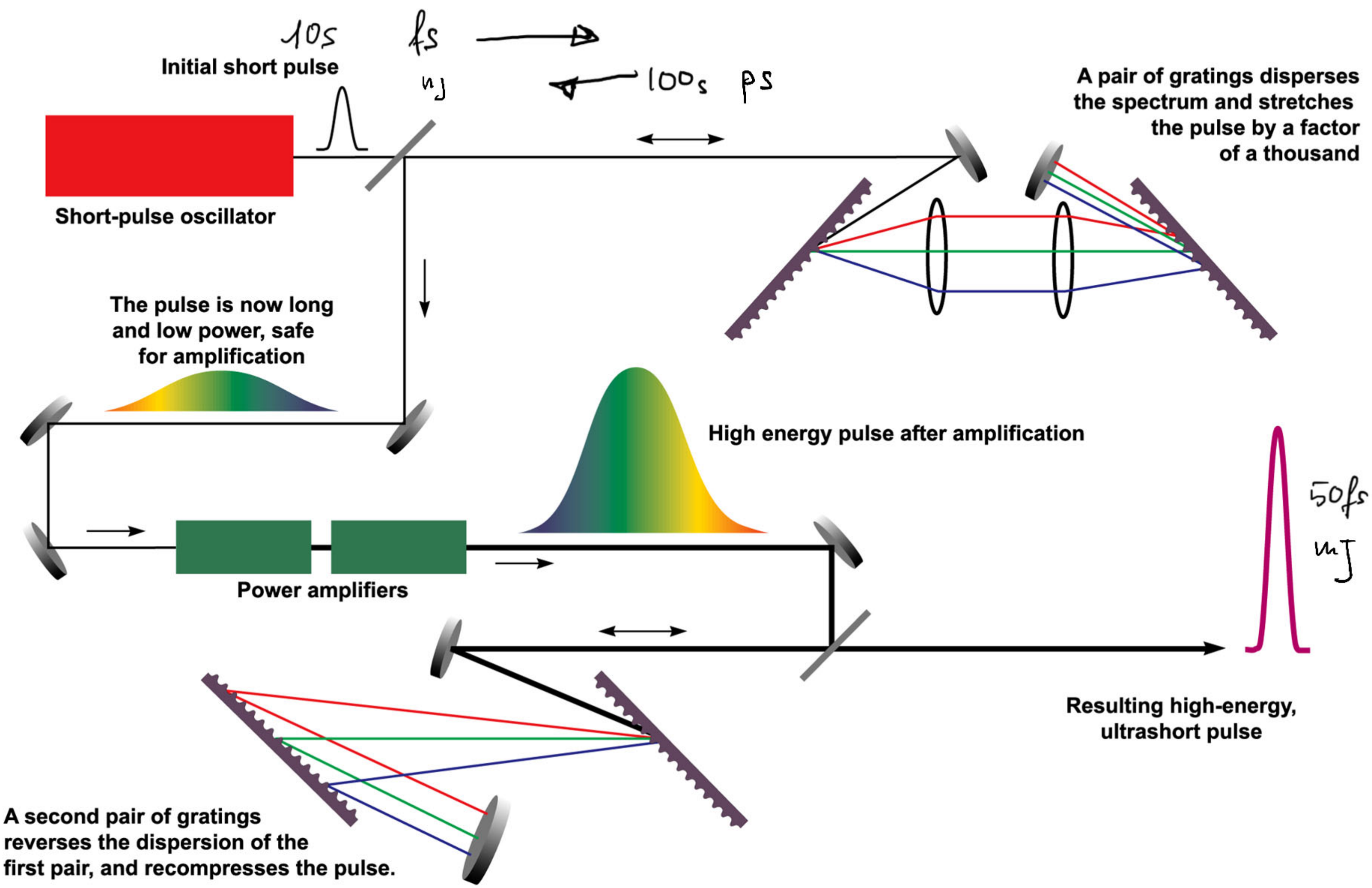
B-integral is a measure of the overall nonlinear phase accumulated. If B is too high, it is almost impossible to recompress the pulse.

Self-focusing might lead to damage of the optics

1. To further improve the laser intensities, the best route is to shorted the pulse duration
2. Self-effects become a fundament limit when amplifying short (picosecond and femtosecond) pulses

CHIRPED PULSE AMPLIFICATION

Chirped pulse amplification

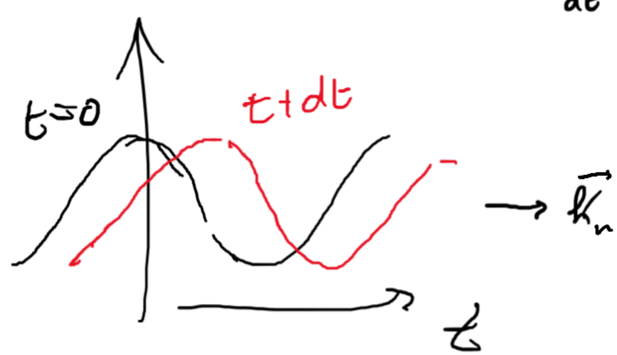


Pulse propagation some concepts:

$$E(z,t) = A\exp(i\phi(z,t)) = A\exp i(\omega t - k_n z) \qquad k_n \to k_n(\omega)$$

Phase front $d\phi = 0 \rightarrow \omega dt - k_n dz = 0$

$$v_{ph} = \frac{dz}{dt} = \frac{\omega}{k_n}$$



Phase velocity of plane wave in a material

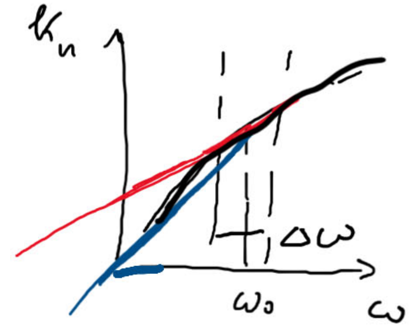
Wavepacket:

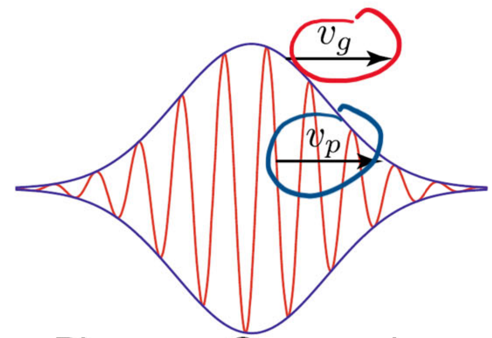
$$E(t,z) = \int_{\Delta\omega} d\omega\, A_\omega\, e^{i(\omega t - k_n(\omega) t)} \longrightarrow E(t,0) = A(t)\, e^{i\omega t}$$

For frequency narrow pulses (e.g. ps) -> small $\Delta\omega$ around ω_0

$$k_n(\omega-\omega_0) = k_n(\omega_0) + k'_n(\omega_0)\,\Delta\omega$$

LINEAR APPROX





Phase vs Group velocity

DIFFERENT

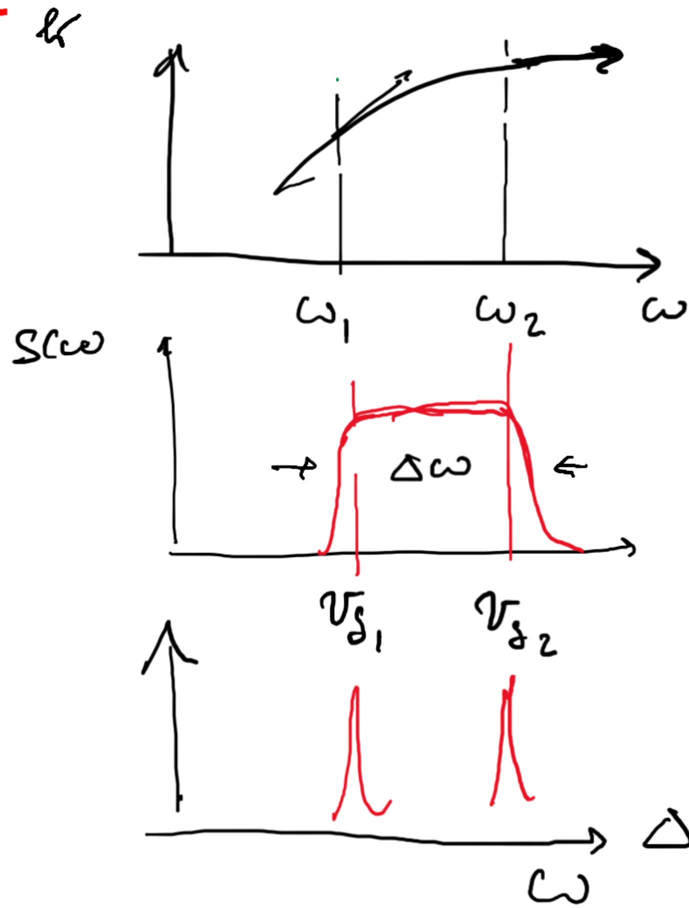
$$E(t,z) \sim A\left(t - z/v_g\right) \exp\left(i\omega t - k_n(\omega_0) z\right)$$

SAME SHAPE, TIME OFFSET

$$1/v_g = \frac{\partial k_n}{\partial \omega} \qquad v_g = \frac{\partial \omega}{\partial k_n}$$

SPECTRAL PHASE $\phi(\omega-\omega_0) = k_n(\omega-\omega_0)\, z$ $\quad \phi'$ "GROUP DELAY"

Broadband frequency pulse propagation:



BANDWIDTH IS LARGE:
AS THE PULSE PROPAGATES
DIFFERENT PORTIONS OF THE SPECTRUM
HAVE DIFFEREN GROUP VELOCITIES.

→ ESTIMATE OF τ AFTER PROP. ?

$$\Delta\tau \sim \ell\,\Delta v_g^{-1} =$$

$$\phi'(\omega_2) - \phi'(\omega_1) \simeq \underbrace{\phi''(\omega_0)}_{\text{GDD}}(\omega_2-\omega_1)$$

GROUP DELAY DISPERSION

$$\Delta\tau \sim \phi''(\omega_0)\,\Delta\omega$$

$$\ell \left| \frac{d^2k_n}{d\omega^2} \right|_{\omega=\omega_0} \Delta\omega \rightarrow$$

GROUP VELOCITY DISPERSION $\left.\frac{d}{d\omega}\left(1/v_g\right)\right|_{\omega=\omega_0}$

General idea: - expand the phase of the pulse around the central frequency
- calculate the temporal shape by inverse Fourier transformation

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Femtosecond pulse propagation : Temporal intensity envelope

REAL E-FIELD

TIME DEP. PHASE

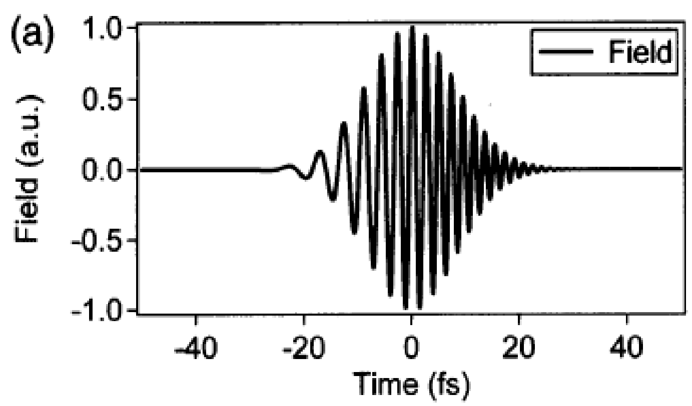
$$\boxed{\mathcal{E}(t) = \tfrac{1}{2}\sqrt{I(t)}\,\exp\{i\,[\omega t - \phi(t)]\} + c.c.}$$

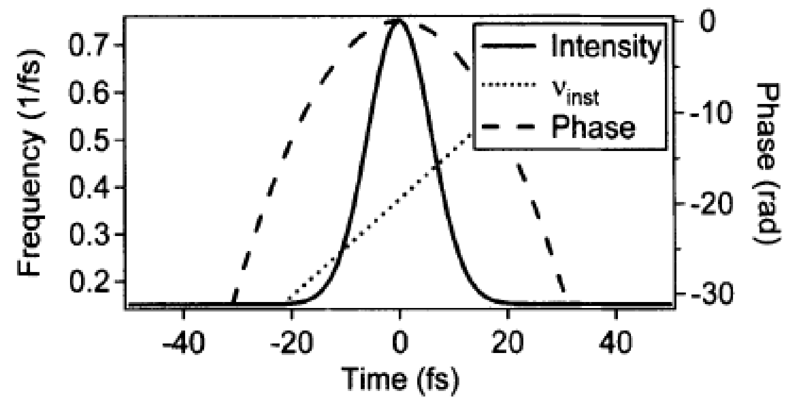
FAST OSCILLATING CARRIER

COMPLEX AMPLITUDE (NO CARRIER)

$$\boxed{E(t) \equiv \sqrt{I(t)}\exp[-\mathrm{i}\phi(t)]}$$

Pulse duration:
FWHM I(t)





$$\omega_{\text{inst}}(t) \equiv \omega_0 - \mathrm{d}\phi/\mathrm{d}t$$

$\varphi(t)$ is the TEMPORAL PHASE: information of frequency vs time

TIME vs FREQUENCY DOMAIN

$$\boxed{\mathcal{E}(t) = \frac{1}{2}\sqrt{I(t)}\ \exp\{i\,[\omega t - \phi(t)]\} + c.c.}$$

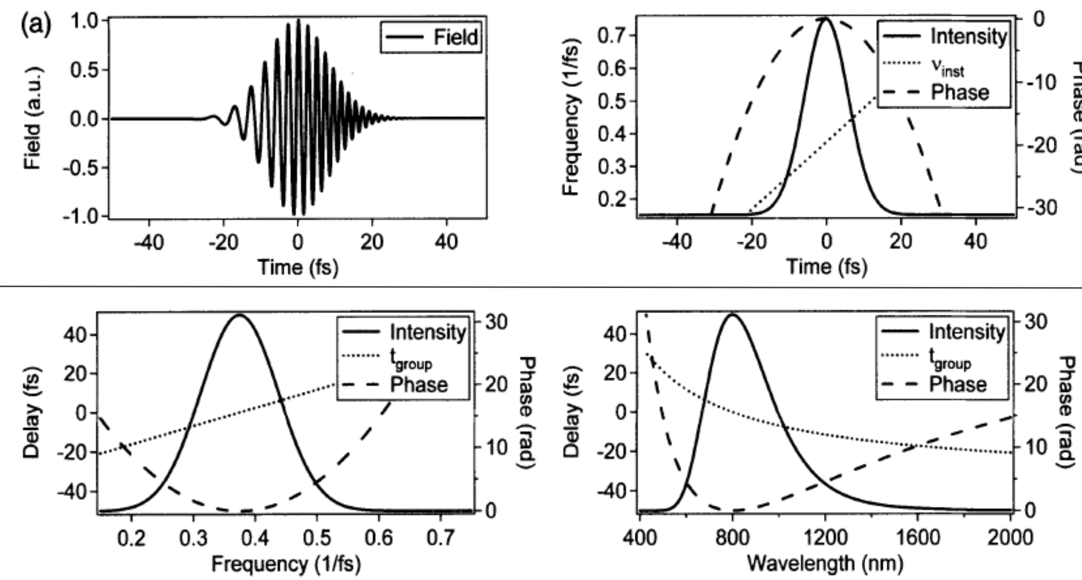
$$\tilde{\mathcal{E}}(\omega) = \int_{-\infty}^{\infty} \mathcal{E}(t)\exp(-\mathrm{i}\omega t)\,\mathrm{d}t$$

FOURIER TRANSFORM

Spectral description:

$$\boxed{\tilde{\mathcal{E}}(\omega) = \sqrt{S(\omega)}\exp[-\mathrm{i}\varphi(\omega)]}$$

COMPLEX AMPLITUDE



$\varphi(\omega)$ is the SPECTRAL PHASE:
information of time vs frequency

Analytical example : Gaussian pulse with complex parameter

$$E(t) = \exp\left[-\Gamma t^2\right] \exp\left[i \omega_0 t\right]$$

$$\Gamma = \Gamma_1 - i \Gamma_2$$

LIN. CHANGE OF FREQ. WITH TIME

$$\phi(t) = \omega_0 t + \Gamma_2 t^2 \Rightarrow \dot{\phi}(t) = \omega_0 + \Gamma_2 t = \omega(t)$$

$$I(t) = |E|^2 \qquad \tau_p = \sqrt{\frac{2 \ln(2)}{\Gamma_1}} \longleftrightarrow g(t) = \exp\left[-4 \ln 2 \left(\frac{t}{\tilde{\tau}_p}\right)^2\right]$$

(FWHM)

$$\tilde{E}(\omega) = \mathcal{F}[E(t)] \propto \exp\left[-\frac{(\omega-\omega_0)^2}{4\Gamma}\right]$$

$\Gamma_2 \rightarrow 0$ Fourier limit

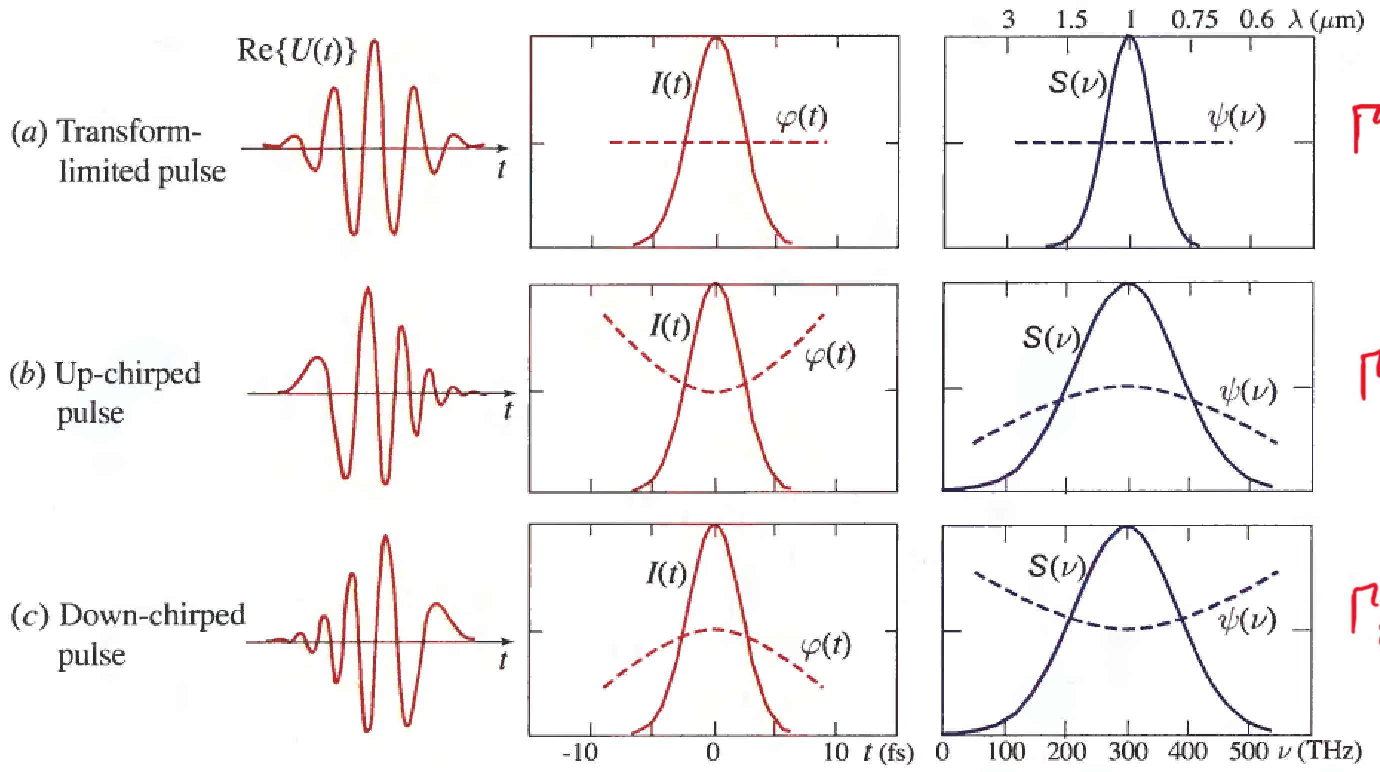
$\Gamma_2 \neq 0$ chirped pulse

$$\Delta \nu_p = \frac{\Delta \omega}{2\pi} = \frac{\sqrt{2 \ln 2}}{\pi} \sqrt{\Gamma_1 \left(1 + \left(\frac{\Gamma_2}{\Gamma_1}\right)^2\right)}$$

SPECTRAL BROAD.

$$\boxed{\tau_p \Delta \nu_p = \frac{2 \ln 2}{\pi} \sqrt{1 + \left(\frac{\Gamma_2}{\Gamma_1}\right)^2}}$$

Chirped pulses



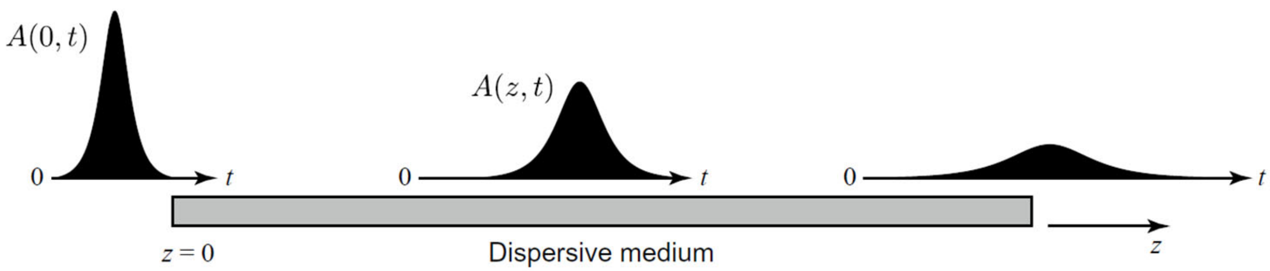
$\Gamma_2 = 0$

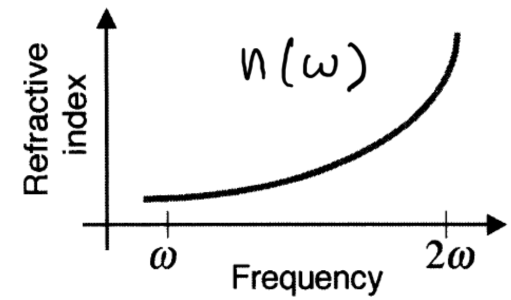
$\Gamma_2 > 0$

$\Gamma_2 < 0$

- A non-flat phase is the result of a non-transform limited pulse: by compensating the phase a shorted pulse could be produced.
- By controlling the phase one can stretch/compress the pulse.

Ultrashort pulse propagation in a **linear** dispersive medium





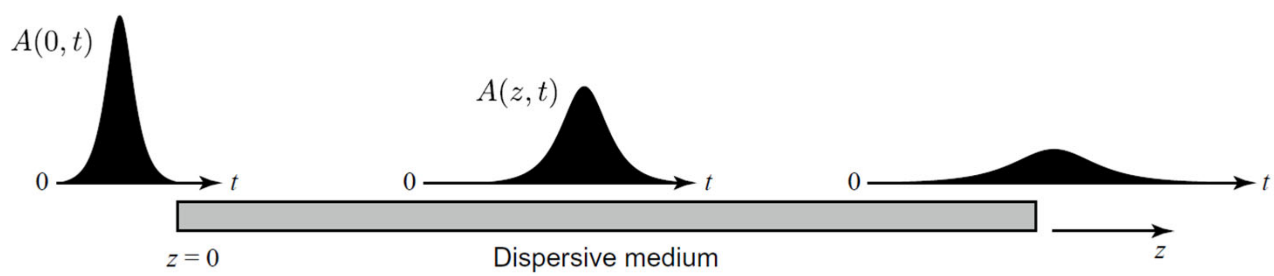
- **linear** medium: no nonlinear effects (e.g. gain or saturable absorption)
- Spectral intensity envelope is unchanged after propagation
- SVEA : Slowly Varying Envelope Approximation

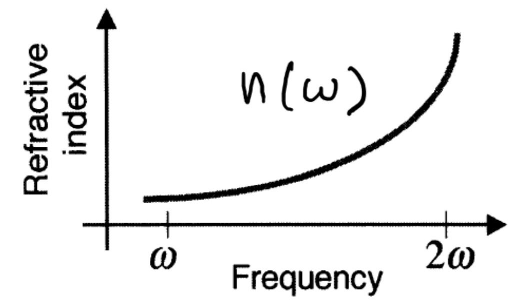
$\rightarrow$ SIMPLIFIED WAVE EQUATION.

$$\left|\frac{\partial^2 \tilde{A}}{\partial t^2}\right| \ll \left|k_n(\omega_0)\frac{\partial \tilde{A}}{\partial z}\right|$$

ENVELOPE DOES NOT CHANGE MUCH OVER A DISTANCE $dz \sim \lambda_n$

Ultrashort pulse propagation in a **linear** dispersive medium





WAVE EQ. IN THE FREQ. DOMAIN →

$$\tilde{A}(z,\omega) = \tilde{A}(0,\omega)\exp[i\varphi(\omega)]$$

$$\varphi(\omega) = k(\omega) z = \frac{\omega n(\omega)}{c} z$$

TAYLOR EXPANSION OF THE SPECTRAL PHASE

$$\boxed{\varphi(\omega) = \varphi_0 + (\omega-\omega_0)\,\varphi_1 + (\omega-\omega_0)^2\,\varphi_2/2 + \cdots}$$

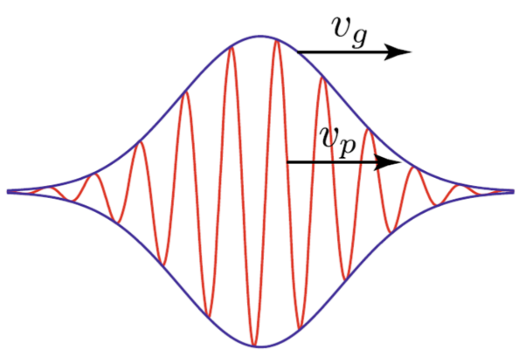
$\frac{\partial \varphi}{\partial \omega}$ GD $\quad \frac{\partial^2 \varphi}{\partial \omega^2}$ GDD $\quad$ -- $\quad \frac{\partial^3 \varphi}{\partial \omega^3}$ TOD

Ultrashort gaussian pulse propagation in a linear dispersive medium

FOR A GAUSSIAN PULSE → ANALITICAL SOLUTIONS (up to 2nd order dispersion)

$$\boxed{\frac{1}{\Gamma(L_\mathrm{d})} = \frac{1}{\Gamma(0)} + 2\mathrm{i}k_n'' L_\mathrm{d}}$$

$$E(L_\mathrm{d}, t) \propto \exp\left[\mathrm{i}\omega_0\left(t - \frac{L_\mathrm{d}}{v_\mathrm{p}(\omega_0)}\right)\right]\exp\left[-\Gamma(L_\mathrm{d})\left(t - \frac{L_\mathrm{d}}{v_\mathrm{g}(\omega_0)}\right)^2\right]$$



Dispersion parameter	Definition
Phase velocity v_p	ω/k_n
Group velocity v_g	$\frac{\mathrm{d}\omega}{\mathrm{d}k_n} = \left(\frac{\mathrm{d}k_n}{\mathrm{d}\omega}\right)^{-1}$
Group delay T_g (first order dispersion)	$\frac{\mathrm{d}\phi}{\mathrm{d}\omega} = L_\mathrm{d}\frac{\mathrm{d}k_n}{\mathrm{d}\omega} = \frac{L_\mathrm{d}}{v_\mathrm{g}}$
Group delay dispersion $\mathrm{d}T_\mathrm{g}/\mathrm{d}\omega$ (second order dispersion)	$\frac{\mathrm{d}^2\phi}{\mathrm{d}\omega^2}$
$\frac{\mathrm{d}T_\mathrm{g}}{\mathrm{d}\lambda}$	$\mathrm{d}\lambda = -\frac{\lambda^2}{2\pi c}\mathrm{d}\omega$
Third order dispersion	$\frac{\mathrm{d}^3\phi}{\mathrm{d}\omega^3}$

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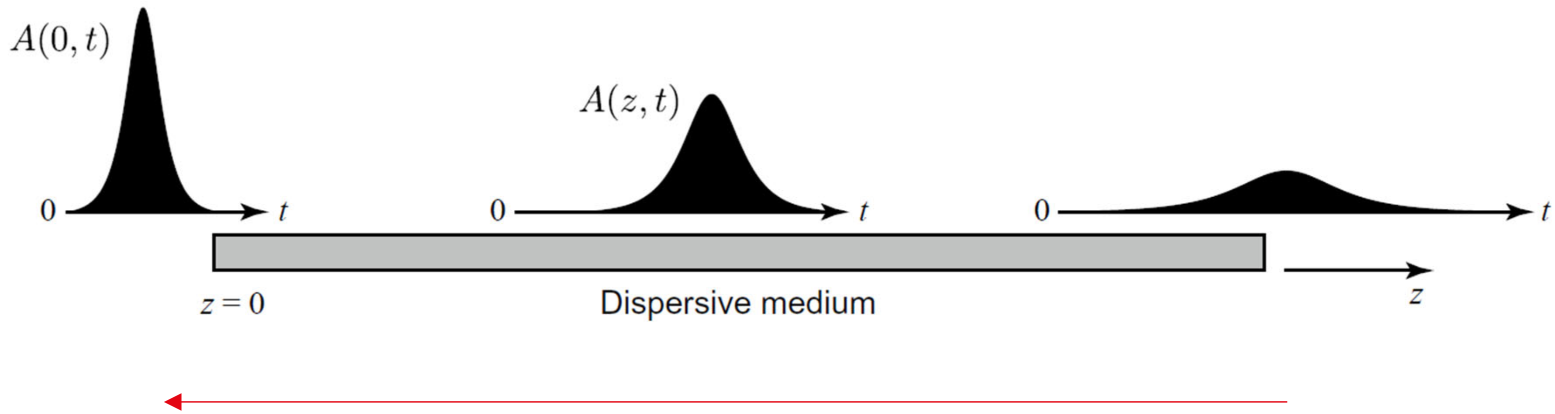
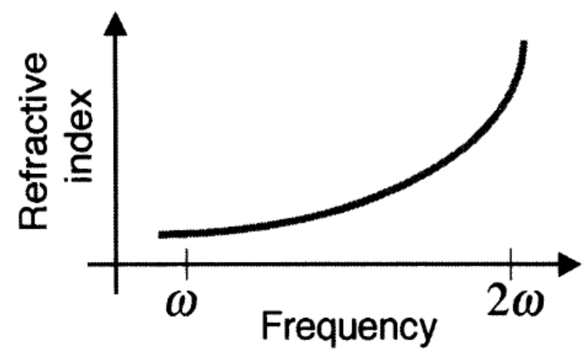
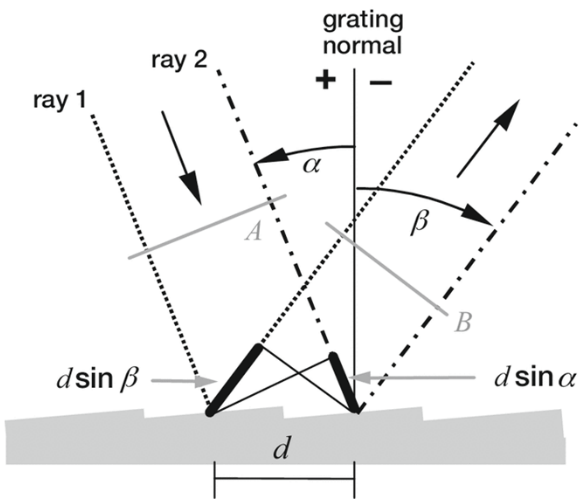


- Most material have “normal” dispersion at VIS-NIR wavelengths
- To compensate linear propagation temporal broadening, we need optical devices with opposite (“anomalous”) dispersion.
- Multiple materials in an amplifier, multiple passes.

Calculate the cumulative GDD (and TOD, and higher orders) -> Compensate with a suitable compressor (e.g. prism compressor, grating compressor..)



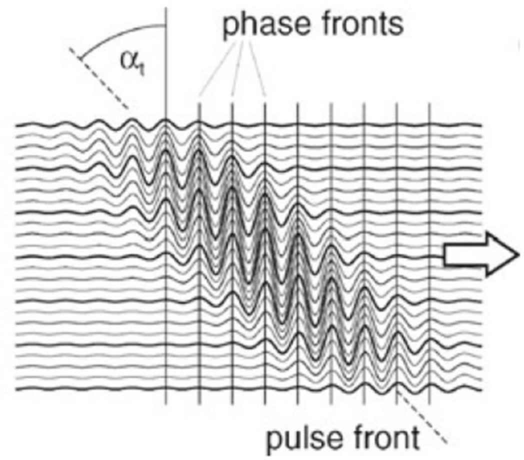
DIFFRACTION GRATING



$$n\lambda = d(\sin\beta - \sin\alpha)$$

$$\Downarrow$$

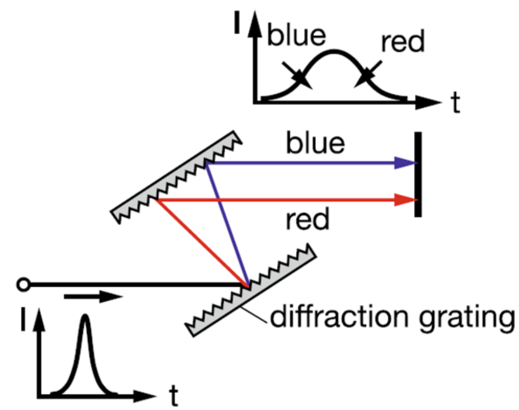
$$\beta = \sin^{-1}\left[\frac{n\lambda}{d} + \sin\alpha\right]$$



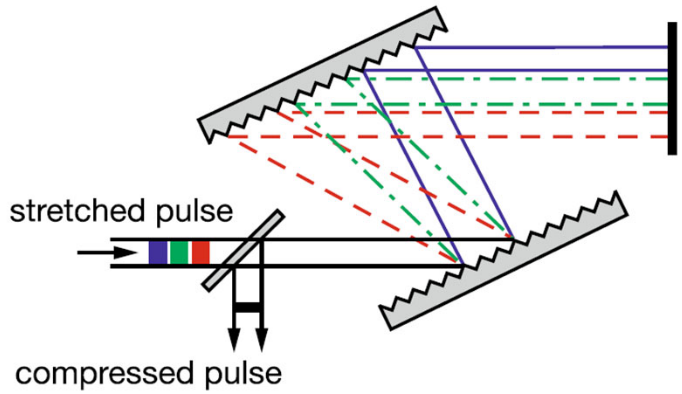
Periodic array of scatterers with spacing d = 1/Λ

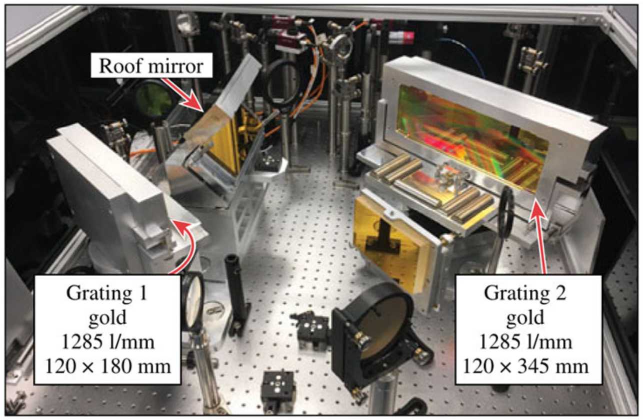
Angular dispersion: wavelength-dependent propagation angle.

Pulse front tilt/spatial chirp: multiple grating configuration

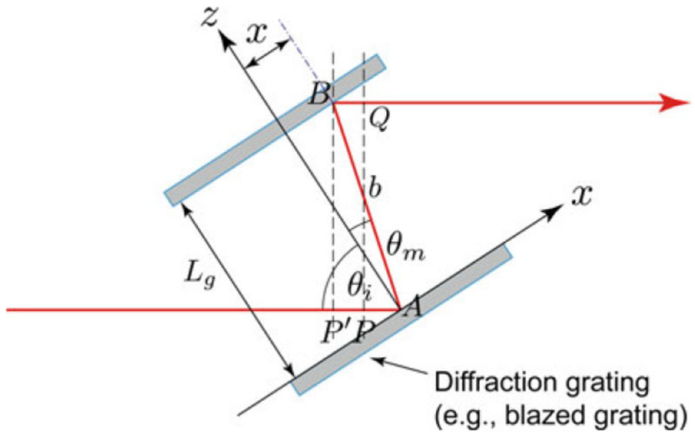


TREACY COMPRESSOR





- 4 gratings are necessary for avoiding pulse front tilt and spatial chirp
- High efficiency is fundamental : transmission η^4
- Blazed gratings, near Littrow condition -> diffraction efficiency in the first order can approach 90%



2x contributions

$$\phi = \frac{\omega}{c} L + \phi_g(x)$$

$$x = L_g \tan\theta_m$$

1. Optical path

$$\frac{L_g}{\cos\theta_m}\left[1 + \cos(\theta_m + \theta_i)\right]$$

2. Spatial dependent phase shift due to diffraction

$$\phi_g(x) = \pi - m\frac{2\pi}{\Lambda}x$$

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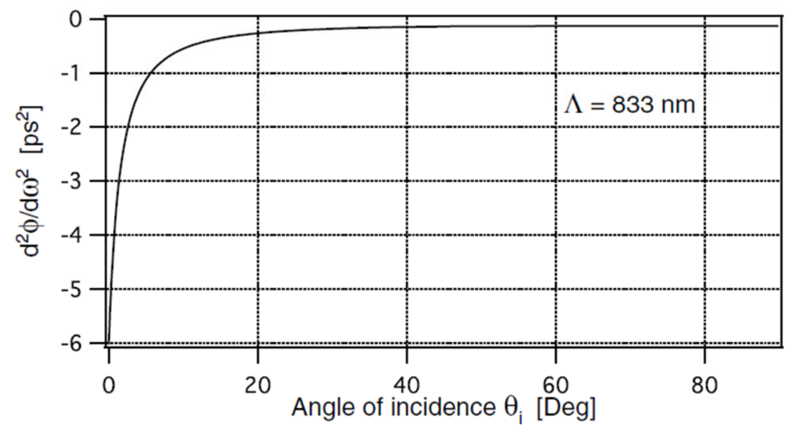
$$\frac{d^2\phi}{d\omega^2} = -\frac{\lambda^3 L_g}{\pi c^2 \Lambda^2}\left[1-\left(\frac{\lambda}{\Lambda}-\sin\theta_i\right)^2\right]^{-3/2}$$

Sapphire	$n\,(800\text{ nm}) = 1.76019$	
	$\left.\frac{\partial n}{\partial \lambda}\right\|_{800\text{ nm}} = -0.0268\frac{1}{\mu\text{m}}$	$\left.\frac{\partial k_n}{\partial \omega}\right\|_{800\text{ nm}} = 5.87\times10^{-9}\frac{\text{s}}{\text{m}}$
	$\left.\frac{\partial^2 n}{\partial \lambda^2}\right\|_{800\text{ nm}} = 0.064\frac{1}{\mu\text{m}^2}$	$\left.\frac{\partial^2 k_n}{\partial \omega^2}\right\|_{800\text{ nm}} = 5.80\times10^{-26}\frac{\text{s}^2}{\text{m}} = 58\frac{\text{fs}^2}{\text{mm}}$
	$\left.\frac{\partial^3 n}{\partial \lambda^3}\right\|_{800\text{ nm}} = -0.377\frac{1}{\mu\text{m}^3}$	$\left.\frac{\partial^3 k_n}{\partial \omega^3}\right\|_{800\text{ nm}} = 4.21\times10^{-41}\frac{\text{s}^3}{\text{m}} = 42.1\frac{\text{fs}^3}{\text{mm}}$



Example: Reg. Ti:Sapphire, 5 mm Crystal, 10 passes -> 58 x 5 x10 = 2.9 x 10^3 fs^2

Grating compressor, 1200 lines/mm, d= 5 cm, 20° incidence -> -0.2 ps^2 = -0.2 x 10^6 fs^2

- Gratings allow for a very large amount of GVD compared to propagation in materials
- In a compact setup from 10 fs -> 500 ps (would require 50 m of fused silica)

Grating compressor -> Negative dispersion: pulse compression during amplification!

$$\boxed{\frac{d^2\phi}{d\omega^2} = -\frac{\lambda^3 L_g}{\pi c^2 \Lambda^2}\left[1-\left(\frac{\lambda}{\Lambda}-\sin\theta_i\right)^2\right]^{-3/2}}$$

$$\frac{d^2\phi}{d\omega^2} = -\frac{m^2\lambda^3 M^2 L_{eff}}{2\pi c^2 \Lambda^2}\left[1-\left(-m\frac{\lambda}{\Lambda}-\sin\theta_i\right)^2\right]^{-3/2}$$

$L < f$ positive GDD

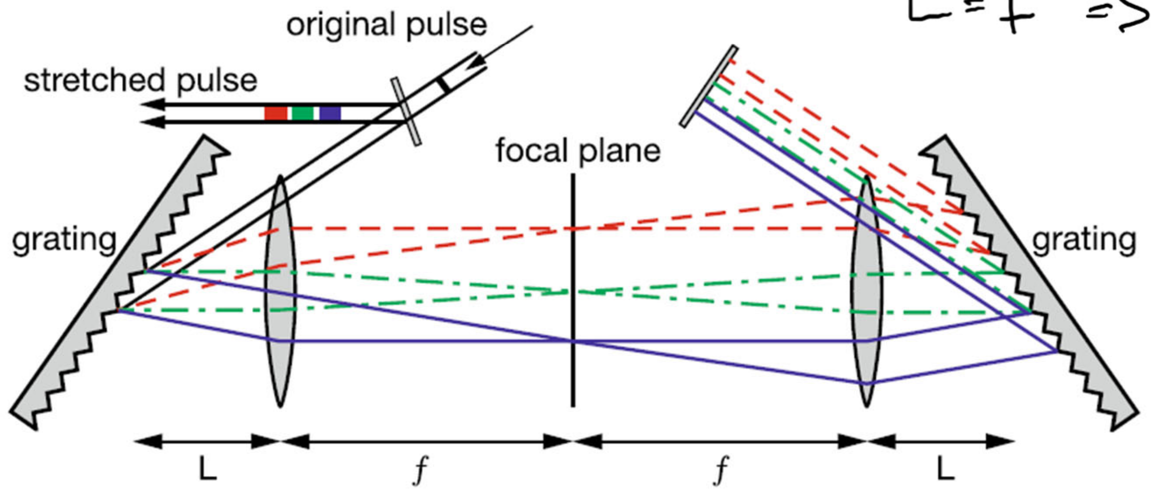
$L > f$ negative GDD

$L = f \Rightarrow$ 4f system

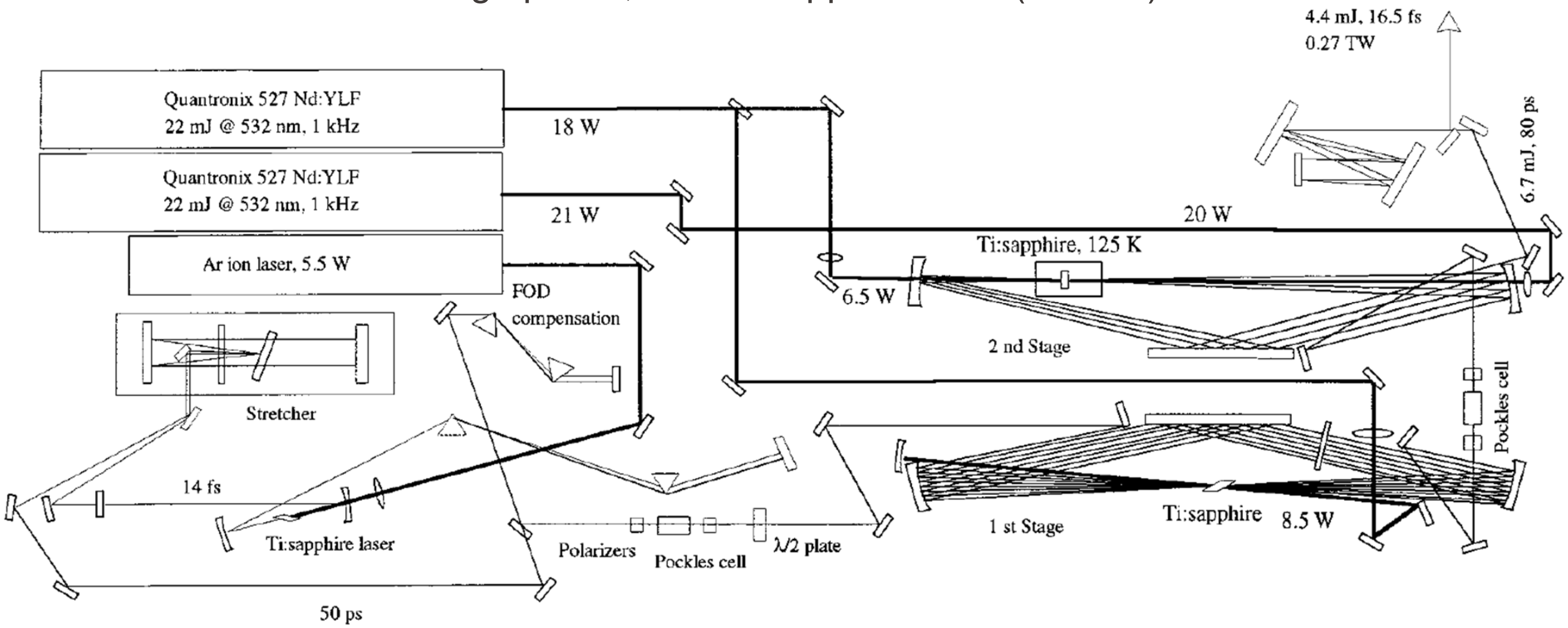
GDD = 0

CAN USE TO "SHAPE" THE PHASE

- Oeffner stretcher



A high power, kHz Ti:Sapphire laser (0.3 TW)



Note: to produce short pulses (<50 fs) the higher order dispersion parameters have to be carefully compensated

Laboratory scale Ti:Sapphire amplifier : 1 kHz, 10-20 W, 30-50 fs (Cryogenically-cooled)

DIFFERENT TECHNOLOGIES ARE NEEDED TO INCREASE THESE PARAMETERS

GAIN NARROWING IN TI:SAPPHIRE

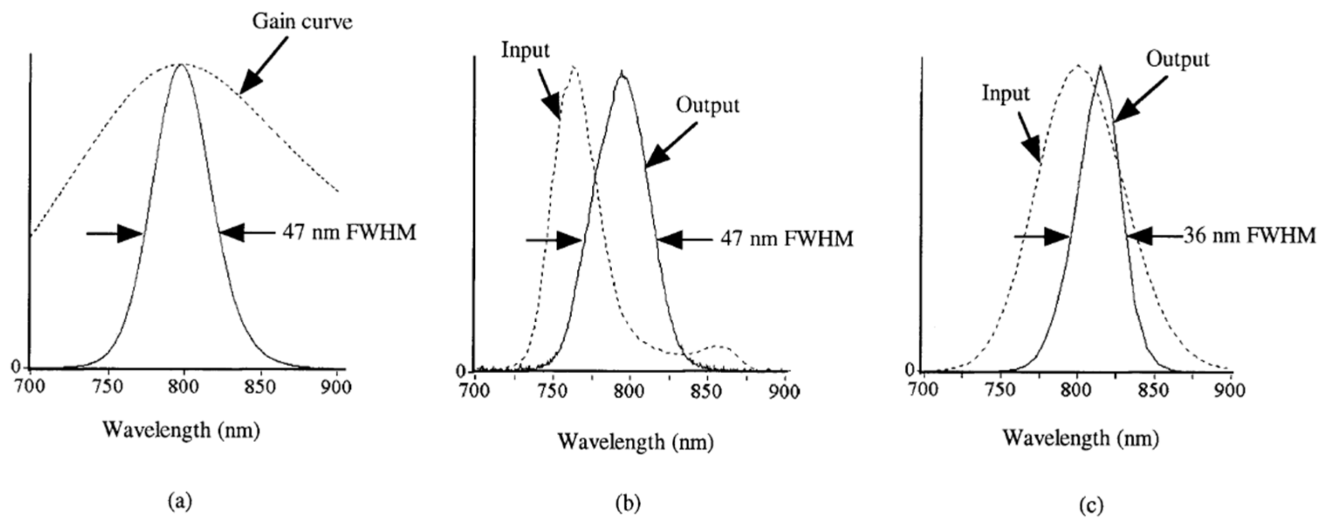


FIG. 5. Gain narrowing for the case of (a) an infinitely broad and flat input spectrum, (b) an optimally offset and shaped input spectrum, and (c) a nonoptimum input spectrum.

GAIN NARROWING IN TI:SAPPHIRE LIMITS THE ACHIEVABLE AMPLIFIED PULSE DURATION