Control and operations of tokamaks
Exercise Session 5 - Free boundary evolution
Solutions

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February 2025

1 Poloidal flux in a tokamak

a,b) Draw a poloidal cross section of a generic tokamak. Sketch the magnetic field generated by the central solenoid.

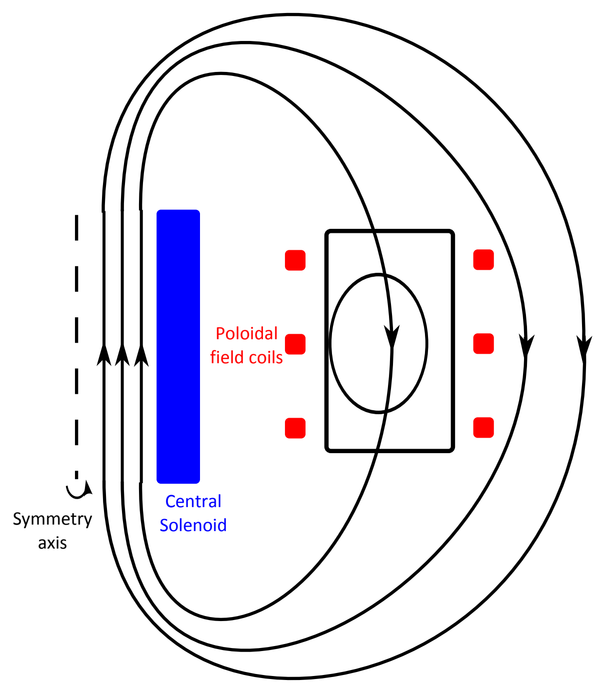

Figure 1: Poloidal cross section of a generic tokamak. Note that the height of the plasma is 1.5 times the width due to $\kappa = 1.5$. The magnetic field is similar to the field produced by a magnetic dipole. Inside the central solenoid the magnetic field is constant. A limited plasma is depicted which touches the wall or limiter.

c) Sketch the magnetic field generated by the solenoid assuming the tokamak has an iron core.

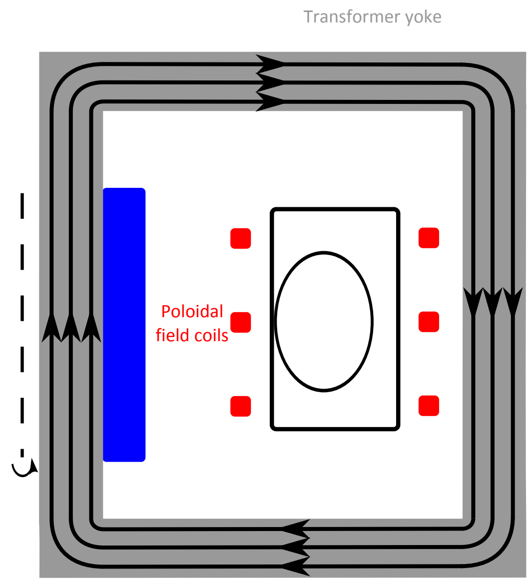

Figure 2: Poloidal cross section of a generic tokamak with a iron transformer yoke. The magnetic field lines are contained within the transformer yoke.

d) Sketch the flux surface distribution inside the vacuum vessel.

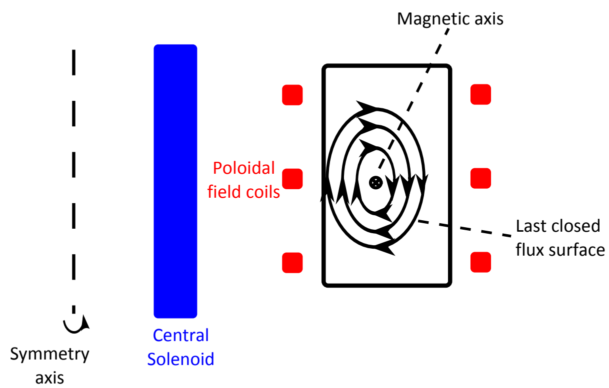

Figure 3: Poloidal cross section of a generic tokamak with flux surfaces. Several flux surfaces are indicated with arrows indicating the direction of the magnetic field lines (projected in the poloidal plane) due to the current in the plasma. The current is directed inside the paper.

e) Sketch the value of the poloidal flux as a function of R. First consider a case without a current in the plasma.

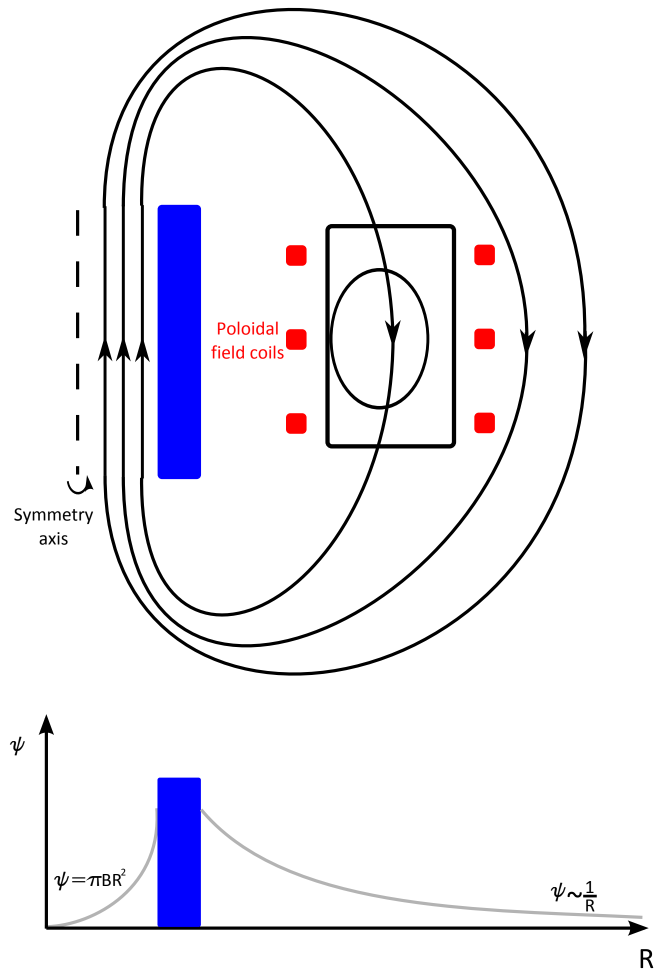

Figure 4: Poloidal cross section of a generic tokamak with the poloidal flux as a function of the major radius R. The magnetic field for small R is constant and, therefore, the poloidal flux increases with R^2. For larger R, the flux decays as $\frac{1}{R}$ and the magnetic field decays as $\frac{1}{R^3}$ (dipole field).

f) Sketch the value of the poloidal flux and take into account a positive plasma current.

g) For the given elongation $\kappa = 1.5$, separately sketch the vertical and quadrupole components of the vacuum field needed to keep the plasma in equilibrium. What does the combination of both look like?

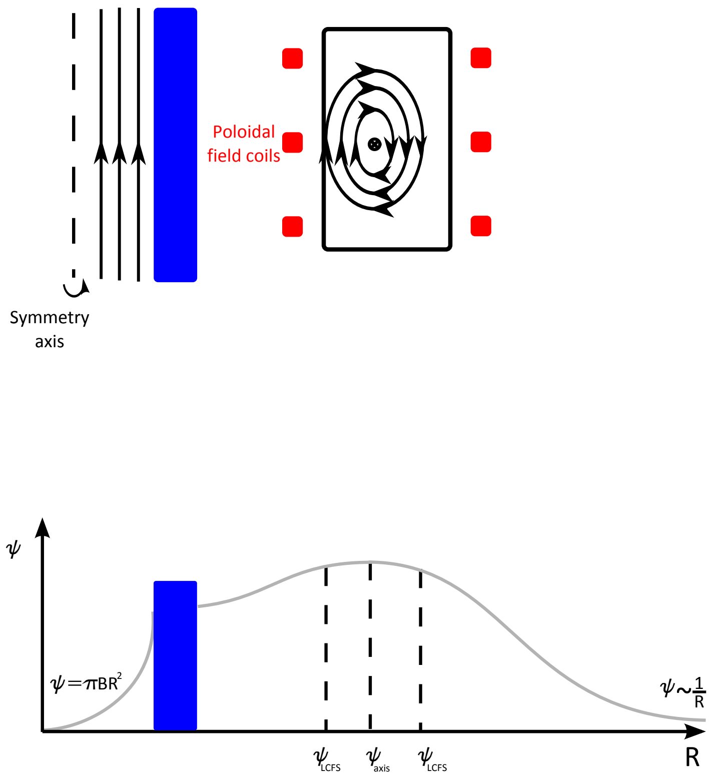

Figure 5: Poloidal cross section of a generic tokamak with the poloidal flux as a function of the major radius R. Due to the magnetic field in positive direction generated by the plasma current, the poloidal flux increases between R and R_0 at the magnetic axis. For R larger than R_0, the magnetic field is in negative direction and the poloidal flux decreases. At very large R the flux also decays as $\frac{1}{R}$ and the magnetic field decays as $\frac{1}{R^3}$ (dipole field).

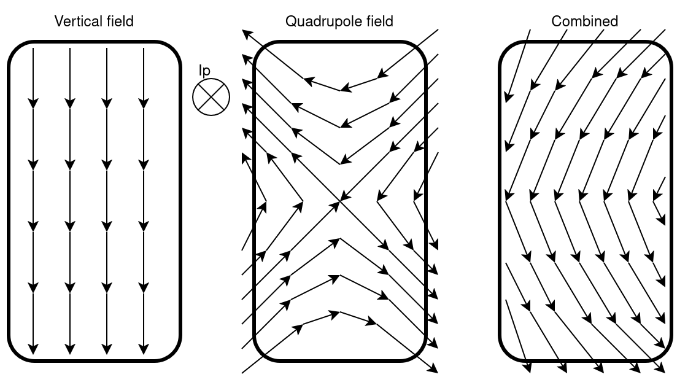

Figure 6: Vacuum fields with Ip going into the page. The vertical field needs to apply an inward force onto the plasma to keep it from going to the LFS. The radial B-field component of the quadrupole field is such that the top and bottom of the plasma are pulled up and down respectively, elongating the plasma. The combination of the two looks like the field presented in the lecture.

2 The q profile

a) What value does q have at the last closed flux surface for a plasma with an X-point?

The last closed flux surface contains the X-point and at this point $B_p = 0$. Therefore, if the safety factor is considered in the midplane at radius R, the limit for the last close flux surface is unbounded.

$$q(R_{LCFS}) = \lim_{R \to R_{LCFS}} q(R) \to \infty$$

b) Derive the expression for the 'engineering q' q^*. This is the value of q at the last closed flux surface assuming a large aspect ratio tokamak for which $R \approx R_0$ and assuming the toroidal magnetic field to be external and constant such that $B_\phi = B_0$. Furthermore it is assumed that the poloidal field is constant on the flux surface and determined by the enclosed plasma current.

Assume a large aspect ratio for which $R \approx R_0$ and that the toroidal magnetic field is external and constant to $B_\phi = B_0$, such that $T(\psi) = R_0 B_0$. Furthermore assume that the poloidal field is constant on the flux surface and determined by the enclosed plasma current

$$|B_p| = \frac{\mu_0 I_p}{c}.$$

For the circumference c, note that $b = \kappa a$ and therefore

$$c = 2\pi \sqrt{\frac{a^2 + b^2}{2}} = 2\pi \sqrt{\frac{a^2(1+\kappa^2)}{2}} = 2\pi a \sqrt{\frac{1+\kappa^2}{2}}.$$

Combining all the elements gives

$$q^* = \frac{T}{2\pi} \oint \frac{1}{R^2} \frac{d\ell}{|B_p|} = \frac{R_0 B_0}{2\pi} \frac{1}{R_0^2} \frac{1}{|B_p|} \oint d\ell = \frac{2\pi a^2 B_0}{\mu_0 R_0 I_p} \left(\frac{1+\kappa^2}{2} \right).$$

c) Estimate the maximum plasma current and compare with typical value for ASDEX Upgrade and TCV.

Using the equation

$$I_p = \frac{2\pi a^2 B_0}{\mu_0 R_0 q^*} \left(\frac{1+\kappa^2}{2} \right).$$

the maximum plasma currents are as follows. TCV: 0.96 MA, ASDEX Upgrade: 2.23 MA, SPARK: 8.5 MA

For ASDEX Upgrade, a typical current of 2 MA is reported which is close to the limited reported in the table above. For TCV, a maximum current of 1.02 MA which also matches the calculated limit.

3 Full magnetic control simulation using MEQ (optional)

4 Full magnetic control simulation using MEQ

a) —

b) At line 50, a time-dependent coil current trajectory is determined to sustain the plasma current. This is then stored in `LX.Ia`. For the vessel currents `LX.Iu`, only a single time slice is computed which describes the vessel behaviour for the whole discharge. Why can `Iu` be assumed to be constant in time but not zero?

Ignoring dynamic effects of the plasma, the `Ia` trajectory is set to a ramp which yields a constant loop voltage inside the plasma domain optimized to drive the desired plasma current. This loop voltage induces currents in the vessel which in steady state are described by $V_{loop} = R_u I_u$. Hence, they are non-zero but also not changing in time.

c) Check whether the time-dependent coil current stored in `LX.Ia` affects the magnetic field evolution by plotting the poloidal flux generated by the time-derivative $\dot{I}_a$. What do you notice about this flux distribution? What is its gradient in the plasma region? And what does the value of the flux represent?

```
%Iadot in LX struct
Iadot = LX.Iadot0;
% or compute it, same result
Iadot = diff(LX.Ia(:, 1:2), [], 2) ./ diff(LX.t(1:2)) ;

% use greenem to get mutual inductance matrices
Brxa = greenem('br', L.rrx, L.zzx, L.G.rw, L.G.zw) * L.G.Twa;
Bzxa = greenem('bz', L.rrx, L.zzx, L.G.rw, L.G.zw) * L.G.Twa;
Mxa = greenem('m', L.rrx, L.zzx, L.G.rw, L.G.zw) * L.G.Twa;

% compute fields and fluxes and reshape
Brx = reshape(Brxa * Iadot, L.nzx, L.nrx);
Bzx = reshape(Bzxa * Iadot, L.nzx, L.nrx);
Fx = reshape(Mxa * Iadot, L.nzx, L.nrx);

% plotting
figure();
contourf(L.rx, L.zx, Fx, 128, 'linecolor', 'none');
meqplotarrows(L.rx,L.zx,Fx,Brx,Bzx);
colorbar;
title('Flux and Field');

figure();
contourf(L.rx, L.zx, Fx, 128, 'linecolor', 'none');
meqplotarrows(L.rx,L.zx,Fx,Brx,Bzx);
contourf(L.rx, L.zx, sqrt(Brx.^2 + Bzx.^2), 128, 'linecolor', 'none');
colorbar;
title('Magnetic field strength');
```

The flux distribution is flat inside the vessel with very small gradients/magnetic fields. The value of this flux map is the value of the change in flux induced by the active coil current ramp and is equal to the loop voltage. In short, the coil current ramp direction is chosen to yield a uniform loop voltage to drive the plasma current without disturbing the plasma geometry.

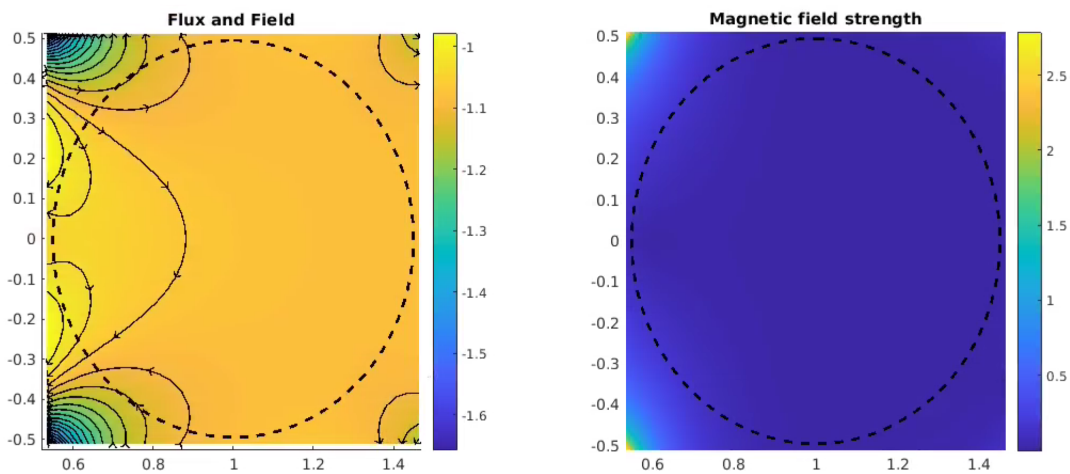

Figure 7: Vacuum flux and mangetic field strength induced by `Iadot0`

d) Later, the structure `ctrlpar` is defined which contains controller parameters. In particular `ctrlpar.KzQ` contains the vertical position controller. Change these gains and see how the vertical stability is affected. What is the minimum and maximum range for the proportional gain? Comment out the (slower) full free-boundary evolution simulations (call to `fget()` on line 101) and do the initial trials only with `rzp` and `fgel` (rigid and linearized GS models). Finally compare your simulations for the various models.

The gain can be changed by adjusting `ctrlpar.KzQ.Kp` somewhere between line 65 and line 99. The default value is `KzQ.Kp=1000`. Choosing `KzQ.Kp=317` is the minimum for the FGEL simulation and `KzQ.Kp=7100` about the maximum.
We observe strong oscillations for too high values of `Kp` and poor following of the reference for too low values of `Kp`. Differences between the models become apparent for the high gain experiment as the RZP model does not oscillate at all while for the full nonlinear FGE model, the oscillations eventually lead to a VDE. For low gains, one can observe some nonlinear behaviour in the FGE model as the plasma changes from being diverted to limited due to the large vertical excursions.

e) Have fun

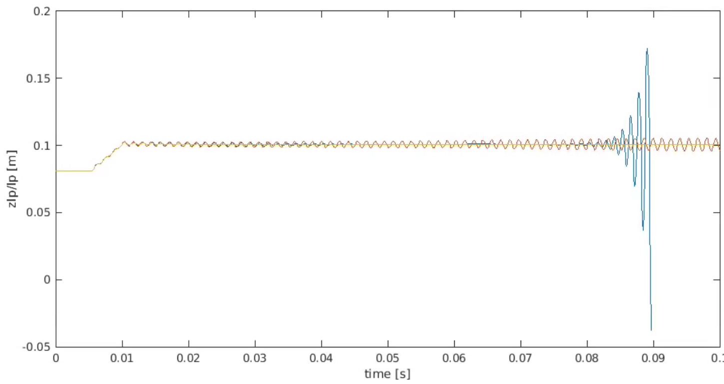

Figure 8: Z position evolution between the tree models: FGE in blue, FGEL in orange and RZP in yellow for `Kp=7100`.

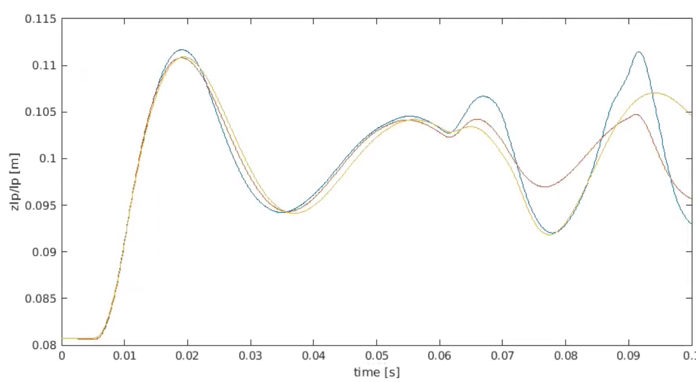

Figure 9: Z position evolution between the tree models: FGE in blue, FGEL in orange and RZP in yellow for `Kp=317`.