Free boundary exercise

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February 2025

1 Poloidal flux in tokamak

a) Draw a poloidal cross section of a generic tokamak with vacuum vessel, 1 central solenoid, 6 poloidal field coils and a limited plasma with $\kappa = 1.5$.

b) Sketch, in the same figure, the magnetic field generated by a positive current in the central solenoid assuming the tokamak has an air core.

c) Sketch the magnetic field generated by a positive current in the solenoid assuming the tokamak has an iron core (iron transformer yoke).

d) Sketch the flux surface distribution inside the vacuum vessel and indicate the magnetic axis and the last closed flux surface. In our sign convention, does the poloidal flux have a maximum or minimum at the magnetic axis?

e) For the air core case, sketch the value of the poloidal flux as a function of R (major radius) on the horizontal symmetry plane of the tokamak passing through the plasma ($Z = 0$). First consider a case without a current in the plasma. Assume the Ohmic coil current is positive (at the beginning of the discharge) and the vacuum vessel does not carry any current.

f) Sketch the value of the poloidal flux (with similar conditions as above), this time including the effects of a positive plasma current.

g) For the given elongation $\kappa = 1.5$, separately sketch the vertical and quadrupole components of the vacuum field needed to keep the plasma in equilibrium. What does the combination of both look like?

2 The safety factor q in practice

In the course, the definition of the safety factor q is given:

$$q = \frac{T}{2\pi} \oint \frac{1}{R^2} \frac{d\ell}{|B_p|} \tag{1}$$

where the integral is evaluated on a flux surface contour in the poloidal plane. Recall $T(\psi) = RB_\phi$

a) If the plasma is diverted, i.e. the last closed flux surface has an $\times$-point, what value does q have at the last closed flux surface?

b) Derive the expression for the ‘engineering q’ q^*. This is the value of q at the last closed flux surface assuming a large aspect ratio tokamak for which $R \approx R_0$ and assuming the toroidal magnetic field to be external and constant such that $B_\phi = B_0$. Furthermore it is assumed that the poloidal field is constant on the flux surface and determined by the enclosed plasma current.

Hint: You can approximate the circumference of an ellipse by $c = 2\pi\sqrt{\frac{a^2+b^2}{2}}$.

c) Estimate the maximum plasma current that can be induced in a tokamak before touching the $q = 2$ ideal MHD limit for TCV, ASDEX Upgrade and SPARK.

- High elongation TCV plasma: $B_0 = 1.5T, a = 0.25m, R_0 = 0.88, \kappa = 2.5$
- Typical ASDEX Upgrade plasma: $B_0 = 3.2T, a = 0.5m, R_0 = 1.6m, \kappa = 1.6$
- Planned SPARK plasma: $B_0 = 12T, a = 0.55m, R_0 = 1.9m, \kappa = 1.6$

3 Full magnetic control simulation using MEQ (optional)

MEQ comes with a tutorial `documentation/tutorials/anamak_rzp_control.m` that contains a full example of closed-loop control of R,Z,I_p and orthogonal PF coil currents.

a) Study the tutorial, possibly inspecting the file `meqctrl` to learn the details of the controller.

b) At line 50, a time-dependent coil current trajectory is determined to sustain the plasma current. This is then stored in `LX.Ia`. For the vessel currents `LX.Iu`, only a single time slice is computed which describes the vessel behaviour for the whole discharge. Why can `Iu` be assumed to be constant in time but not zero?

c) Check whether the time-dependent coil current stored in `LX.Ia` affects the magnetic field evolution by plotting the poloidal flux generated by the time-derivative $\dot{I}_a$. What do you notice about this flux distribution? What is its gradient in the plasma region? And what does the value of the flux represent?

d) Later, the structure `ctrlpar` is defined which contains controller parameters. In particular `ctrlpar.KzQ` contains the vertical position controller. Change these gains and see how the vertical stability is affected. What is the minimum and maximum range for the proportional gain? Comment out the (slower) full free-boundary evolution simulations (call to `fget()` on line 101) and do the initial trials only with `rzp` and `fgel` (rigid and linearized GS models). Finally compare your simulations for the various models.

e) (Optional) Tune the control gains and/or add a feedforward controller to improve the response to the the change in reference current. Compare the error norm in the plasma current error in time to the original solution.