Control and operation of tokamaks
Exercise 3 - Vertical field design and position control

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Ex 3.1: Vertical field design

a) Calculate the vertical field required to balance the radial forces on a TCV plasma of $I_p = 400\text{kA}$ with $l_i = 1$, $\beta_p = 0.5$, $\kappa = 1.6$, $a = 0.25m$, $R = 0.88\text{m}$. The precise equation is found in the slides: *Magnetic Control - Part 3*.

b) Find a combination of E and F coils in TCV that gives the required vertical field in (a). *Hint:* Formulate the problem as a least squares problem trying to get the correct B_z over a large portion of the x grid.

$$\min_{\boldsymbol{I}_a} \|B_z(\boldsymbol{I}_a) - B_{z,\text{required}}\|^2 + \|B_r(\boldsymbol{I}_a)\|^2 \ ,$$

where B_z and B_r are the fields generated by $\boldsymbol{I}_a$ on the xgrid, given by `Bz=G.Bzxa*Ia` and `Br=G.Brxa*Ia`. A file `ex3_1b_provided.m` is provided to get you started.

c) Calculate the vertical field required to balance the radial forces on an ITER plasma of $I_p = 15\text{MA}$ with $l_i = 1$, $\beta_p = 0.8$, $\kappa = 1.6$, $a = 2.0m$, $R = 6.2\text{m}$.

d) Suppose that the plasma beta suddenly decreases by 50% due to a sudden loss of confinement. What consequences does this have for the radial position of the plasma? For a given vertical field, does it move inwards or outwards? What should be done to compensate this?

Ex 3.2: Study of TCV plasma vertical position control

You are given a model of the TCV vertical position dynamics in `Vertical_position_model_TCV.m`. A delay is added to the system, representing the effect of the computational delay in the digital controller plus the power supplies.

a) Plot the Bode diagram of both the system with delay and the system without delay and comment on what you see.

b) In the following, consider only the system with delay. Using Bode diagram combined with the Generalized Nyquist Criterion, determine the minimum and maximum proportional gain that can be used to stabilize the system using negative feedback.

c) Determine the three values of the proportional gain that give, respectively, the maximum phase, gain and modulus margin for the system[1]. For these three cases, plot the Nyquist contours and the closed-loop step responses and comment on what you see.

d) Design a PD controller, of the form $K(s) = K_p(1 + sT_d)$, that has the following properties:

- The modulus margin is larger than 0.5 (for robustness)
- The bandwidth is as high as possible (for performance)

e) Add integral term to your controller to ensure zero steady-state tracking error without affecting the transient performance. To avoid changing the zero location of the PD controller, it is suggested to write the new controller as: $K(s) = K_p(1 + sT_d)(1 + \frac{1}{T_i s})$ What is the shortest integral time constant you can use?

f) To avoid amplification of high-frequency measurement noise, add a second-order roll-off term $1/(T_r s + 1)^2$ to the controller. Determine the largest acceptable value for T_r before the closed-loop performance starts to be significantly affected.

g) Plot the closed-loop step responses, sensitivity and control sensitivity functions for the P, PD, PID, PID + rolloff controllers that you found in the previous exercises. and comment on the results.

[1] The modulus margin m is the minimum distance of the Nyquist contour from the -1 point on the complex plane, and $M_s = 1/m$ is the peak of the closed-loop sensitivity function