Control and operations of tokamaks
Exercise 5 - Kinetic control

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1 Simulating the plasma 0D energy balance

In the lecture we saw the following 0D model for the plasma thermal energy balance

$$\frac{3}{2}\frac{\mathrm{d}p}{\mathrm{d}t} = S_\alpha + S_\Omega + S_{aux} - S_{rad} - S_{cond} \tag{1}$$

where

- $p = 2nT$ is the plasma pressure with T in Joule [1]. We assumed $T_i = T_e = T$ and $n_i = n_e = n$.
- S_α is the power density from the fusion alpha particles. It is given by

  $$S_\alpha = \frac{f_{DT}}{(1+f_{DT})^2} E_\alpha n^2 \langle \sigma v \rangle \tag{2}$$

  where f_{DT} is the Deuterium-Tritium fraction, E_α is in Joules, and $\langle \sigma v \rangle$ can be approximated, in the temperature region of interest, as

  $$\langle \sigma v \rangle = 1 \times 10^{-6} \exp\left(a_{-1}/T_{\mathrm{keV}}^{\alpha} + a_0 + a_1 T_{\mathrm{keV}} + a_2 T_{\mathrm{keV}}^2 + a_3 T_{\mathrm{keV}}^3 + a_4 T_{\mathrm{keV}}^4\right) \mathrm{m}^3\mathrm{s}^{-1} \tag{3}$$

  where the coefficients are given in Table 1

[1] $(T[\mathrm{J}] = q_e T[\mathrm{eV}])$ with q_e the Electron charge $1.602176565 \times 10^{-19}$J/eV

α	a_{-1}	a_0	a_1	a_2	a_3	a_4
0.2935	-21.38	-25.20	-7.101×10^{-2}	1.938×10^{-4}	4.925×10^{-6}	-3.984×10^{-8}

Table 1: Coefficients for $\langle \sigma v \rangle$ approximation, from Hively et al, *Nuclear Fusion* **17**, 873 (1977)

- S_Ω is the ohmic power density from resistive heating due to the plasma current. For the present purposes we can write it as

$$S_\Omega = \frac{1}{V}\left(\frac{5.6 \times 10^4}{1 - 1.31\epsilon^{1/2} + 0.46\epsilon}\right)\left(\frac{R_0 I_{\mathrm{MA}}^2}{a^2 \kappa T_{\mathrm{keV}}^{3/2}}\right) \tag{4}$$

  with $\epsilon = a/R_0$ and the volume $V = 2\pi^2\kappa R_0 a^2$.

- S_{aux} is the power density from the external (auxiliary) heating sources. It is given simply by:

$$S_{aux} = P_{aux}/V \tag{5}$$

- $S_{rad} = 5.35 \times 10^3 Z_{eff} n_{1\mathrm{e}20}^2 T_{\mathrm{keV}}^{1/2}$ is power per unit volume radiated to the first wall,

- S_{cond} is the power per unit volume that reaches the first wall by thermal conduction. It is given by:

$$S_{cond} = \frac{3}{2}\frac{p}{\tau_e} \tag{6}$$

  where τ_e is the confinement time for which we will use the scaling law expression

$$\tau_e = 0.145 I_{p,\mathrm{MA}}^{0.93} R_0^{1.39} a^{0.58} \kappa^{0.78} n_{1\mathrm{e}20}^{0.41} B_{0,\mathrm{T}}^{0.15} A^{0.19} \left(P_{aux,\mathrm{MW}} + P_{\Omega,\mathrm{MW}} + P_{\alpha,\mathrm{MW}}\right)^{-0.69} \tag{7}$$

where $P_\Omega = S_\Omega V$. We will assume a constant density $n = 1 \times 10^{20}\mathrm{m}^{-3}$ for this exercise. The other parameters are:

$$I_p = 15\mathrm{MA}, \quad R_0 = 8\mathrm{m}, \quad a = 2\mathrm{m}, \quad \kappa = 2, \quad B_0 = 7\mathrm{T}, \quad A = 2 \quad Z_{eff} = 1.5, \quad f_{DT} = 0 \tag{8}$$

Also recall that $E_\alpha = 3.5\mathrm{MeV}$. These are parameters close to a DEMO reactor (except for f_{DT}, for now).

a) You are given a Matlab function that accepts as input: T and P_{aux} and returns, as output, $\frac{\mathrm{d}T}{\mathrm{d}t}$. In the demo file it is shown how to integrate this ODE in time with the solver `ode45`. Simulate the response to a staircase-like power input: $P_{aux} =$ 10MW, 25MW, 50MW. Plot the evolution of the pressure, temperature, and all the source and sink terms. For this exercise, there is no power source from fusion reactions.

2 Control of plasma β

Assume $T = T_0 + \delta T$ with $\delta T \ll T_0$ and $P_{aux} = P_{aux,0} + \delta P_{aux}$.

a) Complete the linearisation of the model equations around the operating point corresponding to $P_{aux,0} = 25$MW. Complete the Matlab function *linearise_model.m* by adding the linearisation of the Ohmic heating and the auxiliary power.

b) For this linearised model, write the transfer function between δP_{aux} and δT.

c) Check your solution for the temperature linearisation by investigating a plot of $\partial T/\partial t$ vs T.

d) Design a PID controller for the temperature and test it on the linear model. Requirements: Bandwidth = 0.5Hz (-6dB), zero steady-state error, rejection of input disturbances above 100Hz, and no amplification of measurement noise. Try to keep the controller's response low in order to limit the required control power.

e) Test the PID controller on the original nonlinear ODE model. Plot the response of the system.

Recall the following transfer functions:
- Sensitivity: $S = \frac{1}{1+CG}$
- Disturbance Rejection: $DR = \frac{G}{1+CG}$

3 Burn control

For this final exercise we consider a burning plasma, using the power balance model from the exercise on beta control, and assuming $f_{DT} = 1$.

a) Repeat exercise 1a), now including the nonzero S_α and compare the result.

b) Plot $\frac{\mathrm{d}T}{\mathrm{d}t}$ versus T for the case $P_{aux} = 25$MW. Identify the stationary points where $\frac{\mathrm{d}T}{\mathrm{d}t} = 0$. What do you notice?

c) Linearise the equation including S_α, for all the equilibrium points you found. Analyse the stability of each point.

d) For the burning plasma we can easily measure the output neutron power, e.g. by a neutron detector. This power is equal to $P_\mathrm{n} = 4P_\alpha$. For each equilibrium point, determine the transfer function between the input power P_{aux} and the neutron power $4P_\alpha$

e) Design a linear controller for each operating point that responds to a reference signal for the neutron power. The controllers are to have the same requirements as in the previous exercise.

f) Test the controllers on the original nonlinear ODE model. Plot the response of the system.