Plasma profiles and their control

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Outline I

1. Profile control

2. Tokamak profile dynamics
   - Magnetic field diffusion
   - Kinetic transport
   - Full Tokamak Simulators

3. Profile control
   - Actuators
   - Sensors
   - Control

Section 1

Profile control

Introduction

- Plasma held place by TF+PF coils, magnetic equilibrium described by Grad-Shafranov equation. To determine equilibrium we need to know the internal profiles $p'(\psi)$, $T(\psi) = RB_\phi$.
- But what determines this internal pressure and poloidal current distribution?

Introduction

- Plasma held place by TF+PF coils, magnetic equilibrium described by Grad-Shafranov equation. To determine equilibrium we need to know the internal profiles $p'(\psi)$, $T(\psi) = RB_\phi$.
- But what determines this internal pressure and poloidal current distribution?
- We will discuss *internal* evolution (=transport) of current, particles and energy.
  - 1D problem: transport $\parallel$ **B** is ∞-fast, only transport $\perp$ **B** evolves slowly. (Q:How slowly?)
  - *Plasma profiles*: important for stability and performance.
  - We will (roughly) derive 1D transport equations for j, T, n.
- What are the actuators? How do they affect profile evolution?

Section 2

Tokamak profile dynamics

Subsection 1

Magnetic field diffusion

B field diffusion in resistive medium

- Ohm's law for resistive MHD:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j} \tag{1}$$

assume $\mathbf{v} = 0$, $\eta = cst$ and use $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$

$$\mathbf{E} = \frac{\eta}{\mu_0}(\nabla \times \mathbf{B}) \tag{2}$$

Take $\nabla \times$ both sides and use $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{\eta}{\mu_0} \nabla \times (\nabla \times \mathbf{B}) = \frac{\eta}{\mu_0} \nabla^2 \mathbf{B} \tag{3}$$

Typical time scales

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{\eta}{\mu_0}\nabla^2 \mathbf{B} \tag{4}$$

- Superconducting case: $\eta = 0 \rightarrow \frac{\partial \mathbf{B}}{\partial t} = 0$ (Field frozen in medium)
- Conducting case, typical time scales depend on resistivity and system size $\tau = \frac{\mu_0 L^2}{\eta}$

Time scales w.r.t. shot time

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- Exercise:
  - Calculate typical magnetic diffusion times for TCV, JET and ITER and compare to typical plasma shot times. Recall $\tau = \frac{\mu_0 L^2}{\eta}$ and use $\eta \approx 1.8 \times 10^{-2} T_e^{-3/2} [\mathrm{eV}]$
    - TCV: $L = 0.25\mathrm{m}$, $T_e = 1\mathrm{keV}$, $t_{shot} = 2\mathrm{s}$
    - JET: $L = 1\mathrm{m}$, $T_e = 3\mathrm{keV}$, $t_{shot} = 12\mathrm{s}$
    - ITER: $L = 2\mathrm{m}$, $T_e = 5\mathrm{keV}$, $t_{shot} = 300\mathrm{s}$

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Answer:

- TCV: $0.14\mathrm{s}$
- JET: $11.4\mathrm{s}$
- ITER: $100\mathrm{s}$

NB: Slow internal dissipation of **B** field can not be ignored.

Diffusion of poloidal flux in tokamak

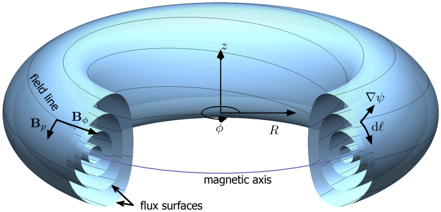
- Sketch of derivation. Full derivations in [1], [2], [3]
- Write Ohm's law (without flows) with conductivity $\sigma = 1/\eta$

$$\underbrace{\mathbf{j}}_{\text{total current}} = \underbrace{\sigma\mathbf{E}}_{\text{inductive/Ohmic current}} + \underbrace{\mathbf{j}_{ni}}_{\text{non-inductive current}} \tag{5}$$

- Project in direction $\parallel$ **B** and average over a flux surface (from Grad-Shafranov equilibrium)

$$\langle\mathbf{j}\cdot\mathbf{B}\rangle = \sigma_{\parallel}\langle\mathbf{E}\cdot\mathbf{B}\rangle + \langle\mathbf{j}_{ni}\cdot\mathbf{B}\rangle \tag{6}$$

Diffusion of poloidal flux in tokamak



- $\psi(R,Z) = \int \mathbf{B} \cdot d\mathbf{A}_Z$, loci of constant ψ define flux surfaces.
- Define toroidal flux through a flux surface:

$$\Phi = \int \mathbf{B} \cdot d\mathbf{A}_\phi = \int B_\phi dA_\phi, \quad (7)$$

associate radial metric $\rho = \sqrt{\Phi}$ and $\rho_N = \rho/\rho_b$ ($_b$ = $_{boundary}$)

Diffusion of poloidal flux in tokamak

Exercise:

- Show that in the case of circular cross section, large aspect ratio ($R/a \to \infty$) and low plasma current ($B_\phi \approx B_0$) it holds that $\rho_N = r/a$, where r is the geometric radius and a is the radius of the LCFS.

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Solution:

- $B_\phi = B_0$, circular cross section, then $\Phi = \pi r^2 B_0$ and $\rho = r\sqrt{\pi B_0}$, $\rho_{edge} = a\sqrt{\pi B_0}$ so $\rho_N = r/a$

Flux surface averaging

Volume:

$$V = \iiint dV = \iiint R \mathrm{d}\phi \frac{\mathrm{d}\psi}{|\nabla\psi|} \mathrm{d}\ell_p = \int \left(\oint \frac{\mathrm{d}\ell_p}{B_p} \right) \mathrm{d}\psi \tag{8}$$

$$\frac{\partial V}{\partial \psi} = \oint \frac{\mathrm{d}\ell_p}{B_p} \tag{9}$$

Flux Surface average of Q:

$$\langle Q \rangle \equiv \frac{\partial}{\partial V} \iiint Q dV = \frac{\partial \psi}{\partial V} \frac{\partial}{\partial \psi} \int \oint Q \frac{2\pi R}{|\nabla\psi|} d\ell d\psi$$

Recalling $\frac{|\nabla\psi|}{2\pi R} = |B_p|$, we get

$$\langle Q \rangle \equiv \frac{\partial}{\partial V} \int Q dV = \oint Q \frac{d\ell_p}{B_p} / \oint \frac{d\ell_p}{B_p} \tag{10}$$

A useful relation

$$
\begin{aligned}
\langle \nabla \cdot \mathbf{F} \rangle &= \frac{\partial}{\partial V} \int (\nabla \cdot \mathbf{F}) dV = \frac{\partial}{\partial V} \int (\nabla \cdot \mathbf{F}) R \mathrm{d}\phi \mathrm{d}\ell_p \frac{d\psi}{|\nabla \psi|} \\
&= \frac{\partial}{\partial V} \oint \mathbf{F} \cdot \frac{\nabla V}{|\nabla V|} R \mathrm{d}\phi \mathrm{d}\ell_p \quad \text{(by Gauss)} \\
&= \frac{\partial}{\partial V} 2\pi \oint \mathbf{F} \cdot \nabla V \frac{\partial \psi}{\partial V} \frac{R d\ell_p}{|\nabla \psi|} \\
&= \frac{\partial}{\partial V} \langle \mathbf{F} \cdot \nabla V \rangle
\end{aligned} \tag{11}
$$

Poloidal flux transport

Consider a surface of constant poloidal flux, of which each point of the boundary is moving with velocity $\mathbf{u}_\psi$. For this surface we can state

$$\frac{\partial \psi}{\partial t} + \mathbf{u}_\psi \cdot \nabla \psi = 0 \tag{12}$$

Now for a general scalar field $F(t, x, y, z)$ define scalar $H(t) = \int_V F dV$ where V is the volume enclosed by a flux surface $\psi = cst$ moving with speed $\mathbf{u}_\psi$, then

$$\frac{\partial}{\partial t} H \bigg|_{\psi = cst} = \int_V \frac{\partial F}{\partial t} dV + \oint_S F \mathbf{u}_\psi \cdot \mathbf{dS}_\psi = \int_V \frac{\partial F}{\partial t} dV + \oint_S F \mathbf{u}_\psi \cdot \frac{\nabla \psi}{|\nabla \psi|} \mathrm{d}S_\psi \tag{13}$$

Toroidal flux evolution

Using this equation, we can express the time rate of change of toroidal flux Φ enclosed by a surface S of constant poloidal flux $\psi = cst$ as:

$$\left.\frac{\partial \Phi}{\partial t}\right|_{\psi=cst} = \frac{1}{2\pi}\frac{\partial}{\partial t}\int_V \mathbf{B}\cdot\nabla\phi \mathrm{d}V$$

$$= \frac{1}{2\pi}\int_V \frac{\partial \mathbf{B}}{\partial t}\cdot\nabla\phi\mathrm{d}V + \frac{1}{2\pi}\oint_S (\mathbf{B}\cdot\nabla\phi)(\mathbf{u}_\psi\cdot\nabla\psi)\frac{\mathrm{d}S}{|\nabla\psi|} \quad (14)$$

Which can rewritten, using Faraday's law and Gauss' therem (details in [3] as:)

$$\left.\frac{\partial \Phi}{\partial t}\right|_{\psi=cst} = -\oint_S (\mathbf{E}\cdot\mathbf{B}_p + B_\phi E_\phi)\frac{dS}{|\nabla\psi|} = -\oint_S (\mathbf{E}\cdot\mathbf{B})\frac{dS}{|\nabla\psi|} \quad (15)$$

$$= -\frac{\partial V}{\partial \psi}\langle \mathbf{E}\cdot\mathbf{B}\rangle \quad (16)$$

Rate of change of poloidal flux

This can be related to the time rate of change of the poloidal flux as follows:

$$\left.\frac{\partial\psi}{\partial t}\right|_{\Phi=cst} = \frac{\partial\psi}{\partial V}\frac{\partial V}{\partial\Phi}\left.\frac{\partial\Phi}{\partial t}\right|_{\psi=cst} \tag{17a}$$

$$\left.\frac{\partial\psi}{\partial t}\right|_{\rho} + \left.\frac{\partial\psi}{\partial\rho}\frac{\partial\rho}{\partial t}\right|_{\Phi} = -\frac{\partial V}{\partial\Phi}\langle\mathbf{E}\cdot\mathbf{B}\rangle \tag{17b}$$

$$\left.\frac{\partial\psi}{\partial t}\right|_{\rho} - \frac{\rho\dot{\Phi}_b}{2\Phi_b}\frac{\partial\psi}{\partial\rho} = -\frac{\partial V}{\partial\Phi}\langle\mathbf{E}\cdot\mathbf{B}\rangle \tag{17c}$$

Ohm's law

Back to the parallel Ohm's law:

$$\langle \mathbf{j} \cdot \mathbf{B} \rangle = \sigma_{||} \langle \mathbf{E} \cdot \mathbf{B} \rangle + \langle \mathbf{j}_{ni} \cdot \mathbf{B} \rangle \tag{18}$$

where $\mathbf{j}_{ni} = \mathbf{j}_{bs} + \mathbf{j}_{cd}$ is the non-inductive component of the current density, decomposed into bootstrap (plasma self-driven) and current drive (externally driven) parts. Equivalently:

$$\sigma_{||} \frac{\langle \mathbf{E} \cdot \mathbf{B} \rangle}{B_0} = \frac{\langle \mathbf{j} \cdot \mathbf{B} \rangle}{B_0} - \frac{\langle \mathbf{j}_{bs} \cdot \mathbf{B} \rangle}{B_0} - \frac{\langle \mathbf{j}_{cd} \cdot \mathbf{B} \rangle}{B_0} \tag{19}$$

$$\text{or } \sigma_{||} E_{||} = j_{||} - j_{bs} - j_{cd} \tag{20}$$

- We have an expression for the LHS term involving $\langle \mathbf{E} \cdot \mathbf{B} \rangle$, from (17c)
- The non-inductive terms can be computed from current drive models directly
- Last piece: expression for $j_{||}$ as a function of $\psi(\rho)$

Parallel current

As a final step we need to rewrite the term $j_{\|} = \frac{\langle \mathbf{j} \cdot \mathbf{B} \rangle}{B_0}$ in terms of ψ:

$$\frac{\langle \mathbf{j} \cdot \mathbf{B} \rangle}{B_0} = \underbrace{\frac{F}{2\pi\mu_0 B_0} \langle \nabla \cdot (\nabla\psi / R^2) \rangle}_{j_\phi B_\phi} + \underbrace{\frac{1}{2\pi\mu_0 B_0} \langle \nabla F \cdot \nabla\psi / R^2 \rangle}_{\mathbf{j_p} \cdot \mathbf{B_p}} \tag{21}$$

$$\frac{\langle \mathbf{j} \cdot \mathbf{B} \rangle}{B_0} = \frac{F^2}{16\pi^4 \mu_0 \Phi_b^2 \rho} \frac{\partial}{\partial\rho} \left(\frac{g_2 g_3}{\rho} \frac{\partial\psi}{\partial\rho} \right) \tag{22}$$

Where we introduced $F(\psi) = RB_\phi$, $g_2 = \left(\frac{\partial V}{\partial\psi}\right)^2 \left\langle \frac{|\nabla V|^2}{R^2} \right\rangle$, $g_3 = \left\langle 1/R^2 \right\rangle$ (*Geometric profiles*)

Flux Diffusion: Sketch of derivation

$$\boxed{\sigma_{||}\left(\frac{\partial\psi}{\partial t}-\frac{\rho\dot{\Phi}_b}{2\Phi_b}\frac{\partial\psi}{\partial\rho}\right)=\frac{T^2}{16\pi^4\mu_0\Phi_b^2\rho}\frac{\partial}{\partial\rho}\left(\frac{g_2g_3}{\rho}\frac{\partial\psi}{\partial\rho}\right)-\frac{B_0V'}{2\Phi_b\rho}j_{ni}} \tag{23}$$

Where:

- $g_2=\left(\frac{\partial V}{\partial\psi}\right)^2\left\langle\frac{|\nabla\psi|^2}{R^2}\right\rangle$, $g_3=\langle 1/R^2\rangle$ (*Flux surface averages*)
- $V'=dV/d\rho$. (*Volume derivative*)
- $T=RB_\phi=R_0B_0+$ small correction due to poloidal currents.
- $\sigma_{||}$ is the conductivity (roughly $\sim T_e^{3/2}$)
- j_{ni} is non-inductive current drive (self-generated, or from auxiliary sources)

Flux Diffusion: Final result

$$\sigma_{||}\left(\frac{\partial\psi}{\partial t}-\frac{\rho\dot{\Phi}_b}{2\Phi_b}\frac{\partial\psi}{\partial\rho}\right)=\frac{T^2}{16\pi^4\mu_0\Phi_b^2\rho}\frac{\partial}{\partial\rho}\left(\frac{g_2 g_3}{\rho}\frac{\partial\psi}{\partial\rho}\right)-\frac{B_0 V'}{2\Phi_b\rho}j_{ni} \quad (24)$$

Recall: (Ohmic **j**) = (Total **j**) - (non-inductive **j**)

Special cases:

- $j_{ni}=0$: "Ohmic" plasma: all current sustained by inductive part (LHS). Requires $\frac{d\psi}{dt}>0\ \forall\ t$.
- $\frac{d\psi}{dt}=$ constant: *stationary state*. $\frac{\partial}{\partial\rho}\frac{\partial\psi}{\partial t}=\frac{\partial}{\partial t}\frac{\partial\psi}{\partial\rho}=0$. Fixed plasma current profile (shape) but not steady-state.
- $\frac{d\psi}{dt}=0$: fully *steady-state* plasma. Plasma current sustained entirely by j_{ni} **desired for reactor**

Boundary conditions for poloidal flux

Boundary conditions:

- Impose total plasma current

$$\left[\frac{g_2 g_3}{\rho}\frac{\partial\psi}{\partial\rho}\right]_{\rho=\rho_b} = I_p(t). \tag{25}$$

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Interpretation:

- To have nonzero I_p we need $\left[\frac{g_2 g_3}{\rho}\frac{\partial\psi}{\partial\rho}\right]_{\rho=\rho_b} \neq 0$, but $\psi(\rho)$ evolution is governed by diffusion equation that evolves to $\frac{\partial\psi}{\partial\rho} = 0$. How do we maintain I_p?

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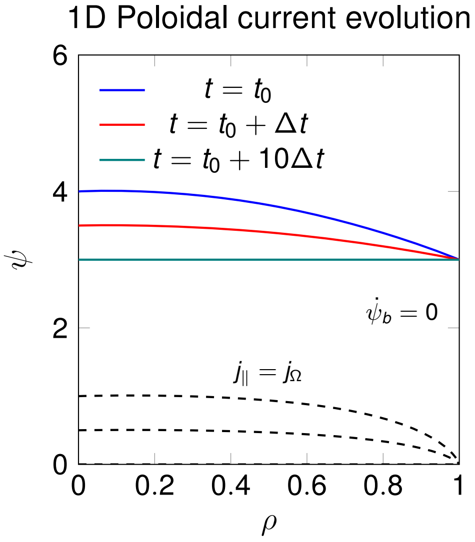
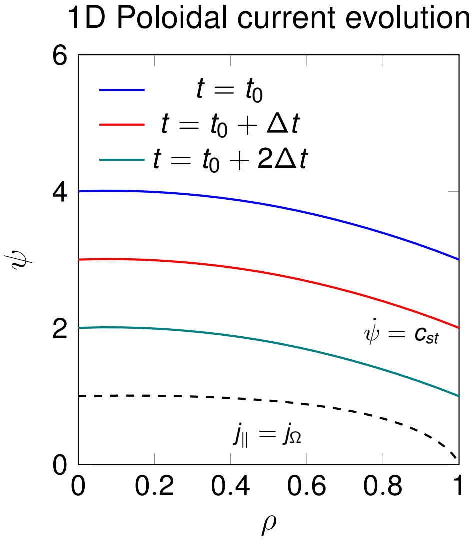
- Two possibilities
    - Time-varying $\psi(\rho_b)$ (induced current)
    - Use source term (non-inductive current)

Flux evolution without inductive current drive



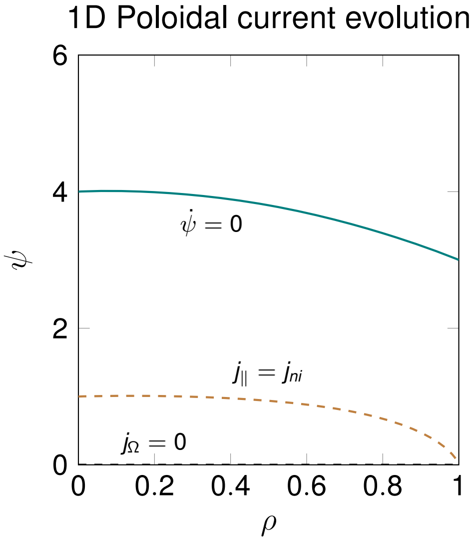
- Start with initial condition for $\psi(\rho)$ s.t. $j_{\|} > 0$
- Diffusion tends to flatten $\psi(\rho)$
- $j_{\|}$ (current) decays in time

Flux evolution for inductive current drive



- Start with initial condition for $\psi(\rho)$ s.t. $j_{\|} > 0$
- If $\dot{\psi} = c_{st}$ current profile stays the same.
- Can maintain a non-flat $\psi(\rho)$ (non-zero $j_{\|}$) despite diffusion.
- Needs time-varying $\dot{\psi}_b$ (Transformer effect!)

Flux evolution for fully-non-inductive current drive



- Start with initial condition for $\psi(\rho)$ s.t. $j_{\|} > 0$
- Assume current is fully carried by non-inductive current $j_{\|} - j_{ni} = 0$
- Can maintain a non-flat $\psi(\rho)$ (non-zero $j_{\|}$) despite diffusion.
- Maintains $\dot{\psi}_b = 0$ (No Transformer!) and $\psi = 0$ *steady state*

Question: response to step in auxiliary current drive

Simplified flux diffusion equation

$$\boxed{\sigma_{||}\frac{\partial\psi}{\partial t} = \frac{F^2}{(2\pi)^4\mu_0 B_0^2\rho}\frac{\partial}{\partial\rho}\left(\frac{g_2 g_3}{\rho}\frac{\partial\psi}{\partial\rho}\right) - \frac{V'}{2\pi\rho}j_{ni}} \tag{26}$$

- Question:
  - Suppose we are in a stationary state with $\frac{\partial\psi}{\partial t} = c$.
  - Qualitatively, what happens if we instantaneously increase j_{ni} using a (spatially) narrow current drive actuator?
  - Does the current density profile change instantaneously?
  - Which physical law does this example represent?

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  - Does the current density profile change instantaneously?
  - Which physical law does this example represent?
- Answer:
  - We can not change the 1st term on the LHS since its time evolution is governed by (26) itself! Thus, instantaneously, only the ohmic current density $\sim \frac{\partial\psi}{\partial t}$ changes.
  - This is an example of Lenz's law: an inductive circuit resists a change in flux. Also called "back-EMF".

Other measures of current distribution

- In practice, physicists like to work with q profile since it is indicative of MHD stability.

$$q(\psi) = \frac{1}{2\pi} \oint \frac{1}{R} \frac{B_\phi}{B_p} d\ell = \frac{T(\psi)}{2\pi} \oint \frac{d\ell}{R^2 B_p} \tag{27}$$

  It can also be shown that $q \equiv \frac{\partial \Phi}{\partial \psi}$.
- Plasma stability, performance, transport is influenced by q and its spatial derivative the *magnetic shear* $s = \frac{\rho}{q} \frac{\partial q}{\partial \rho}$.

Subsection 2

Kinetic transport

Thermal energy transport equation

- Diffusive transport law for energy density $W = k_B T_e n_e$

$$\frac{\partial W}{\partial t} + \frac{1}{\rho}\frac{\partial}{\partial \rho}\Gamma = \text{Sources} \tag{28}$$

with diffusive flux $\Gamma = \rho D \frac{\partial W}{\partial \rho}$ with diffusion coefficient D.

- In toroidal geometry, this is more complicated. For each species:

$$\frac{3}{2}(V')^{-5/3}\left(\frac{\partial}{\partial t}\left[(V')^{5/3} n_\alpha T_\alpha\right]\right) + \frac{1}{V'}\frac{\partial}{\partial \rho}\left(q_\alpha + \frac{5}{2}T_\alpha \Gamma_\alpha\right) = P_\alpha \tag{29}$$

- $V' = \frac{\partial V}{\partial \rho}$ (geometry)
- Similarly for particle transport
- For each species (e^-, i^+, impurities)!

Transport coefficients

- Diffusive/advective fluxes in plasma depend in a complicated way on the plasma itself, nonlinear multi-scale *turbulence*.
- For modeling/control purposes, must resort to simple models.
- Recent breakthrough: Neural Network emulation of Gyrokinetic fluxes from the QuaLiKiz quasilinear gyrokinetic code [4], [5]. Speedup from 1hour to 1ms per time step.

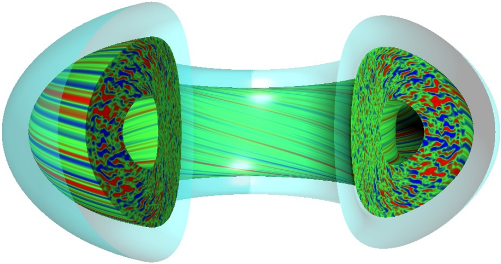


Figure: Gyrokinetic simulations of tokamak turbulence. Source: GYRO/PPPL

Models for diffusive and convective flux

- Transport equation, for species α:

$$\frac{3}{2}(V')^{-5/3}\left(\frac{\partial}{\partial t}\left[(V')^{5/3}n_\alpha T_\alpha\right]\right)+\frac{1}{V'}\frac{\partial}{\partial\rho}\left(q_\alpha+\frac{5}{2}T_\alpha\Gamma_\alpha\right)=P_\alpha \tag{30}$$

where q_α is the diffusive flux and Γ_α is the convective flux.

$$\frac{q_\alpha}{n_\alpha T_\alpha}=-V'G_1\langle(\nabla\psi)^2\rangle\sum_{\beta\in\text{all species}}\left(\chi^T_{\alpha\beta}\frac{1}{T_\beta}\frac{\partial T_\beta}{\partial\rho}+\chi^n_{\alpha\beta}\frac{1}{n_\beta}\frac{\partial n_\beta}{\partial\rho}\right) \tag{31}$$

$$\frac{\Gamma_\alpha}{n_\alpha}=-V'G_1\langle(\nabla\psi)^2\rangle\sum_{\beta\in\text{all species}}\left(D^T_{\alpha\beta}\frac{1}{T_\beta}\frac{\partial T_\beta}{\partial\rho}+D^n_{\alpha\beta}\frac{1}{n_\beta}\frac{\partial n_\beta}{\partial\rho}\right) \tag{32}$$

Qualitative picture of transport coefficients

- Accurate and simple models for thermal transport do not exist (yet). We can only make some qualitative statements.
- Usually diagonal terms are dominant, e.g. χ_{ee} for electrons.
- Usually diffusive terms are assumed dominant. $D_{\alpha\beta} = 0$
- Then T_e diffusion equation becomes, e.g.

$$\frac{3}{2}(V')^{-5/3}\left(\frac{\partial}{\partial t}\left[(V')^{5/3}n_e T_e\right]\right) = \frac{1}{V'}\frac{\partial}{\partial \rho}\left(V' G_1 \chi_{ee} n_e \frac{\partial T_e}{\partial \rho}\right) + P_e \tag{33}$$

Qualitative picture of transport coefficients

- Higher plasma current gives more confinement.
- Transport is 'stiff'. Above a critical $\frac{\nabla T_e}{T_e}$, χ_e increases drastically. Increasing $\frac{\nabla T_e}{T_e}$ above this threshold requires much more power (limited by actuators).
- High magnetic shear and low q are 'good' (low transport). Volume-averaged $\langle s/q \rangle$ [6]
- Shear close to 0 or negative is also good, can give 'internal transport barriers' (ITBs). e.g. [7], [8]

Sources and sinks for thermal energy

- Sources (spatially dependent)
  - Ohmic heating power (resistivity) $\sim \langle \mathbf{j} \cdot \mathbf{E} \rangle$
  - Local heat deposition by auxiliary systems. Compute using specialized codes like TORBEAM (EC) [9], RABBIT (NBI) [10]...
  - Collisional energy exchange with other species (α particles!)
- Sinks (spatially dependent)
  - Radiation losses (line radiation, cyclotron radiation, Bremsstrahlung)
  - Conduction (loss to the wall)
  - Collisional energy exchange with other species.

Subsection 3

Full Tokamak Simulators

Coupling to 2D equilibrium

In reality, 1D transport is tightly coupled to 2D equilibrium (Grad-Shafranov). Geometric factors g_2, g_3 come from 2D $\psi(R, Z)$ distribution

- Given a pressure and current distribution, GS equation determines 2D equilibrium. Very fast (Alfven = $\mu\mathrm{s}$) timescales
- Given a 2D equilibrium geometry, 1D transport equations evolve distribution of current and pressure. Confinement time scales vs current redistribution time scales. ITER: $\tau_E \sim 5\mathrm{s}$, $\tau_{crt} \sim 100\mathrm{s}$.

In practice: strongly nonlinear coupling between two sets of equations.

- Some good simulators exist (ASTRA-SPIDER, DINA, CORSICA, JINTRAC).
- Alternatives: solve transport equations in equilibrium solver directly (NICE)

State-of-the-art of full tokamak simulators

We saw most of the components of a full tokamak simulator:

- Magnetic equilibrium
- Flux + Energy transport + particle transport w Sources/Sinks
- MHD 'events' (sawteeth, NTM) + MHD limits.
- Models for actuators and diagnostics

Several codes or 'code suites' exist to solve these coupled systems of equations representing a 'Full Tokamak Simulator'

- JINTRAC-DINA (ITER) [11]
- FENIX (=ASTRA+SPIDER) (IPP Garching) [12]
- various others in USA, Japan, Korea...

Section 3

Profile control

Subsection 1

Actuators

Actuators

- Poloidal flux (current distribution)
  - I_p (boundary)
  - $\sigma_{||}$ or j_{BS} (heating changes resistivity or *BS* current)
  - j_{aux} (direct source).
  - Shape changes.
- Thermal transport:
  - Heating (direct source)
  - Change of transport via changing turbulence $q, s, \nabla T_e, \nabla n_e, \ldots$.
- Particle transport
  - Gas (edge)
  - Neutral beam (internal source)
  - Change particle transport.

Subsection 2

Sensors

Sensors (diagnostics) for profile control

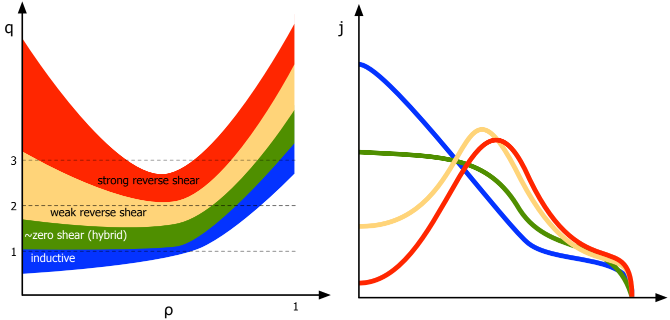
- Kinetic profiles: ECE, Thomson, interferometry, ...
- Magnetic profiles: MSE, Polarimetry, ...
- 2D equilibrium: RT equilibrium reconstruction, ...
- Different sampling rates, varying radial location of measurements, technically challenging diagnostics.
- How to merge information from various sources? *State observers* (Next lecture)

Subsection 3

Control

Control Objectives

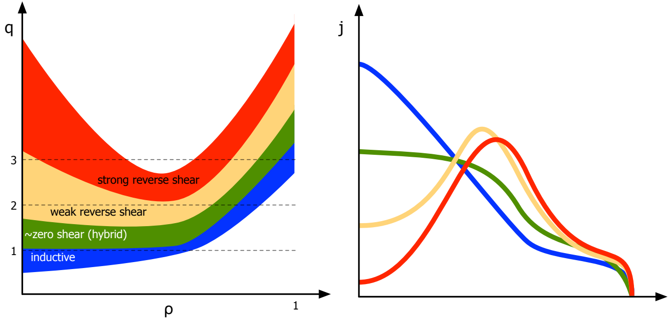
- Obtain plasma with desired profile shapes (“scenario”), possibly in stationary condition, at end of current ramp-up.



- Q: why is the inductive current profile peaked?

Control Objectives

- Obtain plasma with desired profile shapes (“scenario”), possibly in stationary condition, at end of current ramp-up.



- Q: why is the inductive current profile peaked?
- A: $\sigma_{neo} \sim T_e^{3/2}$

Control strategies

- Open-loop vs closed-loop control
  - Open-loop: design actuator time trajectories to get desired plasma state. (*everyday practice for tokamak operators who program plasma discharges*)
  - Closed-loop: attempt to track a reference: reject disturbances, robust against model uncertainties, etc (*done very rarely - desired in the future*)
- Both can benefit from model-based design tools

Model-based open-loop trajectory design

- Solve open-loop trajectory optimization problem
- Include cost function (what you want) and constraints (what is possible)
- Simulation examples in literature: [13] [14], [15] and some have been tested in practice [16].

Examples of closed-loop profile control

- Non model-based
  - Control q_0 and q_{min} in DIII-D ramp-up using NBI [17]
  - Plasma regime control by LH power in Tore Supra [Imbeaux EPS 2009]
- Model-based: system identification of linear model arond operating point
  - Linear optimal feedback control around set point (JET, DIII-D) [18], [19].
- Model-based: first principle models
  - Model-based controllers tested on DIII-D [20], [21], [22]
  - Lyapunov-based controller design [23]
  - Adaptive model based on linearization [24]
  - Model Predictive Control [25], [26]

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