Control and operation of tokamaks
Exercise 0 - Prerequisites

Lecturer: A. Mele (based on notes prepared by F. Felici)
Instructors: R. Coosemans, S, Dubbioso, P. Molina, F. Pastore, J.P. Svantner
EPFL - SPC

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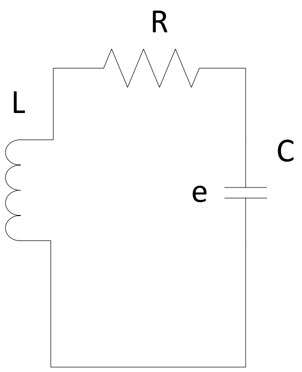
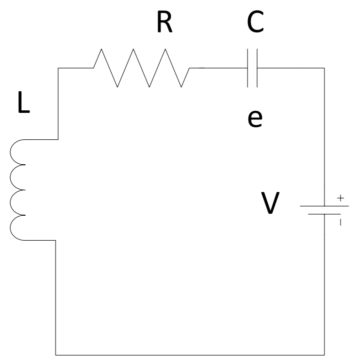
Exercise 1 - State Space Representation



(a) LRC circuit.



(b) LRC circuit with V input.

Figure 1

Consider the unforced LRC circuit of Fig.1a

a) Write down the set of ODEs defining the circuit dynamics.

b) Write them as a first order ODE system in a vector matrix form.

c) Among the variables, identify the states of the system.

d) Write down the state space representation for this system.

Consider now the forced circuit of Fig.1b with $V = 1\,V$, $R = 1\,\Omega$, $C = 0.1\,\mu F$ and $L = 1\,\mu H$.

e) Write down the state space representation of the new system.

e) Determine the stability of the system.

e) Plot the step response for the following inductance values: $L = 1\,\mu H$, $L = 10\,\mu H$ and $L = 100\,\mu H$.

Useful Matlab functions: `ss()`, `step()`, `pole()`, `pzmap()`.

Exercise 2 - Circuit diagram: Frequency and step response

Consider the RL circuit of Fig.2

a) Derive the transfer function from voltage input to current for the RL circuit.

b) Plot the step response for a 1 V step for $R = 1\ \Omega$ and $L = 1\ \mu H$, $L = 10\ \mu H$, and, $L = 100\ \mu H$.

c) Sketch the bode plot for the different inductances. Compare them by using the matlab bode function. Use the Bode diagrams to explain the step responses.

d) Convert the transfer function to a discrete-time plant with Tustin mapping. Note that it is a rational function of z^{-1}. Study its stability with various settings. Recall that a discrete-time systems is stable if for all z-plane eigenvalues $|p_n| \leq 1$.

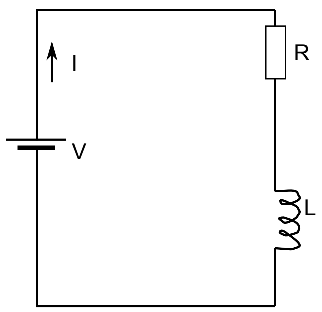


Figure 2: A circuit with a resistor and an inductor.

Useful Matlab functions: `zpk()`, `tf()`, `impulse()`, `bode()`, `c2d()`.

Exercise 3 - P-controller design

In this exercise we will design a proportional controller for a stable dynamical system. The control scheme is given below.

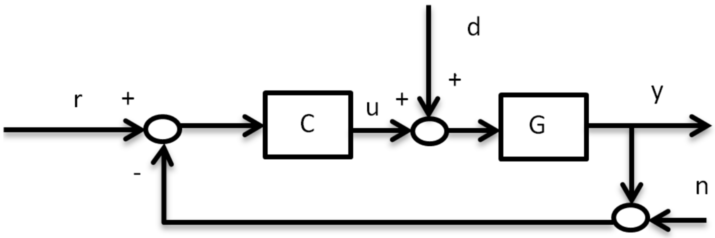


Figure 3: The closed loop for the system $G(s)$ controlled with a controller $C(s)$ in Ex. 3.

The controller computes inputs $u(t)$ to help ensure that the output $y(t)$ follows the reference $r(t)$, even in the presence of input disturbances $d(t)$ and measurement noise $n(t)$.

Robust stability for the closed loop of a stable system $G(s)$ controlled with a controller $C(s)$ can be ensured as follows:

1. In the case of open-loop stable systems, the Nyquist criterion for closed-loop stability requires that in the so-called Nyquist diagram (real vs imaginary plot of the open loop $G(i\omega)C(i\omega)$ for varying frequency ω), the curve $G(i\omega)C(i\omega)$ in the direction of increasing frequencies ω has the point $(-1, 0)$ always on the left hand side.

2. Once stability is obtained, a modulus margin of 6 dB guarantees robustness (stability for a limited variation of the system parameters). The modulus margin is the maximum value of sensitivity function $S = (1 + G(s)C(s))^{-1}$. It also means that the Nyquist plot does not pass through the circle with radius 0.5 around the point $(-1, 0)$.

a) For the plant $G(s)$:
$$G(s) = \frac{0.5}{(s + 0.5)(s + 0.01)}$$
Design a proportional controller $C(s) = K$ with $K \in \mathbb{R}$ that provides a maximum bandwidth (frequency up to which $\mid G(s)C(s) \mid \geq 1$), while ensuring stability.

b) Verify both conditions (Maximum sensitivity function and Nyquist criterion) for your controller.

c) Derive the transfer function between the $u(t)$ and $r(t)$, $y(t)$ and $r(t)$, $y(t)$ and $d(t)$.

d) Plot the step response for the above derived transfer functions.

e) Add a delay to the controller response $u(t)$ (look how to define a delay with InputDelay and pade). Plot the root locus for the closed loop system with and without the delay and study the effect on the stability of the system.

f) Plot the bode diagram of the open loop system, with and without the controller response delay. Why is an infinite bandwidth not possible in practice?

Useful Matlab functions: `feedback`(), `nyquist`(), `margin`(), `rlocus`(), `stepinfo`(), `dcgain`()