Kinetic control of tokamaks

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Outline I

1. Kinetic control
   - 0D model of tokamak confinement
   - Sources of power, particles and current
     - Plasma self-generated sources
     - Auxiliary sources of heat and current
   - Sinks of power and particles
     - Confinement transitions
   - Particle transport
   - Sensors/Diagnostics for kinetic control
   - Current practice for kinetic control

Section 1

Kinetic control

Subsection 1

0D model of tokamak confinement

0D models of tokamaks

- Energy balance equation for plasma pressure $p = (n_e T_e + n_i T_i) \approx 2nT$ (or thermal energy denstiy $w_{th} = \frac{3}{2}p$)

$$\frac{3}{2}\frac{\mathrm{d}p}{\mathrm{d}t} = S_{source} - S_{sinks} \quad (1)$$

where

- $S_{sources}$ is the source power density
- S_{sinks} is the sink power density

See the book by Freidberg for an overview of power balance considerations for reactors, Lawson's criterion, etc. [1]

0D models of tokamaks

- Particles

$$\frac{\mathrm{d}n}{\mathrm{d}t} = S_{source,n} - S_{sink,n} \quad (2)$$

  - $S_{sources,n}$ is the source of particles
  - $S_{sinks,n}$ is the sink of particles
- Global steady-state current balance

$$I_{pl} = I_{p,inductive} + I_{p,non-inductive} = V_{loop} R_{plasma} + I_{p,non-inductive} \quad (3)$$

  - Inductive current: current driven by (induced) loop voltage.
  - Non-inductive current: other sources, self-generated by plasma or auxiliary current drive.

Subsection 2

Sources of power, particles and current

Fusion power

- $\frac{4}{5}$ of each fusion reaction's energy is in the fast 14MeV neutron. To be used by breeder to breed tritium and to extract energy.
- $\frac{1}{5}$ of each fusion reaction is contained in a 3.5MeV α-particle.
- Alpha particle power density (for DT reactions) is

$$S_\alpha = E_\alpha n_D n_T \langle \sigma v \rangle = \frac{f_{DT}}{(1 + f_{DT})^2} E_\alpha n^2 \langle \sigma v \rangle \quad (4)$$

  where $f_{DT} = n_D / n_T$ and we have used $n_T + n_D = n$
- For sustained fusion reaction, significant fraction of plasma thermal energy must come from α-particles.
  $P_\alpha / P_{in} = 1 \leftrightarrow Q = P_{fus}/P_{in} = 5$

Ohmic power

- The plasma current causes resistive heating $P_{oh} = V_{loop} I_p = I_p^2 R_p$.
- 'Ohmic plasmas' are plasmas heated purely by Ohmic power, no auxiliary sources.
- Unfortunately, resistivity decreases with plasma temperature, so P_{oh} decreases. Purely ohmic tokamak plasmas can't reach high enough temperatures for ignition.

Bootstrap current

- Self-generated plasma current due to neoclassical (trapped particle) effects (see [2] for the physical explanation).
- Caused by pressure gradient, proportional to $\frac{\partial p}{\partial \psi}$.
- For steady-state tokamaks, we want the entire plasma current driven non-inductively. But auxiliary current drive is expensive.
- For steady-state tokamaks, we would like a *large bootstrap current fraction*, $I_{BS}/I_p > 50\%$, rest by auxiliary current drive.
- Active field of research in 'advanced scenarios': most current driven by bootstrap + auxiliary.

Neutral beam injection

- Inject high-energy beam of neutral particles, ionize upon entering plasma
- Beam ions thermalize by colliding with plasma ions, kinetic energy of beam becomes thermal energy of plasma
- Also drive electrical current and give momentum (rotation) to the plasma

- Advantages
  - ‘Workhorse’, good for bulk heating and current drive.
- Disadvantages
  - Power can only be on or off for each injector.
  - Technologically difficult for high energy and high power. Need ‘negative ion’ beams.

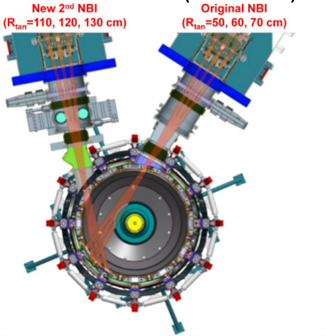
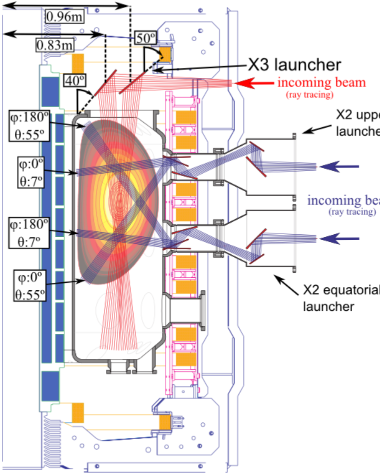


Figure: NSTX

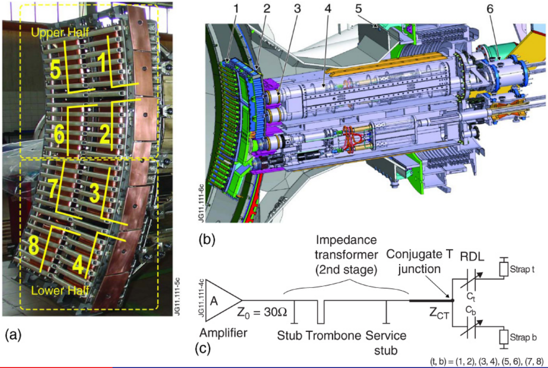
Electron Cyclotron Heating & Current Drive

- Electrons gyrate around field at cyclotron frequency $\omega_c = \frac{eB}{m}$
- EM waves resonantly heat electrons.
- Advantages:
  - Waves propagate through vacuum, no coupling problems.
  - Steerable: highly localized heating/cd location in plasma.
  - Can drive electric current.
- Disadvantages:
  - Heats electrons only.
  - Concentrated stray radiation may damage wall or diagnostics, need protection.
  - Difficult technology for sources ($f \sim 170\mathrm{GHz}$ for $B_0 = 6T$).



Ion Cyclotron Heating and Current Drive

- Advantages:
  - Source technology easy, $f \approx 50\mathrm{MHz}$.
  - Heats the ions: good for fusion.
- Disadvantages:
  - Antennas must be close to plasma to couple power: antenna-plasma interaction and problems with impedance matching



Lower Hybrid Heating and Current Drive

- Advantages:
  - Source and trasmission technology is easy, 1 $- 8\mathrm{GHz}$.
  - Good at driving non-inductive plasma current.
- Disadvantages:
  - Only electron heating.
  - Technologically difficult coupling of antenna to plasma.
  - Hard to tell where the power/current will go.

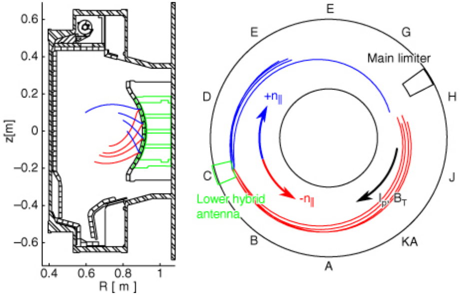


Figure: LHCD wave propagation according to Genray code. From [3]

Subsection 3

Sinks of power and particles

Radiation

- Physical origin
  - Cyclotron: radiation due to centripetal acceleration of gyrating particles.
  - Line radiation: ionization processes involving bound charge states.
  - Bremsstrahlung: radiation due to acceleration (change of velocity direction) of particles during collisions.
- Bremsstrahlung is usually dominant for large hot tokamaks.
  - Approximate model:

$$S_{brem} = 5.35 \times 10^3 Z_{\text{eff}} (n_{e20})^2 T_e^{1/2} \tag{5}$$

  - Here, $Z_{\text{eff}} = \frac{1}{n_e} \sum_j Z_j^2 n_j$ is the effective charge. (j sums over impurity species). Impurities have higher charge and cause electrons to radiate more. Important to keep Z_{eff} low.
- If high-Z impurities (e.g. Tungsten) accumulate in plasma, then impurity line radiation may dominate.

Conductivity losses

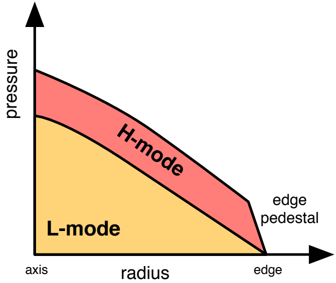
- The temperature gradient from core to edge causes outward thermal diffusion.
- Ultimately, energy flows out of the plasma into the limiter or divertor.
- This is quantified by the energy confinement time. Neglecting all other sources and sinks we have

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -S_{cond} = -\frac{1}{\tau_E} p \tag{6}$$

  so the thermal energy (pressure) decays with characteristic time τ_E.
- Unfortunately, τ_E decreases with increasing input power. Otherwise we could reach any temperature by just heating sufficiently.
- Until very recently, tractable first-principle models for τ_e did not exist: use *scaling laws* instead.

L-H transition

- L-mode: ‘normal’ model of operation for tokamaks.
- H-mode: Transport is locally suppressed near the edge. Edge Transport Barrier gives *edge pedestal*.
    - Pressure pedestal gives extra pressure in entire plasma
    - Occurs in diverted plasmas, after reaching *power threshold*
    - Detailed mechanisms not completely understood, involves complex turbulence and flow.
    - Edge pedestal can repetitively collapse, origin of Edge Localized Mode (ELM).



Subsection 4

Particle transport

Gas valves/pellet injectors

- Gas valves
  - Inject gas into vacuum chamber
  - Neutral gas particles become ionized soon after they enter the plasma.
  - Time delays if valves are far away.
  - Gas stays near edge and does not penetrate into the plasma in certain conditions.

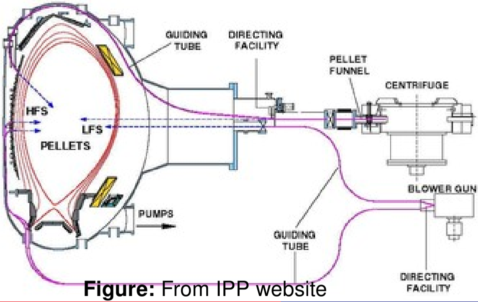


Figure: From IPP website

- Pellet injectors
  - Inject small pellets of frozen D_2 or H_2 ice at high speed.
  - Penetrates into the plasma before ablating.
  - Can be used for localized fuelling deep in the plasma.

0D model of tokamak particle confinement

- Three 'reservoirs': plasma, vacuum, wall.
- Flows:
  - **Ionization**: neutrals in vacuum get ionized when entering the plasma.
  - **Recombination:** ions in plasma get neutralized and leave plasma
  - **SOL losses:** plasma form the *scrape-off layer* exit plasma and impact limiter/diverted.
  - **Recycling:** Particles from wall pushed out by new incoming paticles.

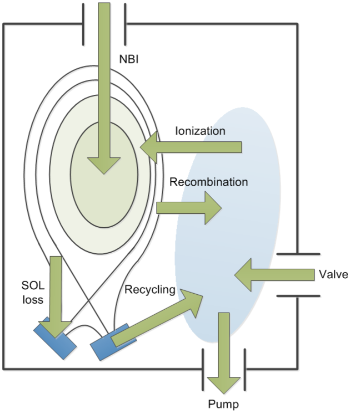


Figure: Particle flows in a tokamak reactor [T. Blanken Fus.Eng.Des 2017]

Subsection 5

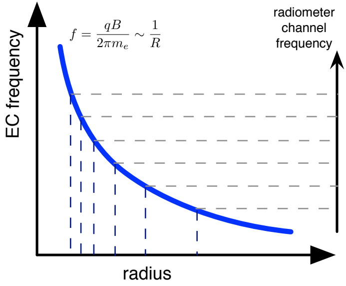
Sensors/Diagnostics for kinetic control

Sensors, aka Diagnostics

- Measurements of high-temperature plasma are not easy.
- Entire courses exist on plasma diagnostics.
- Here we look only at the main diagnostics that are used for real-time control.

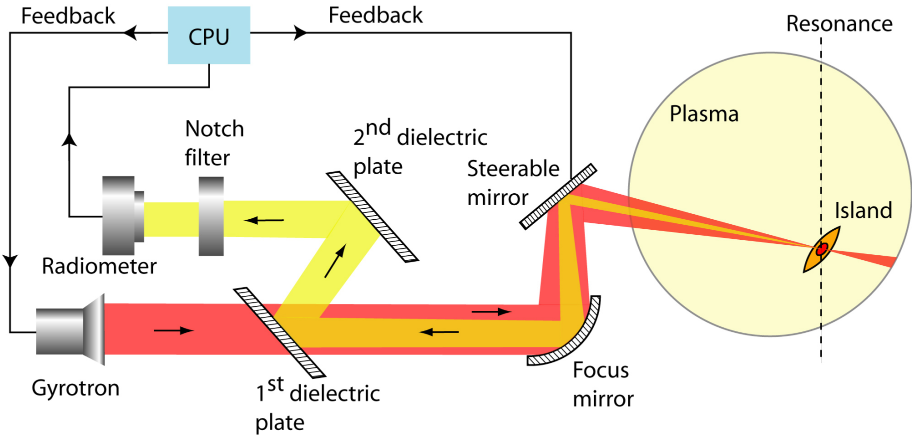
ECE - Electron Cyclotron Emission

- Measure EC radiation leaving the plasma.
- Choose measured frequency $\rightarrow$ nominal emission locadtion inside plasma.
- In practice, thermal, supra-thermal, and relativistic effects play an important role, often source of radiation is not well-localized.
- Advantages
  - High temporal resolution.
  - Viewing angle can be steered (in-line ECE).
- Disadvantages
  - Measurement location depends on B field.
  - Cutoff at high density.



In-line ECE

- ECE line of sight shared with ECRH launcher [4].
- Steerable line of sight, sensing and actuating in same location.



Thomson scattering

- Laser scatters off plasma electrons.
- Intensity of scattered light is proportional to plasma density.
- Broadening of scattered spectrum is proportional to plasma temperature.
- Advantages
  - Localised measurement.
- Disadvantages
  - Laser repetition rate limits temporal resolution.

Radiation measurements

- Soft X-ray
  - Plasma core Bremsstrahlung and line radiation is mostly in the X-ray part of the EM spectrum
  - X-ray detectors can be used to measure line-integrated X-ray radiation in given portion of the spectrum.
  - Can be processed by tomographic inversion techniques to get pictures of plasma position, shape, or internal plasma fluctuations.
- Bolometry
  - Broadband measurement of Visible + UV + X-ray radiation
  - Used to get total radiated power.

Interferometry

- Laser beam follows 2 paths, one through plasma, one not.
- Beam through plasma travels more slowly, phase shift w.r.t. unperturbed beam.
- Interference pattern with other beam gives measure of plasma density.
- Measurement of *line-integrated electron density*: $\Delta\phi = \frac{\lambda e^2}{2\pi\epsilon_0 m_e c^2}\int_L n_e dl$ where L is the path of the chord.

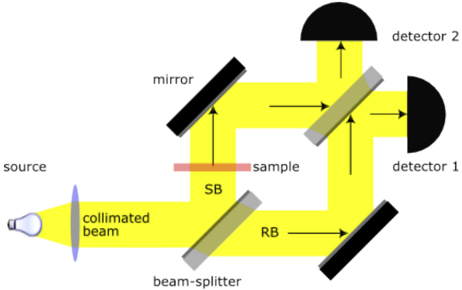


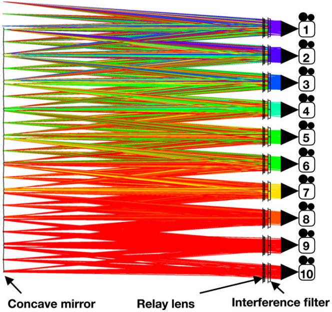
Figure: Schematic diagram of a Mach-Zehnder interferometer

Motional Stark Effect diagnostic

- Neutral particles (from NBI source) passing through magnetized plasma experience $E = v \times B$ field.
- When they ionize the visible light is split under the influence of this E field (Stark effect)
- Polarization of light is aligned with B field: localized measurement.
- Advantages
  - Localized measurement of internal plasma B field: hard to obtain otherwise.
- Disadvantages
  - Technologically difficult, stray polarized light, difficult to calibrate.

Real-time imaging diagnostics

- Real-time camera image processing
- MANTIS [5]: multispectral imaging using filters to extract light at specific wavelengths
- Specific wavelengths correspond to specific atomic processes, linked to temperature and density
- RT-analysis of camera images enabled by ML



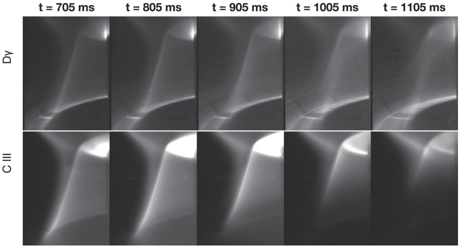


Figure: MANTIS images of *CIII* and D_γ spectral lines (from [5])

Inverting line-integrated measurements

- Given measurements $y_j = \int_{L_j} x(R,Z)dl$ along multiple chords j, how do we reconstruct $x(R,Z)$?
- Tomographic, or Abel inversion. Assume x is parametrized, for example

$$x = \sum f_i(\psi_N(R,Z))b_i \quad (7)$$

where $\psi_N = (\psi(R,Z) - \psi_a)/(\psi_b - \psi_a) \in [0,1]$ and f is some basis function. Then given $\psi_N(R,Z)$ (from equilibrium reconstruction) we can construct $x(R,Z) = \sum_i F_i(R,Z)b_i$ and write

$$\hat{y}(x) = \sum_i b_i \int_L F_i(R,Z)dl \quad (8)$$

and we can solve the least-squares problem

$$\min_b \sum_i \sum_i \left(y_j - \sum_i b_i \int_{L_j} F_i(R,Z)dl \right) \quad (9)$$

New development: state observers

- Tomographic inversion methods are *static*, they treat each measurement sample as independent in time.
- Ongoing work: State observers for interpreting measured signals from plasma including model knowledge.

Standard solution: **static inversion**

$$y = Cx \rightarrow x = C^{+}y \qquad (10)$$

where C^{+} is some inverse of C.

- Simple and robust.
- Does not take time evolution of the system into account.

Observer: **dynamic state estimator**

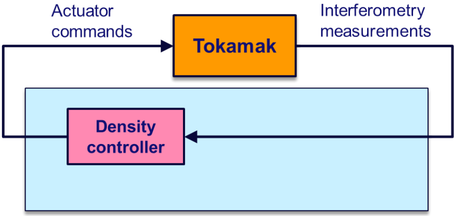
$$\dot{x} = Ax + Bu + K(\hat{y} - y) \qquad (11)$$
$$\hat{y} = Cx \qquad (12)$$

Where K is the *Kalman Gain*. This scheme is known as a **Kalman Filter**.

- K can be designed from knowledge of A, B, C, D and the covariance of the expected noise.
- It can be shown that this is the *optimal* filter for this system, for which $E[(x - \hat{x})^2]$ is minimal.
- See course/book on systems theory or estimation for details.

State observer: example for density control

- Current practice - control of gas valve via single interferometer chord



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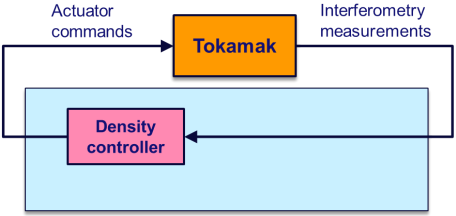
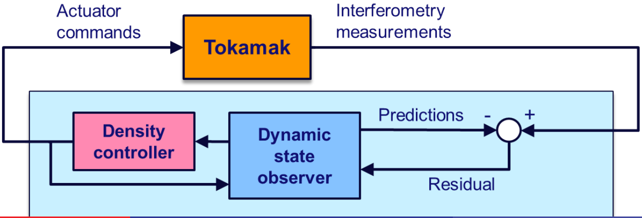


- New approach: model-based observer



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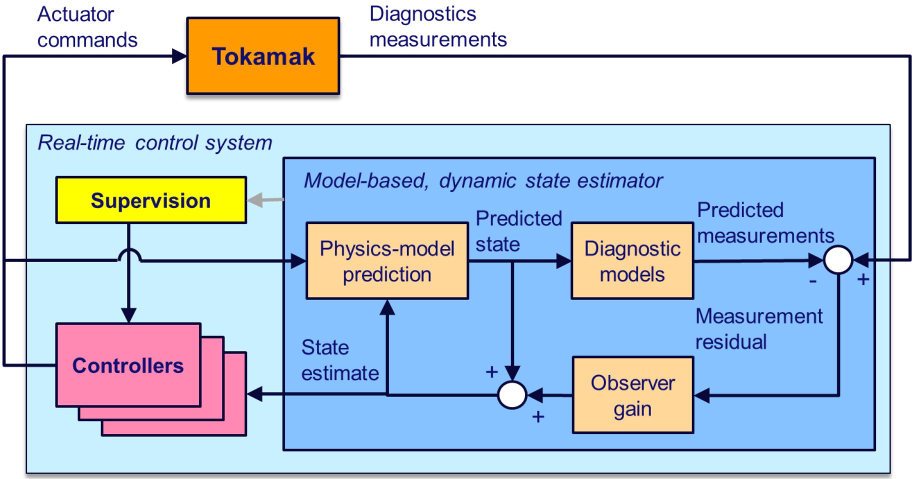

Subsection 6

Current practice for kinetic control

Common practices, challenges for kinetic control

- Series of SISO control loops. Direct link between diagnostic and actuator.
  - Single interferometer chord $\rightarrow$ plasma gas valve
  - Plasma β from magnetic measurements $\rightarrow$ NBI or ECRH power.
  - Individually tuned PID controllers.

Common practices, challenges for kinetic control

- Series of SISO control loops. Direct link between diagnostic and actuator.
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  - Plasma β from magnetic measurements $\rightarrow$ NBI or ECRH power.
  - Individually tuned PID controllers.
- β control usually works quite well.
- Some issues with density controllers:
  - Single-sided actuator: at most can *close* the valve valve, can not extract more particles. Need *anti-windup* compensation.
  - Most simple controllers do not work well for all density regimes. Need to re-tune and re-commission controllers.
  - Gas valve vs pellet fuelling, different efficiency depending on density/temperature

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- Ongoing research topics:
  - Control of radiation fraction and heat flux to mitigate wall loads.
  - Actuator management: RT allocation of actuators to various tasks.
  - Burn control: nonlinear control problem & solution.

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