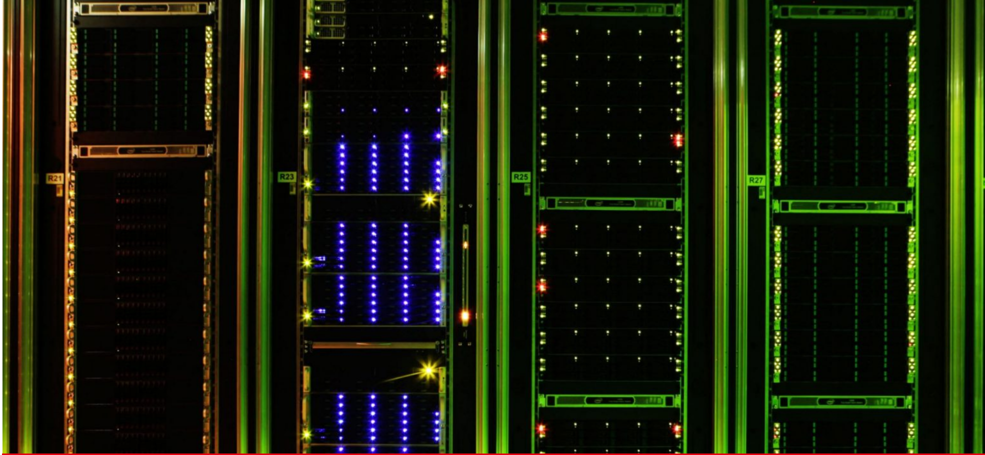

Parallel Programming

Presentation of the exercises

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SCITAS

- The goal of this exercise is to compute an approximate value of π
- To compute π, we will use two identities:

$$\arctan(1) = \frac{\pi}{4} \tag{1}$$

$$\int_0^x \frac{1}{1+y^2}\mathrm{d}y = arctan(x) \tag{2}$$

Then, it follows that

$$\pi = 4\int_0^1 \frac{1}{1+x^2}\mathrm{d}x \tag{3}$$

- Using a Riemann sum, we can approximate the integral in Eq. (2)

$$\pi \approx \frac{4}{n}\sum_{i=0}^{n} \frac{1}{1+(i/n)^2} \tag{4}$$

Poisson

- Let us consider the following Poisson problem

$$\begin{cases} \Delta u = f(x,y) & \text{in} \quad \Omega, \\ u(x,y) = 0 & \text{on} \quad \partial\Omega. \end{cases} \tag{5}$$

- The 2D domain is uniformly discretized using $N+1$ points in each direction:

$$\begin{cases} (x_i, y_j), \\ \{x_i\}, i = 1, ..., N+1, \\ \{j_i\}, i = 1, ..., N+1. \end{cases} \tag{6}$$

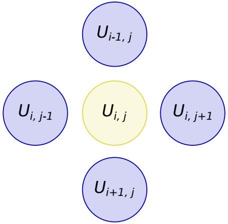
  The grid spacings are $\Delta x = \Delta y = 1/N$ (we assume domains of length one).

- The Laplacian is approximated using second-order centered finite-differences:

$$\begin{aligned} \Delta u(x,y) &= \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \approx \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2} \\ &= \frac{u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j}}{\Delta x^2} \end{aligned}$$

Poisson

- This defines a 4-point stencil



- The system of linear equations is solved using the Jacobi method
- Iterations are done until the error is lower than a prescribed tolerance

Poisson

- Let solve the Poisson problem with

$$u(x, y) = 0 \tag{7}$$

on the boundaries and

$$f = -200\pi^2 \sin(10\pi x)\sin(10\pi y) \tag{8}$$