Introduction to holography - Lecture XIII

Last time : AdS/CFT dictionary relevant for computing **correlation functions**

$$Z_{AdS}[\phi_0] = e^{-W_{CFT}[\phi_0]}$$

- studied about the vacuum, whose isometries det. the str. of 2 & 3 p.f. ✓CFT.
  fixed backgnd
- renormalization was necessary to make sense of both sides (UV/IR conn.)

This time : **finite-temperature** AdS/CFT (non-trivial state)

dictionary : $Z_{AdS}[\text{bnd. cond}] = Z_{CFT}$ ← understand
a gravity

The path-integral approach to quantum gravity

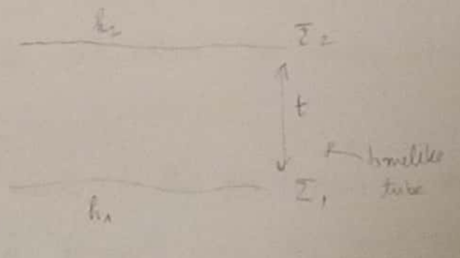
(a few words about)

- generally speaking, path int. compute **transition amplitudes**

$$\langle \phi_2 | e^{-iHt} | \phi_1 \rangle = \int_{\phi(t=0)=\phi_1}^{\phi(t)=\phi_2} \mathcal{D}\phi \, e^{iS[\phi]}$$

- to ensure **convergence** of the above path int, one Wick rotates $t \to -i\tau$
  for standard fields $S_E[\phi] = -i S_L[\phi]$ is ≥ 0

- the GR analogue of this is (for non-compact spt) : choose two spacelike



surfaces $\Sigma_{1,2}$ that are separated by a time interval t @ **infinity** (assume we fixed bnd. cond @ infinity for h_i → well-def notion of AF or aAdS time).

- & specify induced metric on them (up to (allowed) diffeos that leave ∂M invar)

- transition ampl. $= \int_{h|_{\Sigma_1}=h_1}^{h|_{\Sigma_2}=h_2} \mathcal{D}h \, e^{i S_{grav}}$

- $S_{grav} = \frac{1}{16\pi G}\int_{\mathcal{M}} d^{d+1}x\,(R-2\Lambda)\sqrt{g} + \underbrace{\frac{1}{8\pi G}\int_{\partial M} d^dx\,\sqrt{\gamma}\,K}_{\text{var. principle, also glueing}} + \int \mathcal{L}_{matter}\sqrt{g}\,d^{d+1}x$

- oscillatory action contr $\to$ Wick rotate $t \to -i\tau$

- this Wick rot would make the induced metric on the "timelike tube" be pos. def.

  $\Rightarrow$ prescription : path-int over all positive def^n metrics g $\ni$ w/ (pos def^n) induced bnd. met. h.

- the Euclidean action $S_E = -i S_L = -\frac{1}{16\pi G}\int d^{d+1}x\,\underbrace{\sqrt{g_E}(R_E-2\Lambda)}_{\text{not positive def}^n!} - \frac{1}{8\pi G}\int K\,d^dx\sqrt{\gamma_E} + S_{matter}$

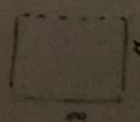
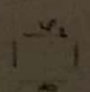
- to see this, consider a conformal transf $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$, under which

$$\tilde{R} = \Omega^{-2}R - 6\Omega^{-3}\Box\Omega \qquad \tilde{K} = \Omega^{-1}K + 3\Omega^{-2}\nabla_a\Omega\, n^a \qquad \text{for } (d+1=4)$$

  $\Rightarrow$ Ω comes w/ a negative kinetic term $\Rightarrow$ euclidean action unbounded from below
  (pot. eng. negative $\Leftarrow$ attractive gravity) ? role b.h?

  - ad-hoc fix : let $\Omega = 1+y$ & rotate $y = i\xi$, int over $\xi \in \mathbb{R}$. $\Rightarrow$ convergent result

  - the grav. action can be split into one w/ fixed R (pos. def. for $R>0$) (equiv. cls of metrics rel via conf. transf) & one for the conf. factor, where the op $-\Box + \frac{1}{6}R$ enters.
    the situation is more complicated if this op. has negative eigenvalues

- coming back to the path int, note that taking the initial state to lie @ $\tau_\epsilon \to -\infty$ projects onto the gnd. state $|\psi_0\rangle =$  $\to$ vacuum b/c field excit. die off
  or $\langle\varphi_2|\psi_0\rangle =$ 

- the trace over the Hilbert sp is obtained by perf. the path int. w/ periodically identif. eucl. time $Z_\beta = \mathrm{Tr}\, e^{-\beta H}$ 

- finally, a cut in an Euclidean path. int. defines a state, also in the Lorentzian th., which we may choose to then evolve in Lorentzian time