Introduction to holography - Lecture V

Last time : 't Hooft large N expansion of gauge th. ≈ perturbative string exp.

renormalizable!

- classical bosonic string : Nambu-Goto $^{[x^\mu]}$, Polyakov $[x^\mu, \gamma_{ab}]$ + gauge fixing $\gamma_{ab} = \eta_{ab}$ using diff + Weyl ⇒ D free bosons, x^μ in 2d Minkowski sp + constraint eqns $T_{\pm\pm} = (\partial_\pm X^\mu)(\partial_\pm X_\mu) = 0$ + bnd. cond. : closed vs. open strings

This time : quantization, target sp. spectrum & low-eng eff action, D-branes

- for a closed string the soln to the e.o.m (periodic $\sigma \in (0, 2\pi)$) can be expanded in left / right-moving modes $X^\mu(\tau,\sigma) = X^\mu_L(\sigma^+) + X^\mu_R(\sigma^-)$ $\qquad \sigma^\pm = \tau \pm \sigma$

$$X^\mu_{\substack{L\\R}}(\sigma^\pm) = \frac{1}{2}\underbrace{x^\mu}_{\substack{\text{C.M.}\\ \text{position}}} + \frac{1}{2}\alpha' \underbrace{p^\mu}_{\substack{\text{C.M.}\\ \text{momentum}\\ \text{(in target sp.)}}} + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n} \underset{\tilde{\alpha}^\mu_n}{\alpha^\mu_n} e^{-in\sigma^\pm} \qquad \underset{\text{reality}}{(\alpha^\mu_n)^* = \alpha^\mu_{-n}}$$

- for an open string w/, say, Neumann bnd. cond $\underbrace{\partial_\sigma X^\mu \Big|_{\sigma = 0,\pi} = 0}_{\text{(no momentum flowing out endpoints)}}$ $\quad (\tilde{\alpha}^\mu_n = \alpha^\mu_n)$

$$X^\mu(\tau,\sigma) = x^\mu + 2\alpha' \underbrace{p^\mu}_{\substack{\text{C.M.}\\ \text{momentum}}} \tau + i\sqrt{2\alpha'} \sum_{n\neq 0} \frac{1}{n} \alpha^\mu_n e^{-in\tau} \underbrace{\cos n\sigma}_{\text{standing waves}}$$

σ → -σ refl by Neum

- factor of 2 due to $\sigma \in (0,\pi)$ for the open string : $p^\mu = -\frac{1}{4\pi\alpha'}\int_0^\pi d\sigma \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu} = \frac{1}{2\pi\alpha'}\int_0^\pi \dot{x}^\mu$

- the constraints are, classically $\qquad \alpha^\mu_0 \equiv \sqrt{\frac{\alpha'}{2}}\, p^\mu$

$$\infty\ \#\ \begin{cases} T_{++} = \partial_+ X^\mu \partial_+ X_\mu = \alpha' \sum_n L_n e^{-in\sigma^+} \underline{= 0} \quad 0 = \boxed{L_n \equiv \frac{1}{2}\sum_m \alpha^\mu_m (\alpha_{n-m})_\mu} \\ T_{--} = \alpha' \sum_n \tilde{L}_n e^{-in\sigma^-} = 0 \quad \text{or} \quad \tilde{L}_n = \frac{1}{2}\sum_m \tilde{\alpha}^\mu_m (\tilde{\alpha}_{n-m})_\mu = 0 \end{cases} \qquad \tilde{\alpha}^\mu_0 = \sqrt{\frac{\alpha'}{2}}\, p^\mu$$

- they are conserved, as cls. conformal invar. of $S_P = \int d^2\sigma\, \partial_+ X^\mu \partial_- X_\mu$ ⇒ $\int d\sigma f(\sigma^\pm) T_{\pm\pm}$ is conserved.
(residual symm. after fixing diff + Weyl).

Quantization

- promote α_n^μ, $\tilde{\alpha}_n^\mu$ to operators w/ canonical comm. rels determined by

$$[X^\mu(\sigma,\tau), \Pi_\nu(\sigma',\tau)] = i\delta^\mu{}_\nu\,\delta(\sigma-\sigma') \qquad [X^\mu, X^\nu] = [\Pi_\mu, \Pi_\nu] = 0$$

which imply that $[\alpha_m^\mu, \alpha_n^\nu] = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = m\,\eta^{\mu\nu}\,\delta_{m+n,0}$ $\qquad [\hat{x}^\mu, \hat{p}_\nu] = i\delta^\mu_\nu$

2 ∞ towers of oscillators

- α_m^μ, $m>0$ are annihilation operators
- α_{-m}^μ, $m>0$ are creation ops, $\alpha_{-m}^\mu = (\alpha_m^\mu)^\dagger$, in standard norm $\frac{\alpha_{-m}^\mu}{\sqrt{m}}$

- define vacuum state $\quad \alpha_n^\mu |0\rangle = \tilde{\alpha}_n^\mu |0\rangle = 0 \quad , \forall n>0$

$$\underset{\text{operator}}{\hat{p}^\mu} |0\rangle = \underset{\text{eigenvalue}}{p^\mu} |0\rangle \quad \Rightarrow \text{better notation } |0;p^\mu\rangle$$

* note $|0;p^\mu\rangle$ is the vacuum state of a single string (not of the spt), which can carry momentum p^μ

- build Fock space of states as in standard QFT
  - excited states $\quad \alpha_{-1}^{\mu_1} \dots \alpha_{-1}^{\mu_n}\,\alpha_{-2}^{\nu_1} \dots\, |0, p^\mu\rangle$
  - classified into representations of the target sp. Lorentz gp.

* note this Fock sp. contains many negative norm states, due to $[\alpha_m^0, \alpha_n^0] = -m\delta_{m+n}$

- impose the constraints on physical states : $L_m |\phi_{phys}\rangle = \tilde{L}_m |\phi_{phys}\rangle = 0$
  - normal ordering ambiguities (in L_0 only)
  - the L_m do not commute QM $\quad [L_m, L_n] = (m-n)\,L_{m+n} + \frac{D}{12}\,m(m^2-1)\,\delta_{m+n}$

- consequently, we will only require that $L_m|\phi_{phys}\rangle = 0$, $\forall\, m>0$

$$\big(\underbrace{L_0}_{N.O.} - \underbrace{a}_{\text{ambiguity}}\big)|\phi_{phys}\rangle = 0$$ where

$$L_0 \equiv \frac{1}{2}\alpha_0^2 + \sum_{n=1}^{\infty} \alpha_{-n}^{\mu}\alpha_{n,\mu}$$

+ RM. for the closed string.

- this is sufficient to ensure that $\langle\psi_{phys}|L_{m_1}\dots L_{m_n}|\phi_{phys}\rangle = 0$ $\forall\, \psi_{ph}, \phi_{phy}$ & $\forall\, m_i$, not necessarily positive. If all negative ones are to the left of pos. ones, then ok, since $\langle\psi_{phys}|L_{-m} = (L_m|\psi_{phys}\rangle)^{\dagger} = 0$; note, however there could be c-# commutators that don't vanish

- note also that $\nexists$ an ordering ambiguity in the target sp angular mom. ops

$$J^{\mu\nu} = x^{\mu}p^{\nu} - x^{\nu}p^{\mu} - i\sum_{n=1}^{\infty}\frac{1}{n}\left(\alpha_{-n}^{\mu}\alpha_n^{\nu} - \alpha_{-n}^{\nu}\alpha_n^{\mu}\right)$$

& $[J^{\mu\nu}, L_n] = [J^{\mu\nu}, \tilde{L}_n] = 0$ $\Rightarrow$ phys. st. cond. are invariant under Lorentz transf

$\Rightarrow$ phys states will form Lorentz multiplets

(this quantiz. method is known as old covariant quantiz.)

- note that the $(L_0 - a)|\psi_{phys}\rangle = 0$ cond. determines $p^2 = -M^2$ $(\text{mass})^2$ of the string ≈ looks pointlike from far away ⇔ mass of the associated particle

in terms of the level $N = \sum_n n N_n$ (eigenval. of modified # op $\hat{N} = \sum_{n=1}^{\infty}\alpha_{-n}^{\mu}\alpha_{n,\mu}$) which counts # osc in the state, weighted by n

- for open strings $(L_0 - a)|\psi_{phys}\rangle = \left(\frac{1}{2}\alpha_0^2 + \hat{N} - a\right)|\psi_{phys}\rangle = 0 \Rightarrow \boxed{M^2 = \frac{1}{\alpha'}(N-a)}$

- for closed strings, need both $(L_0 - a)|\psi_{phys}\rangle = (\tilde{L}_0 - a)|\psi_{phys}\rangle = 0 \Rightarrow$ $M^2 = \frac{4}{\alpha'}(N-a) = \frac{4}{\alpha'}(\tilde{N}$

$\Rightarrow$ $N = \tilde{N}$ level matching (cond. that relates otherwise decoupled LM & RM)

determines spacetime spectrum of the string

- in order to det. spt. spectrum, crucial to det N.O. ct. a

- should also ensure constraints remove the negative norm states (≃ sufficient, as these states are generated by α^0_{-m}, & constr $L_m \sim \underbrace{\alpha_0^\mu}_{p^\mu} \cdot \alpha_{m,\mu} + O(\alpha^2)$ can in principle remove precisely α^0_m for timelike p^μ

- this (tedious) exercise can be done → need $\boxed{a=1}$ and $\boxed{D=26}$

  (more precisely, requiring no negative norm st. in OCQ leads to either $a=1, D=26$ (w/ tons of zero norm states that decouple) or $a \leq 1$, $D \leq 25$. To see inconsistency w/ the latter option, one should go to 1-loop)

- the necessity for this part. choice is easier to see in the (non-covariant) lightcone gauge, $\tau = X^+$ (the reparam. freedom left over in conformal gauge allows setting τ equal to any function that satisfies the free wave eqn., in part X^μ). The constraints then fully det X^- in terms of X^i, $i=1,\dots,D-2$. At level 1 (open string) ∃ D-2 states → not Lorentz multiplet unless $M^2=0$.

The string spectrum

- open string $M^2 = \frac{1}{\alpha'}(N - \underset{1}{\underbrace{a}})$

unstable vac. → ignore (susy)

level	state	mass^2	
$N=0$	$\|0;p^\mu\rangle$	$M^2 = -\frac{1}{\alpha'}$	tachyon
$N=1$	$\alpha^\mu_{-1}\|0,p^\mu\rangle$	$M^2=0$	massless vector
$N>1$	∞ tower of equally spaced massive states		

– Lorentz mult $J_{max} \sim N = \alpha' M^2 + 1$

Regge trajectory

- to see the state w/ $M^2=0$ $|\psi\rangle = \zeta_\mu \alpha^\mu_{-1}|0,p^\mu\rangle$ is indeed a gauge field note the phys. state cond $L_1|\psi\rangle = 0$ $\Rightarrow (\alpha_0^\nu(\alpha_1)_\nu + \sum_{m>0} \alpha_{-m}\cdot\alpha_{m+1})|\psi\rangle = 0$

  $\Rightarrow p^\mu \zeta_\mu = 0$

- the norm of these st. is $\langle\psi|\psi\rangle = \zeta^2 \begin{cases} D-2 \text{ positive norm st. } \perp p^\mu \\ \zeta^\mu \propto p^\mu \text{ zero norm state decouples.} \end{cases}$

- adding to it a spurious st. (⊥ all phys) of the form $L_{-1}|0,p^\mu\rangle = p_\mu \alpha^\mu_{-1}|0,p\rangle$, one can make $\zeta^\mu \to \zeta^\mu + \lambda p^\mu$ (gauge inv)

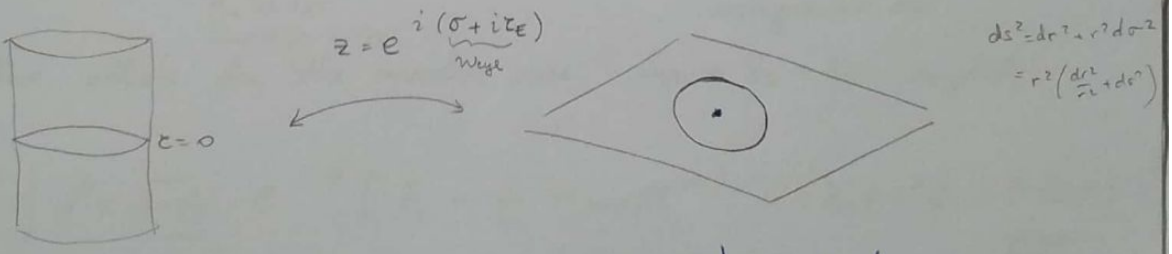
- closed strings

$$M^2 = \frac{4}{\alpha'}(N-a) = \frac{4}{\alpha'}(\tilde{N}-a)$$

level	state	mass^2	
$N=\tilde{N}=0$	$\|0,p^\mu\rangle$	$M^2 = -\frac{4}{\alpha'}$	tachyon
$N=\tilde{N}=1$	$\alpha_{-1}^\mu \alpha_{-1}^\nu \|0,p^\mu\rangle$	$M^2=0$	(*)
$N=\tilde{N}>1$	∞ tower of equally spaced massive states → Lorentz mult.		

- (*) decomposing $\alpha_{-1}^\mu \alpha_{-1}^\nu |0,p\rangle$ in terms of $SO(D-1,1)$ reps.
  - $g_{\mu\nu}$ symmetric traceless spin 2 graviton ! string. th. contains gravity
    (gauge symm visible as above)
  - $B_{\mu\nu}$ anti-symm. "B-field" 2-form gauge field coupling to the string $q\int d\tau\, A_\mu \dot{X}^\mu \overset{\text{pct part}}{\underset{\text{analogy}}{\longleftrightarrow}} \int d\tau d\sigma\, B_{\mu\nu}\, \varepsilon^{ab}\partial_a X^\mu \partial_b X^\nu$
    gauge invar $B_{\mu\nu} \to B_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu$
  - Φ trace : $\alpha_{-1}^\mu (\alpha_{-1})_\mu |0,p\rangle$ spin 0 dilaton

- note all these are states of the free boson CFT on the cylinder. In such th. $\exists$ a state - operator correspondence whereby $\overset{\sigma^\pm \to f(\sigma^\pm)}{}$ state on cyl $\leftrightarrow$ local op. on plane



vertex operator

- in this case, $g_{\mu\nu} \leftrightarrow h_{\mu\nu}\, \partial_a X^\mu \partial^a X^\nu e^{ip\cdot X}$

- exponentiating this ($\approx$ coherent st. of gravitons) $\Rightarrow S_p = \int d^2\sigma\, \eta_{\mu\nu} \partial_a X^\mu \partial^a X^\nu \to$
  $\to S_p + \int d^2\sigma\, h_{\mu\nu}(X) \partial_a X^\mu \partial^a X^\nu \Rightarrow$ the target sp. metric has become dynamical ! the $\underset{\text{vanishing}}{\text{wsheet}}$ β function for $\underset{\text{gen backgnd. metr}}{\underbrace{G_{\mu\nu}}}$ (w-sheet couplings) → Einstein eqn. in target sp !
  ↙ from consistency of string !

- the dilaton appears in the w-sheet action as $\int d^2\sigma \sqrt{\gamma}\, R[\gamma]\, \Phi$

  note that for $\phi = \phi_0 = const$, this term is $\propto \phi_0 \chi$ ← Euler char. of the w-sheet

  ⇒ the string coupling that counts topologies / string interactions

$$g_s = \langle e^{\phi_0} \rangle$$

↰ exp. value of a field in the th. (not an arbitrary param;

↑ ct. mode, usually as r→∞

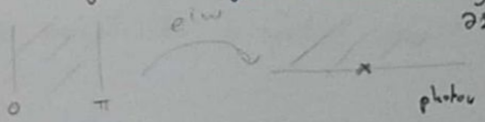
(needs to be small for string pert. th. to be valid)

Lessons : from a target sp. perspective, the various excitations of the string ↔ ∞ tower of massive (higher spin) part. w/ M^2 as above (spacing $\propto 1/\alpha'$)

- they all fit into massless / massive repr. of the Lorentz gp.

- @ the massless level, closed strings ⊃ graviton

  open strings ⊃ gauge boson $\int_{\partial\Sigma} d\tau\, A_\mu \frac{dX^\mu}{d\tau}$ ⇝ exp. photon vertex op.



- @ low energies ($E \ll \frac{1}{\sqrt{\alpha'}}$), the massive string modes can be integrated out

  $R_c \gg \sqrt{\alpha'}$

  needed for approx

  ⇒ effective action for the massless ones $S_{EH} + S_B + S_\phi$ coupled

$$\underset{\text{bosonic}}{S_{\text{closed}}} = \frac{1}{2\kappa_0^2} \int d^Dx\, \underset{\text{string-frame metric}}{\underset{\sim}{\sqrt{-G}}}\, e^{-2\phi} \Big[R - \frac{1}{12} \underset{\text{gauge field strength}}{\underset{\partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}}{H_{\mu\nu\rho}}} H^{\mu\nu\rho} + 4\, \partial_\mu \phi\, \partial^\mu \phi + \underset{\text{higher order corr. suppr. by } \alpha'}{\underset{\sim}{\mathcal{O}(\alpha')}} \Big]$$

κ_0^2 : $\alpha'^{\frac{D-2}{2}}$

- can redefine the target sp. metric as $\tilde{G}_{\mu\nu} = e^{-\frac{4\phi}{D-2}} G_{\mu\nu}$ Einstein frame metric

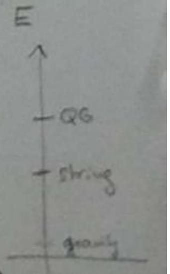
  ∋ the 1st term is standard Einstein - Hilbert.

- effective D-dim'l (D=10) Newton's const $G_D \sim (\alpha')^{\frac{D-2}{2}} g_s^2 \equiv \ell_p^{D-2}$

  ⇒ two scales : $\ell_s = \sqrt{\alpha'}$ effects of massive string modes

  $\ell_p = g_s^{\frac{2}{D-2}} \sqrt{\alpha'}$ quantum gravity effects

- if $\ell_p \ll \ell_s$ if $g_s \ll 1$



pert. string exp $\sum_{g=0}^{\infty} g_s^{2g-2} \mathcal{F}_g(\alpha'/R^2$

- open strings : massless mode → gauge field A_μ w/ low-eng. eff. action $\int d^Dx \left(-\frac{1}{4g_{s\alpha'}} F_{\mu\nu}F^{\mu\nu}\right)$

- if we consider instead superstrings (add fermions on w-sheet) ψ^μ
  - ① can project out the tachyons
  - consistent in $D=10$
  - additional massless fields "RR" (Ramond-Ramond)$(p+1)$-form gauge fields

    $C_{\mu_1 \cdots \mu_p \mu_{p+1}}$ totally antisymm , p even (IIA) / odd IIB

  - + fermions ; has target sp. supersymmetry (low-eng. eff. action highly constrained)

e.g

$$S_{IIB} = \frac{1}{(2\pi)^7 \alpha'^4} \int d^{10}_x \sqrt{G} \left\{ e^{-2\Phi}\left(R + 4(\partial\phi)^2 - \frac{1}{12}H^2_{\mu\nu\rho}\right) - \frac{1}{2}\left(F_\mu^2 + \frac{1}{3!}\tilde{F}^2_{\mu\nu\rho} + \frac{\tilde{F}^2_{\mu\nu\rho\sigma\lambda}}{5!}\right)\right\}$$

$$+ \text{Chern-Simons terms} + \text{fermions} \qquad F_{\mu_1 \ldots \mu_{p+2}} = \partial_{[\mu_1} C_{\mu_2 \ldots \mu_{p+2}]}$$

  - these RR $(p+1)$-form gauge fields naturally couple to p-spatial dim'l objects (they do not couple to the string)

- possible to obtain a non-linear generalization of this action valid for constant field strengths $(\partial F \approx 0)$ by performing path int on the disk (open string) in presence of a backgnd gauge field

$$Z[F] = \frac{1}{g_s}\int \mathcal{D}X^\mu e^{-S_P(X^\mu, A)} = \frac{1}{(4\pi^2\alpha')^5 g_s}\int d^p X_0 \sqrt{\det(\delta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}$$

tree ↗

Born-Infeld action

  - exact in α', but only valid for slowly-varying F
  - non-linear correction to electrodyn. (smoothens out div. near pt-like sources)

    e.g. in $d=4$ $\quad E_r = F_{rt} = \frac{Q}{\sqrt{r^4 + (\alpha' Q)^2}}$

- another interesting generalization : add non-dynamical Chan-Paton d.o.f. to the open string endpoints $\lambda^{i}{}_{j}$ ← fund / ← antifund of $U(N)$ → non-abelian Yang-Mills / BI

D-branes

- all discussion so far was perturbative in g_s

- will now discuss objects that will turn out to be **non-perturbative**

- They led to **huge** progress in string th: 1^{st} microscopic explanation of black hole entropy
  $S = \ln \Omega$ (Strominger-Vafa '96)

  – the discovery of AdS/CFT (Maldacena '97)

- importantly, ∃ **two** ways to think about them

① remember open strings can have either Neumann ($\partial_\sigma X^\mu = 0$) or Dirichlet ($\delta X^\mu = 0$) bnd. cond. @ their endpoints

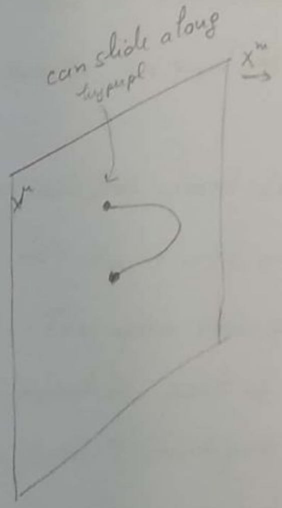
- a **Dp-brane** "Dirichlet membrane": p+1-dim'l hypersurface (p = # spatial dim) is specified by:

  - **Neumann** bnd. cond. **along** the hypersurface

$$\partial_\sigma X^\mu \big|_{\sigma=0,\pi} = 0 \qquad \mu = \{0, \dots, p\}$$

  - **Dirichlet** bnd. cond in the **transverse** directions

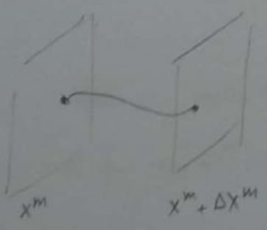
$$\delta X^m \big|_{\sigma=0,\pi} = 0 \qquad m = \{p+1, \dots, D-1\}$$



- mode expansion :

$$X^\mu(\tau,\sigma) = x^\mu + 2\alpha' p^\mu \tau + i\sqrt{2\alpha'} \sum_{n\neq0} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \cos n\sigma$$

$$N: \quad X^\mu_L(\tau,\sigma) = X^\mu_R(\tau,-\sigma)$$



string stretched b/w 2 D-branes

$$X^m(\tau,\sigma) = x^m + \frac{\sigma}{\pi}\Delta x^m + \sqrt{2\alpha'} \sum_{n\neq0} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \sin n\sigma$$

$$D: \quad X_L^m(\tau,\sigma) = -X_R^m(\tau,-\sigma).$$

- the mass shell cond. for such a string is $M^2 = -p^\mu p_\mu = \frac{1}{\alpha'}(N-a) + \frac{|\Delta\vec{x}|^2}{(2\pi\alpha')^2}$

  - massless states when endpts on the same D-brane. $\alpha_{-1}^{\mu}|0,p^\mu\rangle$, $\alpha_{-1}^{m}|0,p^\mu\rangle$

    - massless vector A_μ $\mu = \{0, \dots, p\}$ w/ coupling $\int_{\partial\Sigma} d\tau \, A_\mu \frac{dx^\mu}{d\tau}$

    - D-p-1 scalars Φ^m w/ coupling $\int_{\partial\Sigma} \phi^m \partial_\sigma X_m$

  - low eng. eff action $S \sim \int d^{p+1}x \left(-\frac{1}{4} F_{\mu\nu}^2 + (\partial\phi^m)^2 + \dots\right) \times \frac{1}{g_s \alpha'^{\frac{p-3}{2}}}$

  - also non-abelian generaliz SU(N) Yang-Mills $S = \frac{1}{g_{YM}^2} \int d^{p+1}x \left[-\frac{1}{4} \mathrm{tr}\, F^2 + \mathrm{Tr}(D_\mu\phi^m)\right.$ $\left. + \mathrm{tr}\,[\phi^m, \phi^n]^2 + \dots\right.$

- a priori, a D-brane is just a hyperplane where open strings can end

- however, it should be considered a fully dynamical object that is necessarily part of open + closed string th. $\uparrow$ makes sense in th. of gravity (no rigid objects)

  (this can be seen from T-duality, a symm. of compactified closed (bosonic) string th. under $R \to \frac{\alpha'}{R}$, which is implemented as $X_R \to -X_R$ on the w-sheet : interchanges momentum & winding. For open strings, this interchanges N & D bnd. cond $\Rightarrow$ D-branes must be part of string th. The gauge field $A_\mu \leftrightarrow \phi^m$ as can be seen from T-dualizing in presence of a non-triv. Wilson line. More general configs $\to$ wiggly hyperpl.)

- the scalars ϕ^m should be thought of as the transverse displacement of the Dp-brane ($\phi^m = \text{const} \Rightarrow \delta X^m = \phi_0^m$

- this is similar to the closed string case, where a massless closed string excitation ($h_{\mu\nu}\, \alpha_{-1}^{\mu} \tilde{\alpha}_{-1}^{\nu} |0,p\rangle$) about flat target sp. corresp. to a fluctuation of the target sp. geom.

- here, a massless open string state $\leftrightarrow$ fluctuation of the hypersurf $\Rightarrow$ D-branes are dynamical obj

- the coupling of D-branes to the target sp. fields is

world vol repa + cov

$$S_{DBI} = -\frac{1}{\alpha'^{\frac{p+1}{2}}}\int d^{p+1}x\,\left\{ e^{-\phi}\sqrt{-\det(G_{\mu\nu}+B_{\mu\nu}+2\pi\alpha' F_{\mu\nu})}+\dots\right\}$$

induced metric & B-field $\quad G_{\mu\nu} = \partial_\mu X^M \partial_\nu X^N G_{MN}$, etc.

- for slowly-var $F_{\mu\nu}$, T-duality inven

$T_p = \frac{1}{(2\pi)^p\,\alpha'^{\frac{p+1}{2}}}$ tension very heavy for g_s small

- in superstring theory Dp-branes can be shown to carry RR charge (min quantum).

$\mu_p \int d^{p+1}x\; C^{(p+1)}_{\mu_1\dots\mu_{p+1}}$ & to preserve 1/2 supersymmetry (very important for their identification @ strong coupling) - add coupling: $e^{2\pi\alpha' F + B_2}\wedge\sum_k C_k \Big|_{p+1\text{ form}}$.

- the low-energy effective action = supersymmetric version of SU(N) Yang-Mills

$$S = \frac{1}{g^2_{YM}}\int d^{p+1}x\left[-\frac{1}{4}\,\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{1}{2}\,\mathrm{tr}(\partial_\mu\phi^m\partial^\mu\phi_m) + \mathrm{tr}([\phi^m,\phi^n])^2 + \text{fermions}\right]$$

= dimensional reduction of $\mathcal{N}=1$ SYM in 10 d to p+1 dims

$\mathcal{N}=4$ in 4d

- $g^2_{YM} = g_s\,\alpha'^{\frac{p-3}{2}}$. For $p=3$ the SYM theory is conformally invar (CFT_4) and g_{YM} is an exactly marginal coupling (family of susy CFTs, param by g_{YM} & θ in $\theta\int F_{\mu\nu}\tilde{F}^{\mu\nu}$

$\downarrow$

$C_0\, F\wedge F$.

Summary : D-branes are non-perturbative objects in string th., which must be included if open strings are present (T-duality)

- 1/2 BPS (susic) & charged under the RR p+1-form fields
- described by SYM th. @ low energies $g^2_{YM} = g_s\,\alpha'^{\frac{p-3}{2}}$ - tractable if the 't Hooft coupling $\lambda = N g^2_{YM}$ is small