

# Magnetic modeling and control of tokamaks, Part V: Free boundary equilibrium evolution and control

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# Outline I

- 1 Free boundary evolution modeling
- 2 Plasma shape control
- 3 Plasma discharge evolution, from breakdown to plasma termination
  - Plasma breakdown
  - Ramp-up phase
  - Flat-top
  - Ramp-down

# Section 1

## Free boundary evolution modeling

# 'Forward' Grad-Shafranov equilibrium problem

- In previous lectures we saw how to solve the Grad-Shafranov equation for equilibrium (re)construction problems. We distinguished
  - the *inverse problem* (FBT) where we seek an equilibrium that minimizes a cost function based on the 'desired equilibrium properties' in terms of LCFS location, strike points, etc
  - The *reconstruction problem* (LIUQE) where we seek an equilibrium that minimizes a cost function based on measurements.
- Now consider the *forward problem* of finding an equilibrium given:
  - External currents  $I_e (= [I_a; I_u])$
  - The total plasma current  $I_p$
  - Other constraints equations on moments of the internal plasma profiles (e.g.  $\beta_p, q_A, \ell_i$ )

# Forward Grad-Shafranov equilibrium problem

- Given  $I_e, I_p, c_o$ , we seek a plasma current distribution vector  $I_y$  and basis function coefficients  $a_g$  such that:

$$F_y(I_y, I_e, I_p, a_g, c_o) = 0 \quad \text{residual related to the GS equation} \quad (1)$$

$$F_g(I_y, I_e, I_p, a_g, c_o) = 0 \quad \text{residual of the } I_p \text{ and constraint equations} \quad (2)$$

- How to compute  $F_y$ :

- Given  $I_y^{[n-1]} = j_\phi^{[n-1]} / \Delta S$  from a previous iteration, compute boundary condition  $\psi_b = M_{by} I_y^{[n-1]} + M_{be} I_e$
- Compute new flux by inverting Laplace operator:  $\Delta^* \psi^{[n]} = -2\pi R \mu_0 I_y^{[n-1]} \Delta S$  with boundary condition  $\psi_b$
- Find plasma boundary and domain where  $I_y^{[n]} \neq 0$
- Compute mapping between plasma current and basis function coefficients  $T_{yg}$ , by evaluating basis function expressions on  $\psi^{[n]}$ :  $p' = \sum_i b_i(\psi^{[n]}) a_g^i$ ,  
 $TT' = \sum_j b_j(\psi^{[n]}) a_g^j$
- Compute new plasma current distribution  $I_y^{[n]} = T_{yg}^{[n]} a_g$
- Return *plasma current distribution residual*  $F_y = I_y^{[n]} - I_y^{[n-1]}$

# Forward Grad-Shafranov equilibrium problem

- Residual equations are computed directly from the equilibrium at the present iteration, for example if imposing  $I_p, \beta_p, \ell_i$ :

$$F_g = \begin{bmatrix} I_{p,ref} - \sum_y I_y \\ \beta_{p,ref} - \beta_{p,eq}(I_y, I_e, a_g) \\ \ell_{i,ref} - \ell_{i,eq}(I_y, I_e) \end{bmatrix} \quad (3)$$

# Forward Grad-Shafranov equilibrium problem

- We have a problem of the form

$$F(x) = 0 \quad (4)$$

with unknowns  $x = \begin{bmatrix} I_y \\ a_g \end{bmatrix}$ .

- Solve using Newton method, iterating

$$x^{[n]} = x^{[n-1]} - \left( \frac{\partial F}{\partial x} \right)^{-1} F(x^{[n-1]}) \quad (5)$$

- Construct full Jacobian by Finite Differences or analytical expressions
  - Jacobian-Free Newton-Krylov method [1]: Find Newton step direction by approximating the column space of the Jacobian.
- This is implemented in the FGS code in the MEQ suite

# The plasma equilibrium response matrix

- Once the solution is found, we can construct the *plasma equilibrium response matrices*

$$\frac{\partial I_y}{\partial I_e} \quad \text{response to variation in external currents} \quad (6)$$

$$\frac{\partial I_y}{\partial I_p} \quad \text{response to variation in plasma current} \quad (7)$$

$$\frac{\partial I_y}{\partial \mathbf{c}_o} \quad \text{response to variation in internal constraints} \quad (8)$$

- These can be obtained by finite differences, or if Jacobians are known, by solving

$$0 = \begin{bmatrix} \frac{\partial F}{\partial I_y} & \frac{\partial F}{\partial \mathbf{a}_g} \end{bmatrix} \begin{bmatrix} \delta I_y \\ \delta \mathbf{a}_g \end{bmatrix} + \begin{bmatrix} \frac{\partial F}{\partial I_p} & \frac{\partial F}{\partial \mathbf{c}_o} & \frac{\partial F}{\partial I_e} \end{bmatrix} \begin{bmatrix} \delta I_p \\ \delta \mathbf{c}_o \\ \delta I_e \end{bmatrix} \quad (9)$$



# Free-boundary Grad-Shafranov evolution

- So far we assumed  $I_e$ ,  $I_p$  are given: this makes this a *static* problem.
- In reality  $I_e$ ,  $I_p$  will evolve in response to voltages, following Faraday/Ohm's law
- Add a circuit equation, and discretize

$$M_{ee}\dot{I}_e + R_{ee}I_e + M_{ey}\dot{I}_y = V_e \quad (10)$$

- Add a plasma current evolution equation

$$\frac{I_y^T}{I_p} M_{yy}\dot{I}_y + \frac{I_y^T M_{ye}}{I_p} \dot{I}_e + R_p I_p = 0 \quad (11)$$

- Discretize:

$$M_{ee}(I_e^k - I_e^{k-1}) + \Delta t R_{ee} I_e^k + M_{ey}(I_y^k - I_y^{k-1}) = \Delta t V_e \quad (12)$$

$$\frac{I_y^T}{I_p} M_{yy}(I_y^k - I_y^{k-1}) + \frac{I_y^T M_{ye}}{I_p} (I_e^k - I_e^{k-1}) + \Delta t R_p I_p^k = 0 \quad (13)$$

# Free-boundary Grad-Shafranov evolution

- Extended system

$$F(x^k) = 0 \quad (14)$$

with unknowns  $x^k = \begin{bmatrix} I_y^k \\ a_g^k \\ I_p^k \\ I_e^k \end{bmatrix}$ .

- Solve using similar JFNK or other techniques (Stabilized Picard, etc)
- This is done using the FGE code in the MEQ suite

# Linearized deformable plasma evolution model

- Recall the circuit equation with a generic induction term due to the plasma:

$$M_{ee}\dot{I}_e + R_{ee}I_e + \dot{\psi}_{ep} = V_e \quad (15)$$

with  $\psi_{ep} = \frac{d}{dt}(M_{ey}I_y) = M_{ey}\frac{d}{dt}(I_y)$ . This expression works for any time-varying change of plasma current, not only rigid ones.

- In part III, we parametrized the plasma current distribution using the rigid body assumption as  $I_y = I_y(R_p, Z_p, I_p)$ .
- Instead, we now keep the general form  $I_y = I_y(I_e, I_p, c_o)$  where  $I_e$  are the external currents,  $I_p$  the plasma current, and  $c_o$  any externally imposed profile constraints (e.g.  $\beta_p, q_A, \ell_i$ ).
- We can again linearize using the plasma response matrices (6)-(8)

$$\dot{I}_y = \frac{\partial I_y}{\partial I_e} \delta \dot{I}_e + \frac{\partial I_y}{\partial I_p} \delta \dot{I}_p + \frac{\partial I_y}{\partial c_o} \delta \dot{c}_o \quad (16)$$

# Linearized deformable plasma evolution model

- Similarly to the rigid model, we realize that because we assume  $\dot{l}_{y0} = 0$ , this implies  $\frac{\partial l_y}{\partial l_e} \dot{l}_{e0} = 0$ . Hence  $\frac{\partial l_y}{\partial l_e} \delta \dot{l}_e = \frac{\partial l_y}{\partial l_e} (\dot{l}_{e0}(t) + \delta \dot{l}_e) = \frac{\partial l_y}{\partial l_e} \dot{l}_e$
- Collecting terms yields:

$$(M_{ee} + X_{ee})\dot{l}_e + (M_{ep} + X_{ep})\dot{l}_p + X_{eo}\delta\dot{c}_o + R_{ee}l_e = V_e \quad (17)$$

where:

- $X_{ee} = M_{ey} \frac{\partial l_y}{\partial l_e}$
- $M_{ep} = M_{ey} \frac{\partial l_y}{\partial l_p} = M_{ey} \frac{l_{y0}}{l_{p0}}$
- $X_{ep} = M_{ey} \frac{\partial l_y}{\partial l_p}$
- $X_{eo} = M_{ey} \frac{\partial l_y}{\partial c_o}$

# Generalized rigid plasma evolution model

- Now consider the circuit equation for the plasma current:

$$\dot{\psi}_y + R_{yy} I_y = 0 \quad (18)$$

with

$$\dot{\psi}_y = M_{yy} \dot{I}_y + M_{ye} \dot{I}_e \quad (19)$$

- Again parametrizing  $I_y = I_y(I_e, I_p, c_o)$ , linearizing as in (16), multiplying from the left by  $I_{y0}^T / I_{p0}$ , and assuming plasma resistance does not change with plasma position, yields:

$$\underbrace{\frac{I_{y0}^T}{I_{p0}} M_{yy} \frac{I_{y0}}{I_{p0}}}_{L_{pp} = I_{y0}^T M_{yy} I_{y0} / I_{p0}^2} \dot{I}_p + \underbrace{\frac{I_{y0}^T}{I_{p0}} M_{yy} \frac{\partial I_y}{\partial I_p}}_{\frac{1}{2} \frac{\partial L_{pp}}{\partial I_p} I_{p0}} \dot{I}_p + \underbrace{\frac{I_{y0}^T}{I_{p0}} M_{yy} \frac{\partial I_y}{\partial I_e}}_{\frac{1}{2} \frac{\partial L_{pp}}{\partial I_e} I_{p0}} \dot{I}_e + \underbrace{\frac{I_{y0}^T}{I_{p0}} M_{yy} \frac{\partial I_y}{\partial c_o}}_{\frac{1}{2} \frac{\partial L_{pp}}{\partial c_o} I_{p0}} \dot{c}_o + \underbrace{\frac{I_{y0}^T}{I_{p0}} M_{ye}}_{=M_{pe}} \dot{I}_e + \underbrace{\frac{I_{y0}^T}{I_{p0}} R_{yy} \frac{I_{y0}}{I_{p0}}}_{R_{pp}} I_p \quad (20)$$

# Generalized rigid plasma evolution model

- Hence:

$$(L_{pp} + X_{pp})\dot{I}_p + (M_{pe} + X_{pe})\dot{I}_e + X_{po}\delta\dot{c}_o + R_{pp}I_p = 0 \quad (21)$$

with

- $L_{pp} = I_{y0}^T M_{yy} I_{y0} / I_{p0}^2$
- $X_{pp} = \frac{I_{y0}^T}{I_{p0}} M_{yy} \frac{\partial I_y}{\partial I_p}$
- $M_{pe} = \frac{I_{y0}^T}{I_{p0}} M_{ye}$
- $X_{pe} = \frac{I_{y0}^T}{I_{p0}} M_{yy} \frac{\partial I_y}{\partial I_e}$
- $X_{po} = \frac{I_{y0}^T}{I_{p0}} M_{yy} \frac{\partial I_y}{\partial c_o}$
- $R_{pp} = \frac{I_{y0}^T}{I_{p0}} R_{yy} \frac{I_{y0}}{I_{p0}}$

# Generalized rigid plasma evolution model

- We obtain the complete dynamic model or a rigid plasma

$$\begin{pmatrix} (\mathbf{M}_{ee} + \mathbf{X}_{ee}) & (\mathbf{M}_{ep} + \mathbf{X}_{ep}) \\ (\mathbf{M}_{pe} + \mathbf{X}_{pe}) & (L_{pp} + X_{pp}) \end{pmatrix} \begin{pmatrix} \mathbf{i}_e \\ i_p \end{pmatrix} \quad (22)$$

$$+ \begin{pmatrix} \mathbf{R}_{ee} & 0 \\ 0 & R_{pp} \end{pmatrix} \begin{pmatrix} \mathbf{i}_e \\ i_p \end{pmatrix} = \begin{pmatrix} \mathbf{V}_a \\ 0 \end{pmatrix} \quad (23)$$

- This model has exactly the same structure as the RZIP model, just with more general expressions for  $X_{**}$  terms owing to the deformable plasma response matrix.
- Removing the  $X_{**}$  terms yields the model excluding the effects due to the plasma motion and deformation.
- We can combine this with a measurement equation as shown in part II.

# Summary of plasma equilibrium evolution models

We have seen:

- Conductor-only models. No plasma
- Fixed-plasma models:

$$I_y = I_y(I_p) \quad (24)$$

- Rigid-plasma linearized model:

$$I_y = I_y(R_p, Z_p, I_p) \quad (25)$$

- Deformable-plasma linearized model:

$$I_y = I_{y0} + \frac{\partial I_y}{\partial I_e} I_e + \frac{\partial I_y}{\partial I_p} \delta I_p + \frac{\partial I_y}{\partial c_o} \delta c_o \quad (26)$$

- Full evolution model-plasma model:

$$I_y = I_y(I_e, I_p, c_o) \quad (27)$$



# Examples of equilibria using MEQ

```

addpath ~/matlab/meq/ % adjust to suit your needs
addpath ~/matlab/meq/genlib % adjust to suit your needs

%% RZP model
[L,LX,LY] = rzp('ana',shot,time,'izgrid',true,'cde','OhmTor_rigid');
meas = {'zIp','rIp','Ip'}; % measurements from model
Ts = 0; % sample time: 0=continuous
sys = fgess(L,0,meas); % linearized model for rzp
fprintf('RZP unstable pole growth rate: %2.2f [1/s]\n',max(real(esort(pole(sys)))));

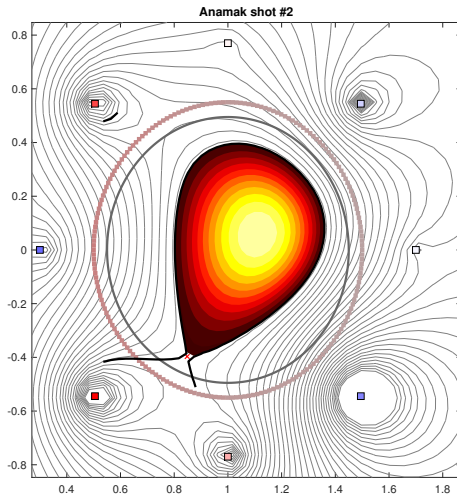
%% FGE: Free boundary Grad-Shafranov Evolution
[L2,LX2,LY2] = fge('ana',shot,time,'izgrid',true,'cde','OhmTor_rigid');
meas = {'zIp','rIp','Ip'}; % measurements from model
Ts = 0; % sample time: 0=continuous
sys = fgess(L,0,meas); % linearized model for fge
fprintf('FGE unstable pole growth rate: %2.2f [1/s]\n',max(real(esort(pole(sys)))));

%% Plot equilibrium
figure(1); set(gcf,'position',[0 0 600 500]); clf;
meqplotfancy(L,LY);
title(sprintf('Anamak shot #%d',shot))
set(gca,'box','on');
set(gcf,'paperpositionmode','auto');
print('-depsc','anamak_eq_2');

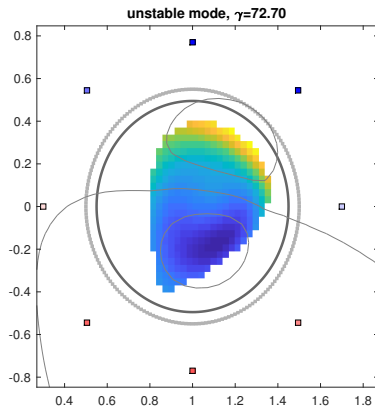
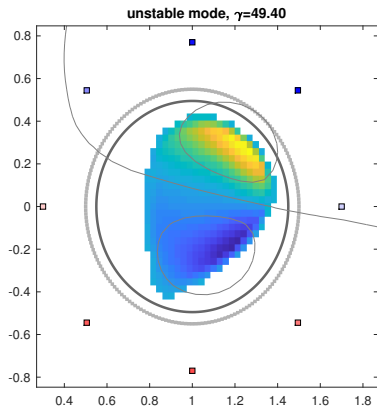
%% Plot eigenmode structures
figure(2); set(gcf,'position',[0 0 800 400]); clf;
subplot(121), fgeploteig(L)
subplot(122), fgeploteig(L2)
set(gcf,'paperpositionmode','auto'); print('-depsc','anamak_growth_rates');

```

# Examples of equilibria using MEQ



# Examples of equilibria using MEQ



## Section 2

# Plasma shape control

# Shape description

- We want the plasma to have a particular shape, since shape affects plasma confinement and stability.
- We also want to avoid part of the plasma touching the wall, so we should stay away from the wall.
- We have seen some parameters of the shape: elongation

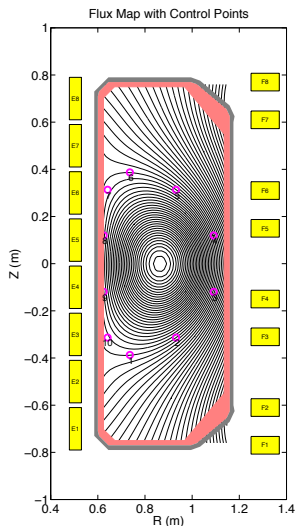
$$\kappa = \frac{Z_{max} - Z_{min}}{R_{max} - R_{min}} \quad (28)$$

- Similarly we can define a 'triangularity'

$$\delta = \frac{R_{max} + R_{min} - (R_{up} + R_{low})}{R_{max} - R_{min}} \quad (29)$$

- Similarly higher-order moments: squareness etc.

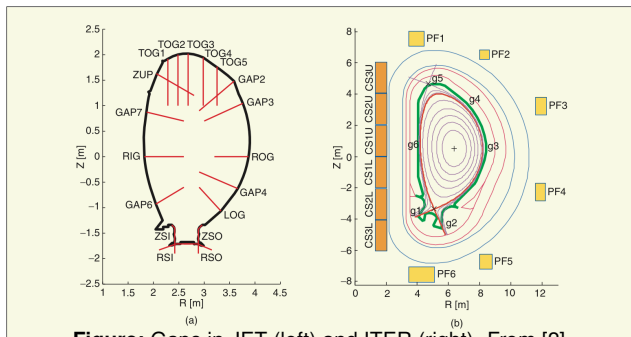
# Flux control



- More generally, we want to prescribe the plasma boundary.
- This can be done by requiring that all points on the boundary have the same flux  $\psi$ .
- $\times$ -point locations can be prescribed by requiring that  $|\nabla\psi| = 0$  there.
- More general constraints on magnetic field values, field angle can similarly be prescribed

# Gap control

- Another approach is to define a set of *gaps*. Distance between LCFS and wall at selected points.
- This has an easy physical interpretation
- These gaps have to be held close to a given reference value (MIMO problem)



**Figure:** Gaps in JET (left) and ITER (right). From [2].

# Control-oriented models for shape control

- Linear controllers are usually designed based on linearized models.
- How do we obtain these linearized models?
- Linearize Grad-Shafranov equation w.r.t. coil currents
  - Obtain sensitivity matrices i.e.  $\frac{\partial I_y}{\partial I_e}$
  - Derive sensitivity to control parameters, e.g.  $\frac{\partial B_{r,xpoint}}{\partial I_e}$ ,  $\frac{\partial B_{z,xpoint}}{\partial I_e}$
  - Pack everything in output matrix yielding static relation between currents and shape errors

$$e_{shape} = C_{shape,e} I_e + C_{I_p} \delta I_p \quad (30)$$

- Optionally combine this with linearized plasma evolution model above to include coupled coil-plasma dynamics effects
- Yields linear state-space model



# Control for overdetermined MIMO systems

- In shape control, there are typically more controlled outputs (control points, gaps) than controlled variables (coil voltages).
- $y = Pu$  where  $P(s)$  is a  $n_y \times n_u$  transfer function, with  $n_y > n_u$ .
- Error can not be controlled to zero for all quantities.
- Instead, control only a particular linear combination of errors:
  - Take SVD of  $P(0)$  to determine steady-state input-output relation

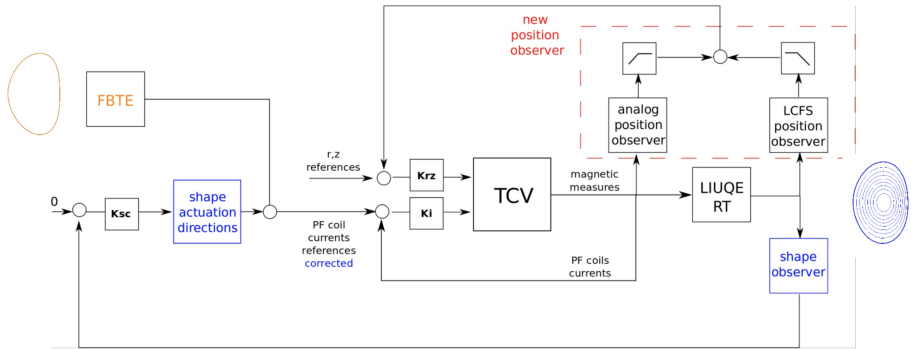
$$USV^T = P(0) \quad (31)$$

- Using the properties of the SVD,  $U = [U_1, U_2]$  where  $U_1 \in \mathbb{R}^{n_y \times n_u}$  is the basis for the steady state output values that can be reached by some  $u \in \mathbb{R}^{n_u}$ , and  $U_2 \in \mathbb{R}^{n_y \times (n_y - n_u)}$  is the basis for the output values that can not be reached.
- We design our controller to only control the component of  $y$  that is in the column space of  $U_1$ . For details, see [2, pp 65-75]

# Common scheme for combining shape and position control

- Close a feedback loop on the vertical position acting on voltages of specific coil sets.
- Close a feedback loop on the PF coil currents.
- Result: Closed-loop *stable* system with as inputs the PF coil reference currents.
- Close other feedback loops on plasma current as well as and shape acting of current references directly.
- *Figure pending*

# Common scheme for combining shape and position control



**Figure:** Scheme for shape control on TCV. Figure: F. Pesamosca

## Section 3

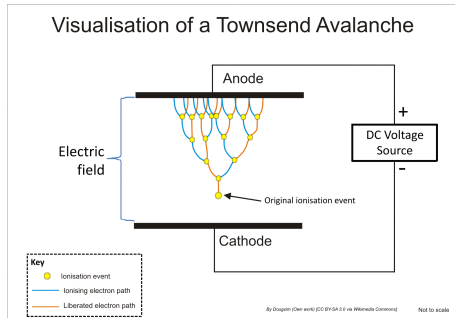
# Plasma discharge evolution, from breakdown to plasma termination

# Subsection 1

## Plasma breakdown

# Plasma breakdown conditions

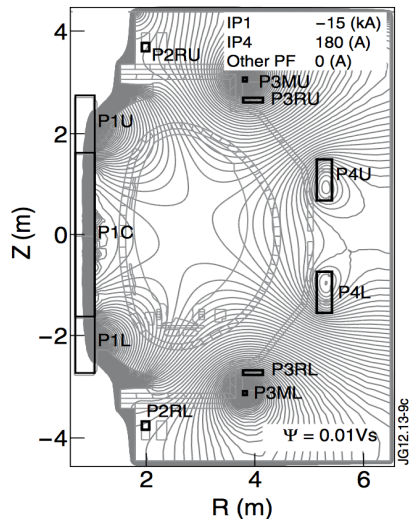
- Plasma breakdown occurs when the gas in the torus chamber ionizes.
  - A single electron is accelerated and collides with a neutral atom, ionizing it.
  - This liberates more electrons
  - These accelerate and collide with other atoms
  - This results in an ionization “avalanche” that quickly ionizes a large part of the gas.



# Setting up the field at breakdown

- Plasma breakdown requires:
  - An electric field to accelerate the electrons.
  - A large *connection length*: distance for electrons to travel along the magnetic field to allow ionization of other atoms.
  - An appropriate pressure (not too many, not too few particles)
- The first two conditions are created by a combination of coils.
  - TF coils generate a toroidal magnetic field.
  - The OH (or CS) coils are ramped to induce a loop voltage (electric field).
  - PF coils are used to create a point with 0 poloidal field at the desired breakdown location and time. The resulting **B** field is locally almost exclusively toroidal - large connection length.

# Example: JET breakdown field



JET poloidal flux at breakdown  
 from: Albanese et al. 2012 *Nucl. Fusion* **52** 123010



# Breakdown design an optimal control problem

## Desiderata for (Ohmic) breakdown

- Field evolution:
  - Before breakdown: vertical field to avoid breakdown.
  - At breakdown: null field maximizing connection length
  - After breakdown: Ramping vertical field to maintain radial force balance + positive curvature for vertical stability.

$$\mathbf{B}_p = B_z(t)\mathbf{e}_z + \mathbf{nullfield}(t) \quad (32)$$

- Loop voltage evolution:
  - Sufficient loop voltage at  $t = 0$  and later, to breakdown, burn-through, and ramp  $I_p$
  - Low loop voltage otherwise to avoid consuming Ohmic coil flux.
- Coil evolution
  - Pre-charge OH coils to have maximum flux swing

## Breakdown design an optimal control problem

- We have linear relations between circuit/passive current evolution and vacuum fields/loop voltage (excluding plasma)

$$B_{r,x}(t) = B_{rxa}i_a(t) + B_{rxu}i_u(t) \quad (33)$$

$$B_{z,x}(t) = B_{zxa}i_a(t) + B_{zxu}i_u(t) \quad (34)$$

$$V_{r,x}(t) = M_{xa}\dot{i}_a(t) + M_{xu}\dot{i}_u(t) \quad (35)$$

- Also linear model linking the circuit current evolution and passive structure evolution

$$M_{uu}\dot{i}_u(t) + R_{uu}i_u(t) + M_{ua}\dot{i}_a(t) = 0 \quad (36)$$

# Breakdown design an optimal control problem

- By discretizing the problem in time and writing out the transfer functions explicitly, we can relate the time-history of fields and fluxes (at spatial points of interest) to time-history of circuit currents:

$$\begin{bmatrix} B_{r,k=1} \\ B_{r,k=2} \\ \vdots \\ B_{r,k=N} \end{bmatrix} = \mathcal{T}_{r,a} \begin{bmatrix} I_{a,k=1} \\ I_{a,k=2} \\ \vdots \\ I_{a,k=N} \end{bmatrix}, \quad \begin{bmatrix} B_{z,k=1} \\ B_{z,k=2} \\ \vdots \\ B_{z,k=N} \end{bmatrix} = \mathcal{T}_{z,a} \begin{bmatrix} I_{a,k=1} \\ I_{a,k=2} \\ \vdots \\ I_{a,k=N} \end{bmatrix} \quad (37)$$

$$\begin{bmatrix} V_{k=1} \\ V_{k=2} \\ \vdots \\ V_{k=N} \end{bmatrix} = \mathcal{T}_{V,a} \begin{bmatrix} I_{a,k=1} \\ I_{a,k=2} \\ \vdots \\ I_{a,k=N} \end{bmatrix} \quad (38)$$

# Breakdown design an optimal control problem

- Constraints on power supply capabilities can also be formulated:

$$M_{uu}\dot{I}_u(t) + R_{uu}I_u(t) + M_{ua}\dot{I}_a(t) = 0 \quad (39)$$

$$I_u(s) = -(sM_{uu} + R_{uu})^{-1} M_{ua}sI_a(s) = 0 \quad (40)$$

$$(M_{aa}s + R_{aa})I_a(s) + M_{au}sI_u(s) = V_a(s) \quad (41)$$

$$(M_{aa}s + R_{aa} - sM_{au}(sM_{uu} + R_{uu})^{-1} M_{ua}s)I_a(s) = V_a(s) \quad (42)$$

- Discretizing, one derives

$$\begin{bmatrix} V_{a,k=1} \\ V_{a,k=2} \\ \vdots \\ V_{a,k=N} \end{bmatrix} = \mathcal{T}_{V_a, I_a} \begin{bmatrix} I_{a,k=1} \\ I_{a,k=2} \\ \vdots \\ I_{a,k=N} \end{bmatrix} \quad (43)$$

# Breakdown design an optimal control problem

- This ultimately allows us to define a *constrained* least-squares problem (see also [3]):

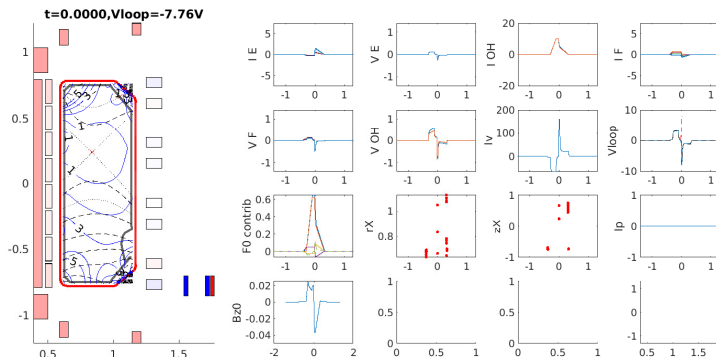
$$\min_x J(x) \text{ s.t. } Cx \leq d \quad (44)$$

with  $J(x) = + \nu_r \| \mathbf{B}_{r,\text{target}} - \mathcal{T}_{r,a} x \|_2^2$  Radial field evolution target  
 $+ \nu_z \| \mathbf{B}_{z,\text{target}} - \mathcal{T}_{z,a} x \|_2^2$  Vertical field evolution target  
 $+ \nu_V \| \mathbf{V}_{\text{target}} - \mathcal{T}_{V,a} x \|_2^2$  Loop voltage evolution target  
 $+ \nu_x \| x \|_2^2$  Regularization term minimizing coil currents

and  $C = \begin{bmatrix} \mathcal{T}_{V_a, I_a} \\ -\mathcal{T}_{V_a, I_a} \end{bmatrix}$ ,  $d = \begin{bmatrix} V_{a,\text{max}} \\ -V_{a,\text{min}} \end{bmatrix}$  Power supply voltage constraints

# Breakdown design an optimal control problem

- Quadratic constrained optimization problem: convex problem and fast solvers exist (e.g. matlab quadprog, fmincon)



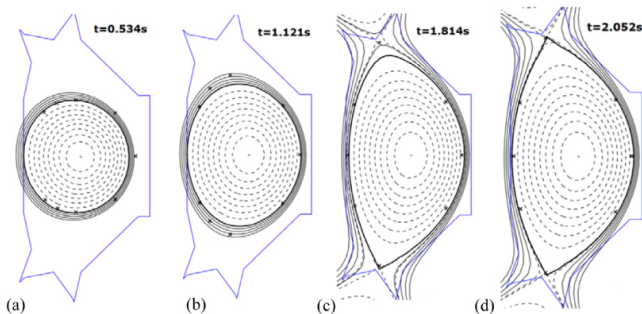
**Figure:** Example of optimized TCV breakdown

## Subsection 2

### Ramp-up phase

## Example: EAST ramp-up phase

- Start with low-current, limited plasma sitting against the wall
- Ramp up current, increase shape and create x-points
- Fully developed shape at start of flat-top.



**Figure:** EAST ramp-up shape evolution, from [4]



# Control issues for plasma ramp-up

- Switching from feedforward-controlled breakdown to feedback control of plasma position and current
- Switching from  $R, Z$  control only to full shape control
- Well-timed formation of x-point.
- Obtain desired  $q$  profile,  $\beta$ ,  $\ell_i$  at the start of the flat-top
- Remain within engineering and physics constraints all the time.

## Subsection 3

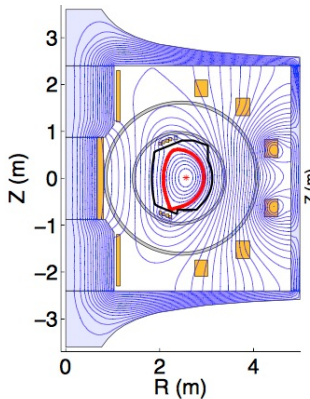
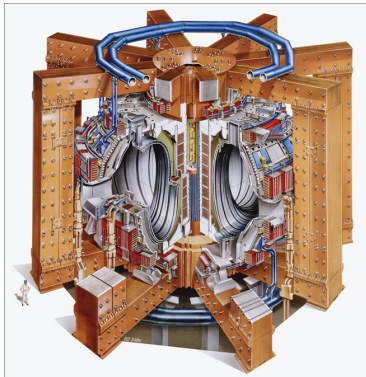
### Flat-top

## Magnetic control issues for plasma flat-top

- Maintain required position, shape and  $I_p$ .
- Compensate from disturbances due to change in  $\beta$ ,  $\ell_i, \dots$
- Compensate for changing stray field due to central solenoid ramp.

## Air core vs. iron core tokamaks

- Iron core: iron *transformer yoke* around tokamak, 'guides' field generated by Central Solenoid.
- Air core: no iron, OH gives 'stray' vertical field. OH coils designed to minimize this field.



# Air core vs. Iron core tokamaks

	<b>Air core</b>	<b>Iron core</b>
<b>Adv:</b>	Circuit equations are LTI, easier reconstruction of fields	Smaller stray field
<b>Disadv:</b>	Need to compensate OH stray field during shot	Field depends on iron magnetization: nonlinear and time-dependent equations.
<b>Examples:</b>	ITER, TCV, AUG, DIII-D	JET, Tore Supra

## Subsection 4

### Ramp-down

## Ramp-down: open research questions

- Ramp down  $I_p$ , decrease shape, etc in a controlled way.
- $I_p$  ramp-down tends to peak current density profile, bad for vertical position stability
- Complex optimisation problem to find optimal timing of  $I_p$  rampdown and heating changes.
- Open research field, but very important for ITER & other large tokamaks: safe plasma termination following unexpected events.

## Magnetic control - summary

- Dynamics of PF coils + vessel, controllers for PF coils ->  $I_a$  control
- Added plasma current model keeping fixed position ->  $I_p$  control
- RZIP model ->  $R_p, Z_p, I_p$  control
- Linearized and nonlinear deformable Grad-Shafranov model -> shape control



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