Spin glasses

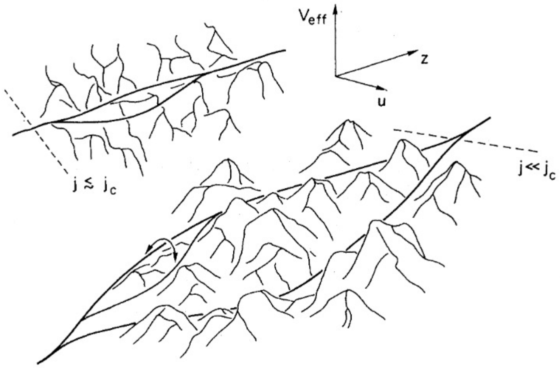
Prototype of a complex system

combining

- Disorder (randomness, no translational invariance)
- Competing interactions

'Glassiness' so far

Directed polymer / elastic interfaces:



Interactions = simple elasticity: favors flat interface

Position dependent disorder: favors roughness

Competition ⟶ metastability: many local energy minima

⟶

- Thermally assisted creep motion over energy barriers between valleys
- Minimal scale L_c where metastability occurs ⟶ *finite* pinning force f_c at T=0
- Non-linear (in f), cooperative motion for $f > f_c$: Depinning transition

'Glassiness' so far

Directed polymer / elastic interfaces:

Interactions = simple elasticity: favors flat interface

Position dependent disorder: favors roughness

Competition → metastability: many low local energy minima

Result: Distortion of the simple flat state
(very much like a ferromagnet in random fields)

Spin glasses: New ingredients

Spin glasses (experimentally discovered in th 1960's)

Interactions are complex/random in themselves!

(there is no simple reference state to distort)

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Of broad interest :

- At low T: Plethora of complex condensed "ordered phases" with no simple pattern
- Special and highly unusual phase transition
- Very interesting properties of the low T glass phase: extremely slow dynamics, non-equilibrium, (memory, "aging")
- New concepts & new tools,   with ...
- ... applications far beyond physics in general complex systems

Spin glasses: Challenging conceptual questions

- Is there an order parameter for glass transitions?
- Is there a broken symmetry?
  What if the Hamiltonian has no symmetry to break? Dynamic transitions?
- Statistical mechanics in these disordered systems: How and what to compute?
- How to handle/describe the many low T minima (phases)?

Spin glasses - An example of complex systems

Spin glasses = Representative of a large class of systems:

Many physical systems share ingredients of disorder and competing interactions:

- Glass forming liquids (where randomness is self-generated through their amorphous structure)
- Electron glasses (doped semiconductors below the metal-insulator transition)
- Neural networks; machine learning
- Complex biological systems (protein folding, gene networks etc)
- Economical systems, markets; societal phenomena

And also:

- Optimization problems

Spin glasses - An example of complex systems

An instance of a more general setting:

Optimization problems:

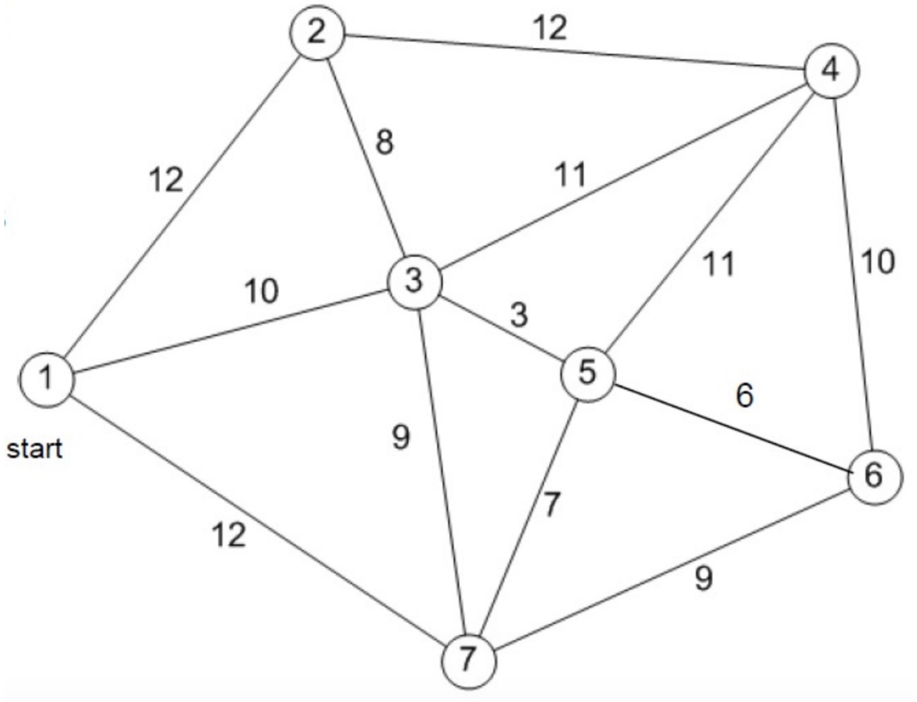
Given a large set X of configurations C (e.g. $\{s_i = \pm 1\}$)
and a cost function E(C)

- Find the optimal $C \in X$ that minimizes E(C)

Or:

- Decide whether there are C's such that $E(C) < E_0$

Travelling salesman problem



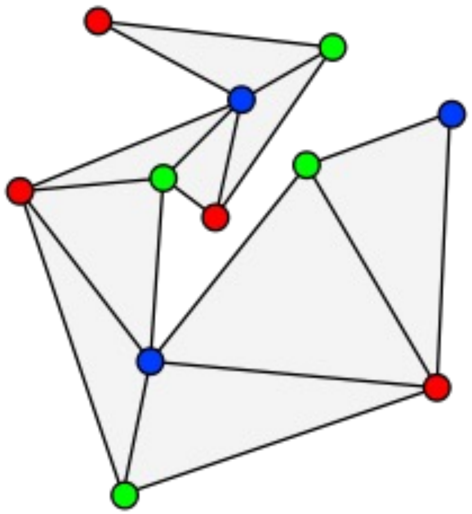
Problem:

Find shortest route through all sites (cities)
Given: distances between two cities

Configuration space:
$N!$ orders in which to visit of the cities

This is hard!

3-coloring



Problem:

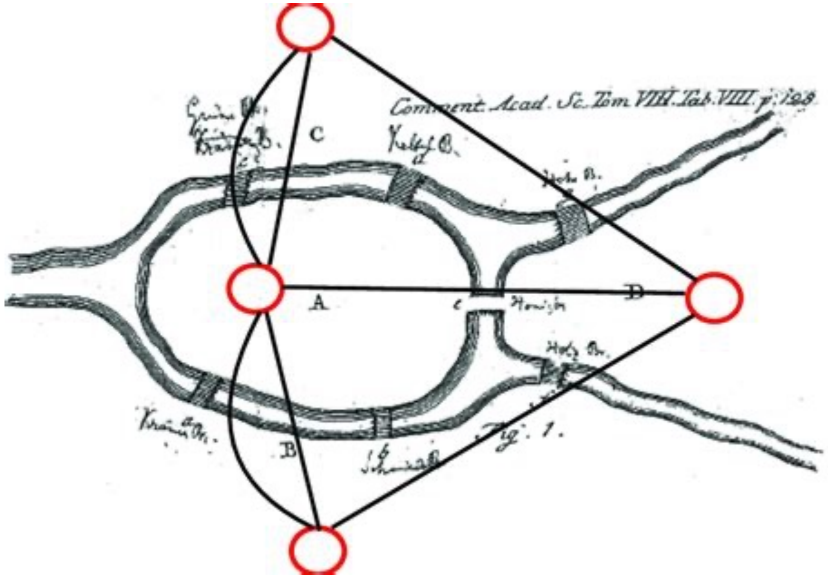
Find coloring of network sites with 3 colors such that no pair of linked sites has the same color!

Configuration space:

3^N color assignments, most of which are bad!

Hard!

Königsberg bridge problem (L. Euler, 1735)



Problem:

Finde a closed path that uses all
7 bridges (links) exactly once

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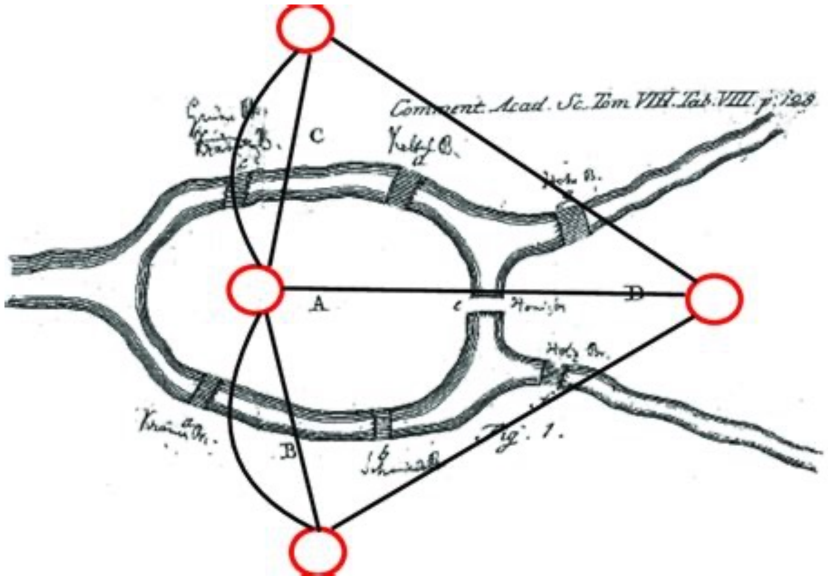

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Easy to prove impossibility!

In general: “Euler circuit” exists (and is easy to construct) iff every node has even degree

(that problem gave birth to graph theory!)

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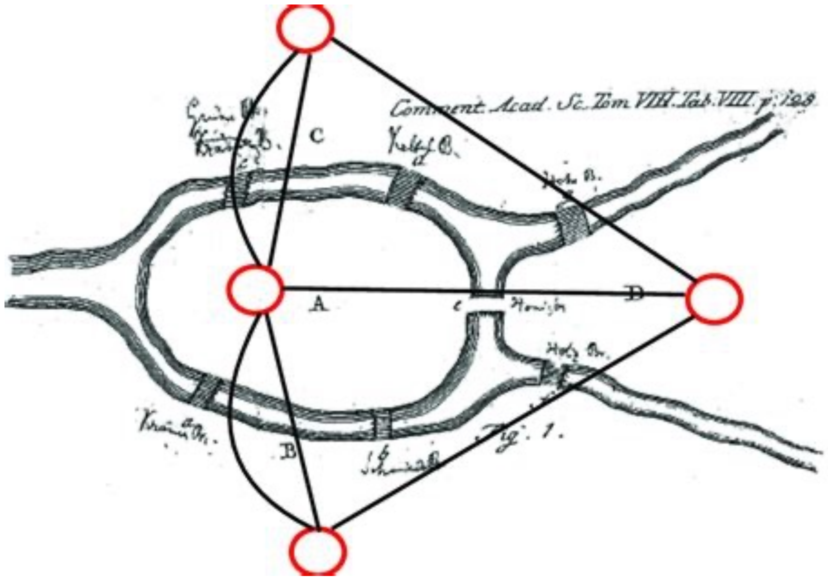

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→ Not every optimization problem is hard!

(Courtesy: L. Zdeborova)

K-satisfiability

Cook 1971

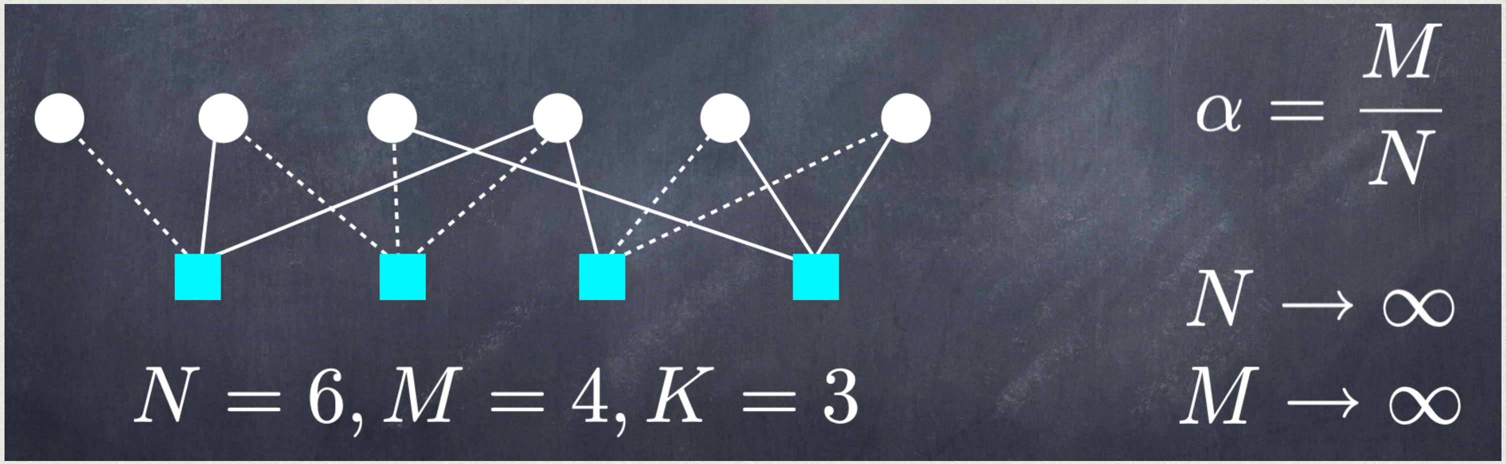
3-SAT on 4 variables with 3 clauses: $x_i \in \{\text{TRUE, FALSE}\}$

$$(\neg x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee x_3 \vee \neg x_4) \wedge (x_2 \vee \neg x_3 \vee x_4)$$

Random K-SAT: N variables, M clauses. Randomly choose a K-tuple of variables for each clause. Negate with probability 1/2.

Variables (N=6)

3-clauses (M=4)



(Courtesy: L. Zdeborova)

K-satisfiability

Boolean constraints can be translated into spin interactions:

$$(\neg x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee x_3 \vee \neg x_4) \wedge (x_2 \vee \neg x_3 \vee x_4)$$

$$x_i = 1, 0 \leftrightarrow s_i = 1, -1$$

$$x_i = (1 + s_i)/2$$

$$(x_1 \vee x_2 \vee \neg x_3) = 1 \leftrightarrow \frac{1 - s_1}{2} \frac{1 - s_2}{2} \frac{1 + s_3}{2} = 0$$

3-spin interaction $\geq$ 0!

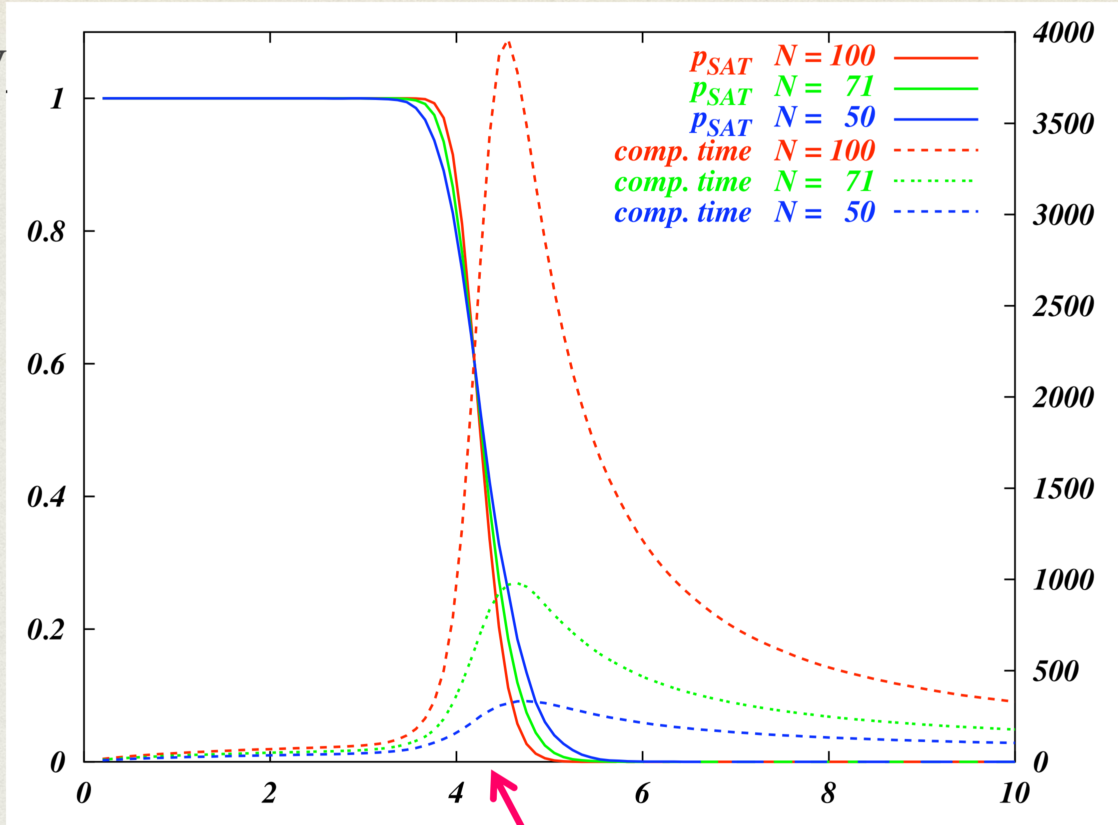
Satisfying assignment of x_i's $\leftrightarrow$ sum of all spin interactions is zero!

Satisfiability $\leftrightarrow E_{GS} = 0 \leftrightarrow$ generalized spin glass problem

(Courtesy: L. Zdeborova)

Random 3-satisfiability: Hardness transition!

satisfiability probability



median time of Davis-Putnam algorithm

Phase transition at a satisfiability threshold $\alpha_s \approx 4.26$!

$\alpha = M/N$ = #clauses per variable

Threshold:
→ hardest samples

random 3-SAT: N variables, M clauses.

(Courtesy: L. Zdeborova)

Suggestion: Hardness related to phase transitions, akin to glasses

Science

2016 Lars Onsager Prize

Analytic and Algorithmic Solution of Random Satisfiability Problems 2002

M. Mézard,[1] G. Parisi,[1,2] R. Zecchina[1,3*]

We study the satisfiability of random Boolean expressions built from many clauses with K variables per clause (K-satisfiability). Expressions with a ratio α of clauses to variables less than a threshold α_c are almost always satisfiable, whereas those with a ratio above this threshold are almost always unsatisfiable. We show the existence of an intermediate phase below α_c, where the proliferation of metastable states is responsible for the onset of complexity in search algorithms. We introduce a class of optimization algorithms that can deal with these metastable states; one such algorithm has been tested successfully on the largest existing benchmark of K-satisfiability.

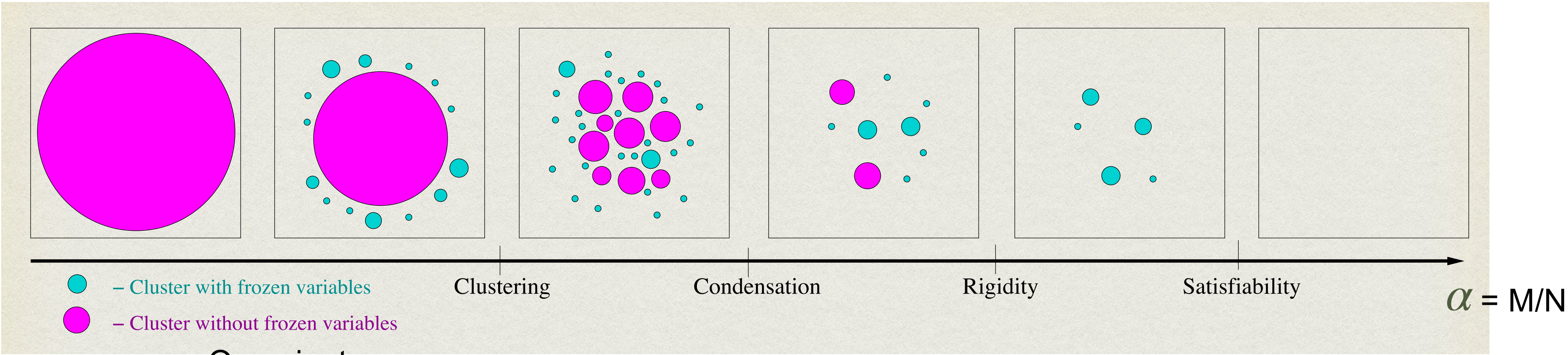
2021 Nobel Prize

Metastability makes the problem hard!

But glass physics insights help in solving them!

(Courtesy: L. Zdeborova)

K-satisfiability: structure of solution space



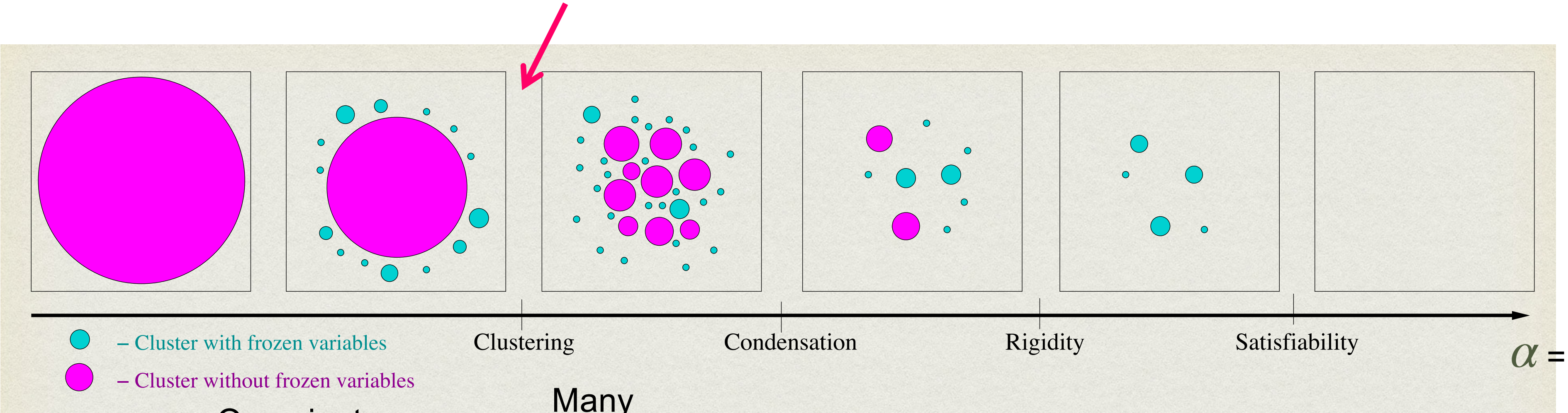
One giant cluster of solutions

Many transitions on approach to satisfiability threshold!
Clusterisation renders solution finding increasingly difficult.
(There are many more clusters of unsatisfiable configurations!)

(Courtesy: L. Zdeborova)

K-satisfiability: structure of solution space

Cluster formation : Akin to dynamic glass transition!



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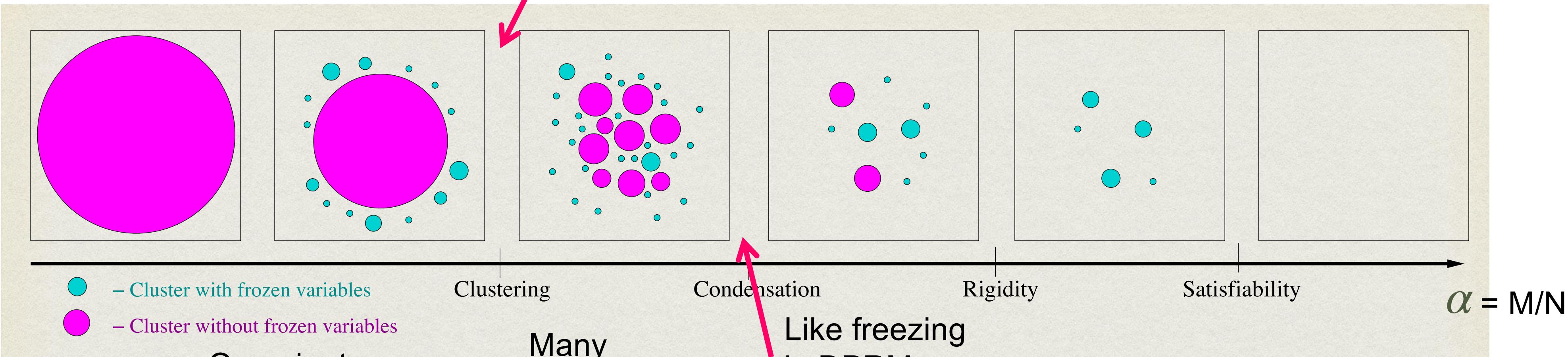

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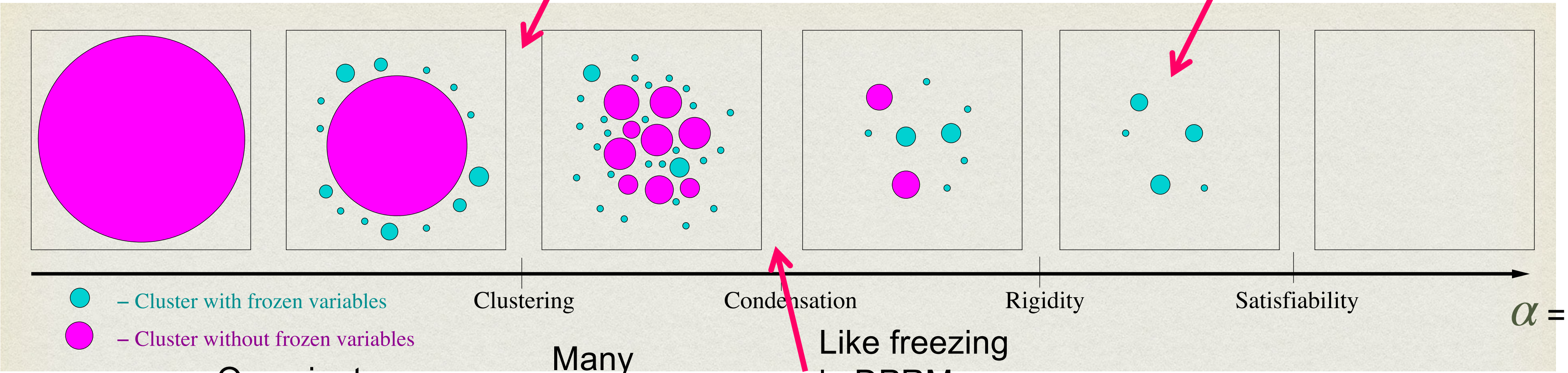
Cluster formation : Akin to dynamic glass transition!

Conjecture: cannot be found in poly time



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Complexity theory

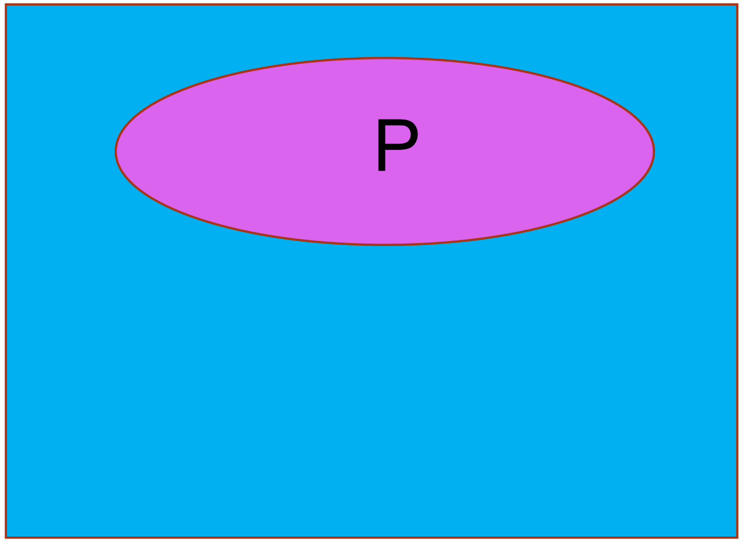
After all these examples:

Systematising hardness of problem solving?

What do we mean by ‘spin glasses are hard problems to solve’?

Complexity theory

Problem space:



- P (polynomial): there exists algorithm which only takes polynomial time for system size *N*, $T(N) \sim N^{\alpha}$
  e.g. DPRM, Euler circuit

= "EASY"

Complexity theory

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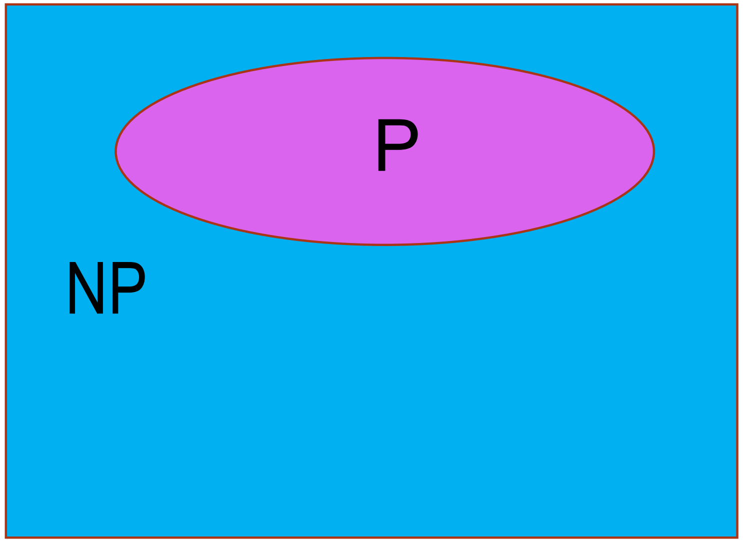

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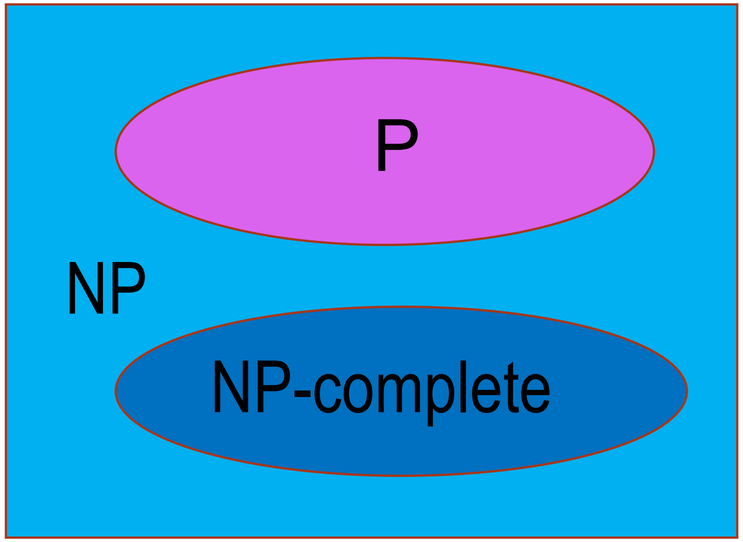

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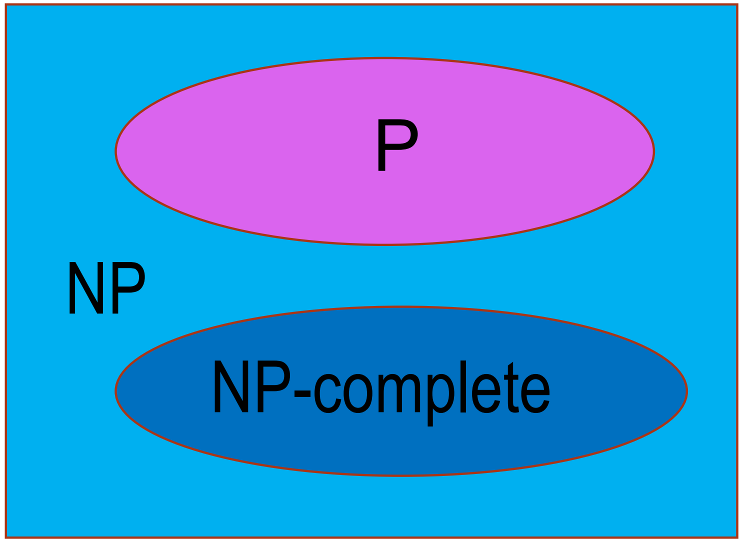

Are there any NP-complete problems? - YES

1971: Cook shows 3-SAT to be NP-complete

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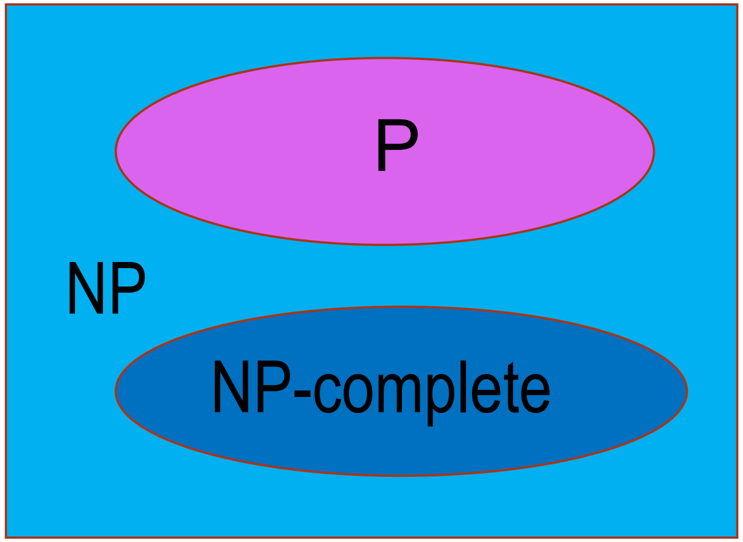

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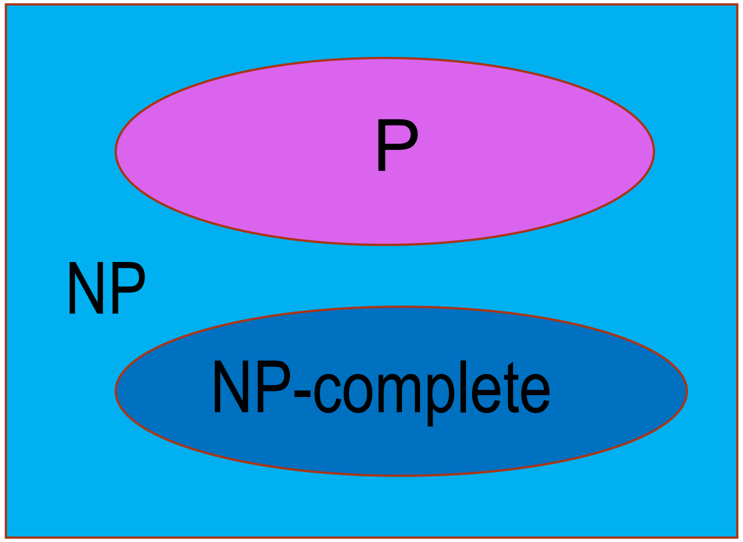

Conjecture / belief:

$$NP \neq P$$

Or: There are problems that are truly harder than others!

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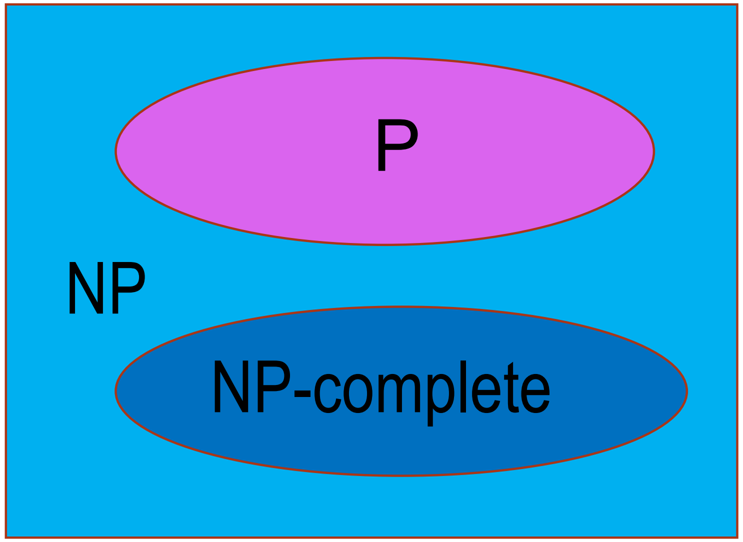

1979: Golman: most instances of a particular structure are easy, only in the worst case they are hard (namely, e.g., close to a phase transition)

But: What is hard (= no good algorithm known) shifts with time - due partly to physics-inspired algorithms

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Upshot:

Spin glasses are clean, physical examples of NP complete problems

Understanding spin glasses gives us insight into many other complex problems

Physics ideas help solving complex problems

A smart problem-solving idea ?

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If spin glasses are NP complete:
Use classical «analogue computer» to solve complex problems:

1. Translate your problem into a spin glass
(and build the glass with all its couplings)

2. Cool the spin system down to low T
(«thermal annealing»)

3. Read out the ground state!

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Why is this idea flawed?

A yet smarter problem-solving idea ?

If spin glasses are NP complete:
Use a quantum analogue computer to solve complex problems:

«Adiabatic algorithm» Kadowaki and Nishimori, 1998

1. Translate your problem into a spin glass

2. Turn on strong quantum fluctuations (transverse field h_x for Ising spins) and cool to low T:
Start in simple paramagnetic ground state

$$H = \sum_{ij} s_i^z J_{ij} s_j^z - h_x \sum_i s_i^x$$

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3. Adiabatically reduce the transverse field $h_x \to 0$

4. Invoke adiabatic theorem: A system stays (with high probability) in the ground state if one changes/anneals parameters adiabatically

$\to$ elegant way to find the ground state !?

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How good is this idea?

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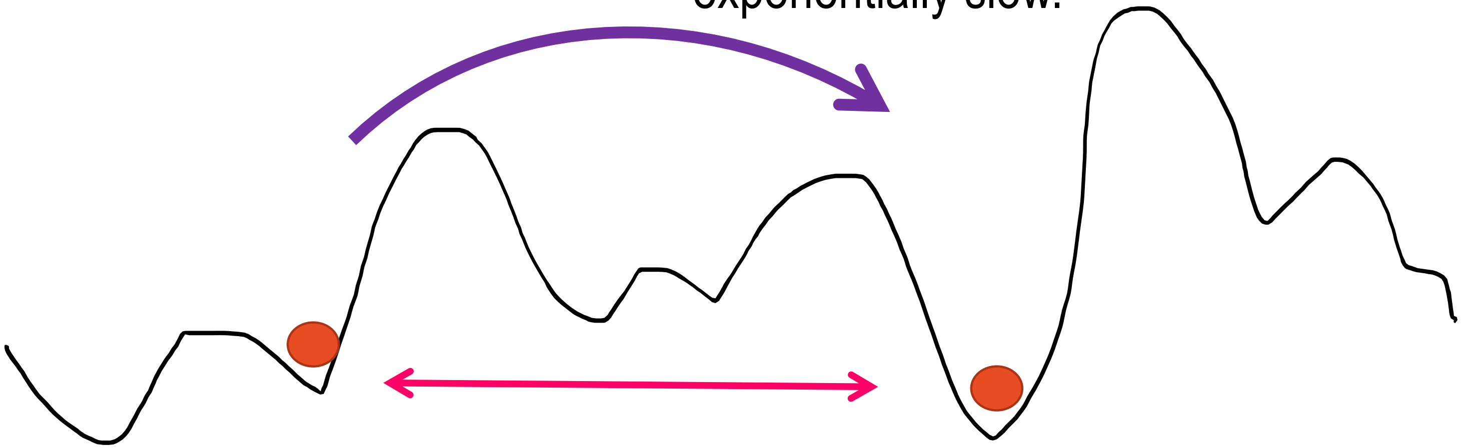
No, both thermal and quantum annealing fail:

In the glass phase (low T, low h_x) : High barriers between minima. Thermal activation and quantum tunneling are both exponentially slow.



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Glasses fall out of equilibrium below Tg and usually do not fall into their ground state on experimentally accessible times

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This is not only bad, it has also very useful sides!

As we will see it manifests itself in interesting ways in experiments.

Manifestations of
out-of-equilibrium behavior in
spin glasses

Spin glasses: protocol dependence of susceptibility χ

$$\chi = \lim_{\mathrm{B} \to 0} \frac{M}{B}$$

Spin glasses: protocol dependence of susceptibility χ

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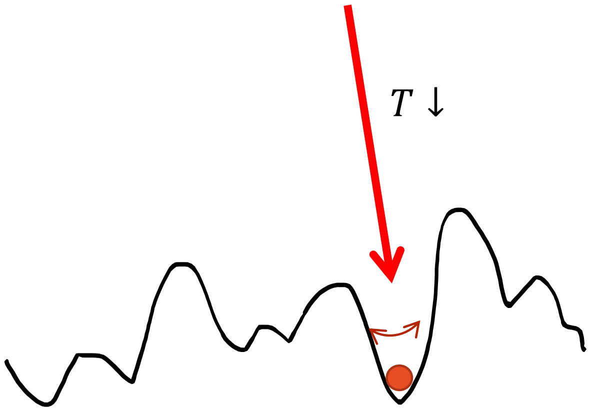

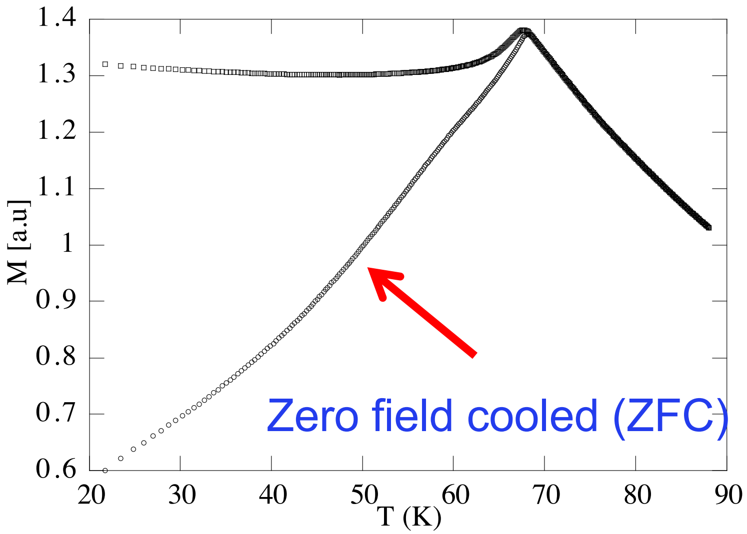

ZFC

- B=0 at T > T_c
- Cool to T < T_c
- Apply finite B

$$\chi_{\rm ZFC} = \frac{1}{N}\sum_i \chi_{ii} = \frac{1}{N}\sum_i \beta(1 - [\langle m_i\rangle^{(\alpha)}]^2) = \beta(1 - q_{\rm EA})$$

Spin glasses: protocol dependence of susceptibility χ

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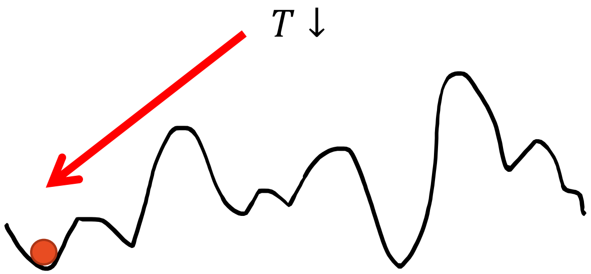

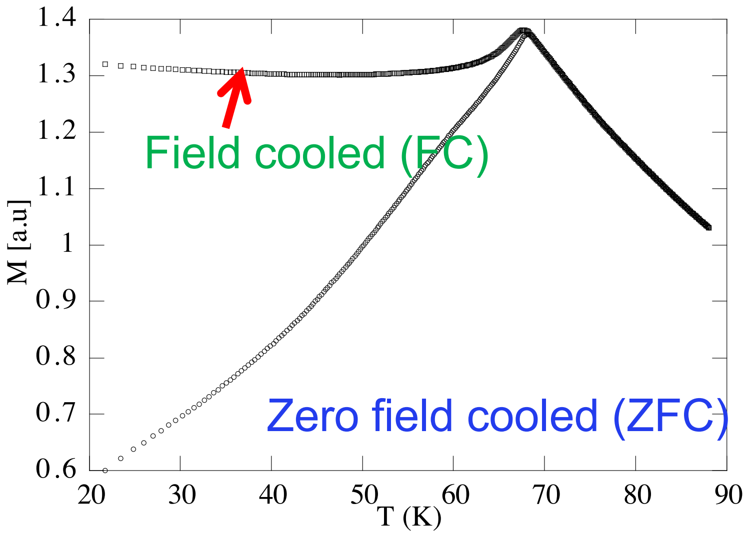

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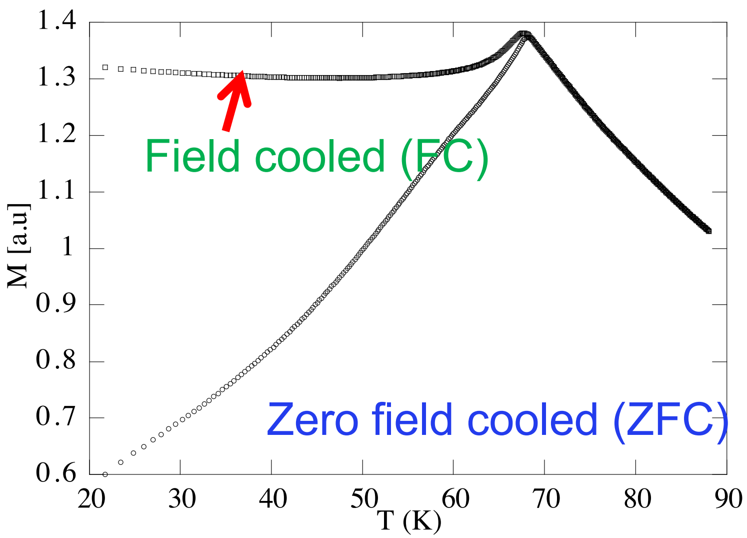

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Final state (M) depends on protocol! → Out of equilibrium, ergodicity is broken!

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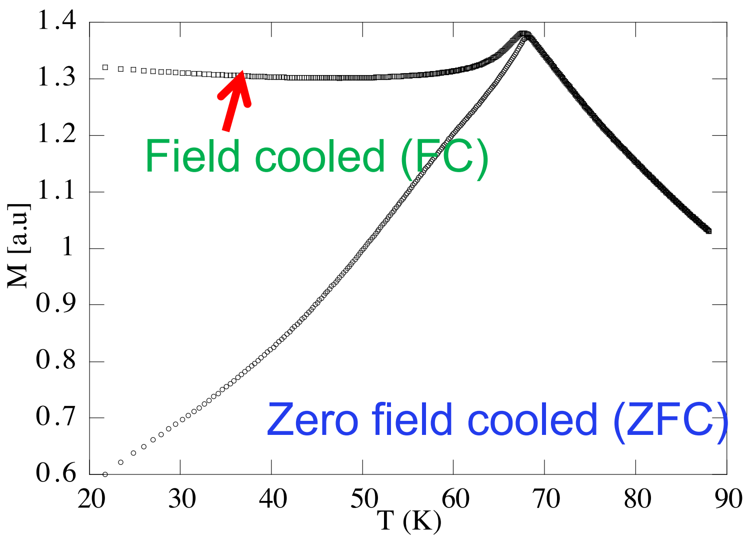

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Difference with hysteresis in ferromagnets?

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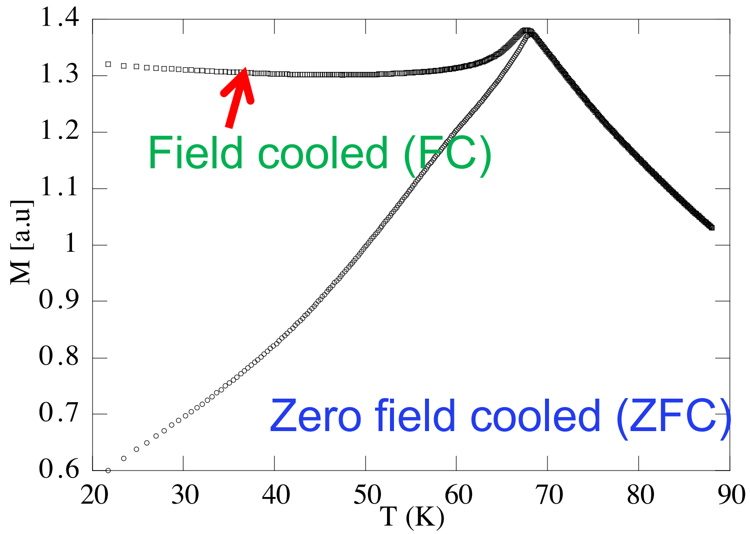

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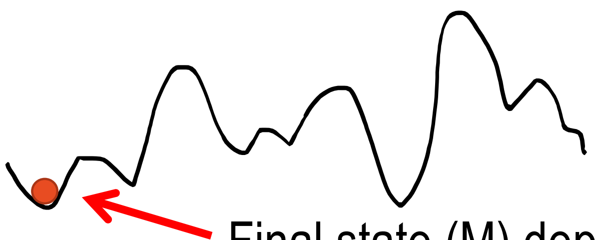

Depends on B

Final state (M) depends on protocol! → Out of equilibrium, ergodicity is broken!

Difference with hysteresis in ferromagnets? $M_{FC} \sim B$, not just $\propto$ sign(B)

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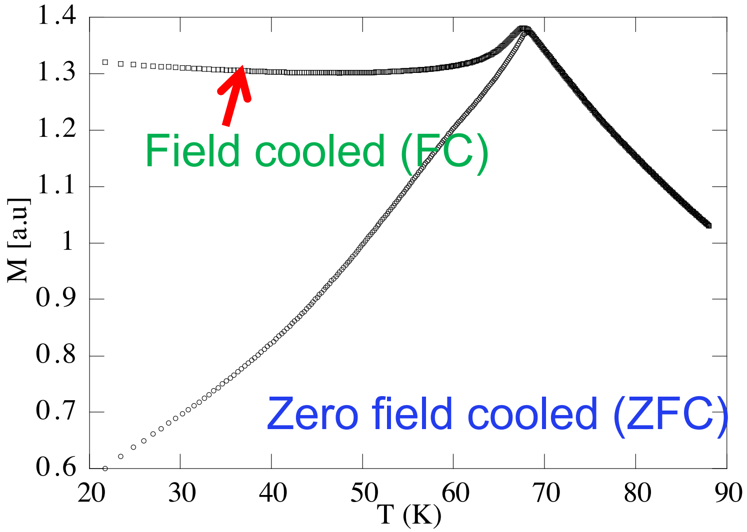

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Final state depends on protocol! → Out of equilibrium, ergodicity is broken!

Interesting: System remembers the past! → Store information!

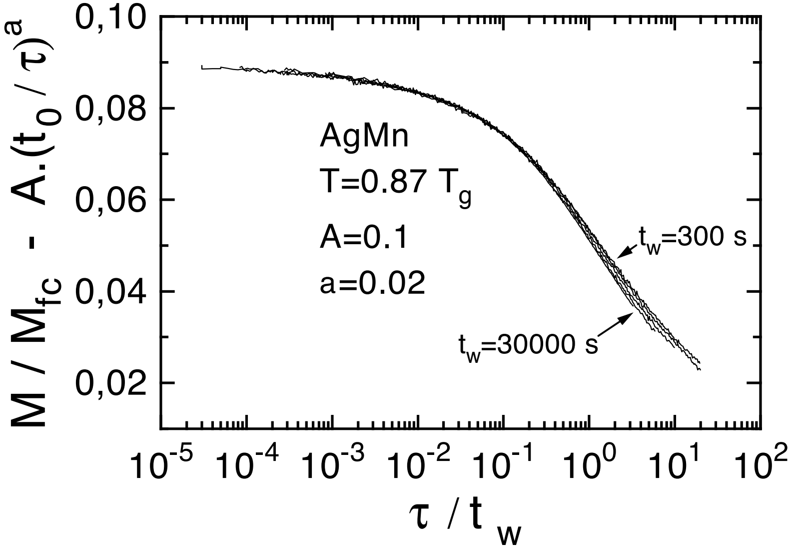
Spin glasses: Aging - Dynamics gets slower with ‘age’

Protocol:

- Apply a field B at high T.
- cool to 9K = T < Tc = 10.4K at t= 0
- Wait for t_w
- Switch off B
- Measure the decay of M

Spin glasses: Aging - Dynamics gets slower with ‘age’

$M_{slow}(\tau)$ =



Protocol:

- Apply a field B at high T.
- cool to 9K = T < Tc = 10.4K at t= 0
- Wait for t_w
- Switch off B
- Measure the decay of M

$$M(\tau) = M_{fast}(\tau) + M_{slow}(\tau)$$

$$M_{fast}(t) = A\left(\frac{t_0}{\tau}\right)^a$$

$$M_{slow}(t) = f\left(\frac{\tau}{tw}\right)$$

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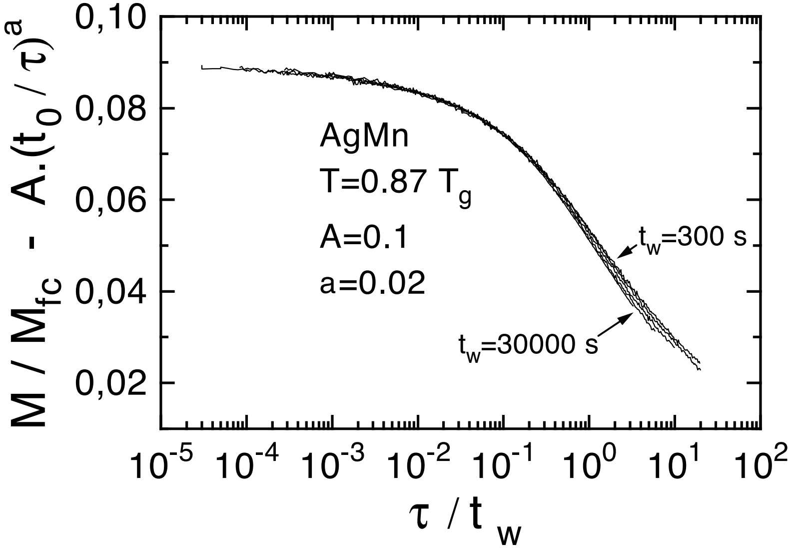

Dynamic time scale grows with t_w: the older the slower
→ the sample is not at equilibrium!

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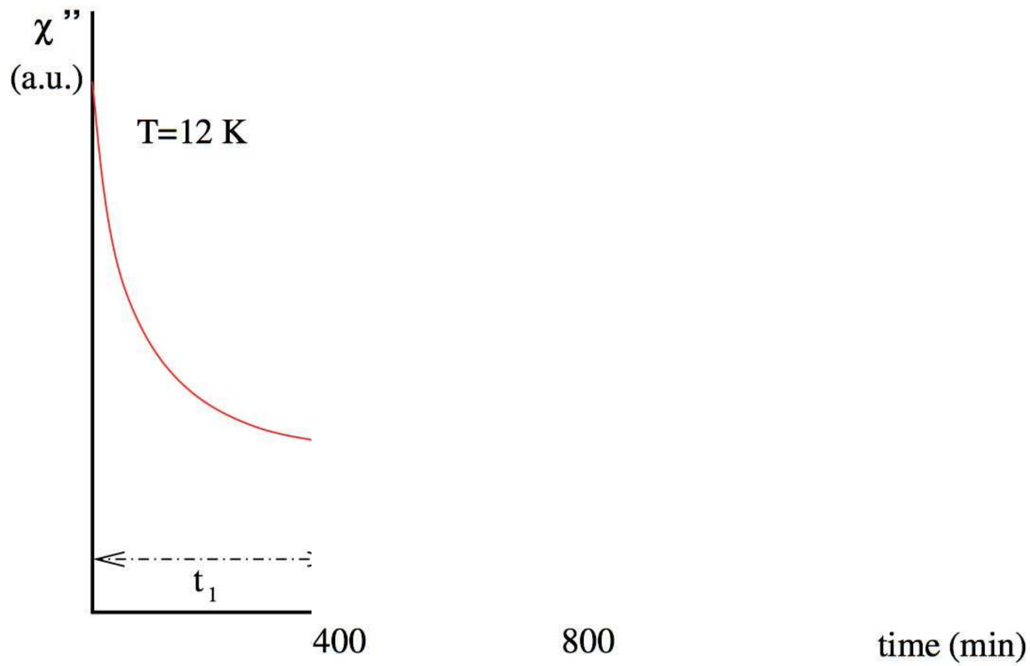
Spin glasses: Aging - Dynamics gets slower with ‘age’

Understanding: Exercise on trap model

Waiting time determines the typical time scale of dynamics and response!

Very different from equilibrium: Waiting longer does not change response

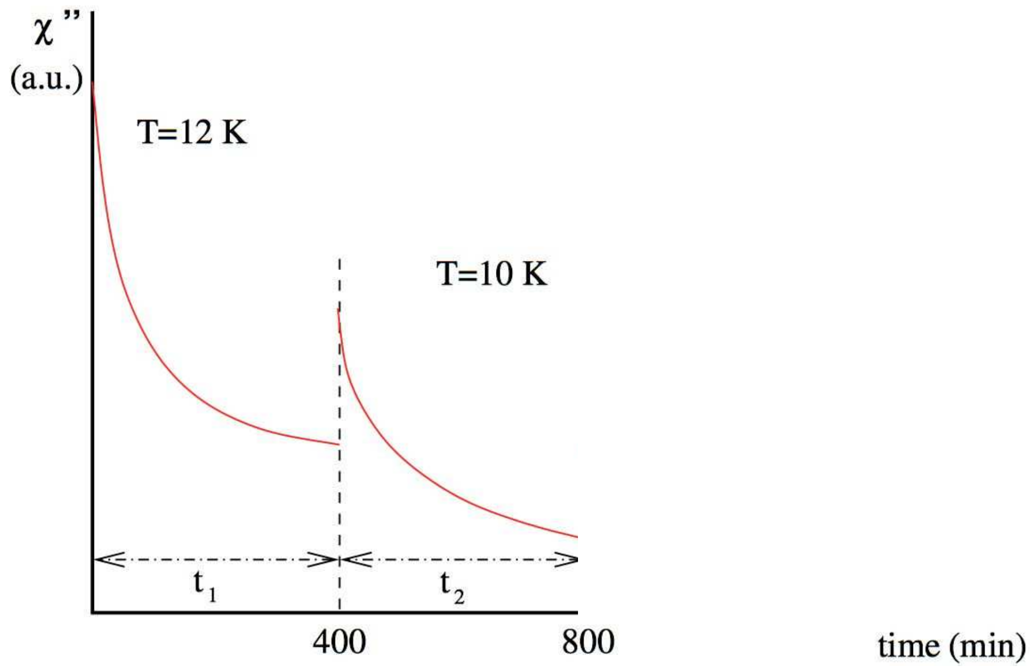
Spin glasses: Rejuvenation



Protocol:

- Cool to 12K < Tc
- Measure χ (still relaxing!)

Spin glasses: Rejuvenation



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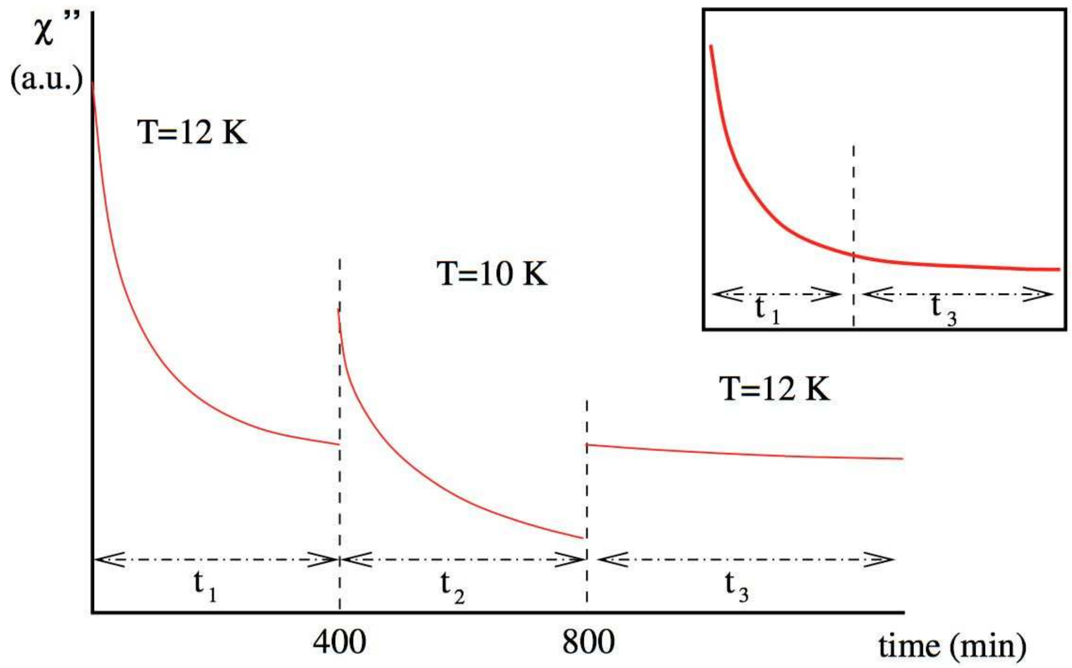
- Measure χ (still relaxing!)
- Cool further to 10K → χ jumps up as if one had directly cooled to 10K (= "rejuvenation")

Spin glasses: Rejuvenation



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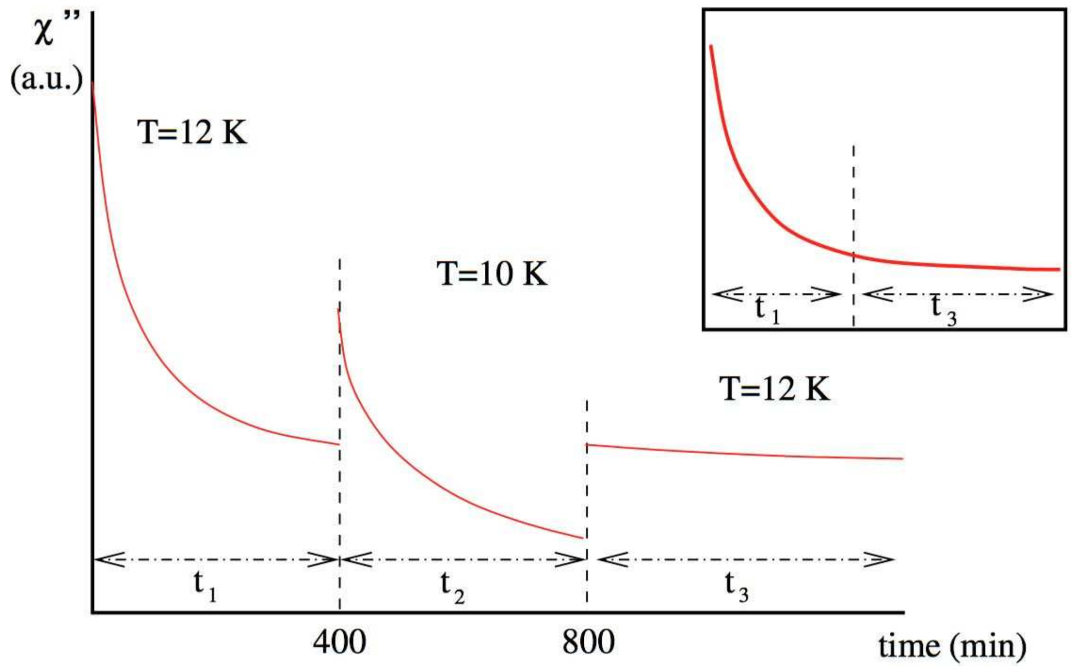
Spin glasses: Rejuvenation



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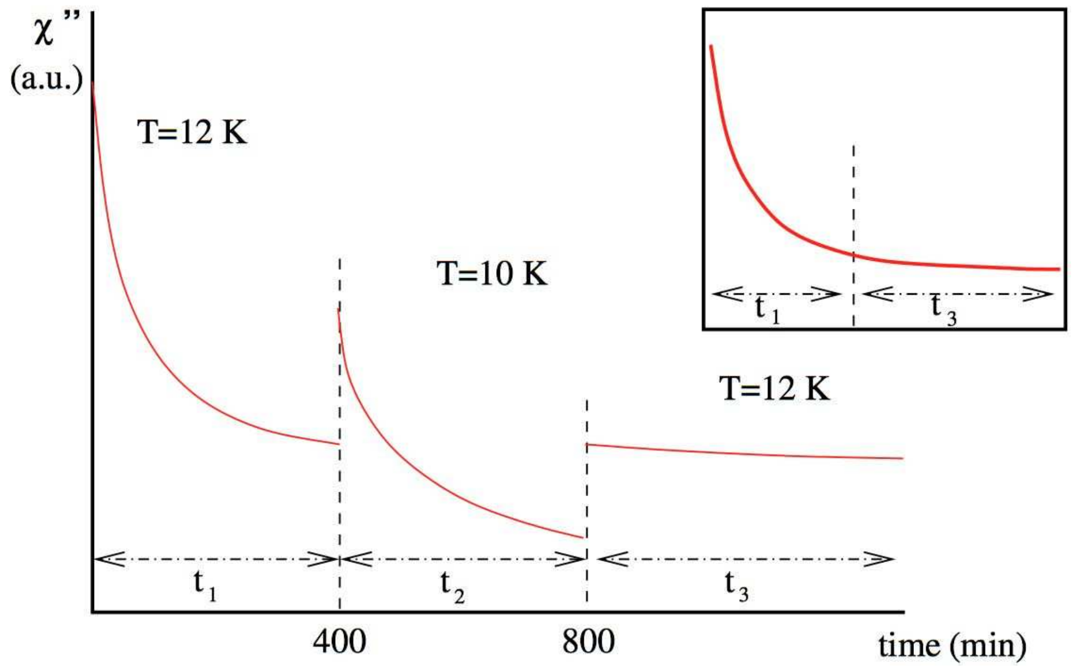
Memory of relaxation at higher T!

Spin glasses: Rejuvenation



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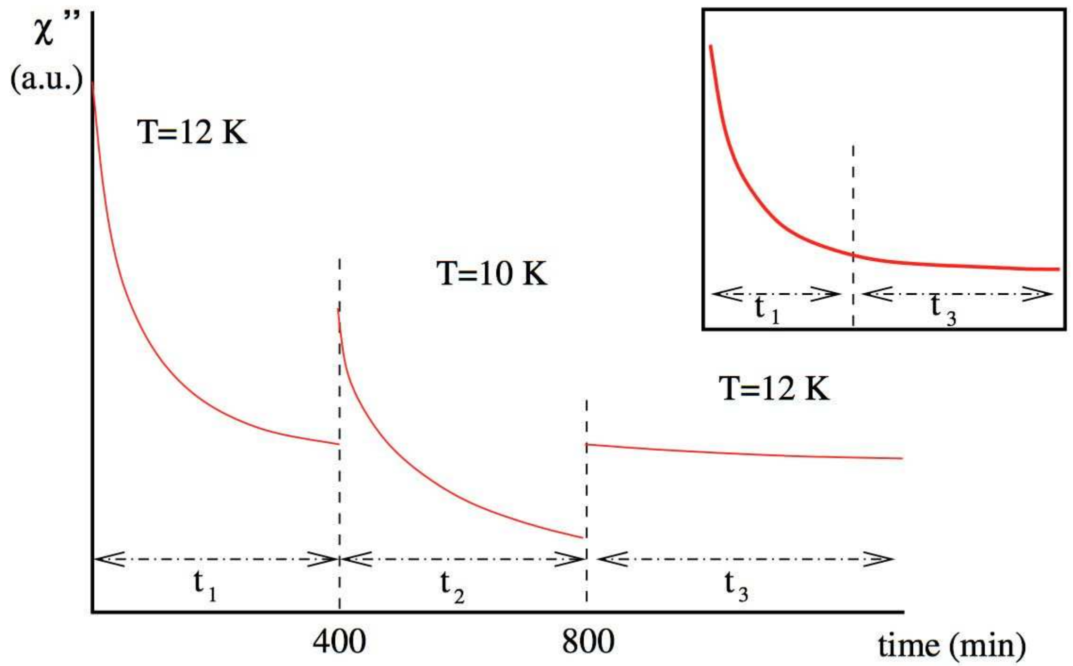
Memory of relaxation at higher T!

Explanation? Landscape and relaxation dynamics at 10K is apparently totally different from that at 12K!

Spin glasses: Rejuvenation



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Memory of relaxation at higher T!

Explanation? Landscape and relaxation dynamics at 10K is apparently totally different from that at 12K!

See later: free energy minima depend (a lot) on T:

Relaxing under T does not imply anything on properties at T'

$$F(\{m_i\}) = F(\{m_i\}; T)$$