Intrinsic Stellar Variability: Pulsations

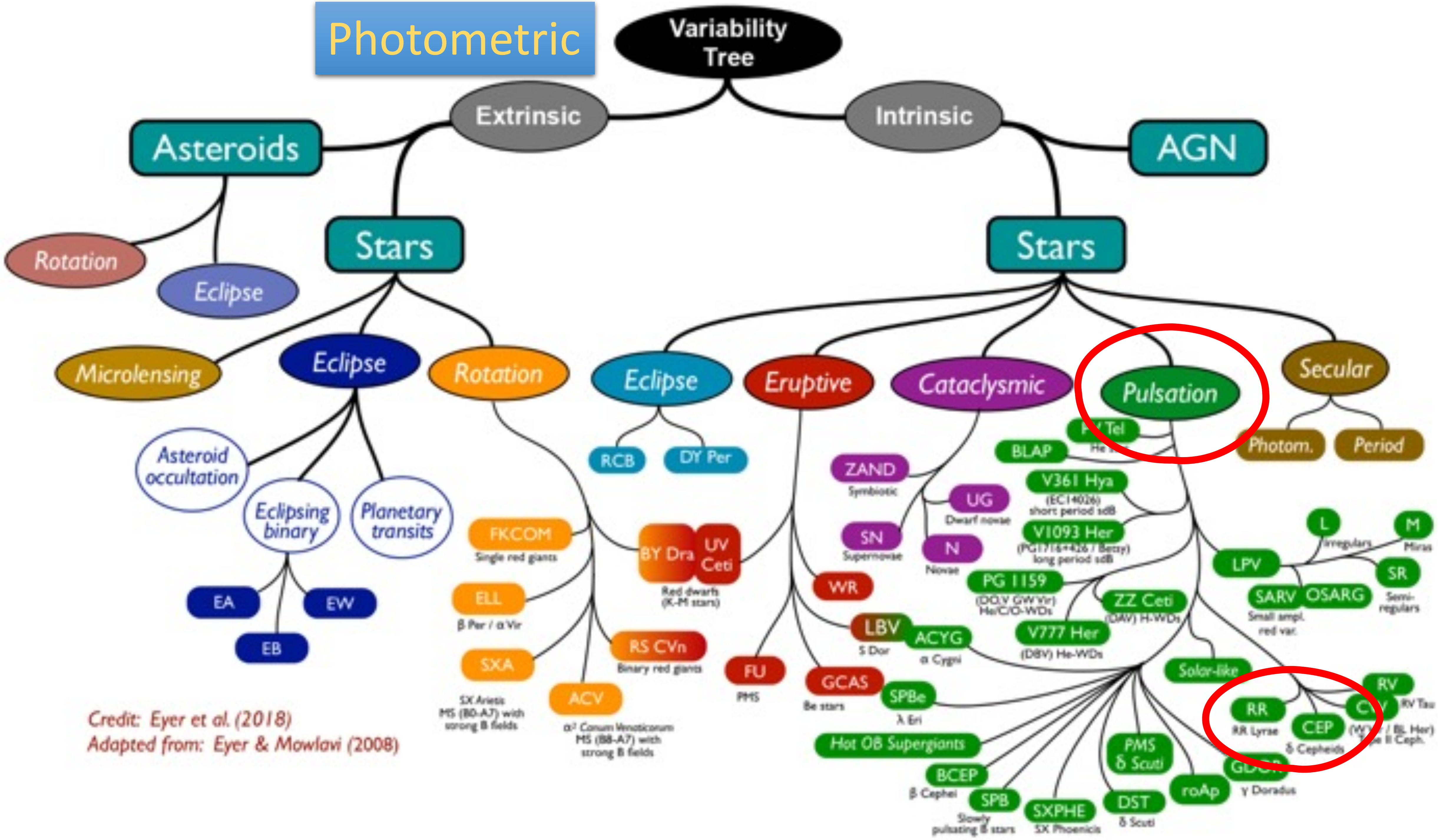
The Variable Universe – Lecture 05
Fall Semester 2022

Richard Anderson
richard.anderson@epfl.ch
022 379 24 25
Sauverny Observatory #265

EPFL

Questions from last week?

Photometric



Credit: Eyer et al. (2018)
Adapted from: Eyer & Mowlavi (2008)

History

- Mira the wonderful: first pulsating star
- Edward Pigott & John Goodricke discover eta Aquilae & delta Cephei
- Henrietta Leavitt (1908): 1777 variables in the SMC (25 periodic)
- "cluster variables" => RR Lyrae variables (globular clusters)
- Baade (1956): two stellar populations
- Delta Cephei types => type-I "classical" Cepheids
- Nomenclature remains confusing:
  - Type-II Cepheids encompass 3 subtypes w/ different evolution paradigms
  - Anomalous Cepheids even less understood; nothing like type-I Cepheids

Establishing stellar pulsations

- Doppler : cluster variables change color -> fast orbital motion?
- Hertzsprung 1912: 3-peaked distribution (0.5, 7, 300 d)
- Shapley (1914): pulsation hypothesis
- Eddington (1917): heat engines
- Christy (1962), Baker & Kippenhahn (1962): first detailed models of pulsations

The Calculation of Stellar Pulsation*

ROBERT F. CHRISTY

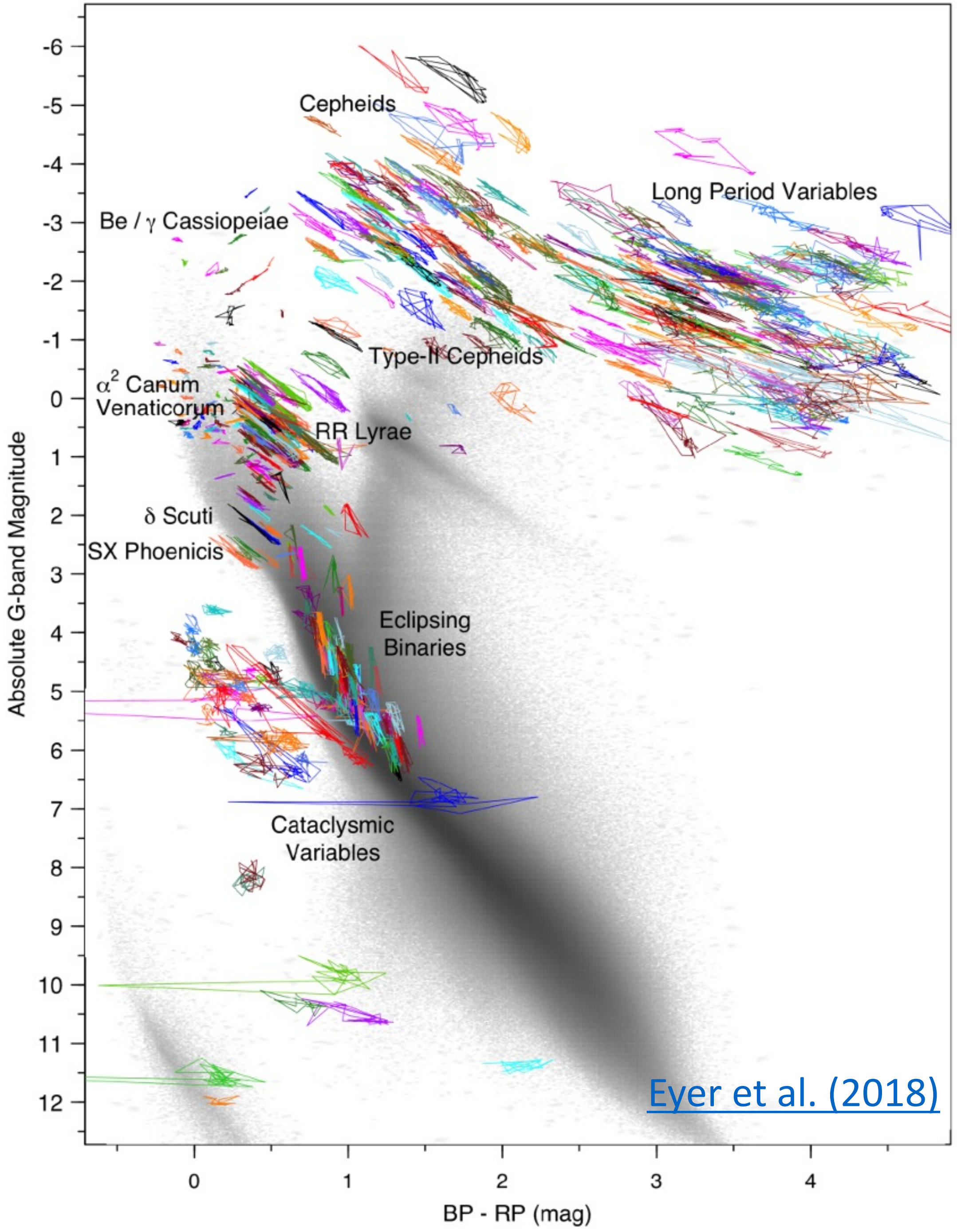
1962, ApJ 136, 887

ACKNOWLEDGMENT

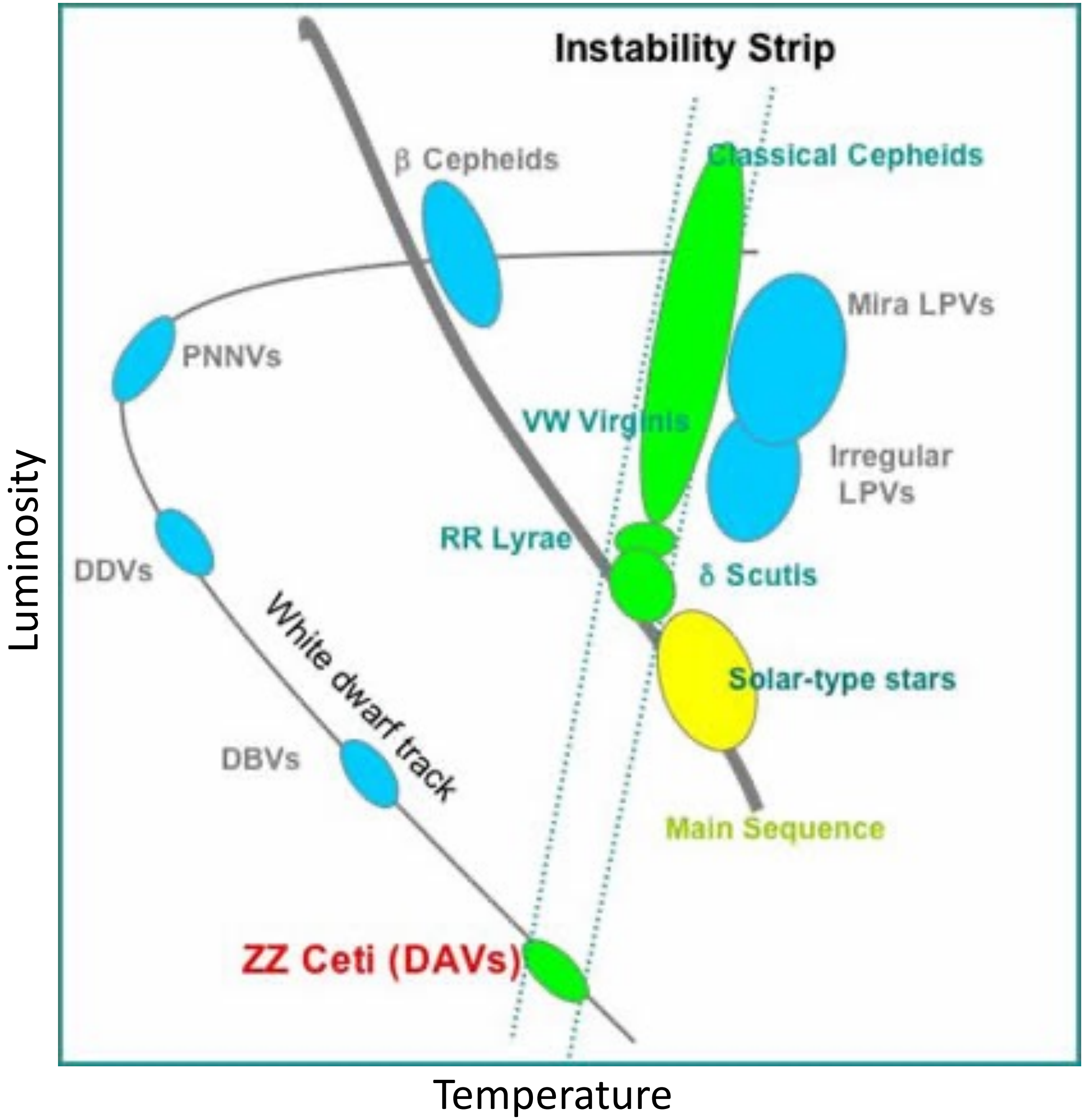
It is appropriate that this paper should be part of a salute to J. R. Oppenheimer. […] It is particularly gratifying to note that techniques, developed for the design of atomic weapons, are here employed to study the dynamics of stars.

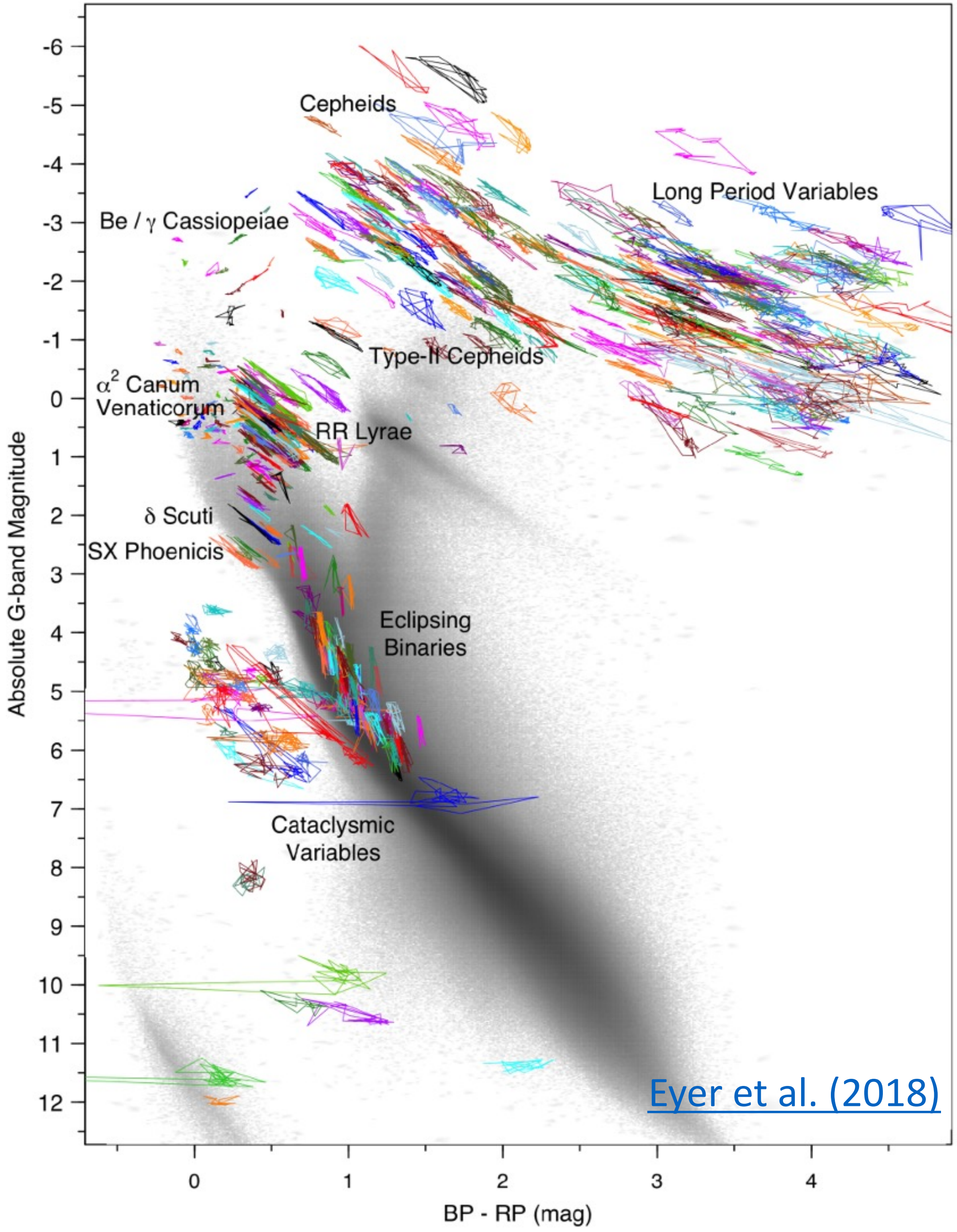
Pulsating stars across the Hertzsprung Russell Diagram

- Classes of pulsating stars clump in HRD
- Certain regions devoid of pulsations
- Pulsating stars all move diagonally: why?
- HRD relates luminosity, temperature, radius
- Pulsation intimately linked with stellar structure



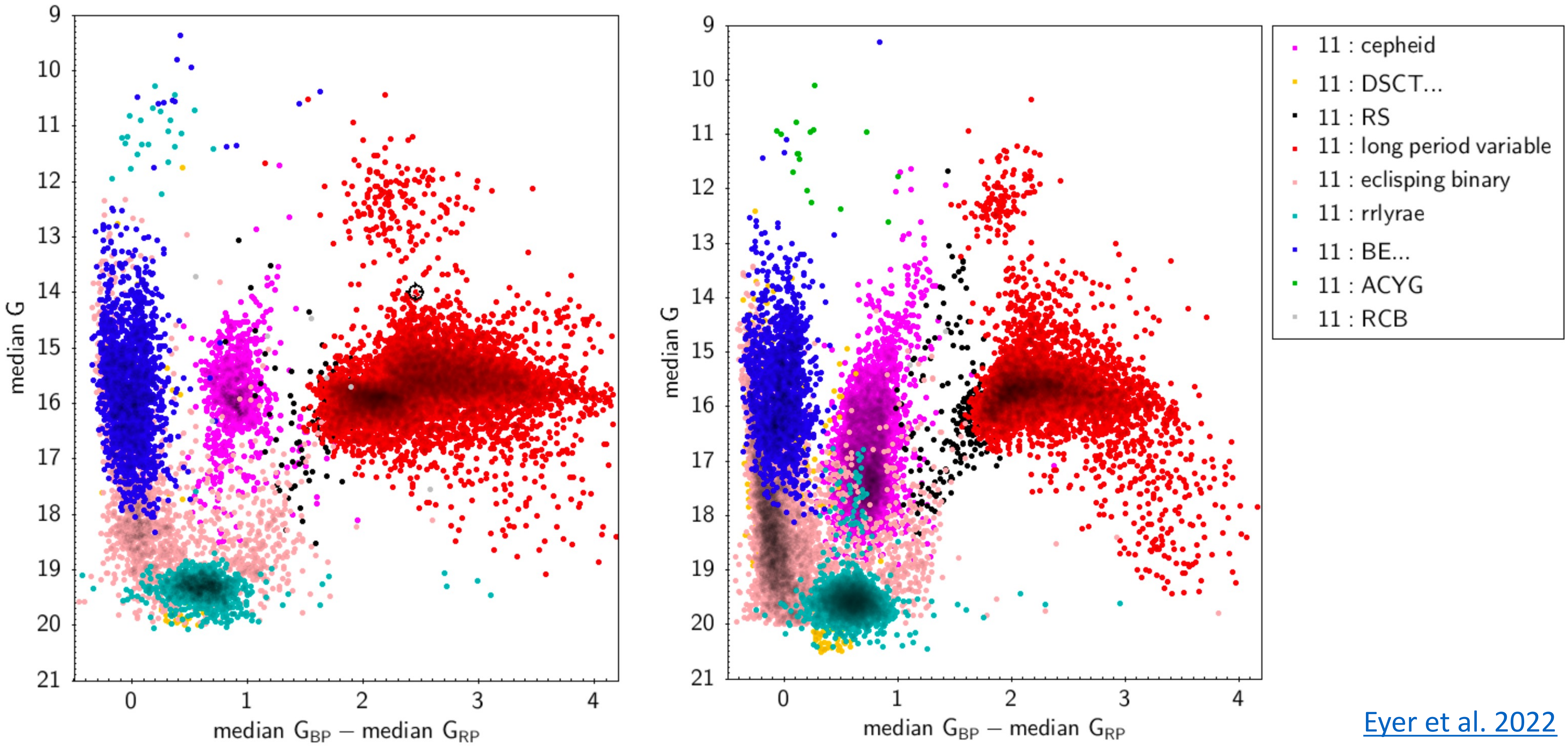
Instability regions in the HRD





Eyer et al. (2018)

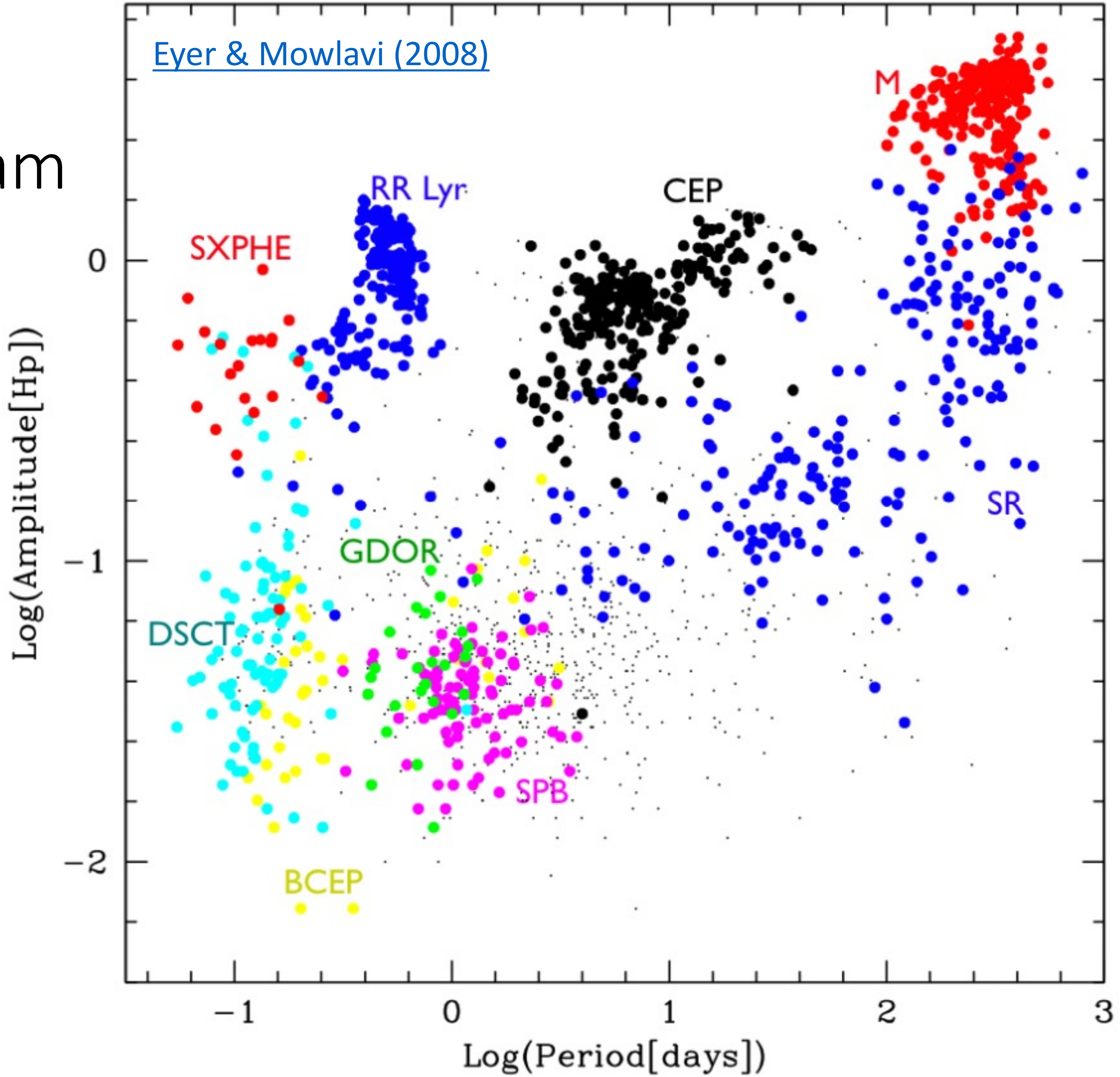
Gaia Pulsating stars in LMC and SMC



[Eyer et al. 2022]

Classifying Pulsations: Period-amplitude diagram

- How to distinguish classes of variable stars?
- CMD is useful, but: overlaps & needs color information
- Pulsations group according to amplitude and timescales
- Can be used with single filter

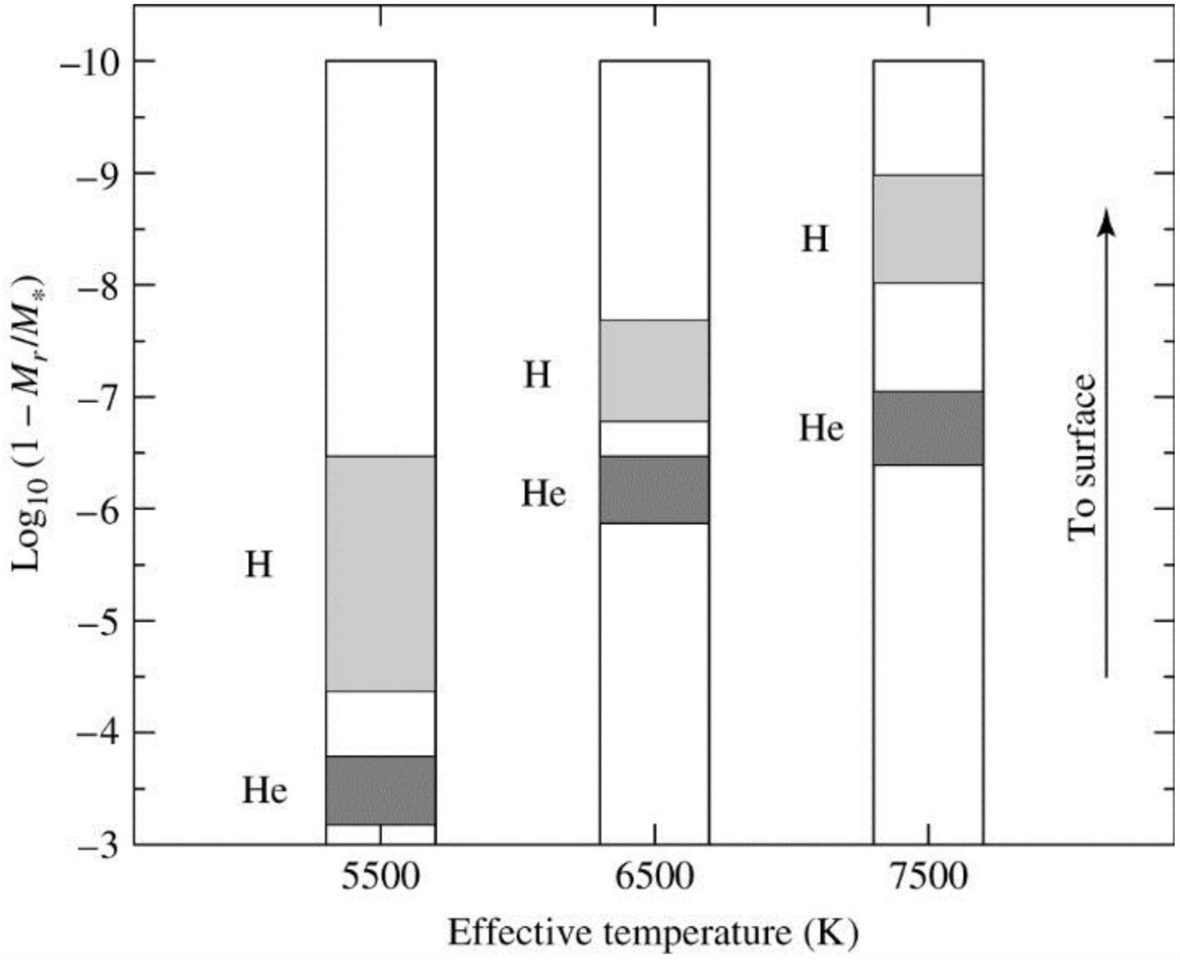


Pulsation mechanisms

- Kappa mechanism (opacity stores thermal energy)
- Epsilon mechanism (change of nuclear energy generation rate)
- Gamma mechanism (adding heat by conduction)
- Verbal, non-equation description of pulsations: http://astro.physics.uiowa.edu/~kgg/teaching/astrophysicsII/pulsation.html

The κ mechanism (opacity)

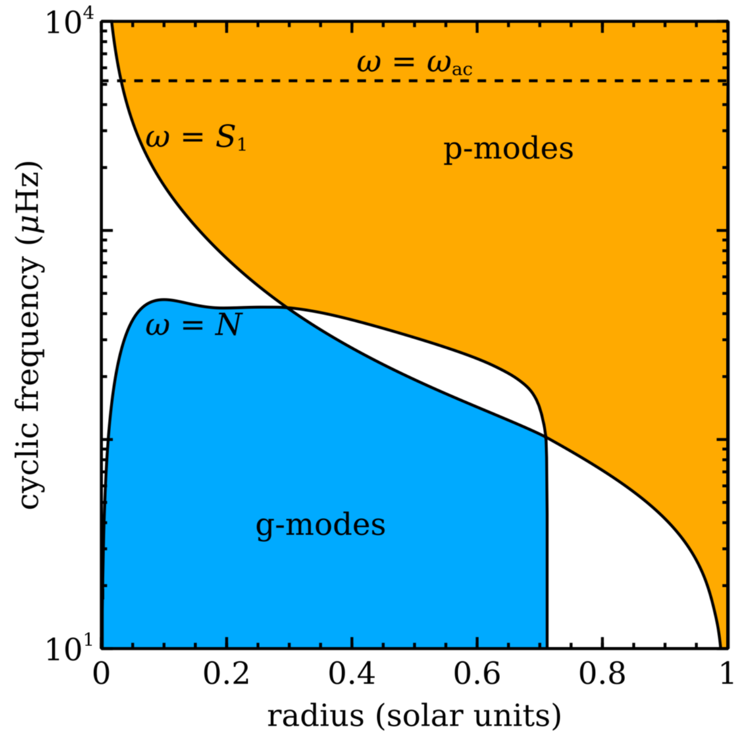
- Partial Helium ionization zone
- Stores compression energy by ionizing $He^{+} \rightarrow He^{++} + e^{-}$
- Ionization *increases* opacity
- Contraction raises temperature, increases radiation pressure
- Outward push until gas cools and He recombines
- Drop in opacity lets radiation pass
- Gravity causes outer layers to recontract and cycles starts anew



[Carrol & Ostlie (2006)]

Pressure and gravity modes

- Restoring force: brings system back to equlibribum
- Most commonly observed: p- and g-modes
- p-modes: acoustic oscillations mostly outer parts (lowest ℓ reaches deepest)
- g-modes: buoancy, confined to core regions



Displacement eigenfunction

$$\frac{d^2\xi_r}{dr^2} = \frac{\omega^2}{c_s^2}\left(\frac{S_l^2}{\omega^2} - 1\right)\left(1 - \frac{N^2}{\omega^2}\right)\xi_r$$

Cowling approximation:
Gravitational potential constant
Stellar structure varies slowly

$$S_l^2 = \frac{\ell(\ell+1)c_s^2}{r^2}$$

Lamb frequency

p-modes if dominant

$$N^2 = g\left(\frac{1}{\Gamma_1 P}\frac{dP}{dr} - \frac{1}{\rho}\frac{d\rho}{dr}\right)$$

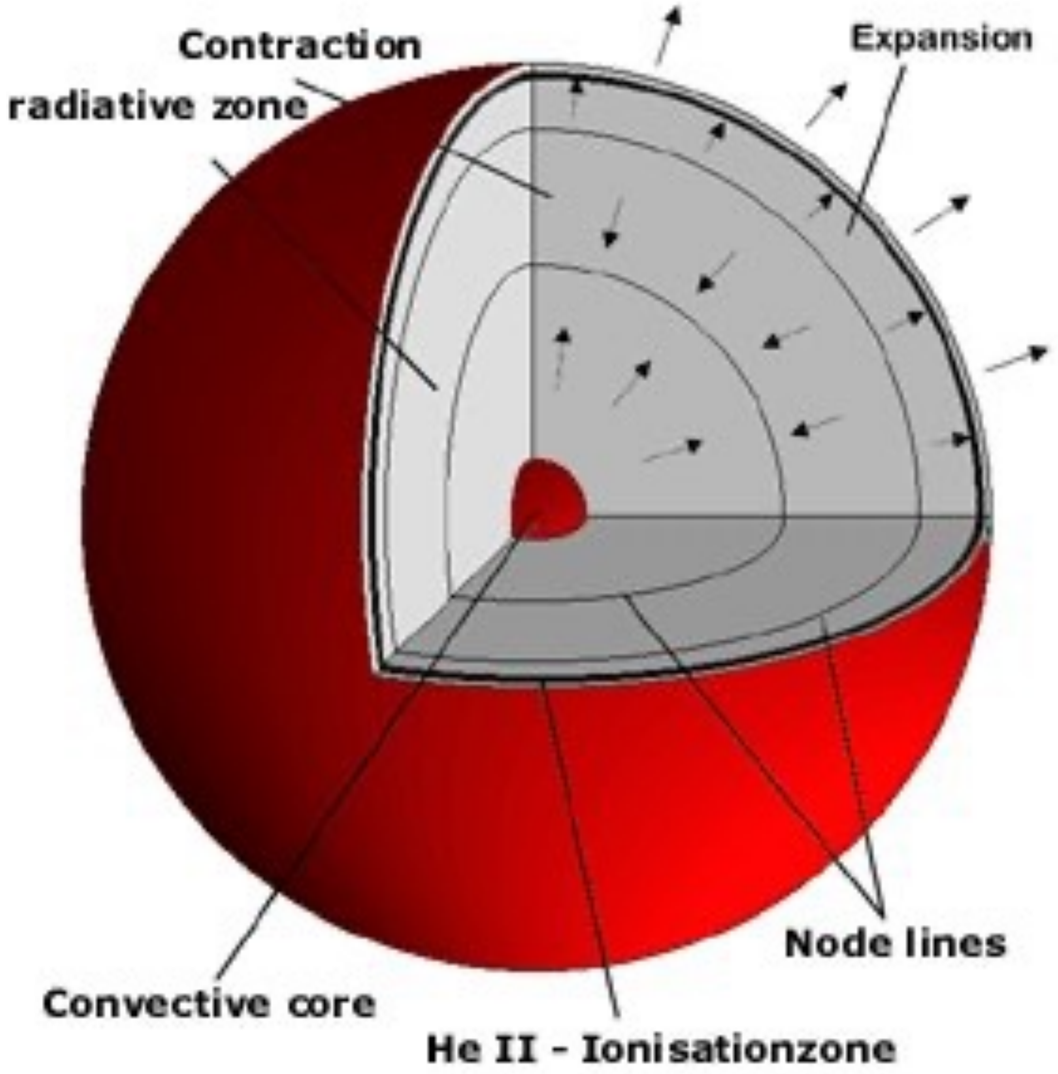
Brunt-Väisälä frequency

g-modes if dominant

$$\Gamma_1 = \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_S$$

Radial pulsations: Fundamental vs overtone

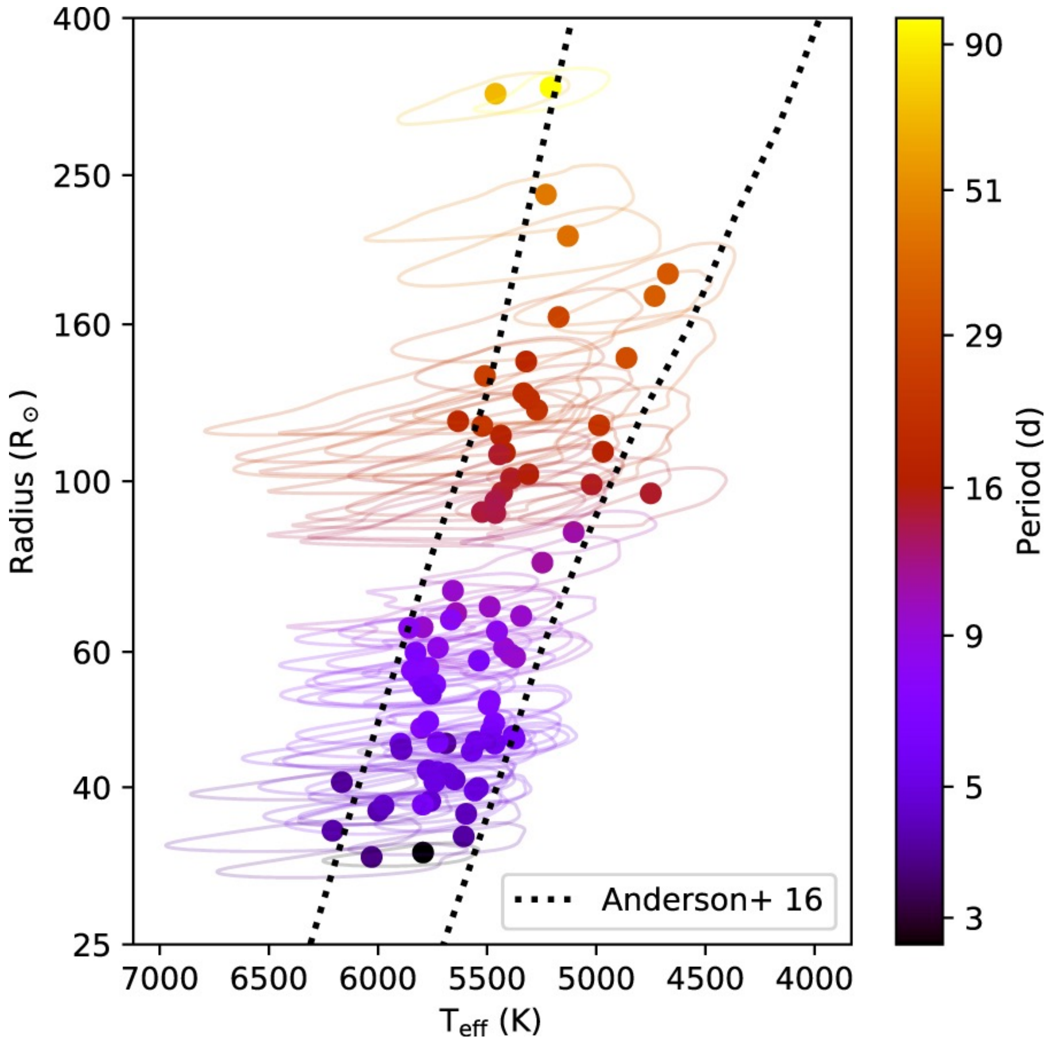
- Special case of non-radial oscillation
- Spherical harmonics quantum numbers n, l, m
- Radial order: n
- n=0 : fundamental mode
- n=1 : first overtone
- Special feature: radial oscillations change size while maintaining shape



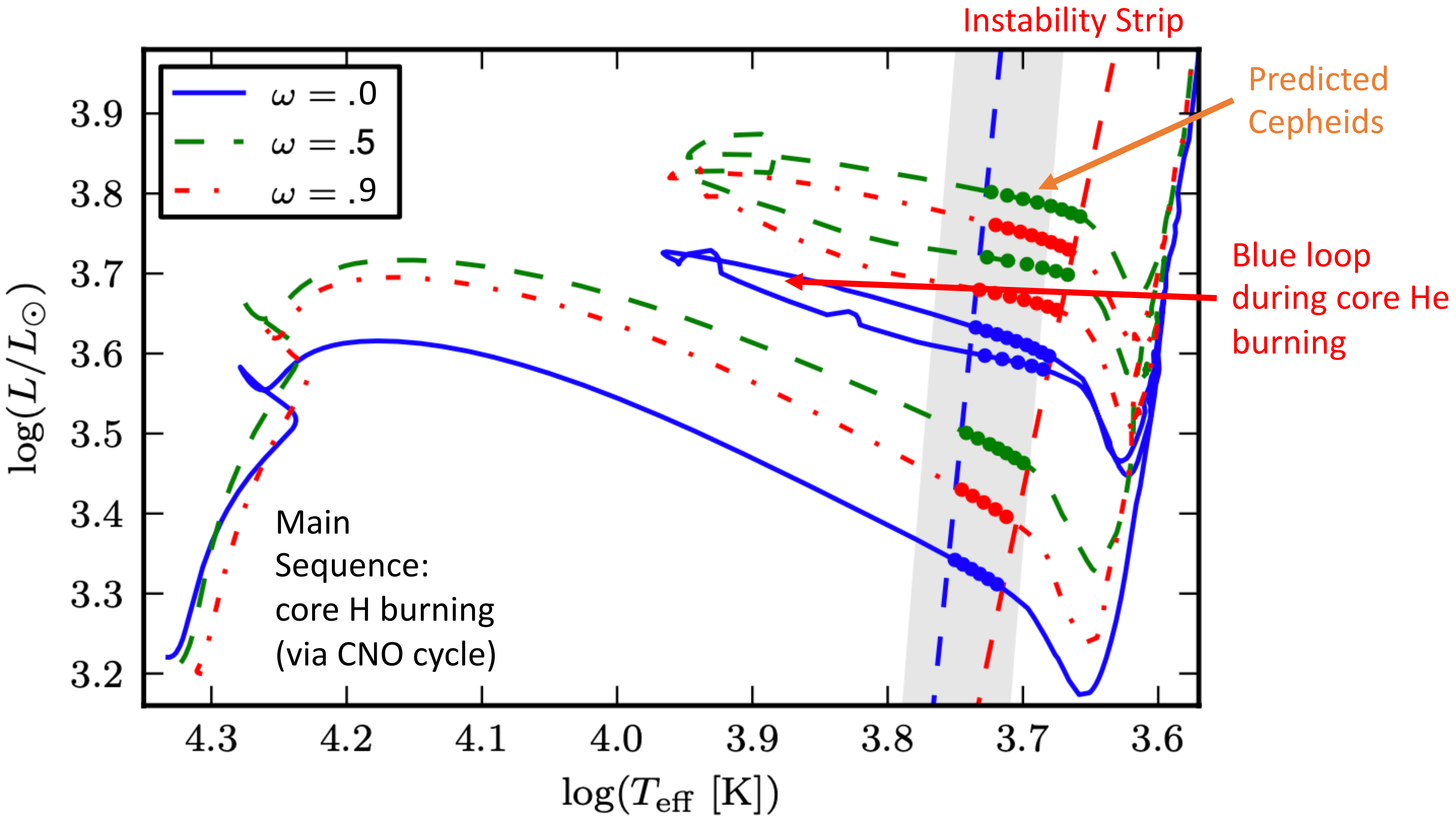
The classical instability strip

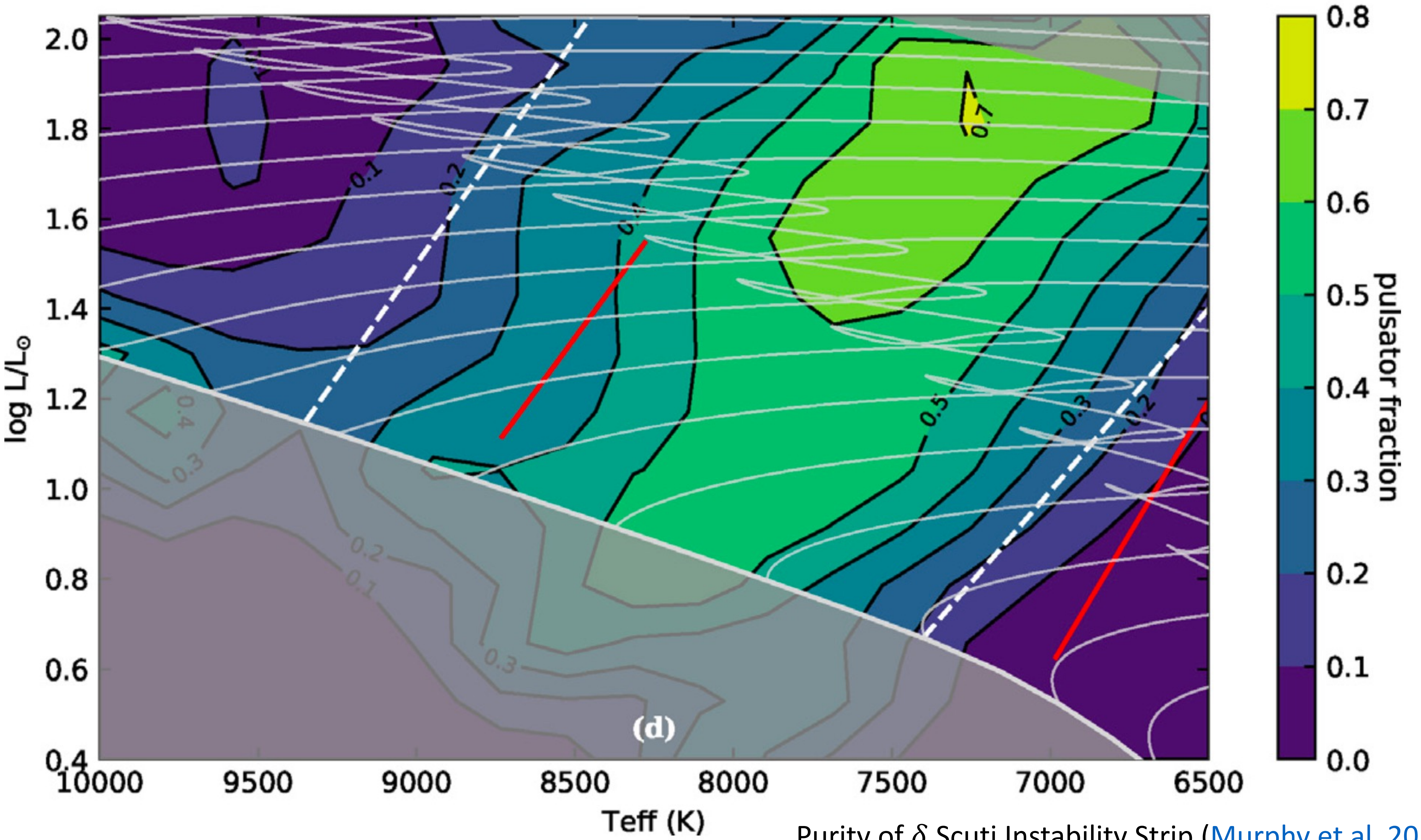
- κ mechanism requires PIZ to operate
- PIZ requires temperature “just right”
- Too hot: Helium fully ionized
- Too cool: Convection dampens pulsations
- Narrow temperature range where this can work
- Near vertical region in HRD
- Location of boundary depends on stellar structure

Javanmardi et al. (2021)



On the purity of instability strips

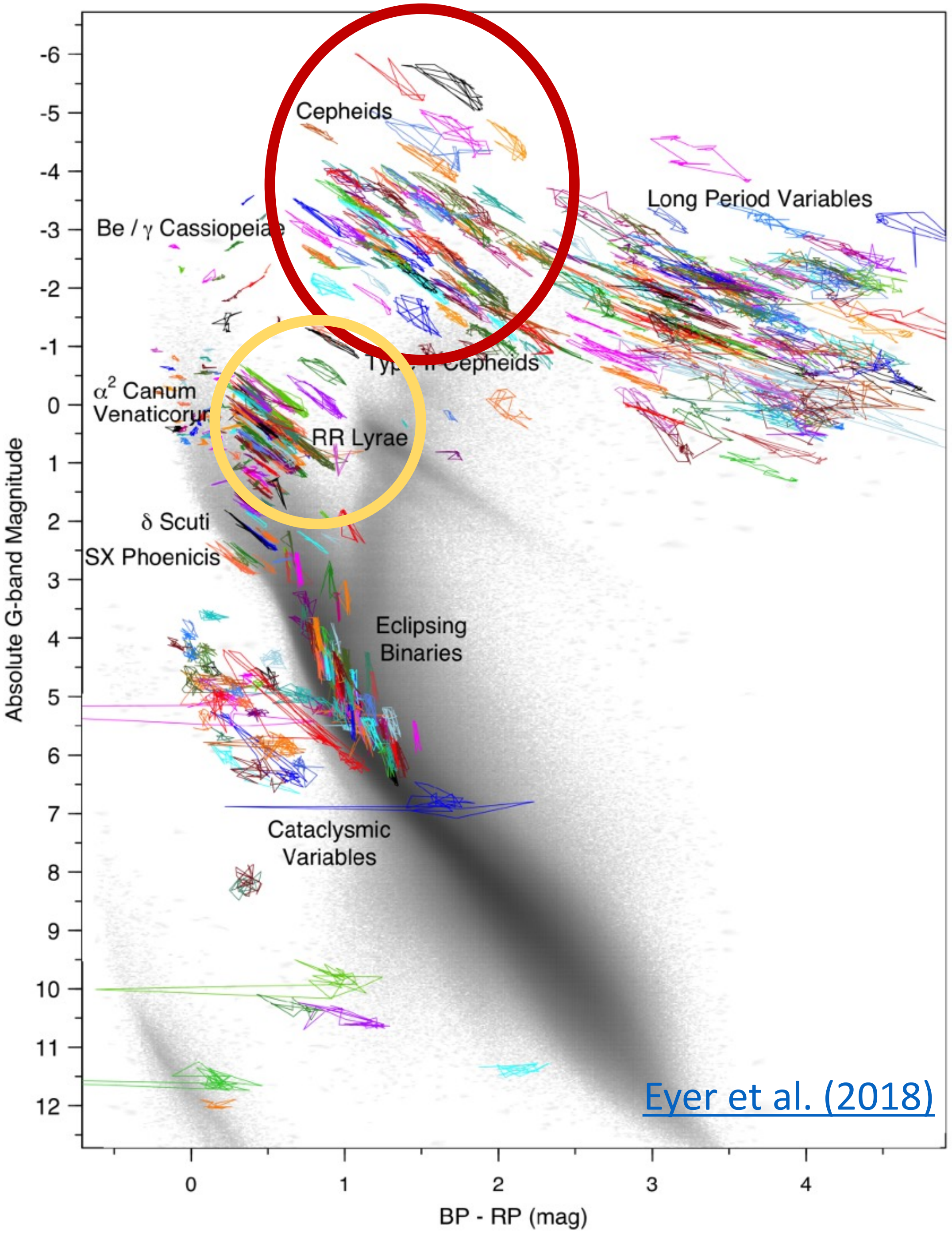




Purity of δ Scuti Instability Strip (Murphy et al. 2019)

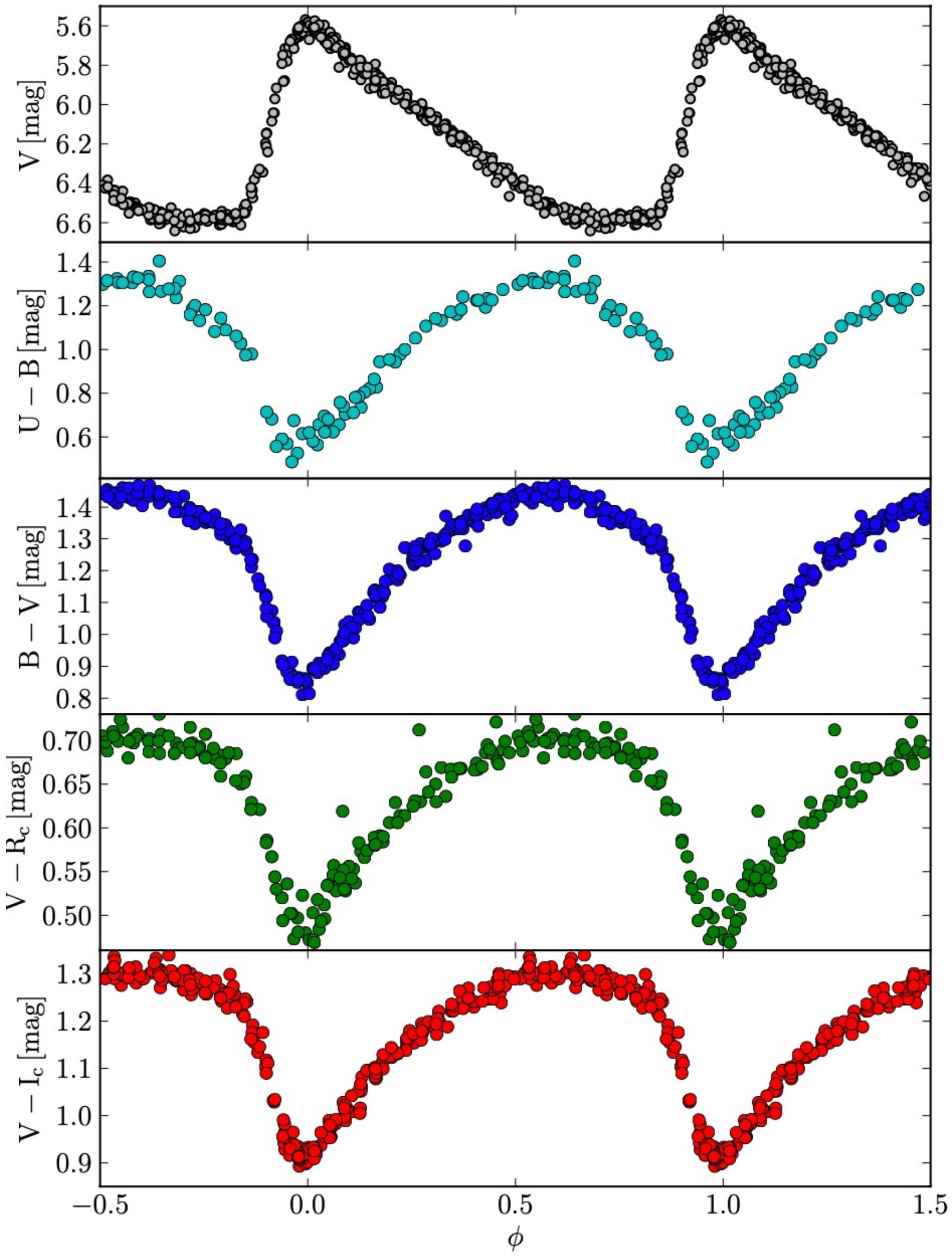
Classical Cepheids

(and a bit on RR Lyrae stars): high-amplitude radial pulsations in classical instability strip



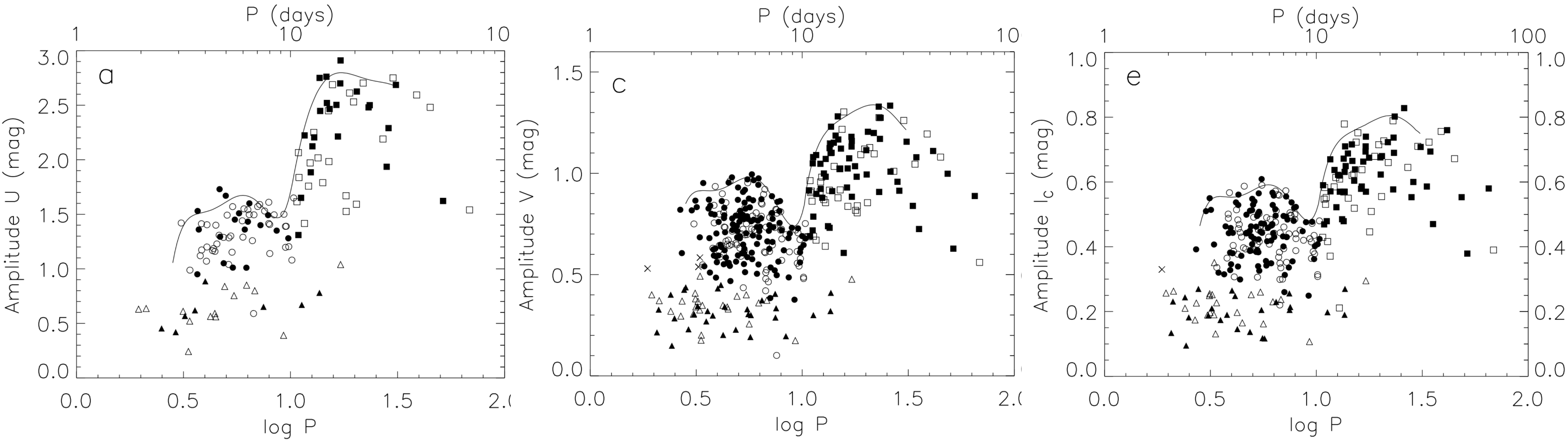
Chromaticity of pulsational variability

- Pulsations are chromatic!
- When is a pulsating star brightest?
- Temperature dominates variability
- Wavelength dependent phase offsets
- Amplitudes decrease with wavelength



Anderson (2013) using data by Berdnikov (2008)

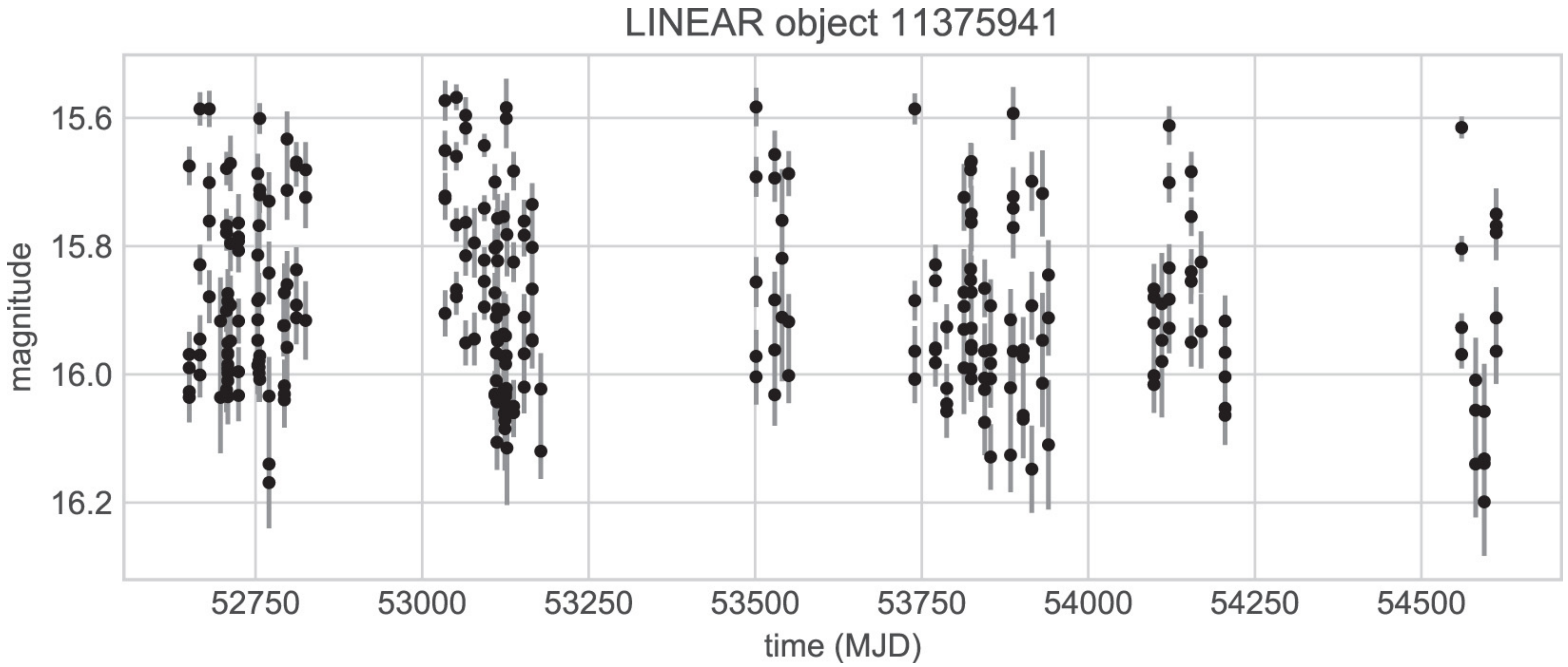
Chromaticity of pulsations



[Klagyivik & Szabados (2009)]

Oh pulsation, where art thou?

VanderPlas et al. (2007)

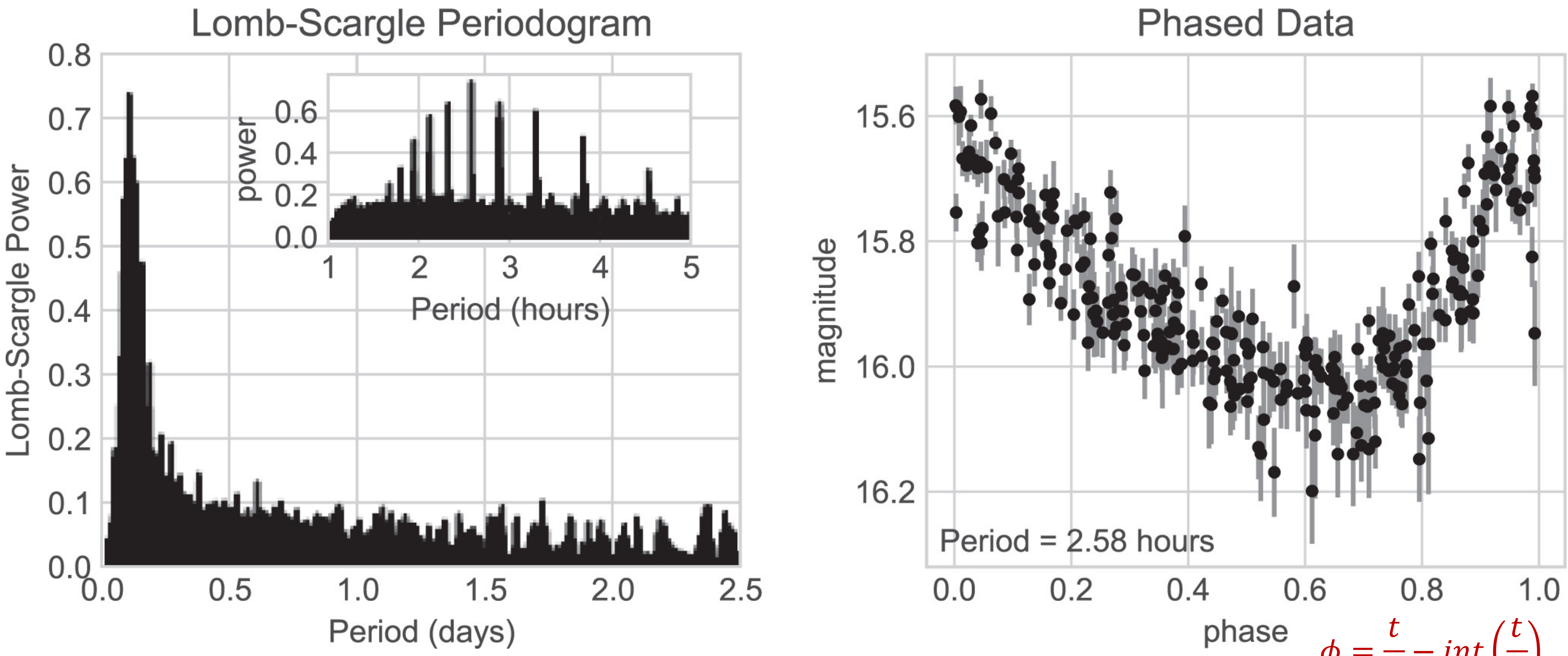


Time-series analysis: T0, P, phase, O-C

- Time Series Analysis:
  - characterizes observed temporal correlation among measurements (including estimates of significance)
  - forecasts future behavior
- Pulsations usually (quasi-)perdiodic
- T0 : precisely measurable reference time; often max light (arbitrary?)
- P : the period of the signal = 1/f if f is frequency
- Phase : $\phi = \frac{t}{P} - int\left(\frac{t}{P}\right)$
- $O - C$: observed minus calculated, tracks variations in period (e.g. apsidal motion, light time effect)

Period search to allow phase folding

VanderPlas et al. (2007)

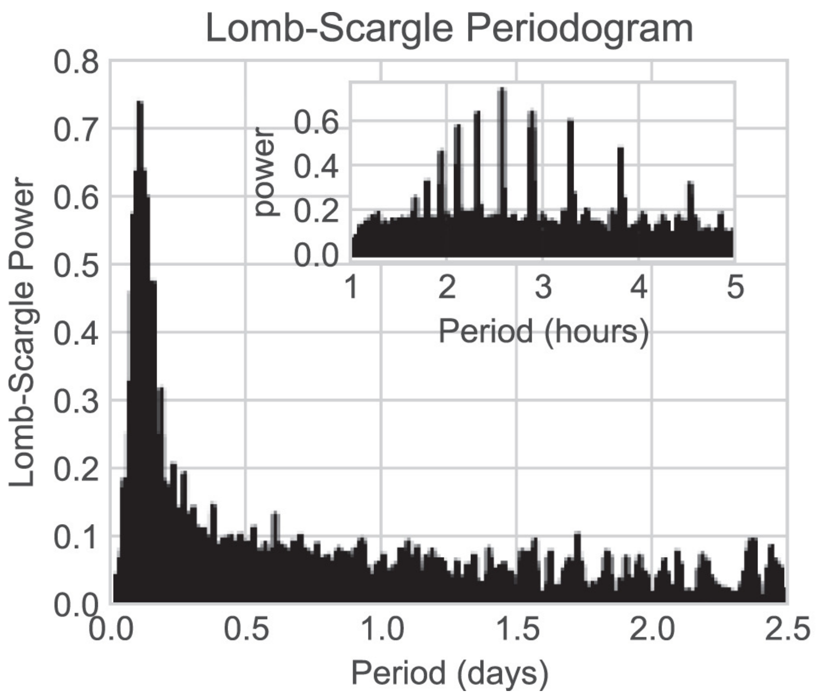


$$\phi = \frac{t}{P} - int\left(\frac{t}{P}\right)$$

The Periodogram

- Searches for *strictly periodic* variations
- Fourier transform of the time series: $H(f) = \int h(t)e^{-i2\pi \mathrm{ft}}dt$
- Produces the power spectrum of frequencies in the dataset
- Power spectral density (PSD) for $0 \leq f \leq \infty$:
$$PSD(f) = |H(f)|^2 + |H(-f)|^2$$
- PSD tells power in frequency interval $f + df$
- Different implementations
  - Fast Fourier Transform (FFT) cannot be used on irregularly sampled data
  - Lomb-Scargle periodogram: irregular sampling & heteroscedastic errors!

The Lomb-Scargle Periodogram



- Uses single sinusoid model per frequency
- $P_{LS}(\omega) = \frac{1}{V}\left[\frac{R^2(\omega)}{C(\omega)} + \frac{I^2(\omega)}{S(\omega)}\right]$
  - $\bar{y} = \sum_{j=1}^{N} w_j y_j$: weighted mean
  - $V = \sum_{j=1}^{N} w_j \left(y_j - \bar{y}\right)^2$: variance
  - $w_j = \frac{1}{W} 1/\sigma_j^2,\ \ W = \sum_{j=1}^{N} 1/\sigma_j^2$
  - $R(\omega) = \sum_{j=1}^{N} w_j (y_j - \bar{y}) \cos[\omega(t_j - \tau)],\quad \mathrm{I}(\omega) = \sum_{j=1}^{N} w_j (y_j - \bar{y}) \sin[\omega(t_j - \tau)]$
  - $\mathrm{C}(\omega) = \sum_{j=1}^{N} w_j (y_j - \bar{y}) \cos^2[\omega(t_j - \tau)],\ \ \mathrm{S}(\omega) = \sum_{j=1}^{N} w_j (y_j - \bar{y}) \sin^2[\omega(t_j - \tau)]$
- $\omega = 2\pi f = 2\pi P^{-1}$
- Many local maxima: grid search within frequency range e.g. $\omega_{min} = 2\pi/\Delta T_{obs}$ and $\omega_{max} = \pi/\overline{\delta t}$ (median sampling)

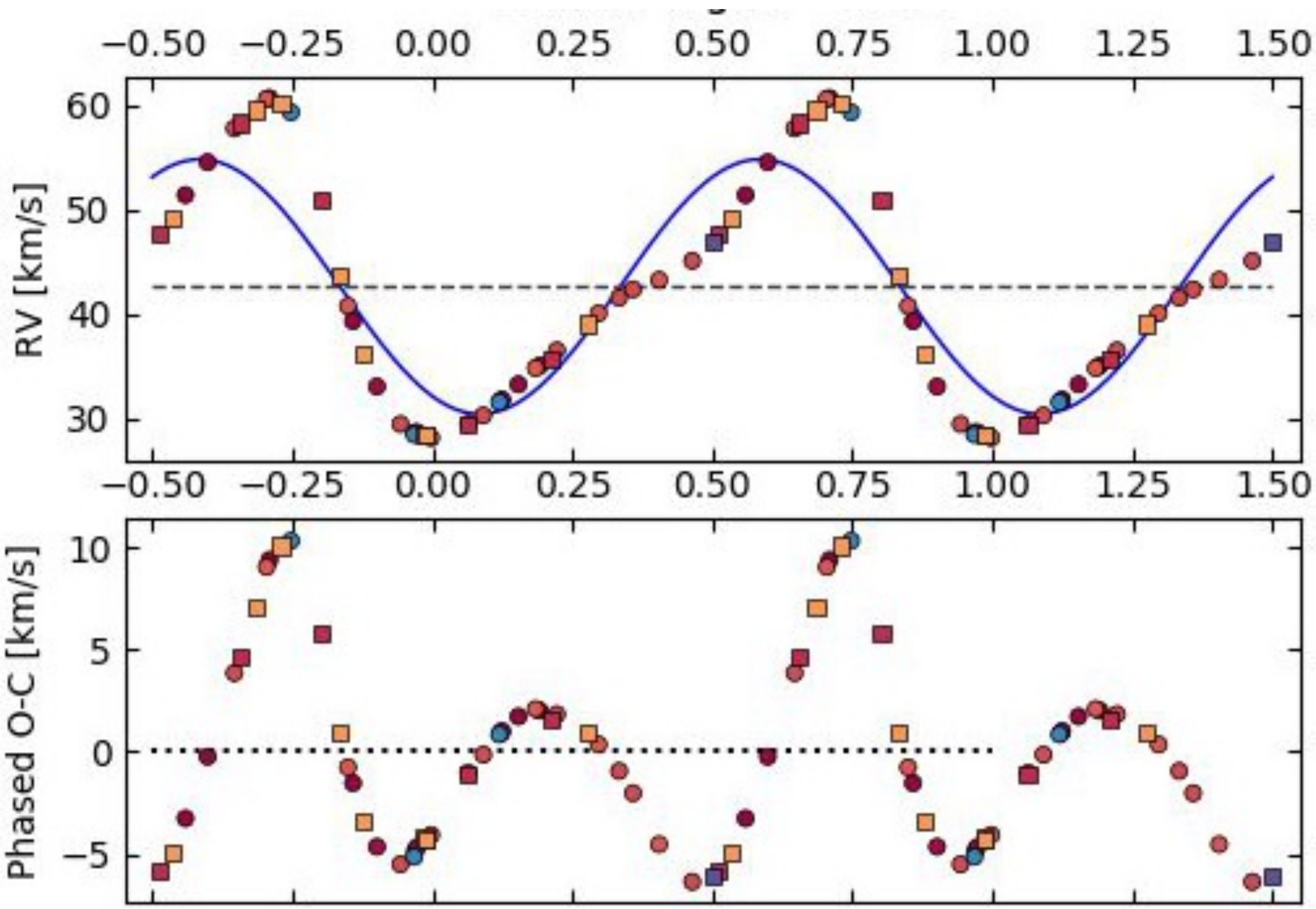
Often: frequency in [cycles/day]

Generalized LS periodogram (Zechmeister & Kürster 2009) improves on LS by generalizing relation to Chi^2 fitting, can also deal with Keplerians

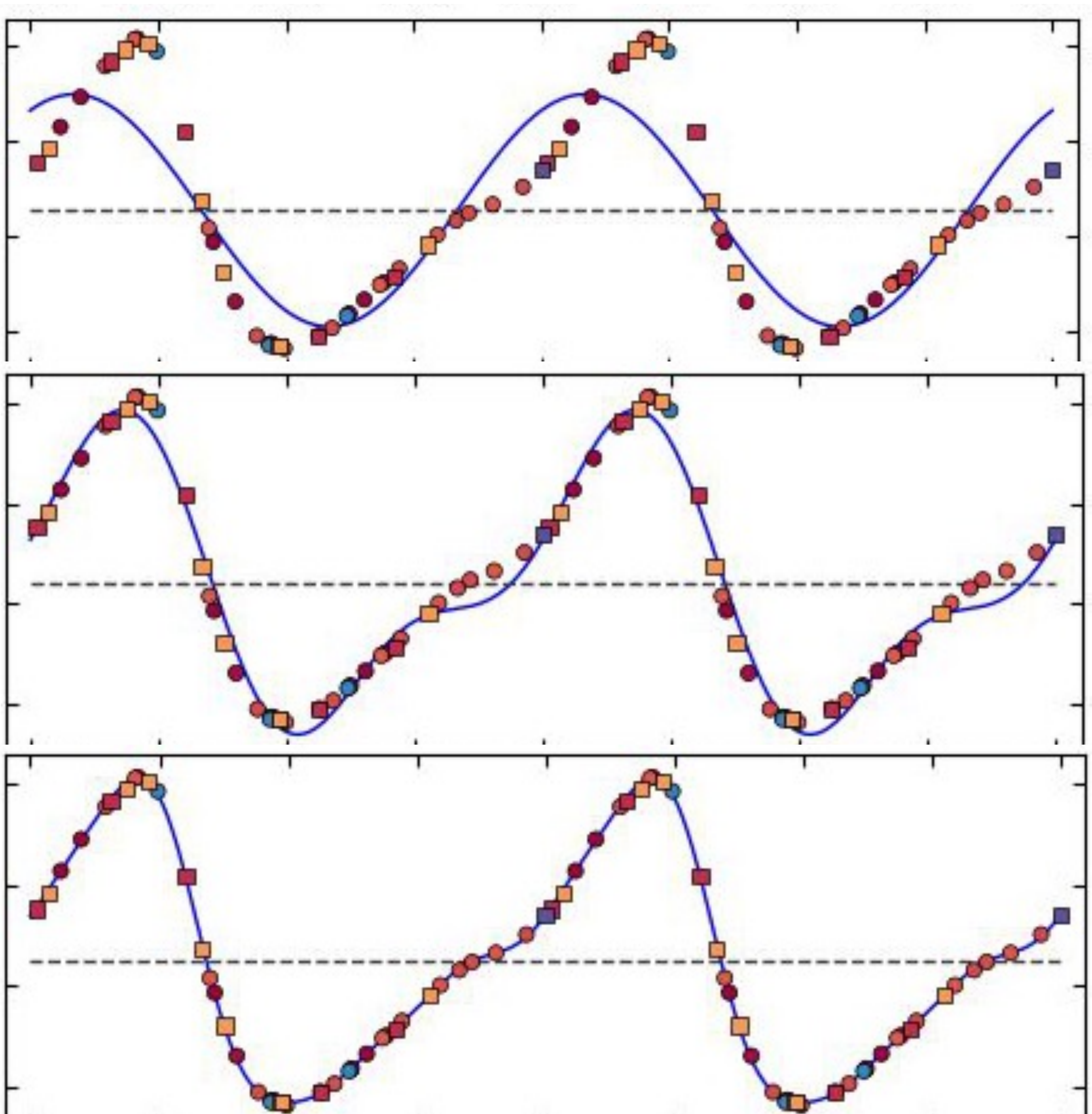
What if the signal is not a single sinusoid?

- Multi-harmonic Fourier series to the rescue
- $y(t) = b_0 + \sum_{m=1}^{M} a_m \sin(m\omega t) + b_m \cos(m\omega t)$
- $P_M(\omega) = \frac{2}{V}[\sum_{m=1}^{M} R_m^2(\omega) + I_m^2(\omega)]$
- $I_m(\omega) = \sum_{j=1}^{N} w_j y_j \sin(m\omega t_j)$
- $R_m(\omega) = \sum_{j=1}^{N} w_j y_j \cos(m\omega t_j)$
- For large N (observations), median a posteriori values:
  $a_m = 2I_m(\omega)/N, \quad b_m = 2R_m(\omega)/N, \quad b_0 = \bar{y}$

Choosing the right model for phased data



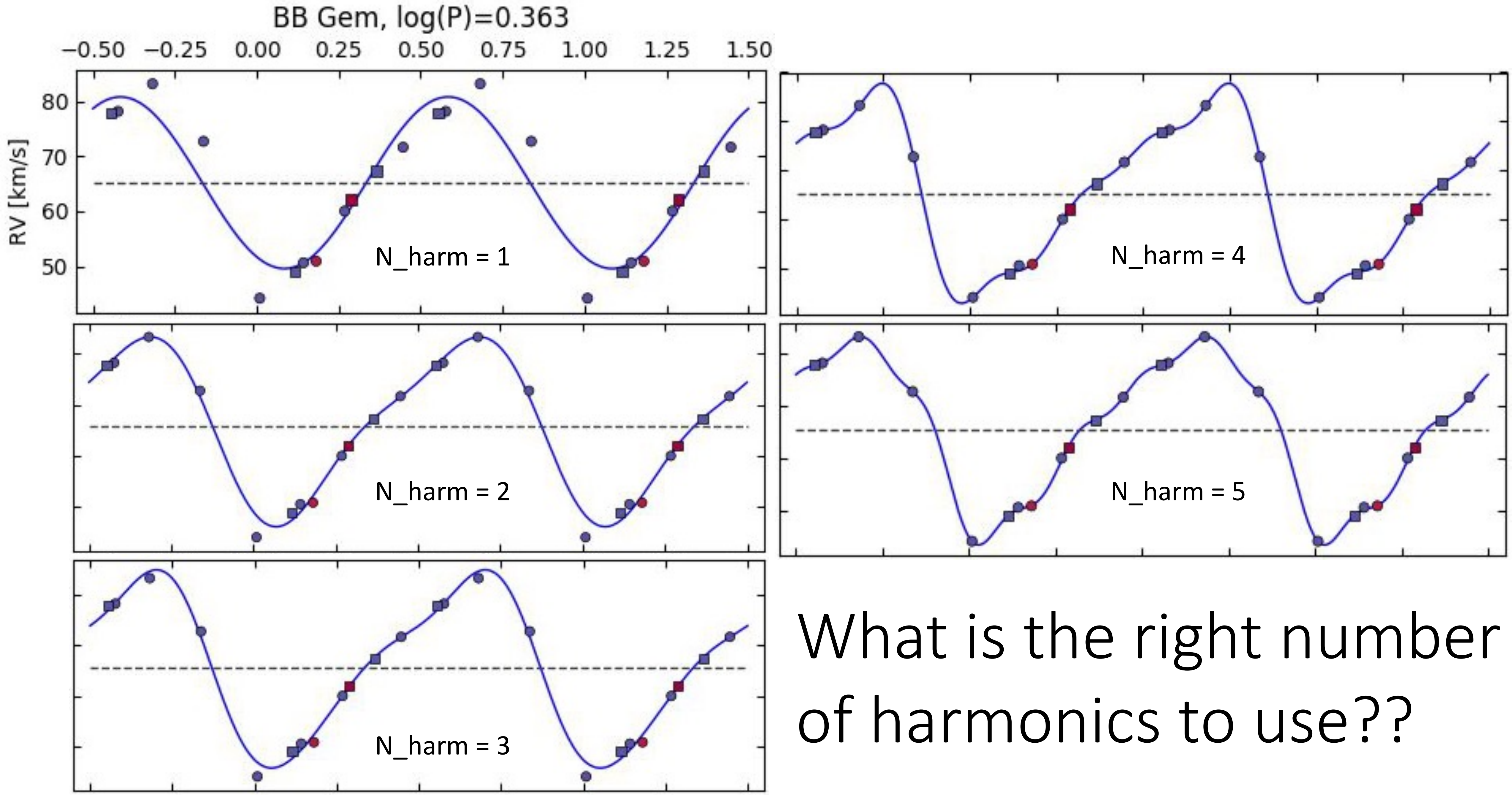
Adding harmonics



N_harm = 1

N_harm = 2

N_harm = 5



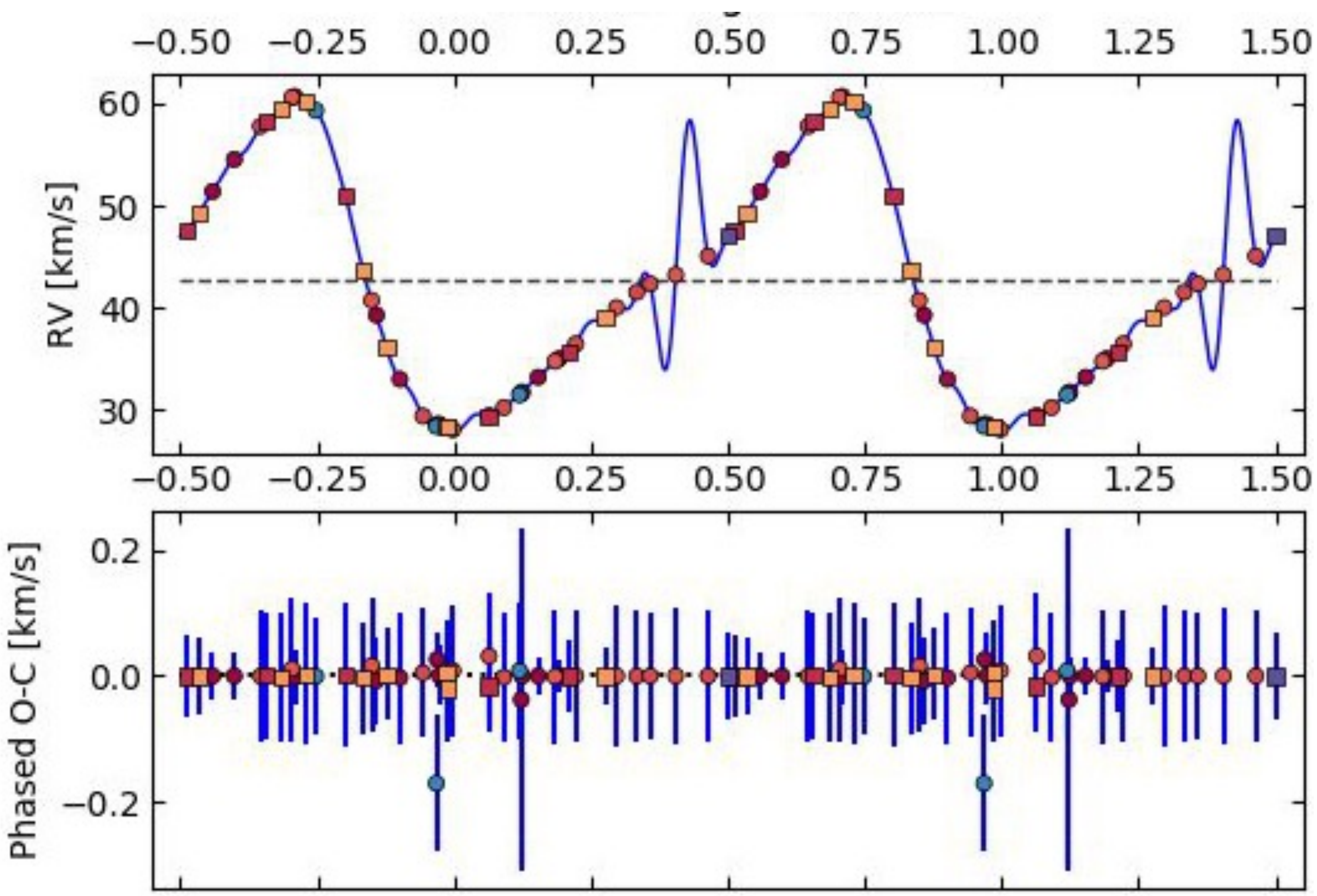
What is the right number of harmonics to use??

N_harm = 18

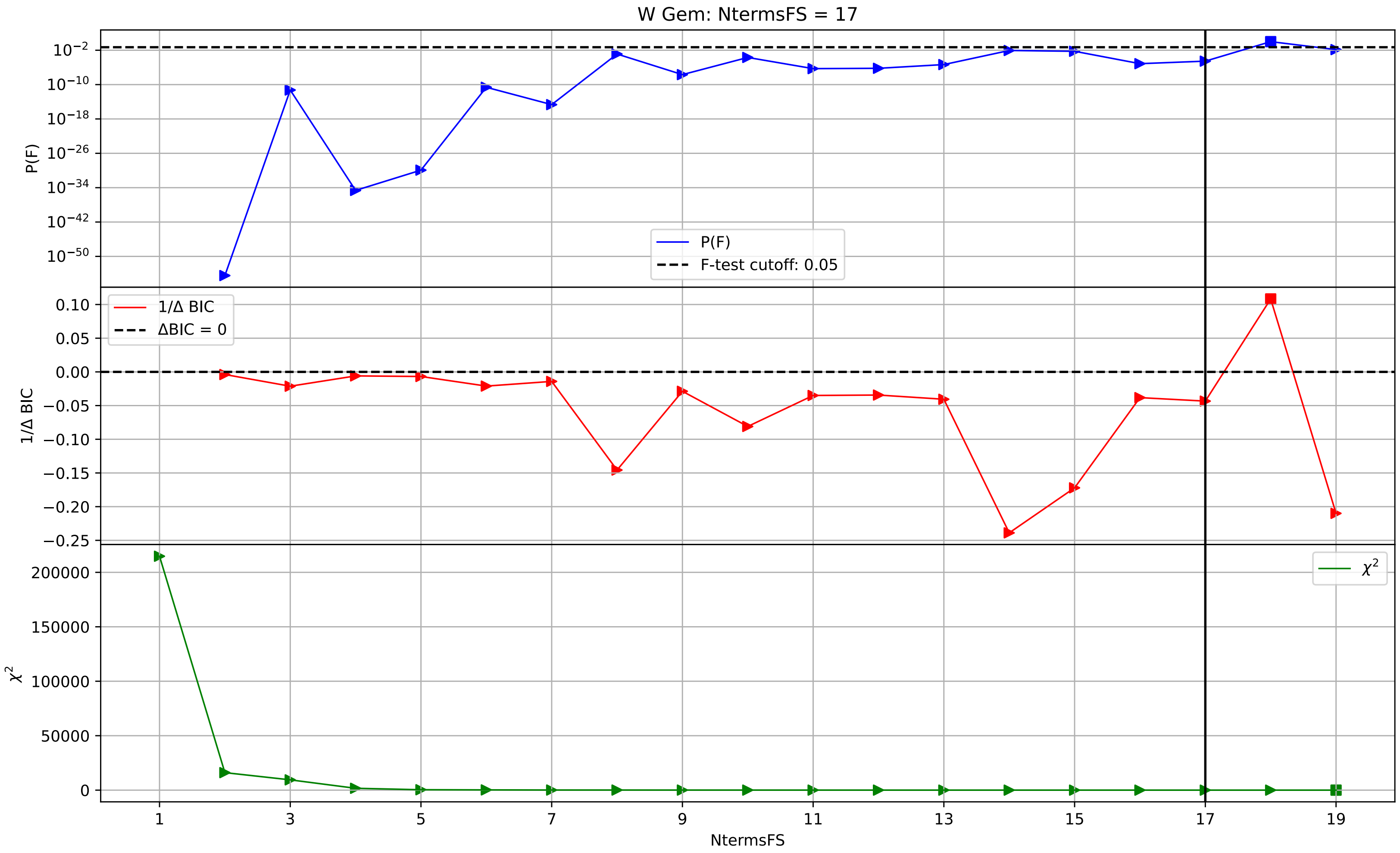
Knowing when to stop (adding harmonics)

- Finding the right N_m can be challenging
- Consider sampling and complexity of variability
- Overfitting can lead to false predictions
- Tools for model selection
  - F-test compares gain in χ^2 to cost in N_{dof} (hypothesis test)
  - Bayesian information criterion (BIC): similar with likelihoods

$$BIC = k \ln n - 2 \ln \hat{L}$$



Model comparison & brute force method



Accept more complex model if

F-Test

P(F) < p (=0.05)

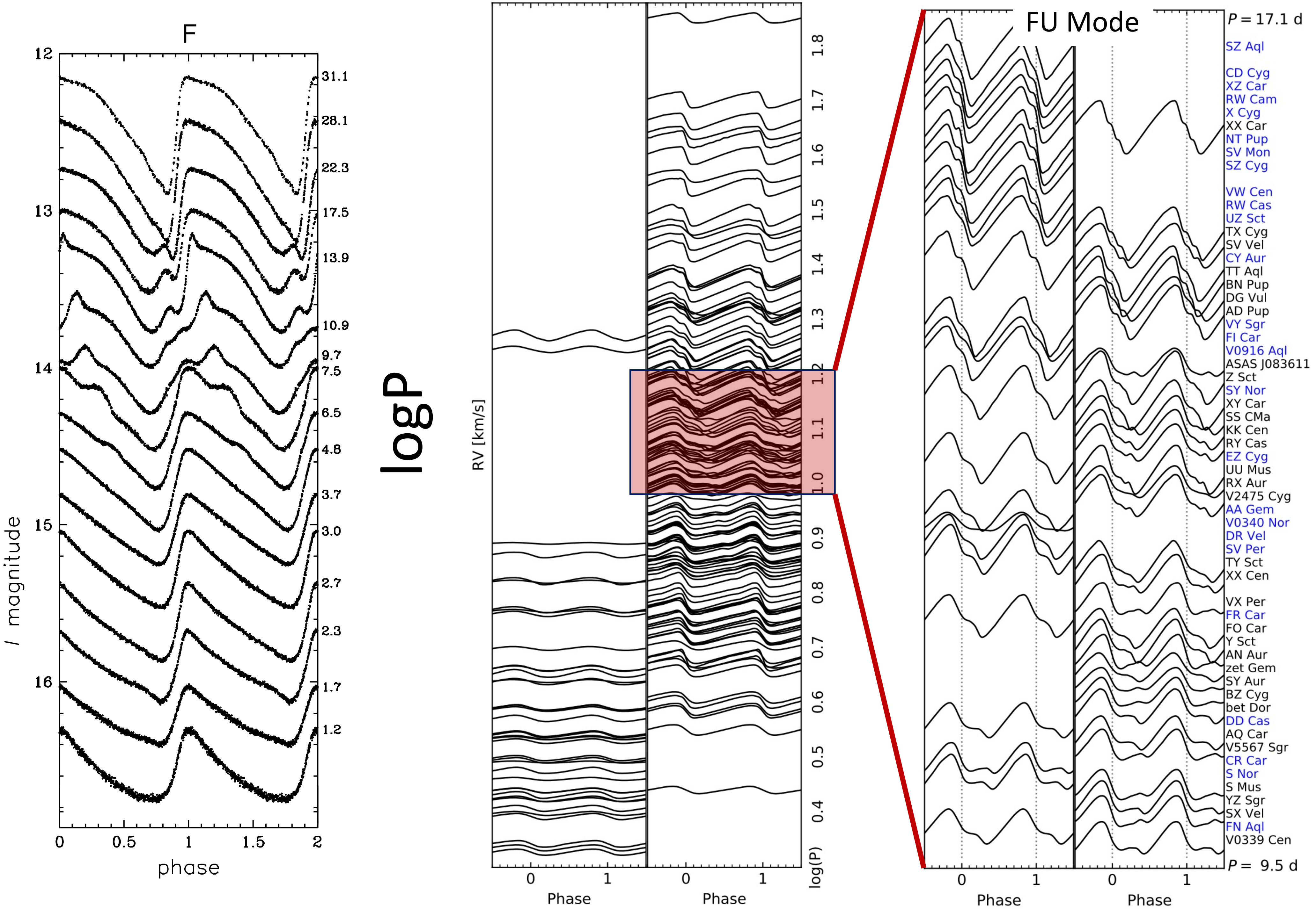
OR/AND

BIC

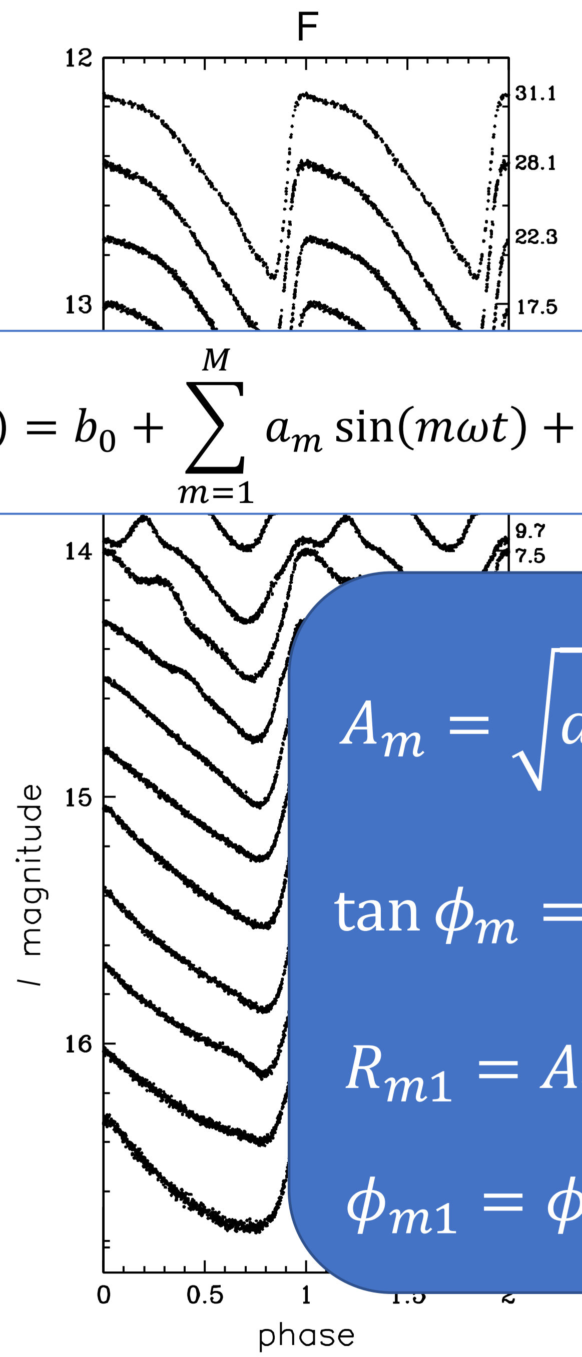
1/ΔBIC < 0

$$\mathrm{BIC} = k\ln(n) - 2\ln(\widehat{L})$$

The Hertzsprung Progression



The Hertzsprung Progression



$$\omega = 2\pi/P$$

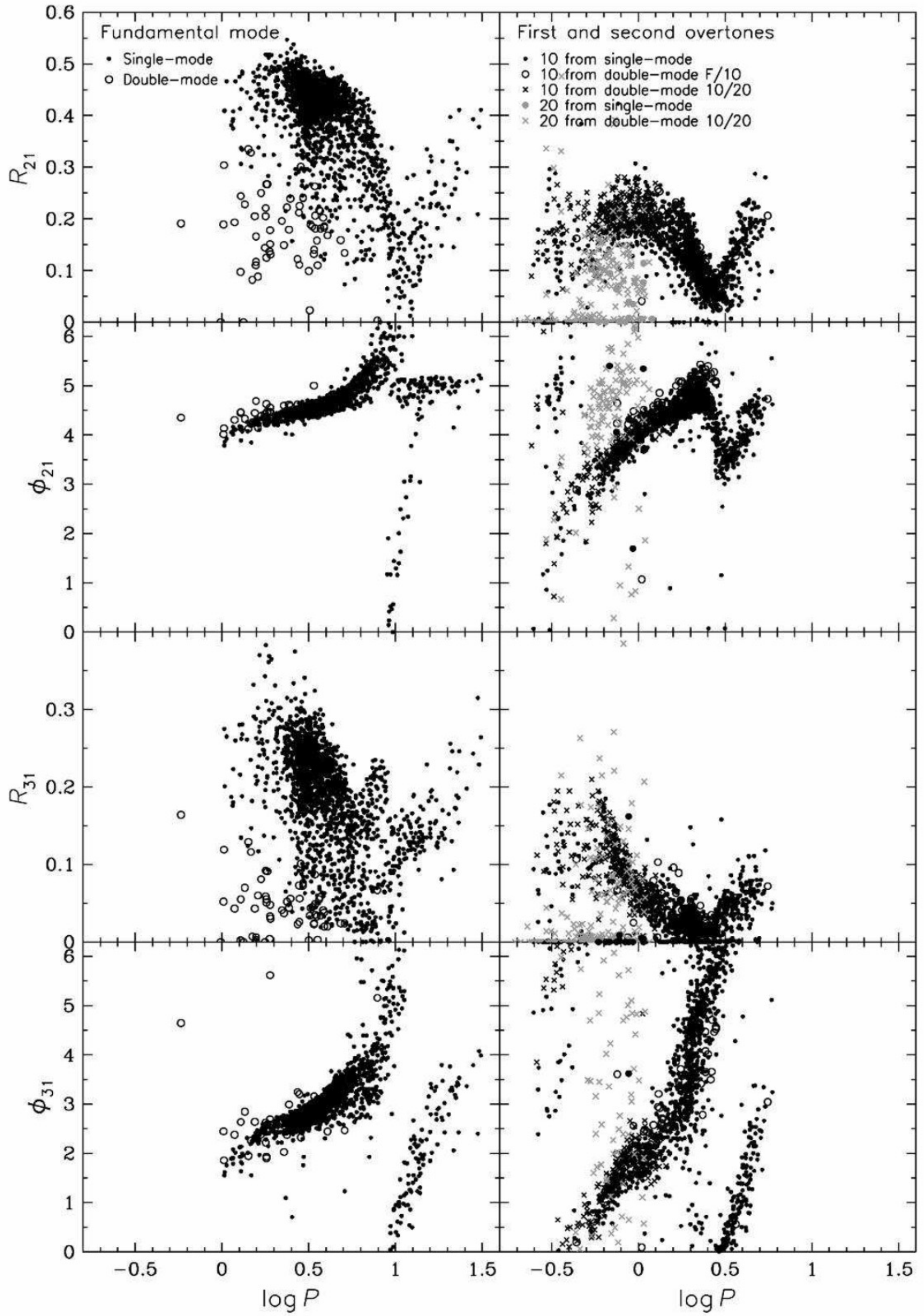
$$y(t) = b_0 + \sum_{m=1}^{M} a_m \sin(m\omega t) + b_m \cos(m\omega t)$$

$$A_m = \sqrt{a_m^2 + b_m^2}$$

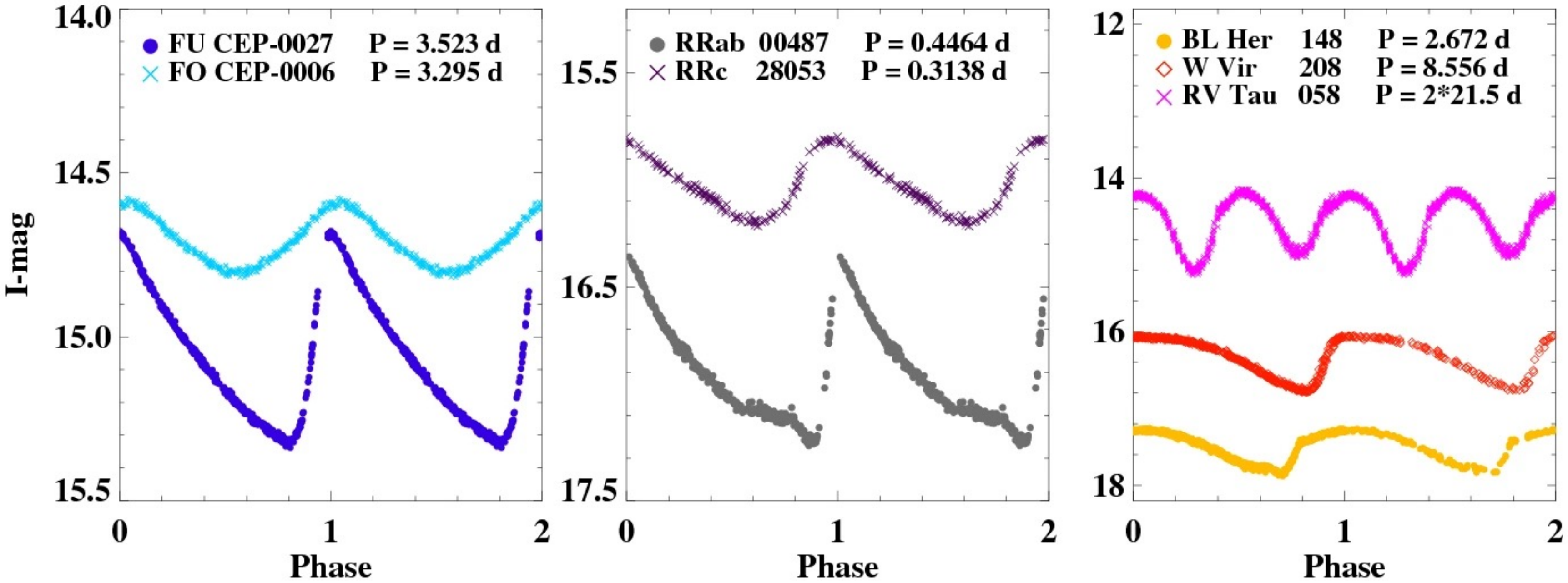
$$\tan \phi_m = \frac{b_m}{a_m}$$

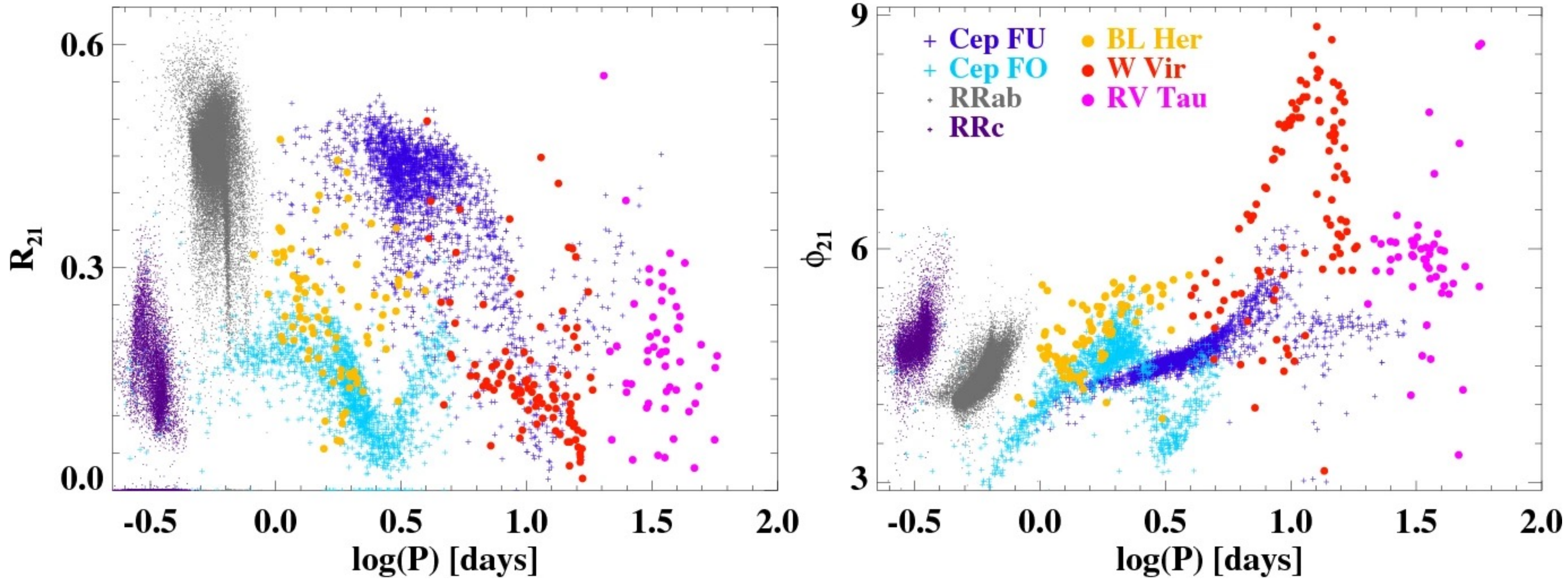
$$R_{m1} = A_m / A_1$$

$$\phi_{m1} = \phi_m - i \cdot \phi_1$$



Fourier decomposition

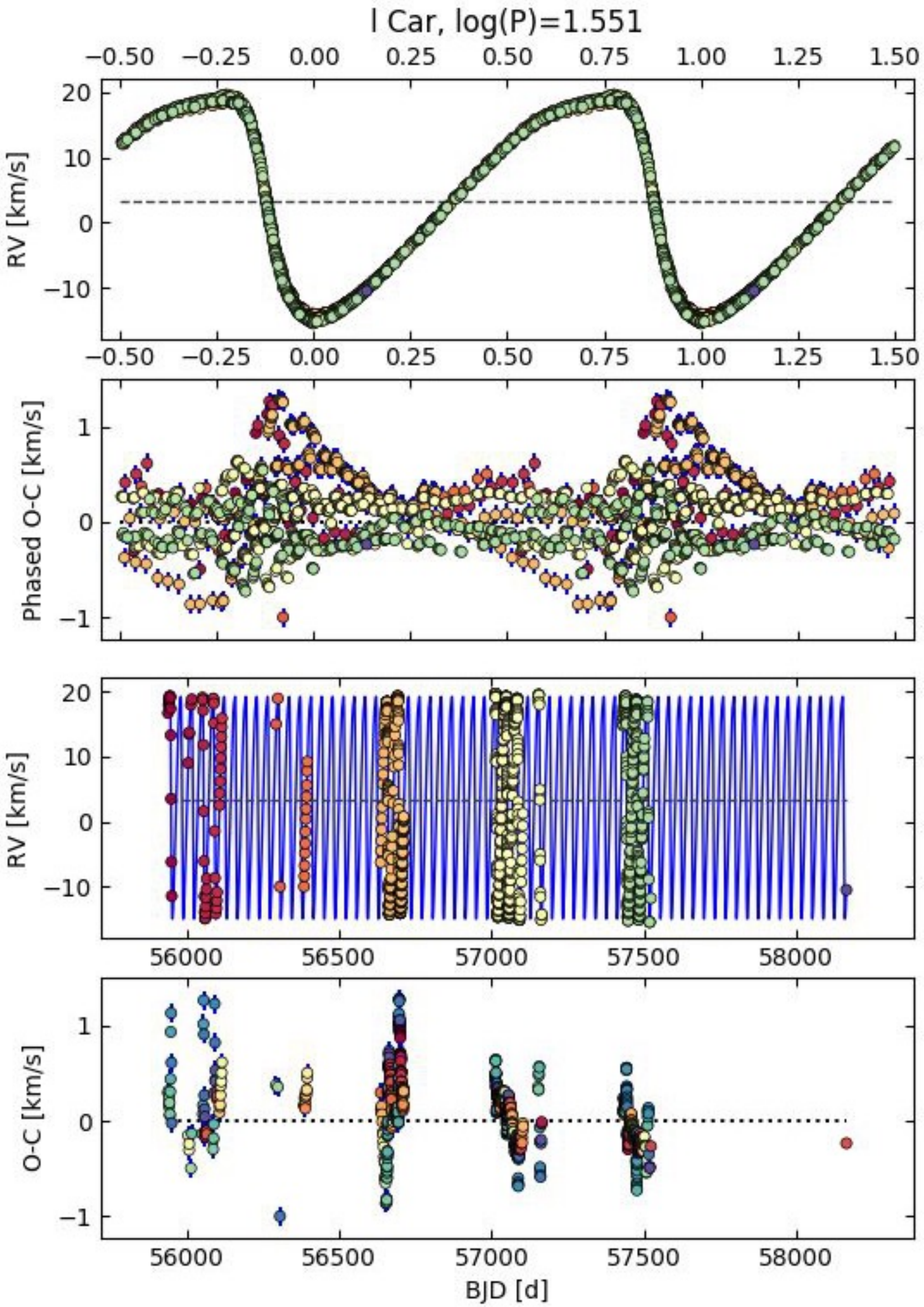




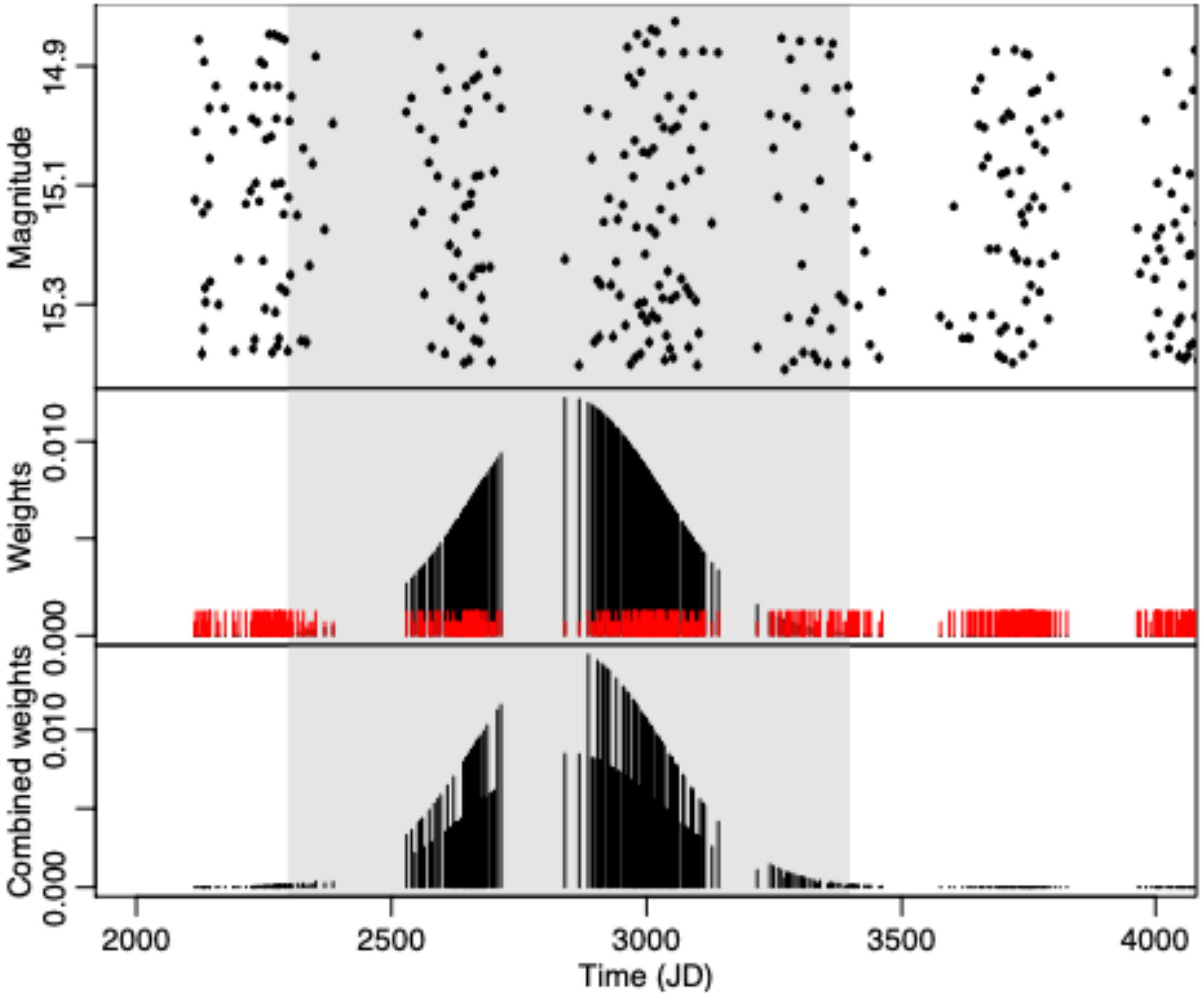
Bhardwaj (2020)

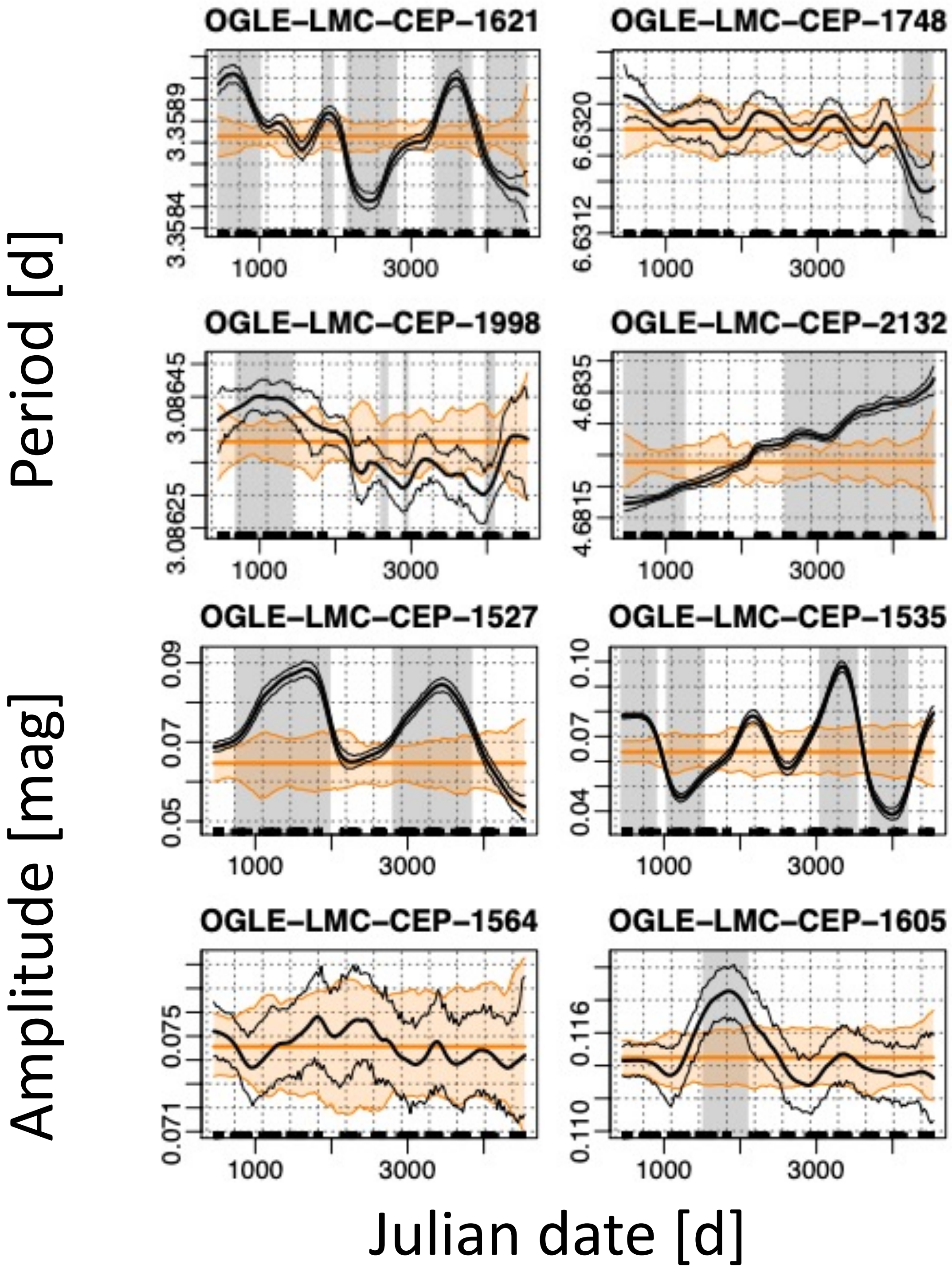
Not all pulsations periodic

- Fourier analysis requires periodicity
- Residuals showing structure:
  - Additional oscillation modes?
  - Other noise?



Time-variable variability

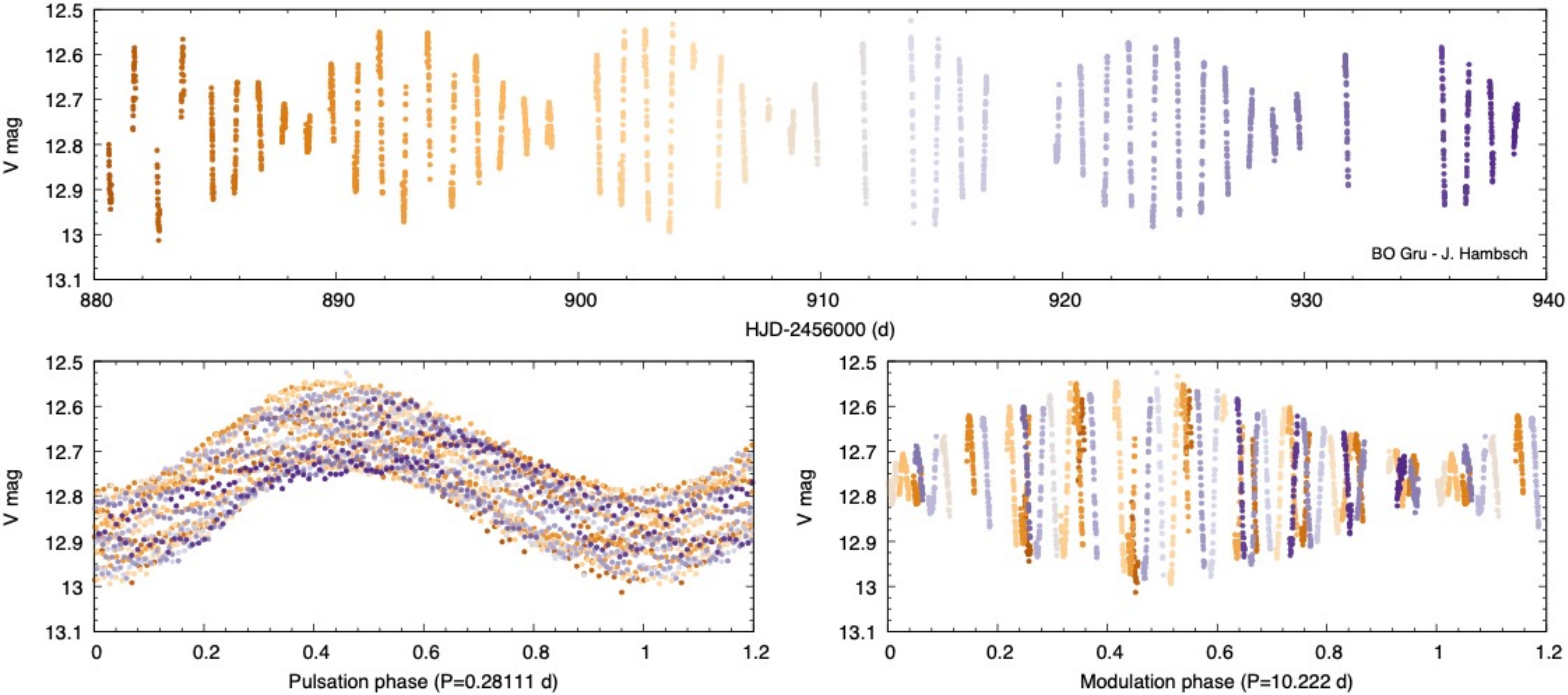




Süveges & Anderson (2018)

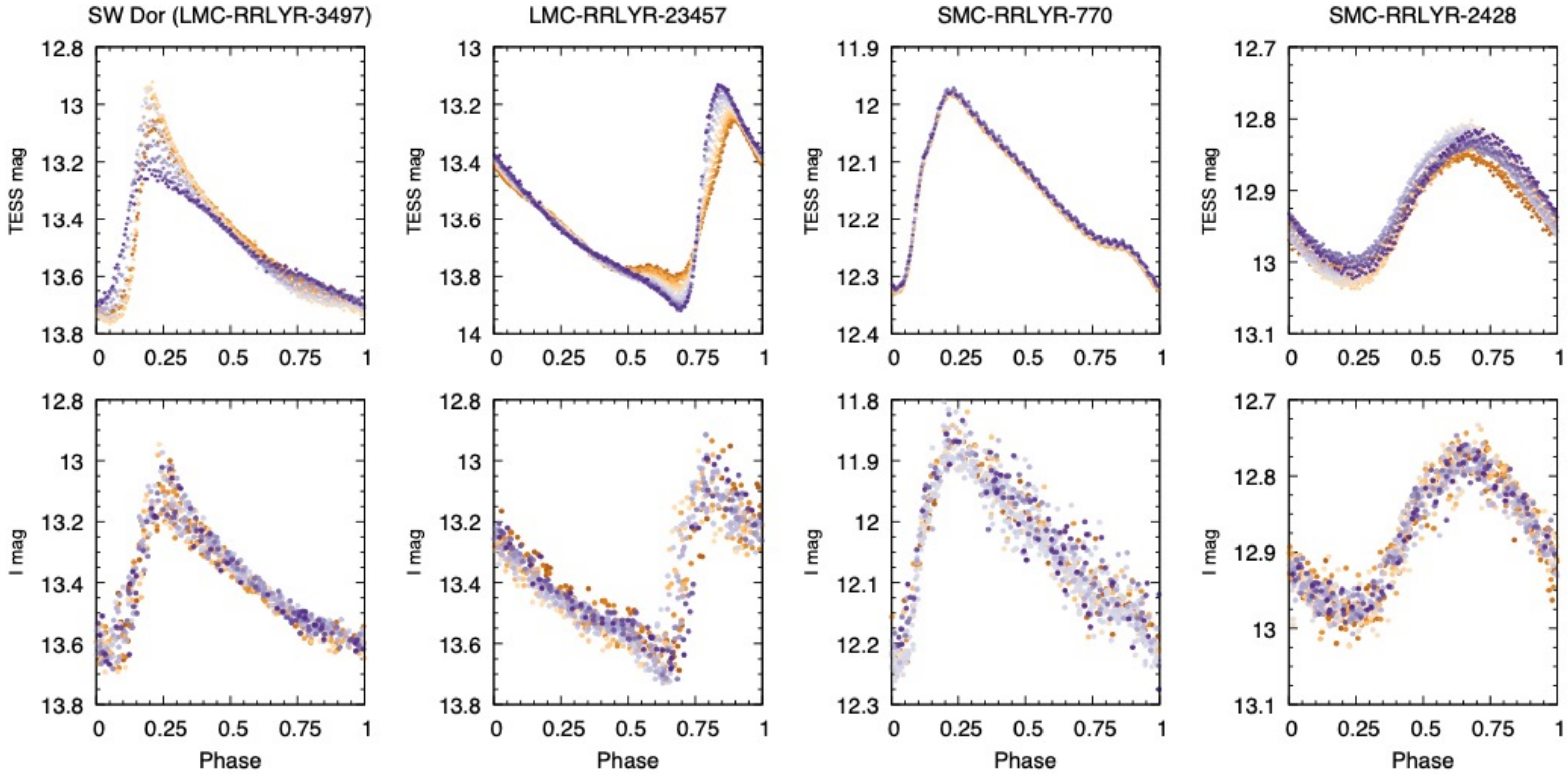
Strongly modulated variability

Molnár et al. (2021)



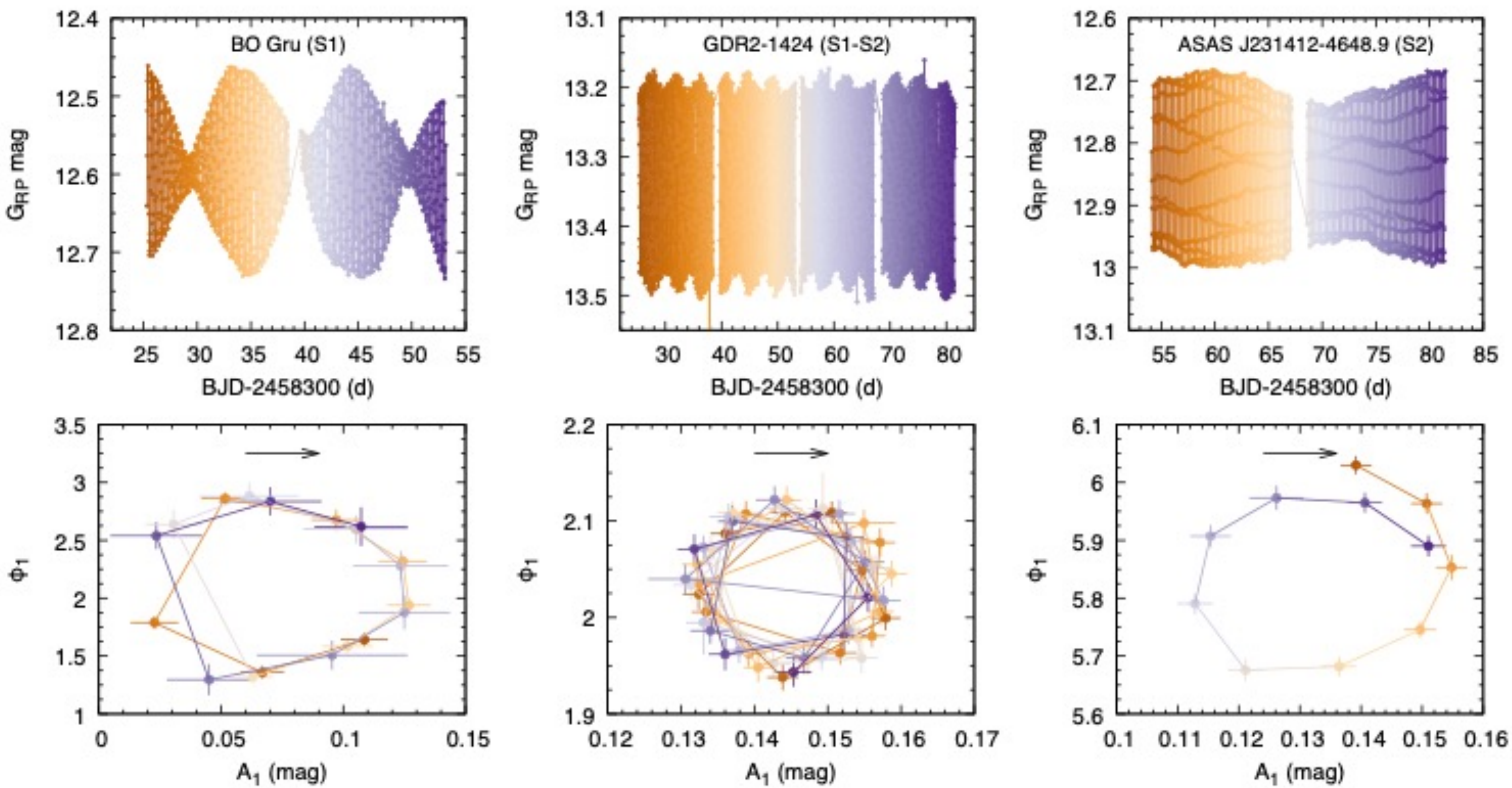
TESS vs OGLE: a matter of precision

Molnár et al. (2021)



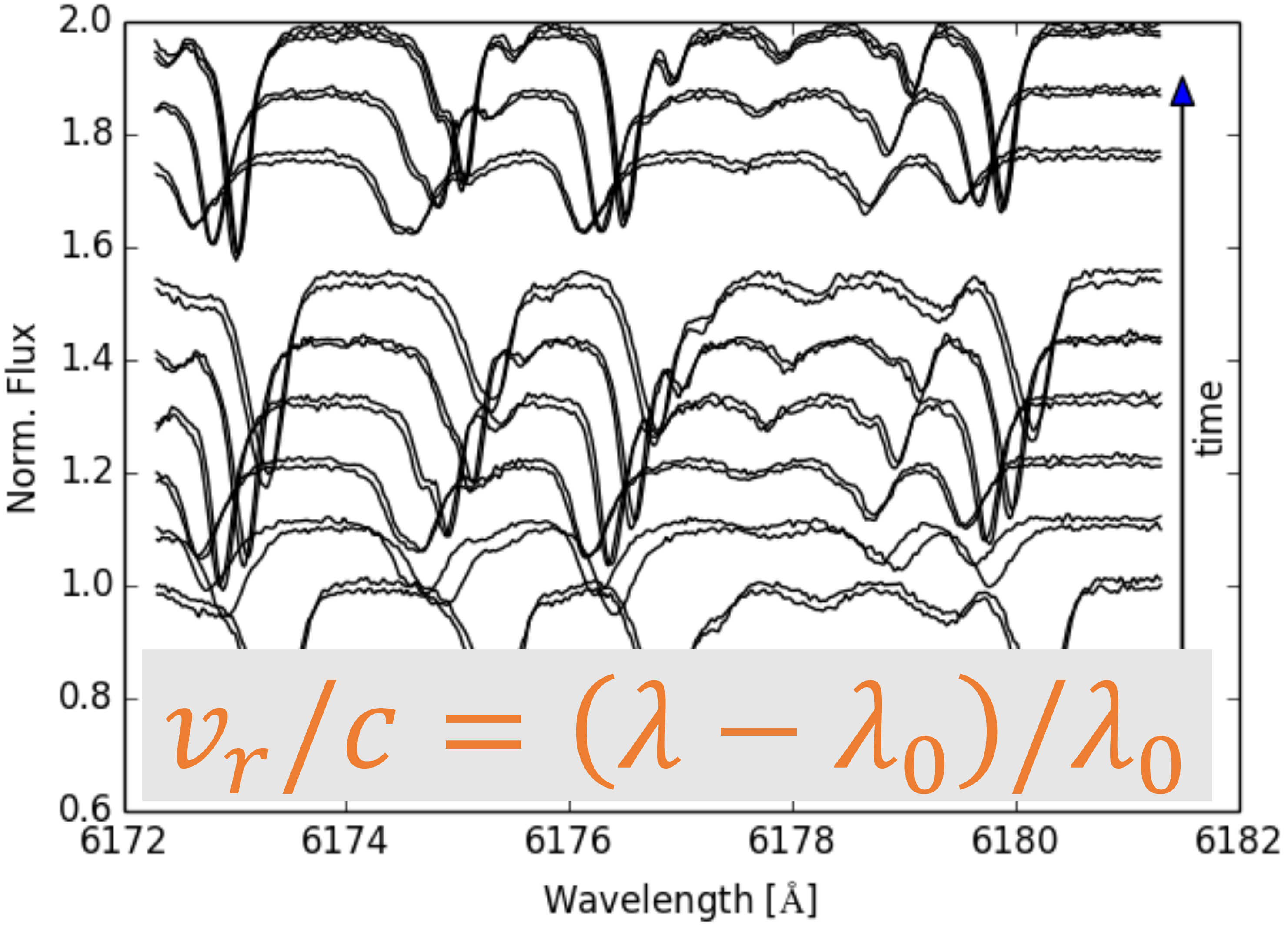
The Blazhko effect

Molnár et al. (2021)



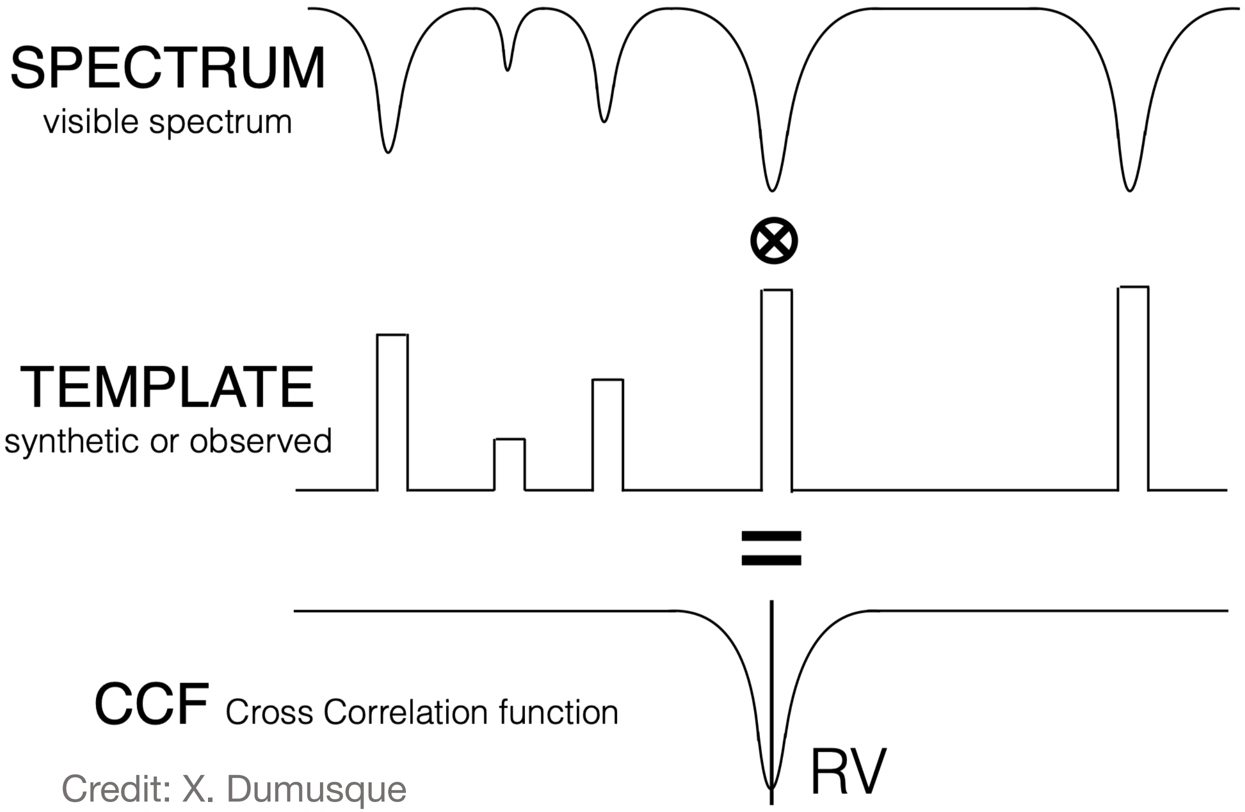
Resolving changing radius along line of sight

Tracing pulsations with Doppler Spectroscopy

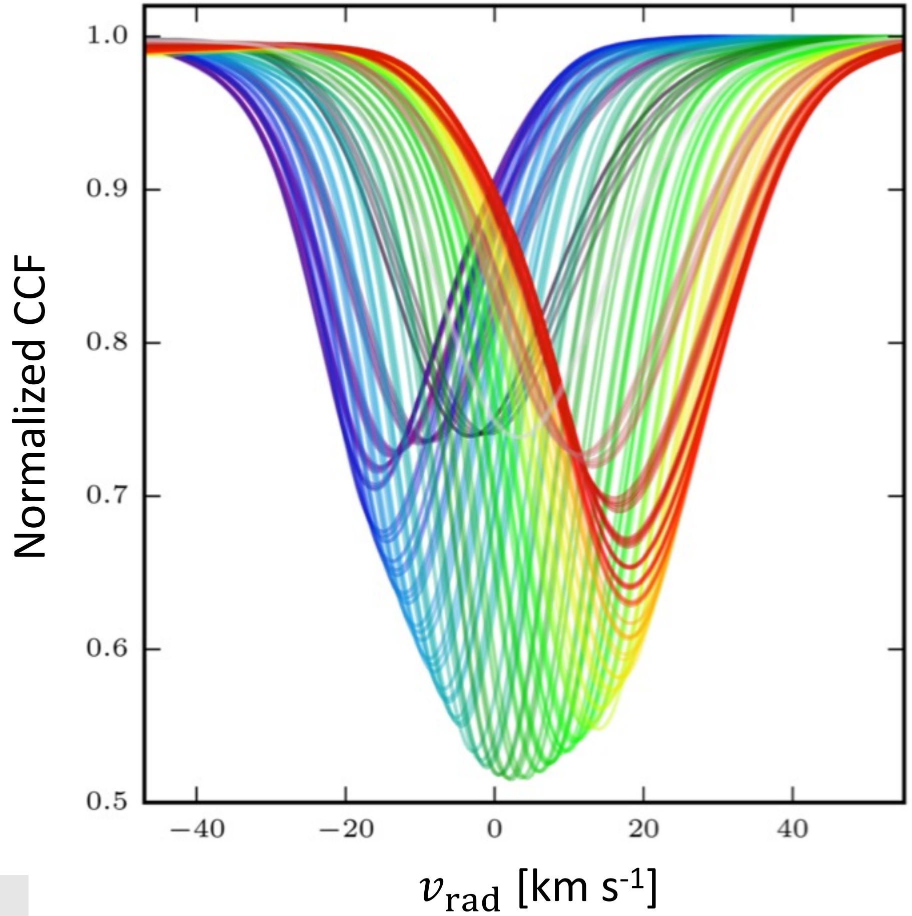


$$v_r/c = (\lambda - \lambda_0)/\lambda_0$$

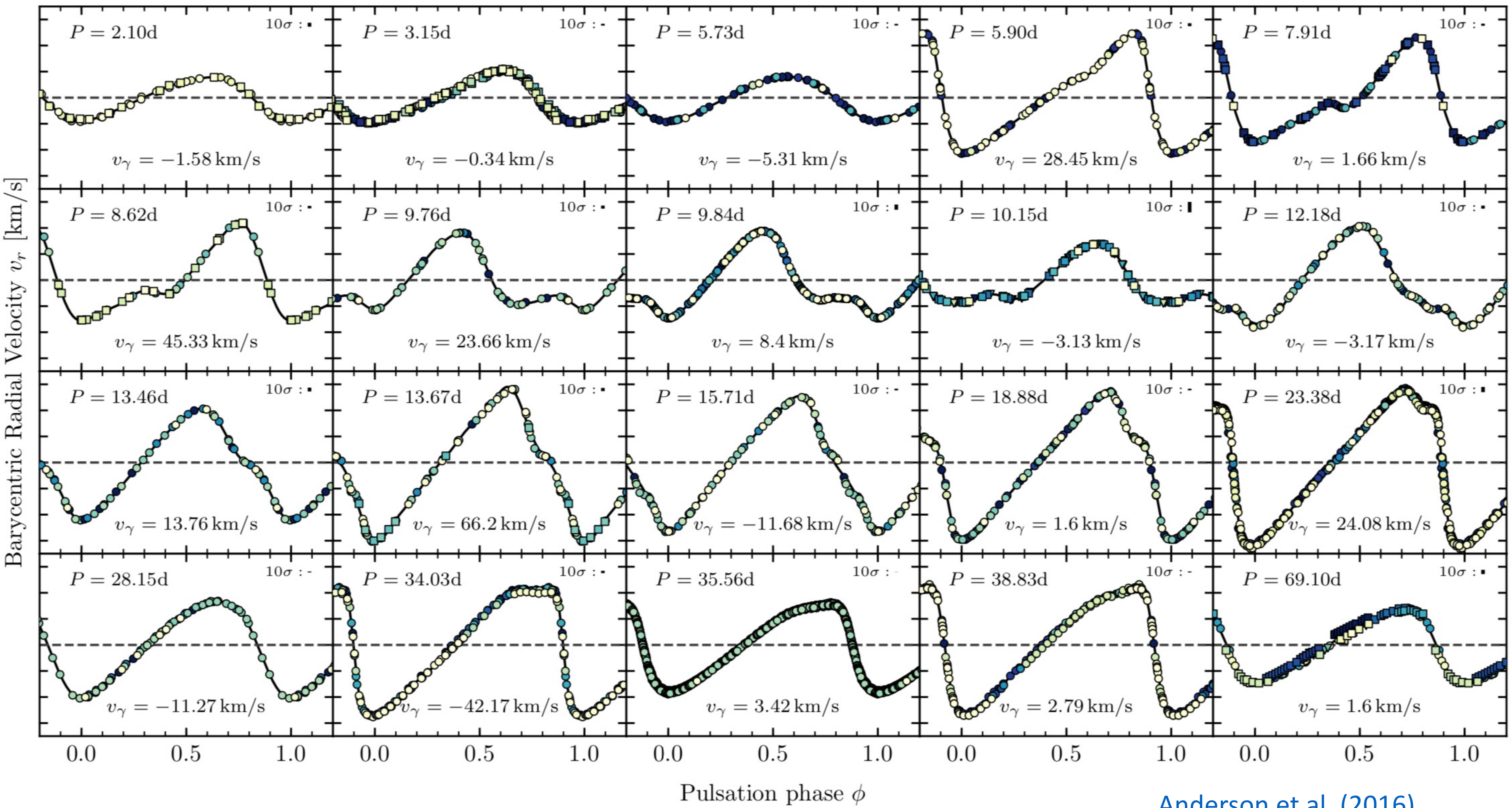
Cepheid radial velocity measurements



Credit: X. Dumusque



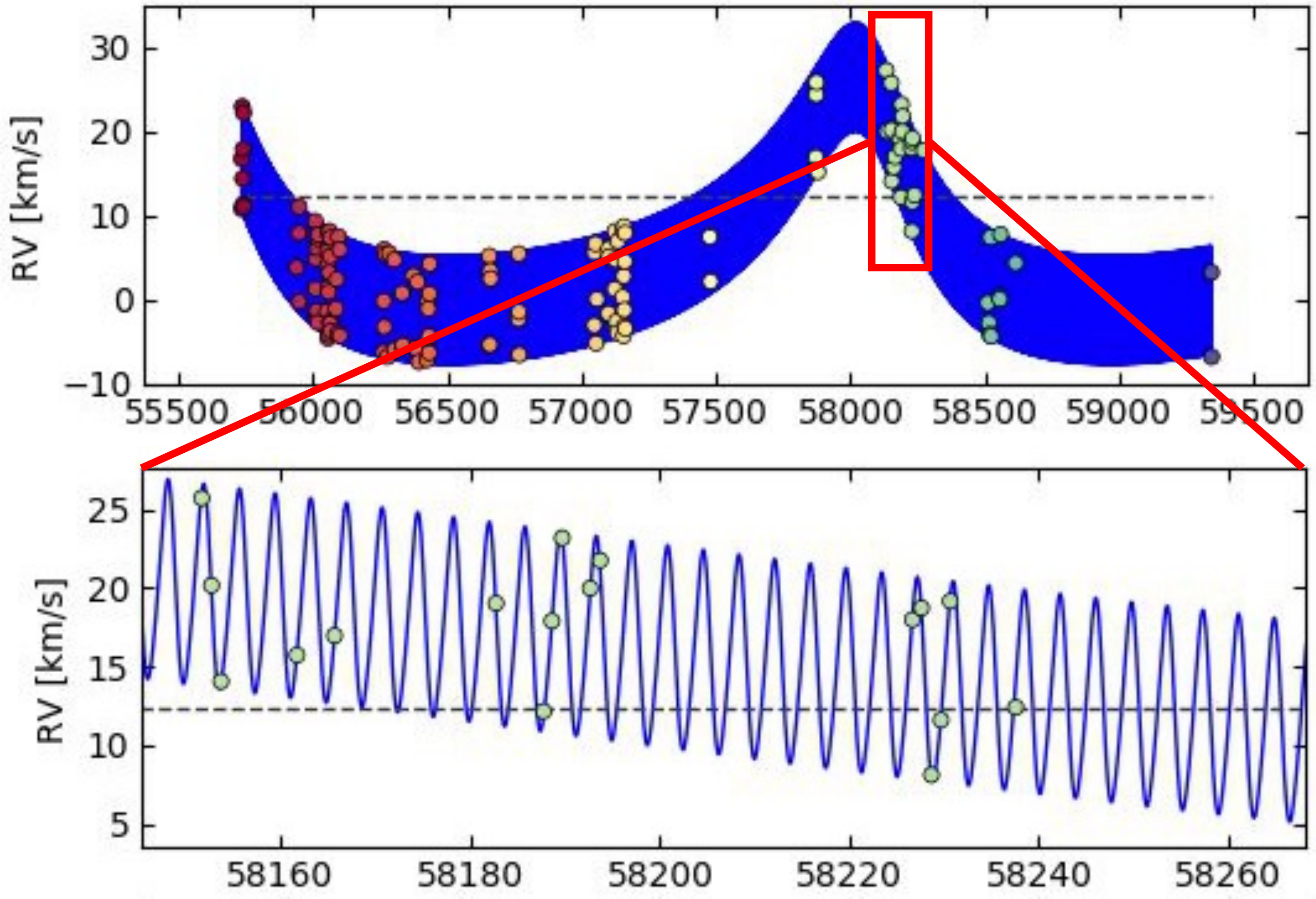
$$v_r/c = (\lambda - \lambda_0)/\lambda_0$$



Anderson et al. (2016)

Cepheids in binary systems

- Pulsations and orbital motion superposed in radial velocities
- Timescales generally well-separated: pulsations < 2 months, orbits > 1 year
- Amplitudes can be similar; either can be larger



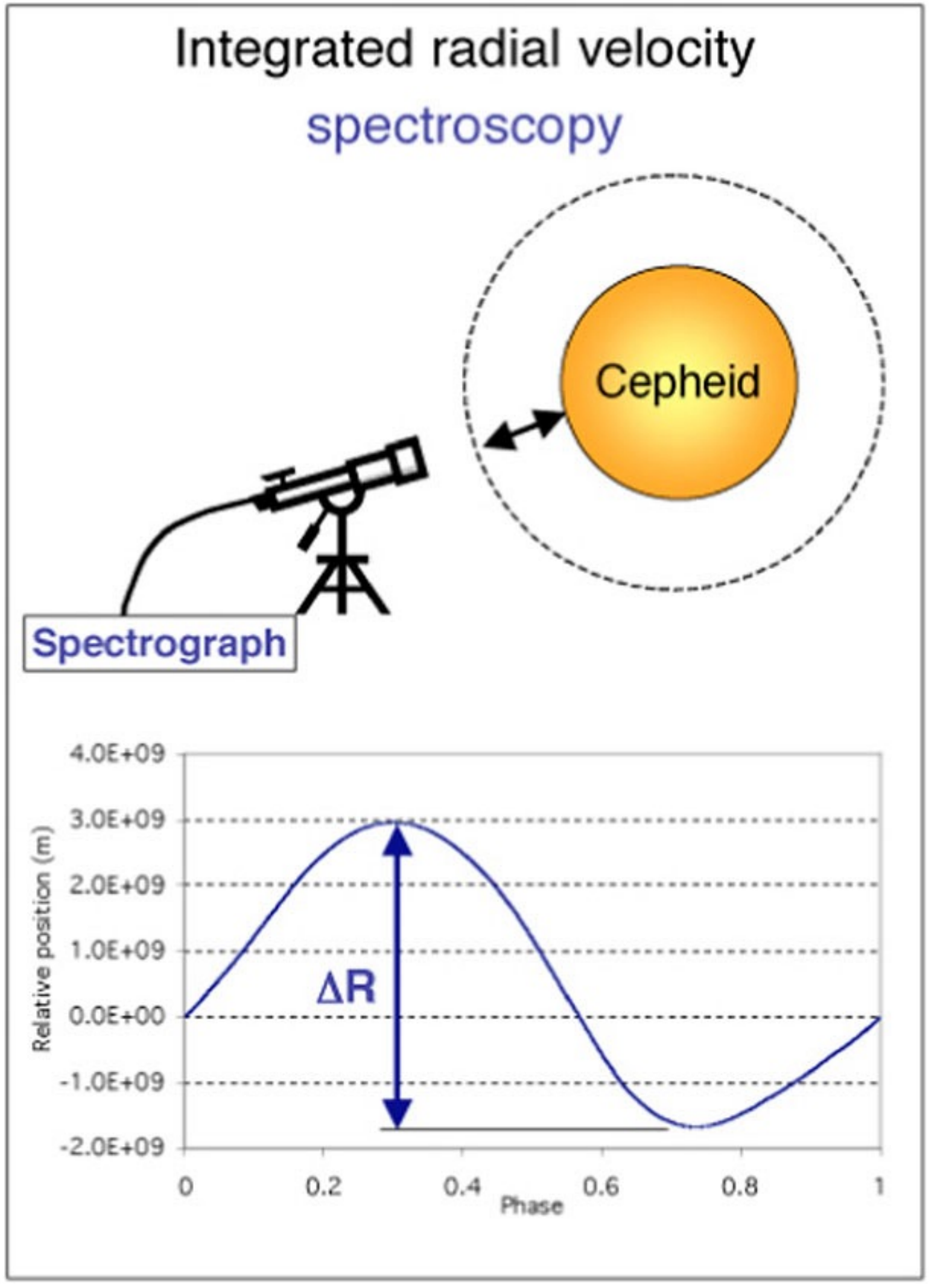
Cepheids in multiple star systems

- Very frequent: up to ~80% are binaries (or higher order)
- Multiplicity fraction among B-stars > 1. What happens to > 20% of B-stars before they become Cepheids?
- Principal source of mass measurements
- Problem: to get accurate mass you need SB2 systems
- If you also resolve the orbit, you get distance!
- This week’s paper by Pilecki et al. (2021) reports discovery of large population of SB2 Cepheids : prime targets for mass measurements

Kervella et al. (2004)

Linear radius variations

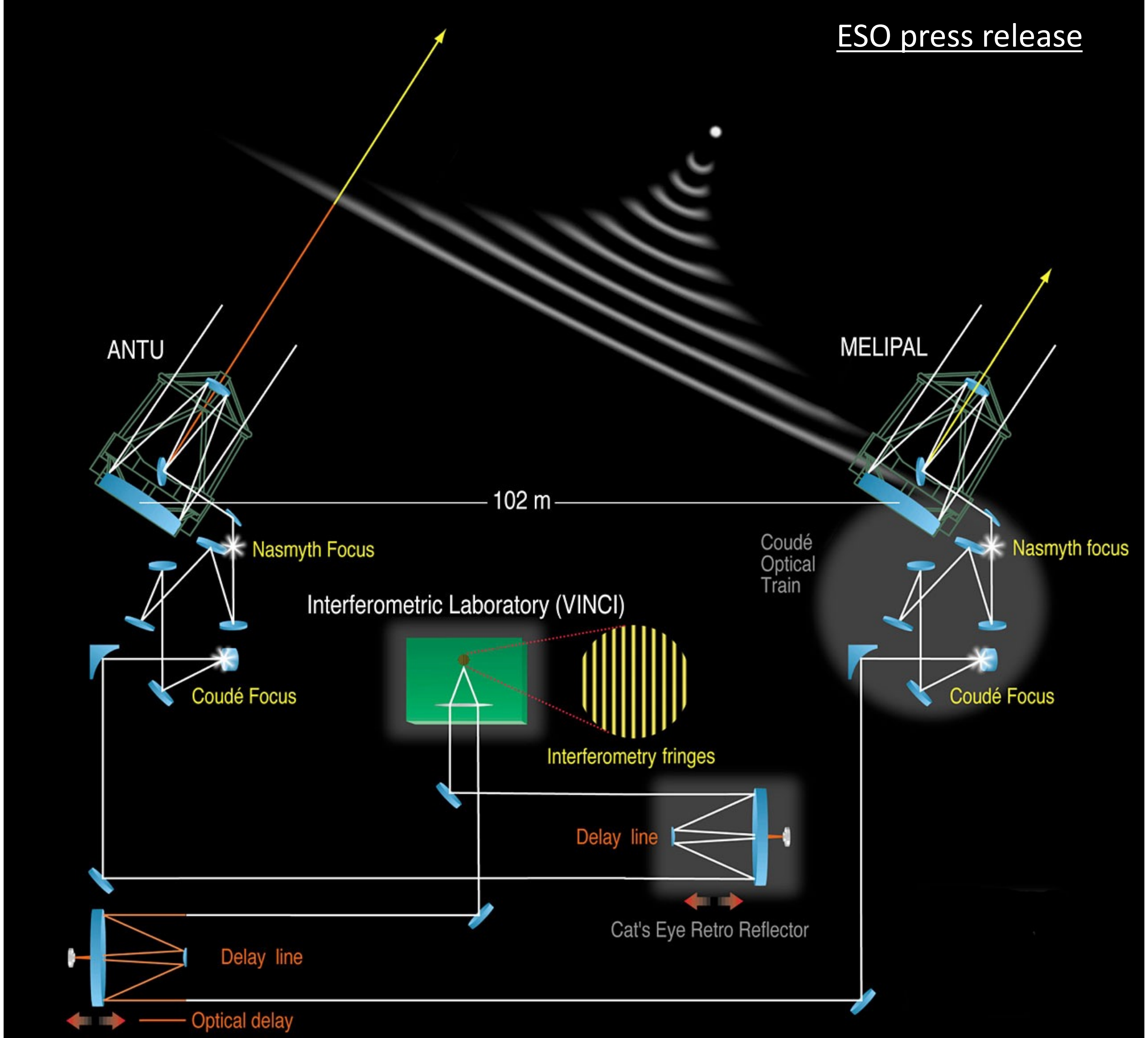
- Integral of velocity curve yields distance traveled by surface
- $\Delta R = p \int v_r dt$
- Projection factor p fairly complicated
- p depends on:
  - surface intensity distribution & geometry
  - Velocity gradients in atmosphere
- Does p depend on?
  - Stellar parameters (period, mass, temperature)
  - Time : e.g. pulsation-convection coupling
- Current limitation: p is known to ~10%



Resolving pulsations spatially

Resolving Pulsations with Interferometry

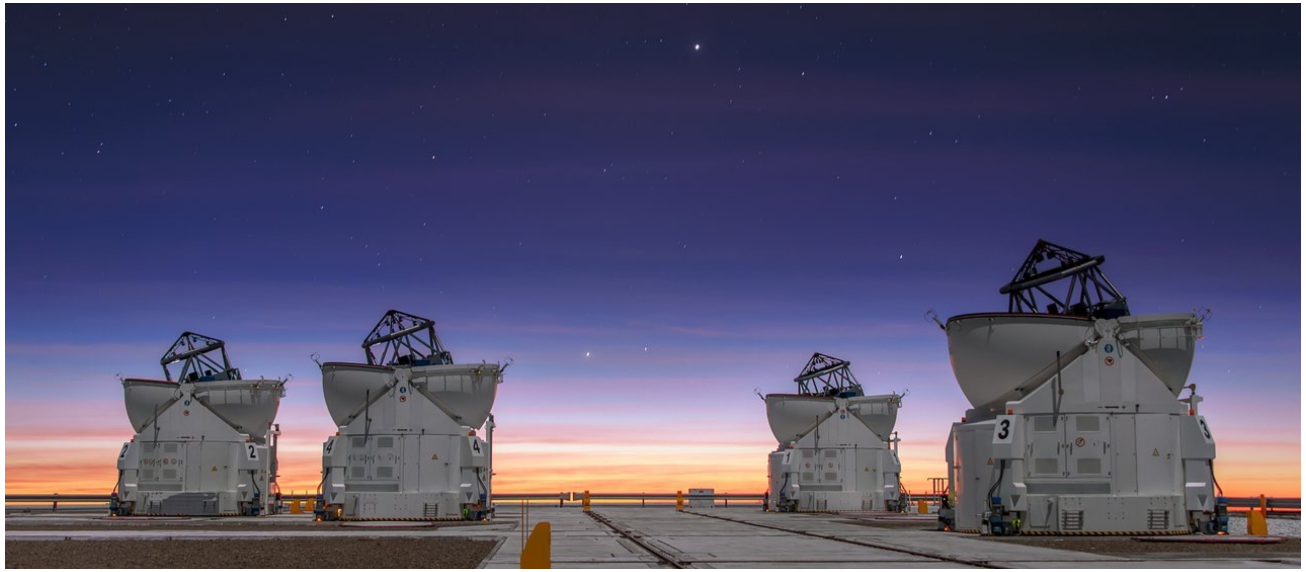
- $\frac{\lambda}{D_{tel}} \sim \frac{1.2\mu m}{130m} \approx 10^{-8}$
- $\theta_{res} = \frac{\lambda}{D_{tel}} \cdot d$
- $1pc \approx 4.44 \times 10^{7} R_{sol}$
- $\theta_{res} \approx 0.41 \times d[pc]$
- **$R_{*} \sim 100\ R_{sol}@500pc$**

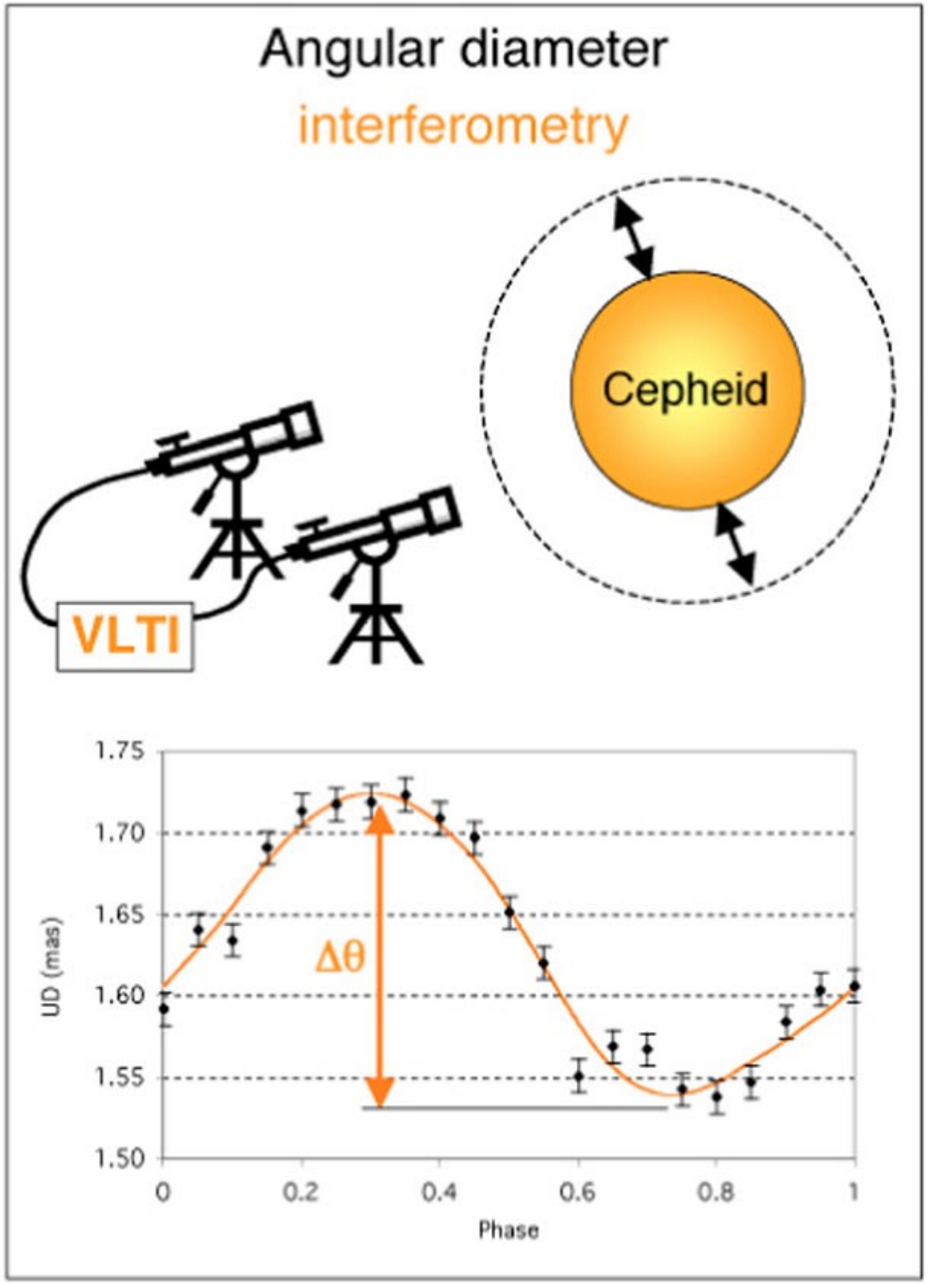


Kervella et al. (2004)

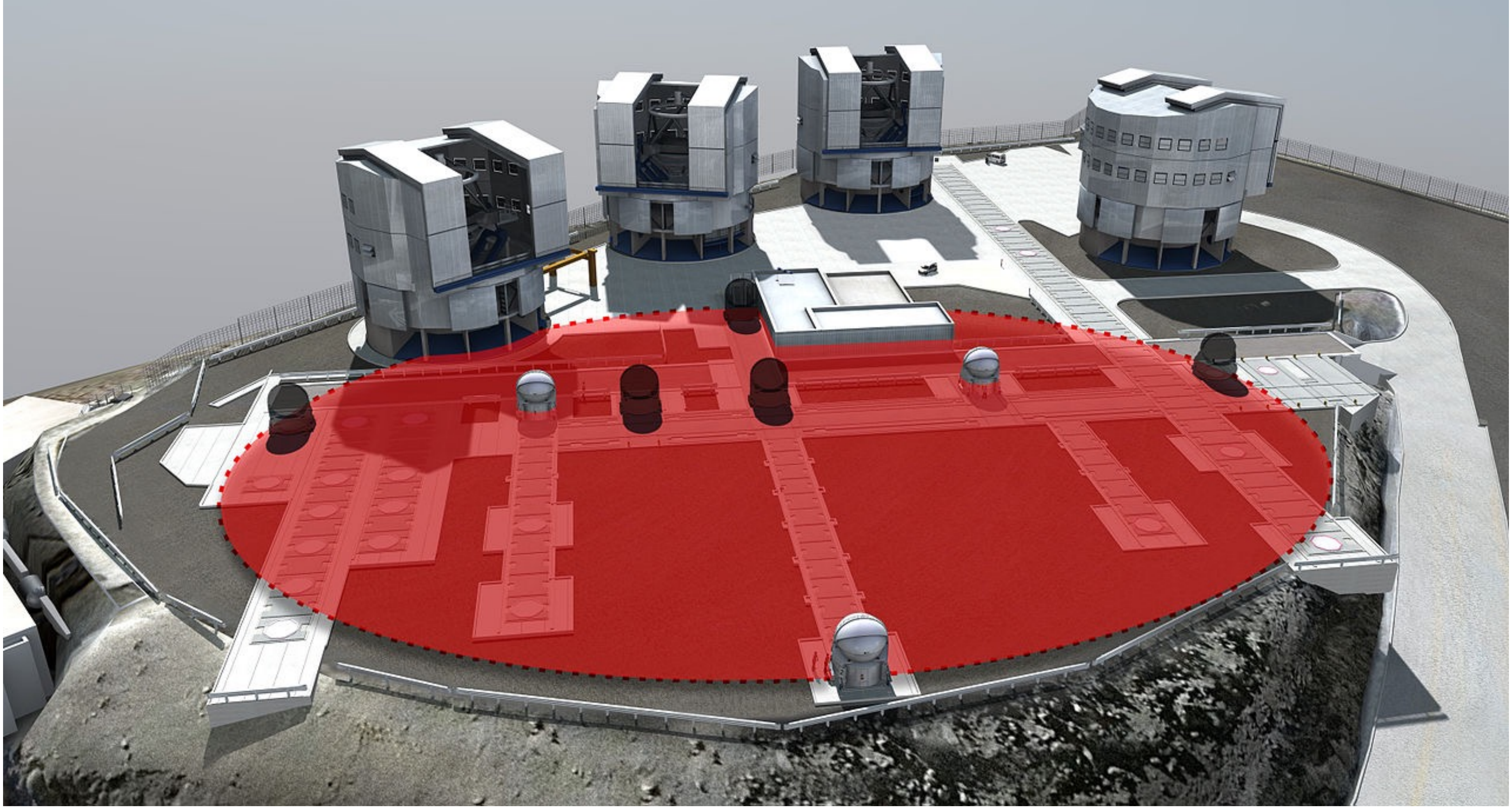
Interferometrically resolved pulsations

- About a dozen Cepheids are close enough to measure diameter changes
- Typical angular sizes ~ 1-3 mas
- Bright & distant: Use the ATs instead of UTs





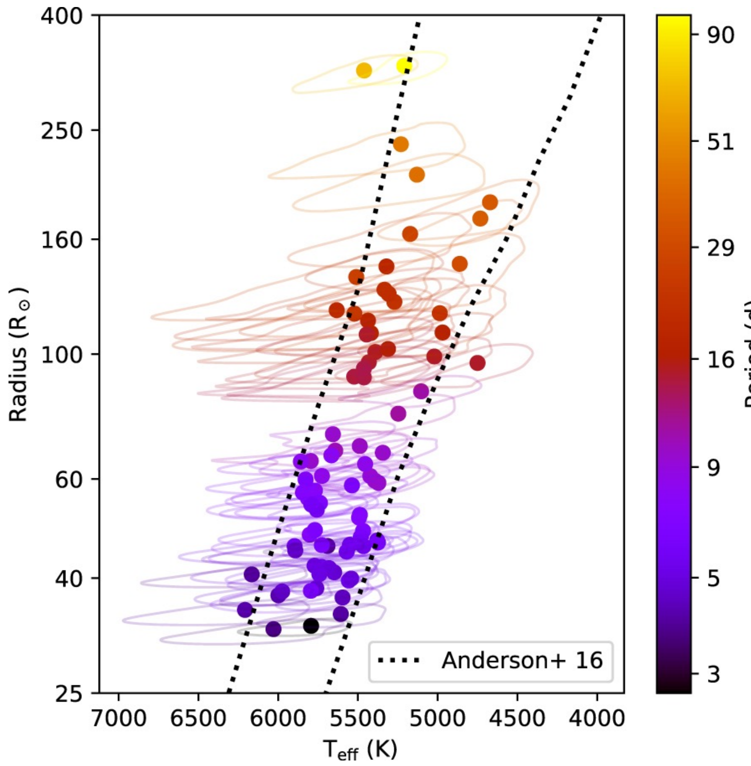
https://www.eso.org/sci/facilities/paranal/telescopes/vlti/tuto/tutorial_interferometry.html

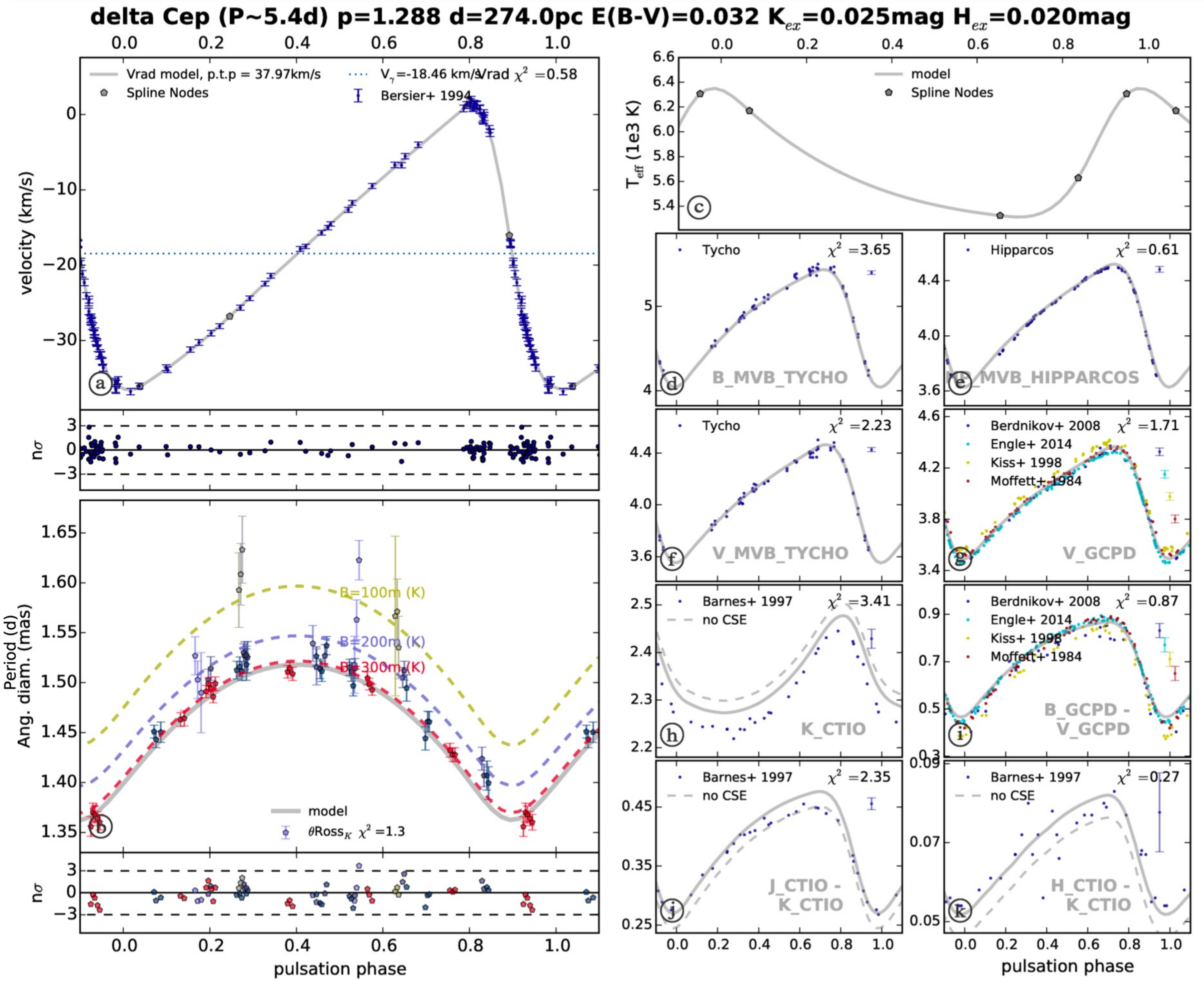


Putting it all together

Mérand et al. (2015)

Javanmardi et al. (2021)





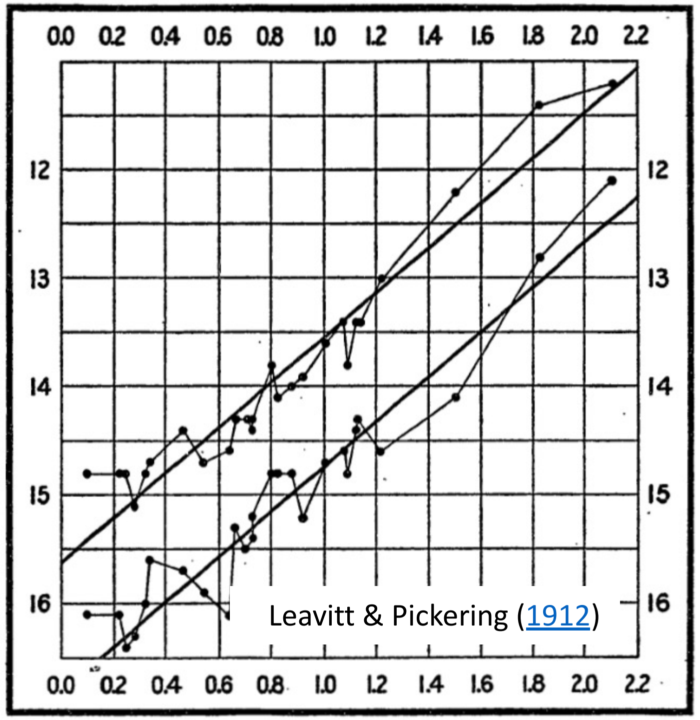
Leavitt's law

The period-luminosity relation

Source: US library of Congress

Leavitt's law makes Cepheids standard candles

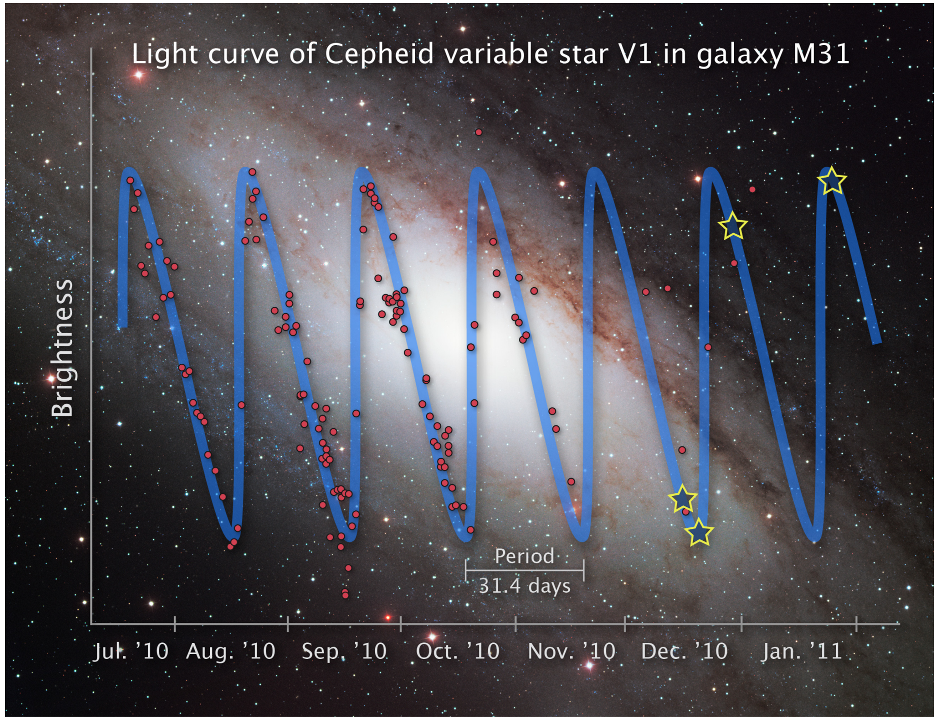
Period indicates true brightness



brightness

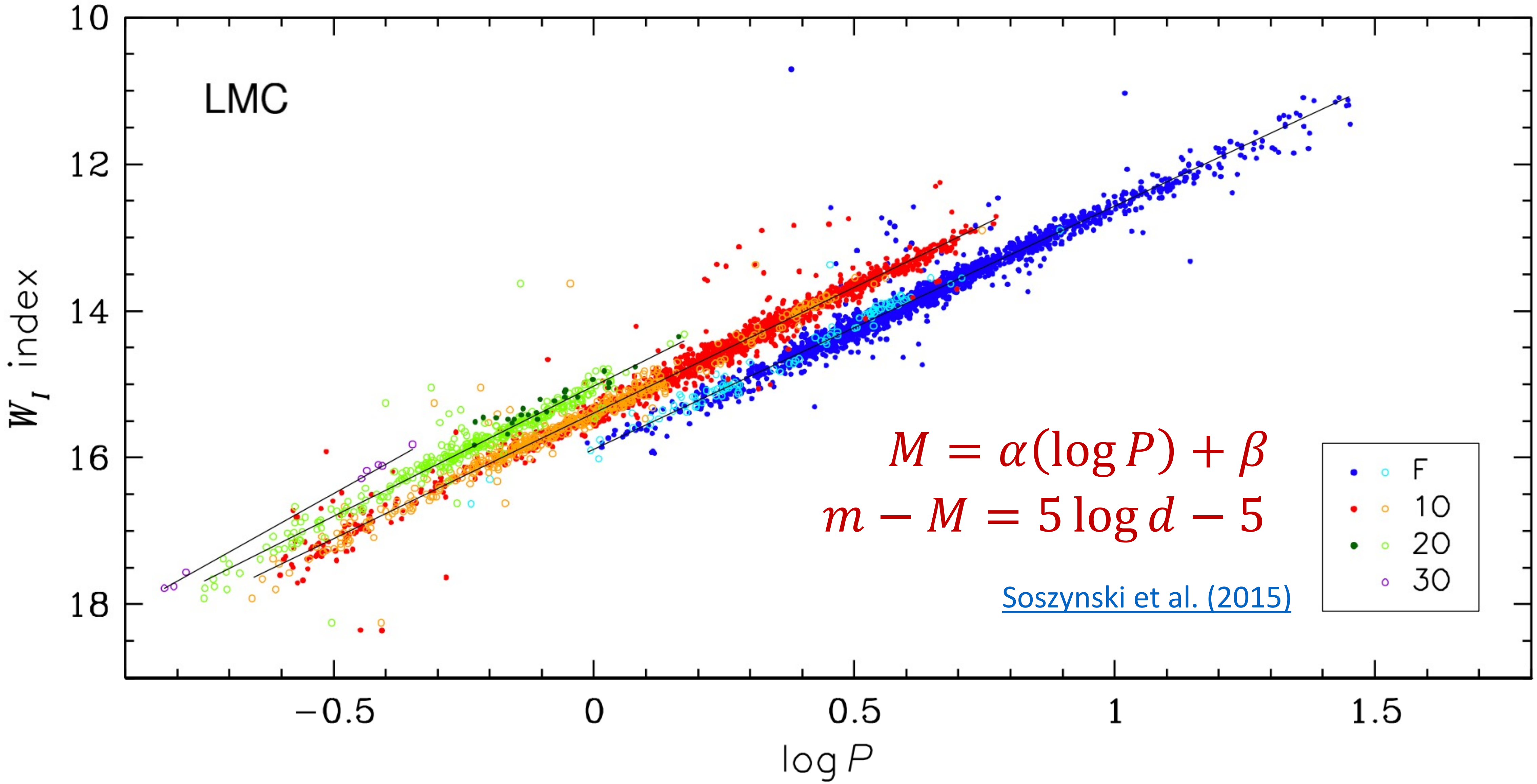
log(period [d])

Periodic & characteristic light variations



Flux decreases as distance squared

OGLE-IV LMC Leavitt Law based on 4620 CCs



$$M = \alpha(\log P) + \beta$$

$$m - M = 5\log d - 5$$

Soszynski et al. (2015)

Questions?