EPFL

Superconducting Magnets: Exercise 3 - Solutions

Kamil Sedlak

25/03/2025

SWISS
PLASMA
CENTER

Dimensioning of a superconducting solenoid

Exercise 1
- Requirements
- Calculate the overall current
- Suggest number of turns and operating current

Exercise 2
- Calculate the self inductance
- Calculate the hoop load
- Estimate the need of structural support

Exercise 3
- Discuss the discharge requirement in case of quench
- Discuss the hot spot temperature
- Discuss an option for graded conductor

Requirement and input data

- Generation of 4 T inside the solenoid
- Bath cooling (4.2 K)
- Use NbTi superconductor (scaling law -> current density)
- Free bore of the solenoid, ø = 50mm
- Length of the solenoid λ = 500mm
- Thin (single?) layer winding
- NbTi composite: cu:non-cu = 2, σ_y = 300 MPa
- Suggested criteria for engineering margins:

$$\Delta T = 0.5\ K \qquad \sigma_{op} \leq 2/3\ \sigma_y \qquad T_{hot\ spot} \leq 150\ K$$

Calculate overall current

- *Apply Ampere law to find the overall current*
- *Use "long solenoid" approximation*

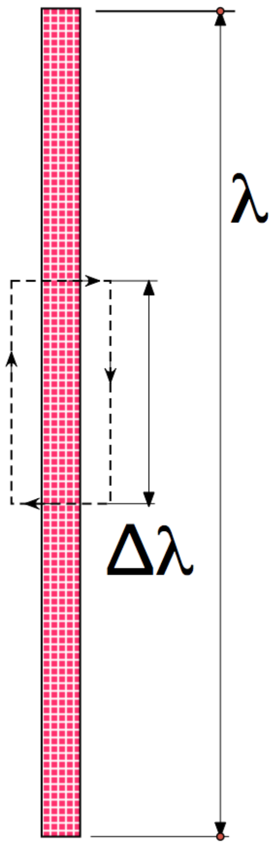
The "long solenoid" approximation tell us that the central field, B_c, is homogeneous and vertical inside the solenoid. The flux lines close at infinite, i.e. the field is 0 outside the solenoid.

Applying the Ampere law on the dotted path in the sketch, which include the current $I_{tot}(\Delta\lambda/\lambda)$ the two horizontal segments give 0 contribution (90° orientation of path and field). The outer segment gives also 0 contribution (because of 0 field).

$$\oint B \cdot d\ell = \mu_0 I \Rightarrow B_c \cdot \Delta\lambda = \mu_0 \frac{\Delta\lambda}{\lambda} I_{tot}$$

$$\Downarrow$$

$$I_{tot} = \frac{B_c \cdot \lambda}{\mu_0} = 1.59 MA$$



Calculate the current density at operating conditions

- *Retain approximately B_c as B_{op} for the conductor*
- *Retain $T = T_{bath} + \Delta T = 4.7$ K*
- *Calculate from scaling law J_{NbTi}*
- *Normalize J to strand area*
- *Calculate total strand area*

- $b = 4 / 14.61 = 0.2738$
- $t = 4.7 / 9.03 = 0.5205$
- $J_{c\ NbTi}$ (4T, 4.7K) = 3043 A/mm²
- $J_{c\ Strand}$ (4T, 4.7K) = 1014 A/mm²
- $J_{op\ Strand}$ (4T, 4.2K) ≈ 1000 A/mm²
- $A_{strand} = I_{tot}/J_{op} = 1590\ mm^2$

Number of turns and operating current

- *Discuss the implications of the selections*
- *Is a single layer realistic?*

The total current I_{tot} can be obtained by many combinations of number of turns, n, and operating current I_{op}. The criteria for a sound selection are technology and common sense. Let consider two extreme cases:

- *n=1, I_{op}= 1.59 MA The conductor should be a 500 mm x 3.18 mm slab, wrapped to a cylinder. Problems about the current injection (where are the terminals), the current leads (huge heat load in the cryostat) and the power supply (1.59 MA converter)…*
- *n=1000 000, I_{op}= 1.59 A The NbTi composite would have a diameter (non-insulated) of ≈ 45 μm (impossible to handle). The inductance would be in the range of kHy with very large voltage requirements…*

A sound solution aims at a reasonable current (avoid kA range for power supply and current leads). The sound range is between 100A and 300A, say 159A / 10 000 turns. (non-insulated diameter ≈ 0.45 mm)

A single layer is not possible with a round strand. A rectangular conductor would be necessary, to be wound on the short edge of 500mm/10 000 = 0.05mm.

Calculate the self inductance

Kamil Sedlak

- *Use the flux definition and the Faraday law to extract the self inductance*

$$\Phi = B \cdot \pi R^2 = \frac{\mu_0 n I_{op}}{\lambda} \pi R^2$$

$$V = n\dot{\Phi} = L\dot{I}_{op}$$

$$L = \mu_0 \frac{n^2 \pi R^2}{\lambda} = 494\, mHy$$

$$E_{st} = \frac{L I_{op}^2}{2} = 6.24\, kJ$$

Suggest the conductor size and number of layers

- *Add electrical insulation (varnish, 20-30 µm thick) for final strand diameter.*
- *Check number of layers, i.e. adjust the number of turns (Nt) and operating current (I_{op}) for an integer number of layers.*

$A_{comp} = 0.159\ mm^2$
$Ø_{bare} = 0.45\ mm$
$Ø_{ins} = 0.5\ mm$

- *Varnish insulation ≈ 25 µm thick*
- *1000 turns per layer -> 10 layers*
- *The thickness of the solenoid is s = 5 mm*
- *No need of adjustment!!*

Calculate the hoop load

- *Calculate the Lorentz force for the operating conditions*
- *Discuss limits and approximation*
- *Estimate the average load from the winding geometry*

$$F = I_{tot} \int B \times d\ell$$

Over a cross section, the tension in the winding is $T = I_{tot}B(R)R$.

Assuming a field distribution, linear from 4T to 0 from the inner radius to the outer radius, and an average radius of 27.5 mm, the average hoop load is 79.5 kN over the winding pack.

For the innermost single turn, the tension is $I_{op}B_{max}R_{min}$ = 15.9 N

Kamil Sedlak

Estimate the stress

- *Estimate the peak stress is at the innermost turn.*
- *Discuss the need of an additional mechanical support (i.e. can the superconducting strand alone withstand the hoop stress, or is there a need to include an additional mechanical support?*

The peak stress is at the innermost turn, where the tension per strand is 15.9 N, is:

$$\sigma_{max}^{strand} = \frac{15.9}{A_{comp}} = 100MPa$$

Need of structural support?

- *The stress of the winding pack is well within the allowable, including the peak stress on a “free standing” innermost turn*
- *A detailed analysis, e.g. FE, including the electrical insulation properties (glass epoxy impregnation) and the shear stress at the insulation between layers, will provide a map of the stress distribution with actual peak lower than estimated here.*

Discuss the discharge requirement in case of quench

Kamil Sedlak

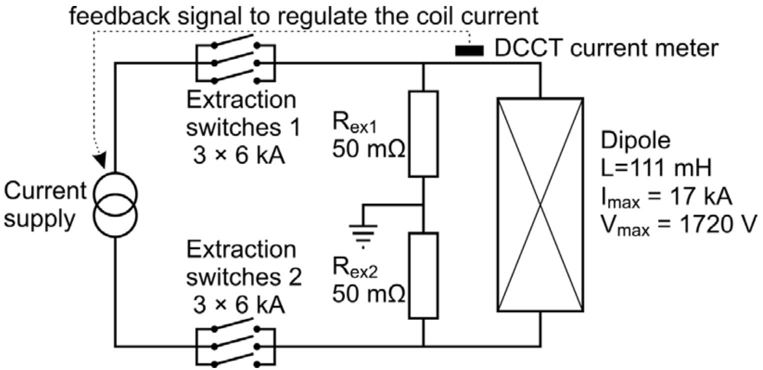
- *Using the scheme similar to Lecture 1 and retaining a maximum voltage at the terminal of 100 V, calculate for the "thin solenoid" the*
  - *Dump resistor* R_{dump} = 0.63 Ω
  - *Decay time constant* τ = 0.78 s

$$I = I_{op}\, e^{-t/\tau}$$

$$V_{max} = L \cdot \dot{I}_{max} = \frac{0.494}{\tau} I_{op}\, e^{-t/\tau}\Big|_{t=0}$$

$$R_{dump} = V / I_{op}\Big|_{t=0}$$

$$\tau = \frac{L}{R_{dump}} = \frac{0.494\ H}{0.63\ \Omega} = 0.78\ s$$



Estimate the hot spot temperature

- *Calculate the dissipated ohmic energy at the hot spot, assuming a delay time for quench detection, t_{delay} = 0.5 s and the calculated dump time constant*
- *Under adiabatic conditions (no heat exchange from the strand to the surrounding), estimate the temperature rise at the hot spot*

$$\rho_{copper} = 1.6\cdot 10^{-10}\ \Omega m$$

$$\rho_{NbTi} = 2.1\cdot 10^{-8}\ \Omega m \text{ (non-superconducting mode)}$$

$$\eta_{copper} = 8.9 \cdot 10^{3}\ kg/m^{3}$$

$$\eta_{NbTi} = 6.5 \cdot 10^{3}\ kg/m^{3}$$

$$Cp_{copper} = 0.0011T^{3}+0.011T\ \ J/K\cdot kg$$

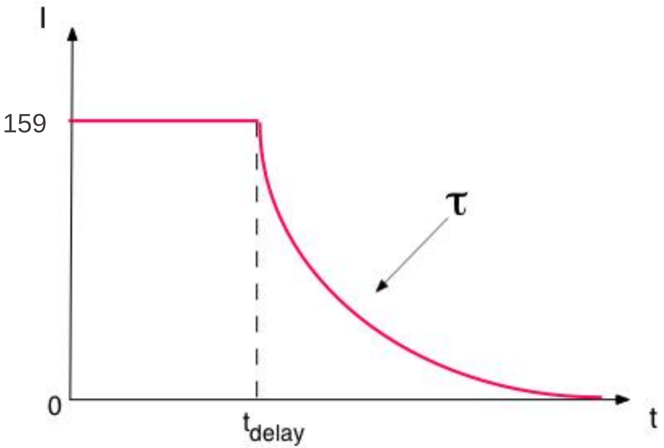
$$Cp_{NbTi} = 0.0023T^{3}+0.145T\ \ J/K\cdot kg$$

Estimate the hot spot temperature -1

- *Calculate the dissipated energy at the hot spot, assuming a delay time for quench detection, t_{delay} = 0.5 s and the calculated dump time constant*

$$\rho_{copper} = 1.6\cdot10^{-10}\ \Omega m$$

$$\rho_{NbTi} = 2.1\cdot10^{-8}\ \Omega m$$



The contribution of NbTi to conductivity (above Tc) is negligibly low and can be ignored. The Cu resistance (assuming no T dependence) in strand is $\rho_{copper}/A_{copper} = 1.53\ m\Omega/m$

The energy deposited at the end of the dump is

$$E = \int I^2(t)Rdt = R\cdot I_{op}^2\left(t_{delay} + \frac{\tau}{2}\right) = 34.5\ J/m$$

This estimate is not conservative. If the hot spot temperature turns out to exceed 20-30K, the actual R(T) function must be used. On the other hand, at temperature >100K, the NbTi also contributes to conductivity.

Estimate the hot spot temperature - 2

- *Under adiabatic conditions (no heat exchange from the strand to the surrounding), estimate the temperature rise at the hot spot, $T_{hotspot}$*

$$Cp_{copper} = 0.0011T^3+0.011T \;\; J/K\cdot kg$$

$$Cp_{NbTi} = 0.0023T^3+0.145T \;\; J/K\cdot kg$$

The enthalpy of the strand up to $T_{hotspot}$ (neglecting the enthalpy at 4.2 K)

$$H_{hotspot} = \int_{4.2}^{T_{hotspot}} \left(C_p^{Cu} A_{Cu}\eta_{Cu} + C_p^{NbTi} A_{NbTi}\eta_{NbTi}\right) dT = \int_{4.2}^{T_{hotspot}} \left(1.8\cdot 10^{-6} T^3 + 5.95\cdot 10^{-5} T\right) dT \approx 4.5\cdot 10^{-7} T^4 + 3\cdot 10^{-5} T^2 \; J/m$$

above 15K, only the fourth power term matters

$$H_{hotspot} \approx 4.5\cdot 10^{-7} T_{hotspot}^4 \; J/m$$

Setting the $H_{hotspot}$ equal to the dissipated energy,

$$T_{hotspot} = \sqrt[4]{\frac{34.5}{4.5\cdot 10^{-7}}} = 93.6\,K$$

Discuss an option for graded conductor

- *Accounting for the layer winding and the field distribution in radial direction, propose a two-grades winding*
- *Propose a layout for the low grade conductor and discuss the implications for hot spot temperature*

The field inside the winding (thin solenoid) decreases linearly from the innermost to the outermost layer. i.e. the five outer layers see a field ≤ 2 T and can use a strand with reduced NbTi cross section, 0.04 mm^2

Option 1, maintain the Cu:non-Cu=2.

The strand diameter is reduced by 10% in the low grade, the 10^{th} layer is not full, the overall size of the coil is unchanged, the overall strand cost is reduced by 20%, the dissipated power increases by 20% and the enthalpy decreases by 20%. As a result, the hot spot temperature increases from 93K to 103 K.

Option 2, maintain the strand diameter.

The Cu:non-Cu increases to 2.9, the winding geometry is unchanged, the overall strand cost remains almost identical, the dissipated power decreases by 10% and the enthalpy is almost identical. As a result, the hot spot temperature decreases from 93K to 88K.