EPFL

Superconducting Magnets: Exercise 1 - Solutions

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Dimensioning of a superconducting solenoid

Exercise 1

- Requirements
- Calculate the overall current
- Suggest number of turns and operating current

Exercise 2

- Calculate the self inductance
- Calculate the hoop load
- Estimate the need of structural support

Exercise 3

- Discuss the discharge requirement in case of quench
- Discuss the hot spot temperature
- Discuss an option for graded conductor

Requirement and input data

- *Generation of 4 T inside the solenoid*
- *Bath cooling (4.2 K)*
- *Use NbTi superconductor (scaling law -> current density)*
- *Free bore of the solenoid, ø = 50mm*
- *Length of the solenoid λ = 500mm*
- *Thin (single?) layer winding*
- *NbTi composite: cu:non-cu = 2, σ_y = 300 MPa*
- *Suggested criteria for engineering margins:*

$$\Delta T = 0.5\ K \qquad \sigma_{op} \leq 2/3\ \sigma_y \qquad T_{hot\ spot} \leq 150\ K$$

Calculate overall current

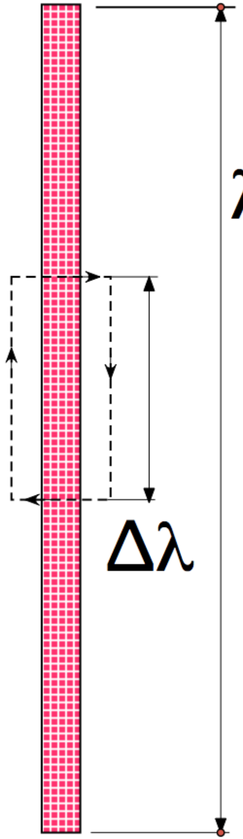
- *Apply Ampere law to find the overall current*
- *Use "long solenoid" approximation*

The "long solenoid" approximation tell us that the central field, B_c, is homogeneous and vertical inside the solenoid. The flux lines close at infinite, i.e. the field is 0 outside the solenoid.

Applying the Ampere law on the dotted path in the sketch, which include the current $I_{tot}(\Delta\lambda/\lambda)$ the two horizontal segments give 0 contribution (90° orientation of path and field). The outer segment gives also 0 contribution (because of 0 field).

$$\oint B \cdot d\ell = \mu_0 I \Rightarrow B_c \cdot \Delta\lambda = \mu_0 \frac{\Delta\lambda}{\lambda} I_{tot}$$

$$I_{tot} = \frac{B_c \cdot \lambda}{\mu_0} = 1.59 MA$$



Calculate the current density at operating conditions

- *Retain approximately B_c as B_{op} for the conductor*
- *Retain $T = T_{bath} + \Delta T = 4.7\ K$*
- *Calculate from scaling law J_{NbTi}*
- *Normalize J to strand area*
- *Calculate total strand area*

- $b = 4 / 14.61 = 0.2738$
- $t = 4.7 / 9.03 = 0.5205$
- $J_{c\ NbTi}$ (4T, 4.7K) = 3043 A/mm²
- $J_{c\ Strand}$ (4T, 4.7K) = 1014 A/mm²
- $J_{op\ Strand}$ (4T, 4.2K) ≈ 1000 A/mm²
- $A_{strand} = I_{tot}/J_{op} = 1590\ mm^2$

Number of turns and operating current

- *Discuss the implications of the selections*
- *Is a single layer realistic?*

The total current I_{tot} can be obtained by many combinations of number of turns, n, and operating current I_{op}. The criteria for a sound selection are technology and common sense. Let consider two extreme cases:

- *n=1, I_{op}= 1.59 MA The conductor should be a 500 mm x 3.18 mm slab, wrapped to a cylinder. Problems about the current injection (where are the terminals), the current leads (huge heat load in the cryostat) and the power supply (1.59 MA converter)…*
- *n=1000 000, I_{op}= 1.59 A The NbTi composite would have a diameter (non-insulated) of ≈ 45 µm (impossible to handle). The inductance would be in the range of kHy with very large voltage requirements…*

A sound solution aims at a reasonable current (avoid kA range for power supply and current leads). The sound range is between 100A and 300A, say 159A / 10 000 turns. (non-insulated diameter ≈ 0.45 mm)

A single layer is not possible with a round strand. A rectangular conductor would be necessary, to be wound on the short edge of 500mm/10 000 = 0.05mm.