Initialisation

Physical constants

S. Alberti, 10 March 2017
The problem is given below in the section “Problem in cryogenics” and the solution on the following section “Problem solution”.
The Mathematica program is available on the website: https://crpp-www.epfl.ch/plasmatech/exercices.html

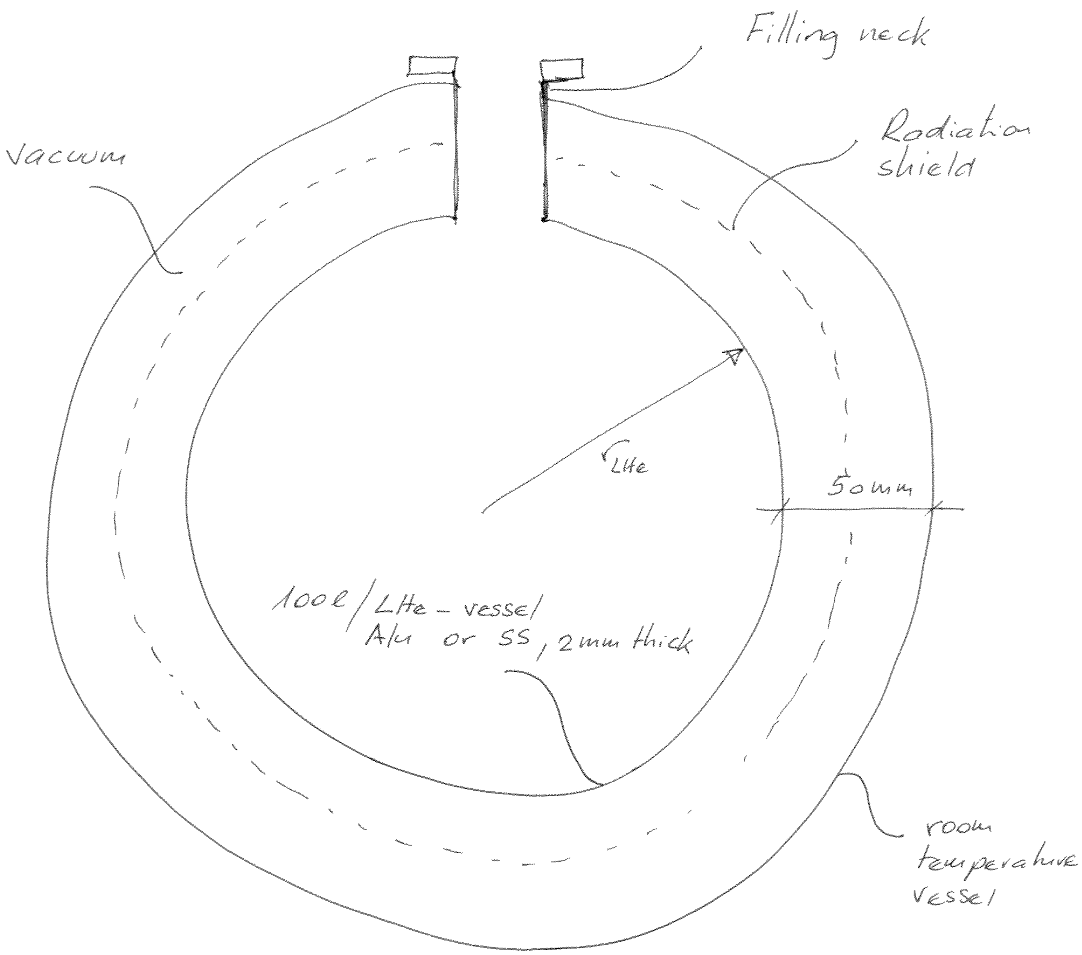
Problem in cryogenics

Generic LHe cryogenic vessel

The main goal of this problem is to discuss a variety of physical aspects relevant to the design and best practicies when using a liquid helium (LHe) cryogenic vessel which can be for example a dewar, a superconducting magnet, etc..

Let consider a spherical LHe vessel with a nominal capacity of 100liters.
The LHe vessel operated at atmospheric pressure consists of a spherical container made of a metallic plate, Aluminum (Alu) or Stainless Steel (SS), with 2mm thickness.
The room temperature (RT) vessel (300K) is also of spherical shape and has a radius 50mm larger than the LHe vessel.
The space between the LHe and RT vessel is evacuated and a radiation shield can be placed in between. Three cases are considered:
a) no radiation shield,
b) radiation shield actively cooled at liquid nitrogen temperature (LN),
c) a passive mono-layer MLI (Multi-Layer Insulation) radiation shield.

The filling neck consists of a 1mm-thickcylinder with 50mm inner diameter and 50mm length connecting the LHe-vessel to the RT-vessel. The filling neck is either of aluminum or teflon.
A sketch of the dewar geometry is shown below.



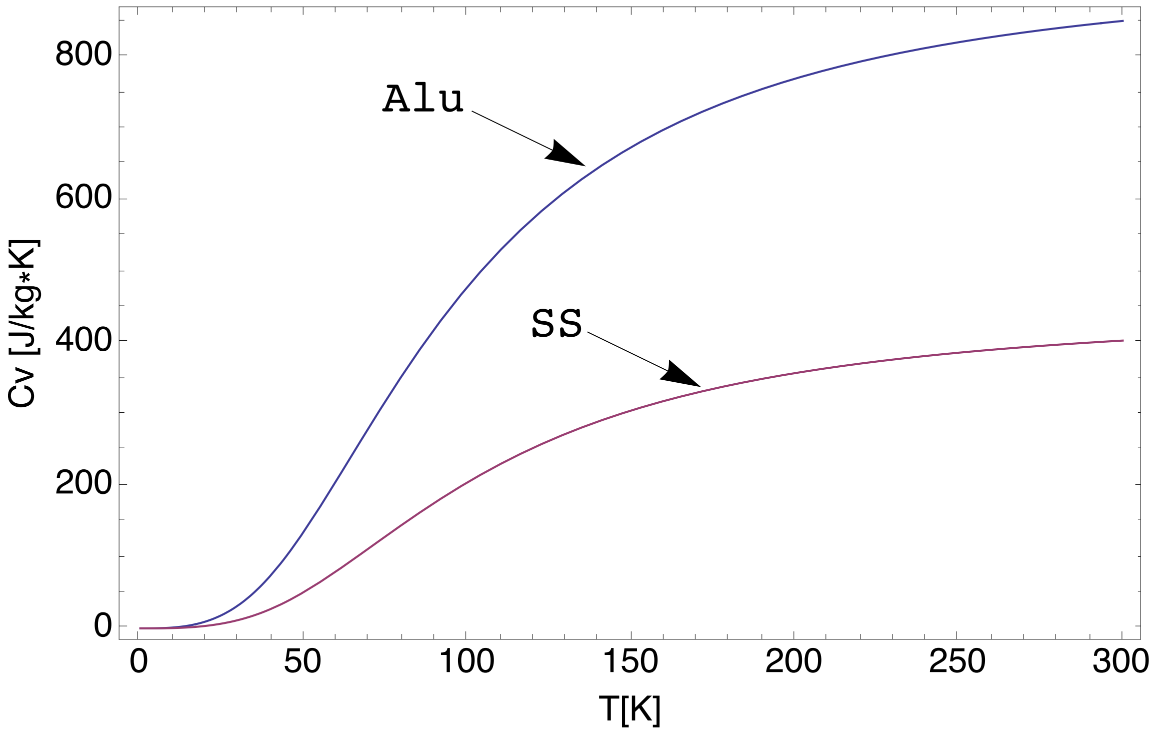
Based on the different material characteristics at cryogenic temperature calculate the following properties:

Cool-down and filling of the LHe-vessel

Considering the two materials (Alu or SS) of the LHe-vessel, calculate the amount of LHe needed for cooling down the vessel:
i) cooling with LHe from 300K to LHe temperature by only taking into account the latent heat of evaporation
ii) same as i), but consider a pre-cooling with LN followed with a cooling with LHe
iii) Considering a cost of LHe of 13CHF/l calculate the cool-down costs for the above mentioned cases
iv) If one can take advantage of the He-gas enthalpy, the LHe needed for case ii) is approximately 10 times less, calculate the LHe needed and cost in this case. Why is it difficult to take advantage of the He-gas enthalpy?

For calculating the above mentioned quantities, you will nee the following properties:

	L[J/kg]	C_p[J/kg*K]	ρ[kg/l]	Cost[CHF/l]
Helium	20 900.	4480.	0.125	13.
Nitrogen	199 300.	2050.	0.808	0.5



Alu
SS
800
600
400
200
0
Cv [J/kg*K]
0
50
100
150
200
250
300
T[K]

T[K]	Alu-C_V[J/kg*K]	Alu-Int Cv/R[J/kg]	SS-C_V/R[J/kg*K]	SS-Int Cv/R[
$1. \times 10^{-7}$	-3.55549×10^{-14}	-0.0944115	-1.16651×10^{-14}	-0.0339617
5.	0.151704	0.0952188	0.054571	0.0342521
10.	1.21363	2.93967	0.436568	1.05746
15.	4.09602	15.2656	1.47342	5.49135
20.	9.70885	48.4507	3.49253	17.4287
25.	18.9535	118.41	6.82038	42.5983
30.	32.6549	245.428	11.772	88.3415
35.	51.3753	453.352	18.6103	163.469
40.	75.2072	767.731	27.4792	277.838
45.	103.714	1213.24	38.3533	441.613
50.	136.047	1811.26	51.0384	664.397
55.	171.144	2578.32	65.2145	954.485
60.	207.914	3525.48	80.497	1318.39
65.	245.366	4658.58	96.4914	1760.64
70.	282.678	5978.9	112.833	2283.88
75.	319.21	7484.06	129.209	2889.02
80.	354.499	9168.92	145.366	3575.6
85.	388.233	11026.5	161.11	4341.99
90.	420.221	13048.3	176.303	5185.78
95.	450.368	15225.6	190.848	6103.94
100.	478.649	17548.9	204.689	7093.08
105.	505.087	20009.	217.796	8149.6
110.	529.741	22596.8	230.163	9269.81
115.	552.691	25303.6	241.798	10450.
120.	574.03	28121.	252.721	11686.6
125.	593.858	31041.4	262.961	12976.1
130.	612.276	34057.3	272.549	14315.1
135.	629.385	37161.9	281.521	15700.5
140.	645.281	40349.1	289.914	17129.4
145.	660.057	43612.9	297.763	18598.8
150.	673.8	46947.9	305.104	20106.1
155.	686.591	50349.3	311.973	21649.
160.	698.505	53812.4	318.401	23225.1
165.	709.611	57333.	324.421	24832.4
170.	719.975	60907.3	330.061	26468.7
175.	729.654	64531.6	335.348	28132.4
180.	738.702	68202.7	340.308	29821.7
185.	747.169	71917.7	344.965	31535.
190.	755.1	75673.5	349.341	33270.8
195.	762.537	79467.8	353.455	35027.9
200.	769.516	83298.1	357.326	36805.
205.	776.074	87162.3	360.973	38600.8
210.	782.24	91058.2	364.41	40414.4
215.	788.044	94984.1	367.652	42244.6
220.	793.513	98938.1	370.713	44090.6
225.	798.671	102919.	373.605	45951.4
230.	803.539	106924.	376.34	47826.4
235.	808.138	110954.	378.928	49714.6
240.	812.488	115005.	381.38	51615.4
245.	816.604	119078.	383.703	53528.2
250.	820.503	123171.	385.907	55452.2
255.	824.199	127283.	388.	57387.1
260.	827.706	131413.	389.987	59332.1
265.	831.036	135560.	391.877	61286.8
270.	834.2	139723.	393.675	63250.7
275.	837.21	143901.	395.386	65223.4
280.	840.073	148095.	397.016	67204.4
285.	842.801	152302.	398.571	69193.4
290.	845.4	156522.	400.053	71190.
295.	847.878	160756.	401.468	73193.8
300.	850.244	165001.	402.82	75204.6

Boil-off rate

Once the vessel is full of LHe, calculate the boil-off rate associated to radiation losses and conduction losses in the filling neck

Radiation losses

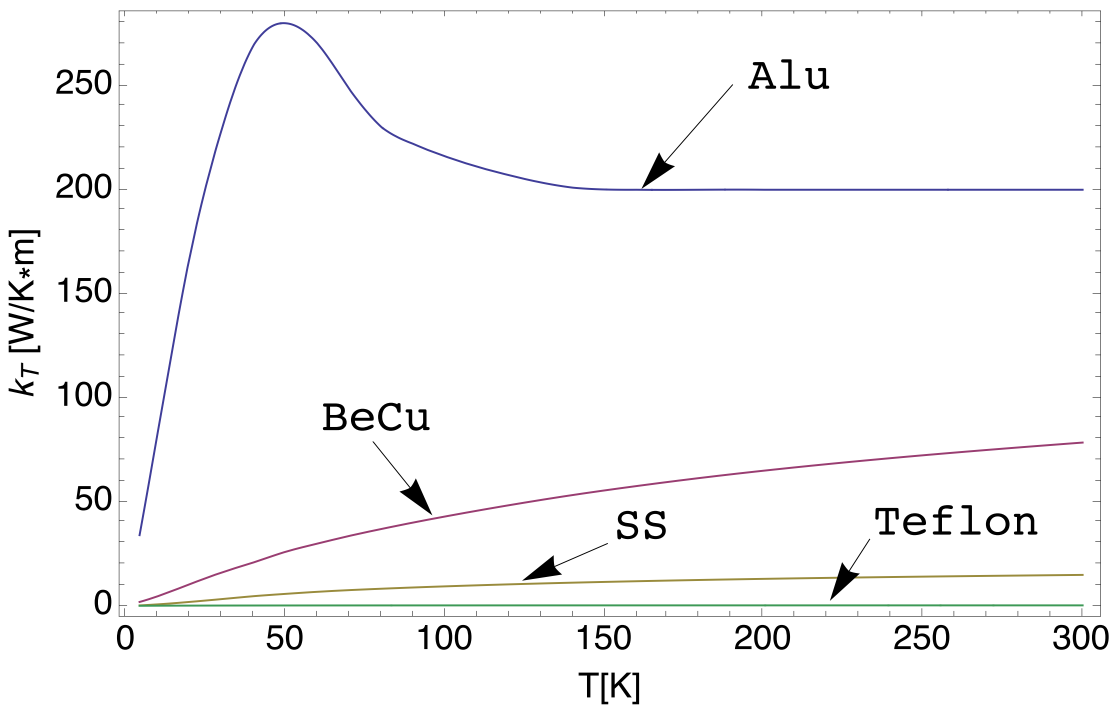
For the following cases consider the emissivities of the radiation surfaces of $\epsilon = 0.1$:

v) Without MLI which implies that the LHe vessel “sees” directly the RT vessel. Assume that the equivalent radiation surface is the average of the cold and warm surfaces which are separated by a distance d = 50mm.
vi) With an actively cooled radiation shield at the LN temperature.
vii) With a passive radiation shield consisting of a mono-layer of MLI.

Conduction losses

Considering a filling neck connecting the LHe-vessel and the RT-vessel composed either by viii) Aluminum or ix) Teflon, calculate the conduction losses.

For these calcutaions use the following tables:



T[K]	Alu kT[W/m*K]	Alu-Int kT[W/m]	Teflon kT[W/m*K]	Teflon-Int kT[W/m]
5.	42.4519	31.2473	0.0552981	0.0410027
10.	86.	351.772	0.096	0.426207
15.	129.51	891.183	0.12274	0.977734
20.	170.	1641.84	0.141	1.63948
25.	202.75	2576.11	0.159125	2.39109
30.	230.	3660.17	0.174	3.22531
35.	253.125	4870.33	0.184625	4.12307
40.	270.	6181.	0.193	5.06781
45.	278.125	7554.18	0.201	6.05323
50.	280.	8951.84	0.208	7.07614
55.	277.	10346.2	0.213875	8.1312
60.	270.	11715.2	0.219	9.21364
65.	259.5	13039.8	0.22375	10.3207
70.	248.	14308.5	0.228	11.4503
75.	238.125	15523.2	0.231688	12.5997
80.	230.	16692.7	0.235	13.7666
85.	225.25	17830.	0.238188	14.9497
90.	222.	18947.7	0.241	16.1478
95.	218.797	20049.5	0.243188	17.3585
100.	216.	21136.3	0.245	18.5791
105.	213.403	22209.7	0.246731	19.8085
110.	211.05	23270.8	0.2483	21.0461
115.	208.922	24320.6	0.249719	22.2912
120.	207.	25360.3	0.251	23.5431
125.	205.141	26390.6	0.252188	24.8011
130.	203.5	27412.1	0.25325	26.0647
135.	202.109	28426.	0.254188	27.3334
140.	201.	29433.6	0.255	28.6064
145.	200.438	30437.1	0.255648	29.8831
150.	200.125	31438.4	0.256188	31.1627
155.	200.	32438.7	0.256633	32.4448
160.	200.	33438.6	0.257	33.7289
165.	199.945	34438.5	0.257305	35.0147
170.	199.938	35438.2	0.257563	36.3019
175.	199.961	36437.9	0.257789	37.5903
180.	200.	37437.8	0.258	38.8797
185.	200.039	38437.9	0.258211	40.1703
190.	200.063	39438.2	0.258438	41.4619
195.	200.055	40438.5	0.258695	42.7547
200.	200.	41438.6	0.259	44.0489
205.	200.	42438.6	0.259223	45.3445
210.	200.	43438.6	0.259429	46.6411
215.	200.	44438.6	0.259613	47.9387
220.	200.	45438.6	0.259771	49.2372
225.	200.	46438.6	0.259902	50.5364
230.	200.	47438.6	0.26	51.8361
235.	200.	48438.6	0.260063	53.1363
240.	200.	49438.6	0.260086	54.4367
245.	200.	50438.6	0.260066	55.7371
250.	200.	51438.6	0.26	57.0373
255.	200.	52438.6	0.260021	58.3373
260.	200.	53438.6	0.260034	59.6375
265.	200.	54438.6	0.26004	60.9377
270.	200.	55438.6	0.26004	62.2379
275.	200.	56438.6	0.260036	63.5381
280.	200.	57438.6	0.260029	64.8382
285.	200.	58438.6	0.26002	66.1384
290.	200.	59438.6	0.260011	67.4384
295.	200.	60438.6	0.260004	68.7385
300.	200.	61438.6	0.26	70.0385
305.	200.	62438.6	0.26	71.3385

Boil-off rate and running costs

x) Considering the radiation and conduction losses for the cases above, v)-ix), calculate the boil-off rate and running cost.

xi) How would one design the Lhe-vessel for taking advantage of the He-gas enthalpy during cool-

down as well as for lowering the boil-off rate?
xii) Describe a few techniques for measuring the boil-off rate.

Problem solution

Cryogenic vessel geometry and cryogenic properties

Vessel geometry:
V: volume
dr: thickness
R: inner radius

```
V = 0.1 (*[m³]*);
dr = 0.002;
R[V_] := (3*V/(4*Pi))^(1/3);
```

Material properties:
ρ : specific weight [kg/m3]
θ : Debye temperature [K] from Barron, Cryogenic Systems, 2nd edition, OxfordUniversity Press, 1985, p.26, Table 2.2
M: atomic weight [kg/mol]

```
ρAlu = 2.7*10^3; θDAlu = 390; MAlu = 27*10^-3; (* Kg/mol *)
ρSS = 7.9*10^3; θDSS = 430;  MSS  = 56*10^-3;
```

LHe vessel mass depending on material (Alu, SS)

```
M[V_, dr_, ρ_] := 4*Pi*R[V]^2*dr*ρ

MVesAlu = M[V, dr, ρAlu];
MVesSS = M[V, dr, ρSS];
Print["Mass Aluminum Vessel, MAlu = ", MVesAlu, "[kg]"]
Print["Mass StainlessSteel Vessel, MSS = ", MVesSS, "[kg]"]
```

```
Mass Aluminum Vessel, MAlu = 5.62615[kg]

Mass StainlessSteel Vessel, MSS = 16.4617[kg]
```

Vessel thermal heat capacity

Cv: Debye specific heat from Barron, Cryogenic Systems, 2nd edition, OxfordUniversity Press, 1985, Eq. 2.6 (p.24)

```
Cv[T_, θD_] := 9*(T/θD)^3*NIntegrate[x^4*Exp[x]/(Exp[x]-1)^2, {x, 0, θD/T}]
```

```
Plot[{Cv[y, θDAlu], Cv[y, θDSS]}, {y, 0, 300}, FrameLabel → {"T[K]", "Cv/R"}]

Table[{{T}, Rμ / MAlu * Cv[T, θDAlu]}, {T, 1., 300., 1}];
CvAlu = ListInterpolation[%];
Table[{{T}, Rμ / MSS * Cv[T, θDSS]}, {T, 1., 300., 1}];
CvSS = ListInterpolation[%];
```

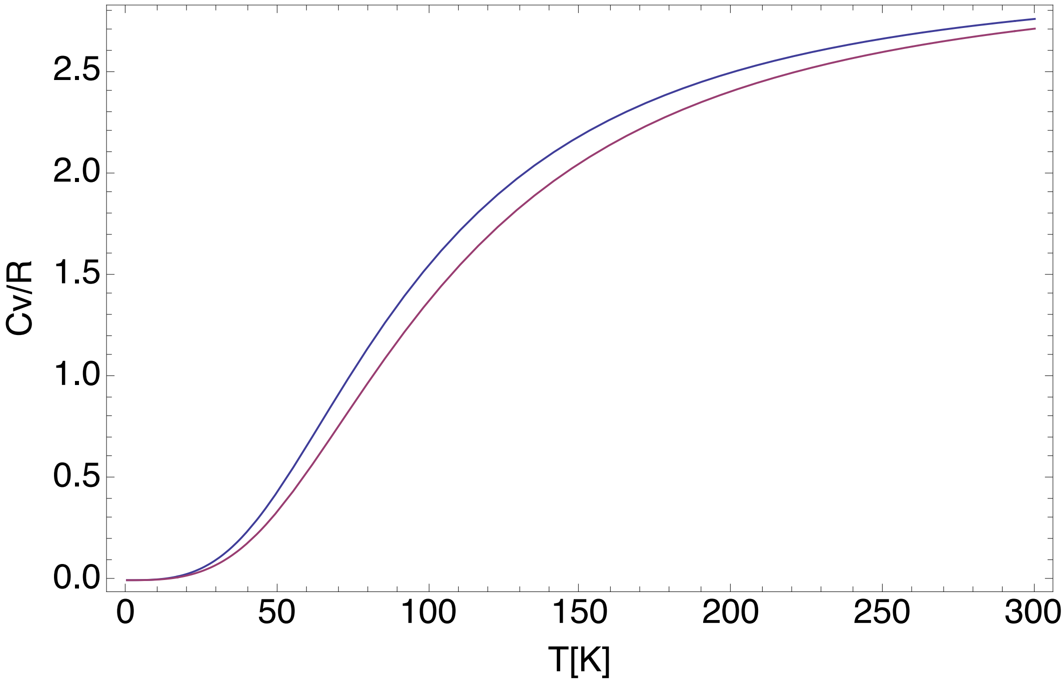
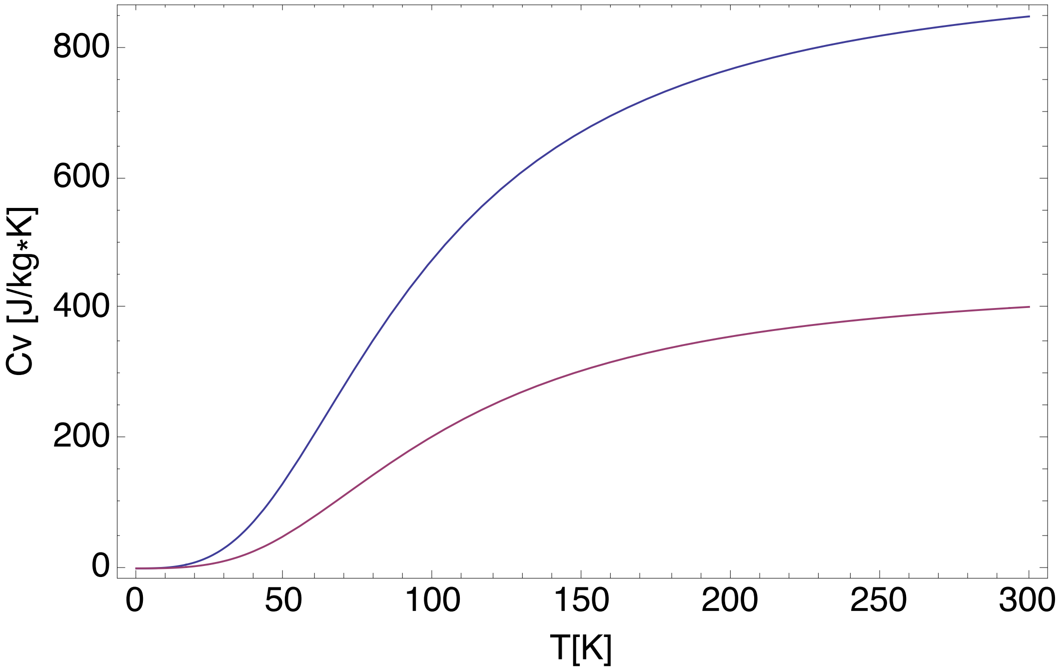


Table form of the thermal heat capacity and the integrated values
Note: the negative values at “zero” temperature are a numerical artifact due to the used interpolating function

```
Tv = Table[i, {i, 0.0000001, 305, 5}];
CvAluv = Table[CvAlu[Tv[[i]]], {i, 1, Length[Tv]}];
CvAluIntv  = Table[NIntegrate[CvAlu[x], {x, 4.2, Tv[[i]]}], {i, 1, Length[Tv]}];
CvSSv = Table[CvSS[Tv[[i]]], {i, 1, Length[Tv]}];
CvSSIntv  = Table[NIntegrate[CvSS[x], {x, 4.2, Tv[[i]]}], {i, 1, Length[Tv]}];
TableForm[Transpose[{Tv, CvAluv, CvAluIntv, CvSSv, CvSSIntv}],
 TableHeadings → {None, {"T[K]", "Alu-Cv[J/kg*K]",
    "Alu-Int Cv/R[J/kg]", "SS-Cv/R[J/kg*K]", "SS-Int Cv/R[J/kg]"}}]
```

T[K]	Alu-C_v[J/kg*K]	Alu-Int Cv/R[J/kg]	SS-C_v/R[J/kg*K]	SS-Int Cv/R[
$1.\times10^{-7}$	-3.55549×10^{-14}	-0.0944115	-1.16651×10^{-14}	-0.0339617
5.	0.151704	0.0952188	0.054571	0.0342521
10.	1.21363	2.93967	0.436568	1.05746
15.	4.09602	15.2656	1.47342	5.49135
20.	9.70885	48.4507	3.49253	17.4287
25.	18.9535	118.41	6.82038	42.5983
30.	32.6549	245.428	11.772	88.3415
35.	51.3753	453.352	18.6103	163.469
40.	75.2072	767.731	27.4792	277.838
45.	103.714	1213.24	38.3533	441.613
50.	136.047	1811.26	51.0384	664.397
55.	171.144	2578.32	65.2145	954.485
60.	207.914	3525.48	80.497	1318.39
65.	245.366	4658.58	96.4914	1760.64
70.	282.678	5978.9	112.833	2283.88
75.	319.21	7484.06	129.209	2889.02
80.	354.499	9168.92	145.366	3575.6
85.	388.233	11026.5	161.11	4341.99
90.	420.221	13048.3	176.303	5185.78
95.	450.368	15225.6	190.848	6103.94
100.	478.649	17548.9	204.689	7093.08
105.	505.087	20009.	217.796	8149.6
110.	529.741	22596.8	230.163	9269.81
115.	552.691	25303.6	241.798	10450.
120.	574.03	28121.	252.721	11686.6
125.	593.858	31041.4	262.961	12976.1
130.	612.276	34057.3	272.549	14315.1
135.	629.385	37161.9	281.521	15700.5
140.	645.281	40349.1	289.914	17129.4
145.	660.057	43612.9	297.763	18598.8
150.	673.8	46947.9	305.104	20106.1
155.	686.591	50349.3	311.973	21649.
160.	698.505	53812.4	318.401	23225.1
165.	709.611	57333.	324.421	24832.4
170.	719.975	60907.3	330.061	26468.7
175.	729.654	64531.6	335.348	28132.4
180.	738.702	68202.7	340.308	29821.7
185.	747.169	71917.7	344.965	31535.
190.	755.1	75673.5	349.341	33270.8
195.	762.537	79467.8	353.455	35027.9
200.	769.516	83298.1	357.326	36805.
205.	776.074	87162.3	360.973	38600.8
210.	782.24	91058.2	364.41	40414.4
215.	788.044	94984.1	367.652	42244.6
220.	793.513	98938.1	370.713	44090.6
225.	798.671	102919.	373.605	45951.4
230.	803.539	106924.	376.34	47826.4
235.	808.138	110954.	378.928	49714.6
240.	812.488	115005.	381.38	51615.4
245.	816.604	119078.	383.703	53528.2
250.	820.503	123171.	385.907	55452.2
255.	824.199	127283.	388.	57387.1
260.	827.706	131413.	389.987	59332.1
265.	831.036	135560.	391.877	61286.8
270.	834.2	139723.	393.675	63250.7
275.	837.21	143901.	395.386	65223.4
280.	840.073	148095.	397.016	67204.4
285.	842.801	152302.	398.571	69193.4
290.	845.4	156522.	400.053	71190.
295.	847.878	160756.	401.468	73193.8
300.	850.244	165001.	402.82	75204.6

```
Plot[{Rμ / MAlu * Cv[y, ΘDAlu], Rμ / MSS * Cv[y, ΘDSS]},
 {y, 0, 300}, FrameLabel → {"T[K]", "Cv  [J/kg*K]"}]
```



LHe boil-off in for cool-down from room temperature or LN2 temperature

Thermal properties and cost of helium, He, and Nitrogen, N2
L: latent heat capacity
Cp: heat capacity at constant pressure
ρ: density
Cost: price per liter

```
LHe = 20.9 * 10^3 (*[J/kg]*); CpHe  = 4.48 * 10^3 (*[J/kg]*);
LN2  = 199.3 * 10^3 (*[J/kg]*); CpN2  = 2.05 * 10^3 (*[J/kg]*);
ρHe  = 0.125 (*[kg/l]*);  ρN2  = 0.808 (*[kg/l]*);
CostHe  = 13. (*[CHF/l]*);
CostN2  = 0.5(*[CHF/l]*);
```

```
TableForm[Transpose[{{LHe, LN2}, {CpHe, CpN2}, {ρHe, ρN2}, {CostHe, CostN2}}],
 TableHeadings →
  {{"Helium", "Nitrogen"}, {"L[J/kg]", "Cp[J/kg*K]", "ρ[kg/l]", "Cost[CHF/l]"}}]

T1 = 4.2; T2 = 77; T3 = 290;
NIntegrate[MVesAlu * CvAlu[x] / (LHe + CpHe * (x - T1)), {x, T1, T2}] //
 Print["Alu Vessel, mLHe with gas enthalpy = ", #, "[kg]", ", T1 =", T1,
   "[K],", " T2 =", T2, "[K]", ", LHe cost =", # / ρHe * CostHe, "CHF"] &
NIntegrate[MVesAlu * CvAlu[x] / (LHe), {x, T1, T2}] //
 Print["Alu Vessel, mLHe without gas enthalpy = ", #, "[kg]", ", T1 =", T1,
   "[K],", " T2 =", T2, "[K]", ", LHe cost =", # / ρHe * CostHe, "CHF"] &
NIntegrate[MVesAlu * CvAlu[x] / (LHe), {x, T1, T3}] //
 Print["Alu Vessel, mLHe without gas enthalpy = ", #, "[kg]", ", T1 =", T1,
   "[K],", " T2 =", T3, "[K]", ", LHe cost =", # / ρHe * CostHe, "CHF"] &
Print[""]
NIntegrate[MVesSS * CvSS[x] / (LHe + CpHe * (x - T1)), {x, T1, T2}] //
 Print["SS Vessel, mLHe with gas enthalpy = ", #, "[kg]", ", T1 =", T1,
   "[K],", " T2 =", T2, "[K]", ", LHe cost =", # / ρHe * CostHe, "CHF"] &
NIntegrate[MVesSS * CvSS[x] / (LHe), {x, T1, T2}] //
 Print["SS Vessel, mLHe without gas enthalpy = ", #, "[kg]", ", T1 =", T1,
   "[K],", " T2 =", T2, "[K]", ", LHe cost =", # / ρHe * CostHe, "CHF"] &
NIntegrate[MVesSS * CvSS[x] / (LHe), {x, T1, T3}] //
 Print["SS Vessel, mLHe without gas enthalpy = ", #, "[kg]", ", T1 =", T1,
   "[K],", " T2 =", T3, "[K]", ", LHe cost =", # / ρHe * CostHe, "CHF"] &
```

	L[J/kg]	C_p[J/kg*K]	ρ[kg/l]	Cost[CHF/l]
Helium	20 900.	4480.	0.125	13.
Nitrogen	199 300.	2050.	0.808	0.5

Alu Vessel, m_{LHe} with gas enthalpy = 0.182521 [kg], T1 =4.2[K], T2 =77[K], LHe cost =18.9822CHF

Alu Vessel, m_{LHe} without gas enthalpy = 2.19038 [kg], T1 =4.2[K], T2 =77[K], LHe cost =227.799CHF

Alu Vessel, m_{LHe} without gas enthalpy = 42.1348 [kg], T1 =4.2[K], T2 =290[K], LHe cost =4382.02CHF

SS Vessel, m_{LHe} with gas enthalpy = 0.204879 [kg], T1 =4.2[K], T2 =77[K], LHe cost =21.3074CHF

SS Vessel, m_{LHe} without gas enthalpy = 2.48418 [kg], T1 =4.2[K], T2 =77[K], LHe cost =258.355CHF

SS Vessel, m_{LHe} without gas enthalpy = 56.0722 [kg], T1 =4.2[K], T2 =290[K], LHe cost =5831.5CHF

Radiation losses

Radiating surfaces

One assumes that the equivalent radiation surface is the average of the cold and warm surfaces which are separated by a distance d = 50mm without MLI

```
S1 = 4 * Pi * R[V] ^2;
d = 50 * 10^-3;
S2 = 4 * Pi * (R[V] + d) ^2;
S = 1 / 2 * (S1 + S2);
Print["Average radiating surface, S = ", S, "[m²]"]
```

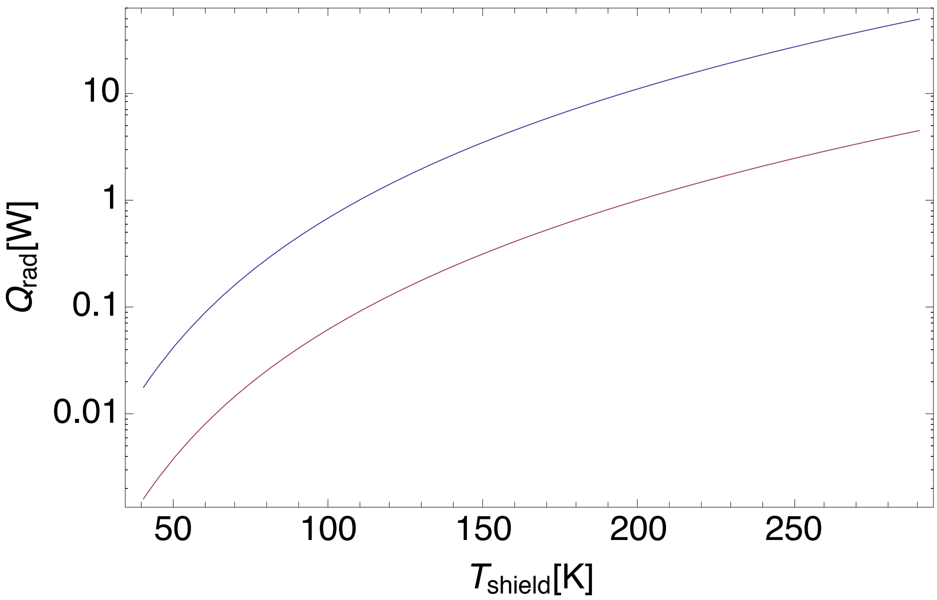
```
Average radiating surface, S = 1.23851[m²]
```

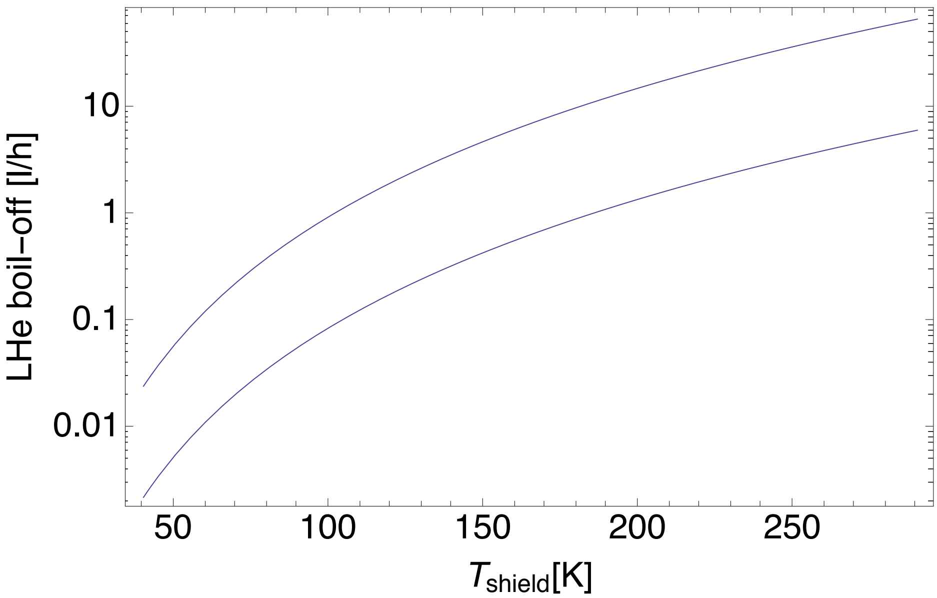
Radiation losses including the effect of multi-layer insulation (MLI)

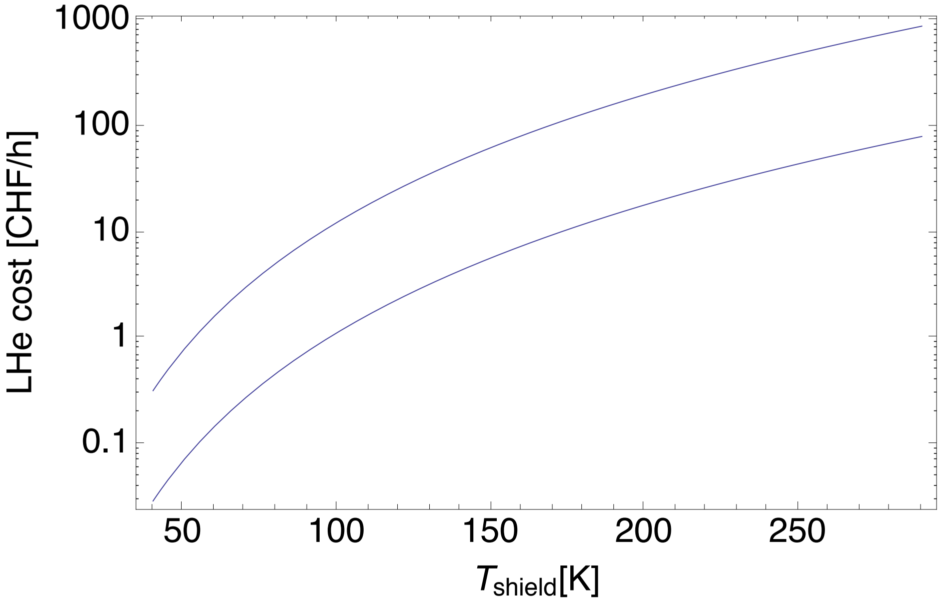
```
Qrad[T1_, T2_, ϵ_, S_, n_] := 1 / (n + 1) * σSB * ϵ * S (T2^4 - T1^4)
```

The radiation losses shown in the figures below are for no layers of the MLI (blue curves) and 10 layers (red curves)

```
ϵ = 0.1; (* Figure of merit of emissivity for Alu & SS *)
LogPlot[{Qrad[T1, x, 0.1, S, 0], Qrad[T1, x, 0.1, S, 10]},
 {x, 40, T3}, FrameLabel → {"Tshield[K]", "Qrad[W]"}]
LogPlot[{Qrad[T1, x, 0.1, S, 0], Qrad[T1, x, 0.1, S, 10]} / LHe / ρHe * 3600,
 {x, 40, T3}, FrameLabel → {"Tshield[K]", "LHe boil-off [l/h]"}]
LogPlot[{Qrad[T1, x, 0.1, S, 0], Qrad[T1, x, 0.1, S, 10]} / LHe / ρHe * CostHe * 3600,
 {x, 40, T3}, FrameLabel → {"Tshield[K]", "LHe cost [CHF/h]"}]
```







Note: the typical boil-off of a 100l dewar without LN2 radiation shield is of 1l/day. In order to reach this level of boil-off rate, the number of layers for the MLI is more than 10 and, more importantly the material choice is such that the emissivity is significantly lower than 0.1. See for example “Aluminum film 400A on Mylar”

Table 5: Hemispheric emissivity for a selection of materials

	Temperature (K)			
	4	20	80	300
Copper mechanically polished	0.02		0.06	0.1
Copper black oxidised				0.8
Gold			0.01	0.02
Silver	0.005		0.01	0.02
Aluminium electropolished	0.04		0.08	0.15
Aluminium mechanically polished	0.06		0.1	0.2
Aluminium with 7 μm oxide				0.75
Magnesium				0.07
Chromium			0.08	0.08
Nickel			0.022	0.04
Rhodium			0.08	
Lead	0.012		0.036	0.05
Tin	0.012		0.013	0.05
Zinc			0.026	0.05
Brass, polished	0.018		0.029	0.035
Stainless steel, 18-8	0.2		0.12	0.2
Glass				0.94
Ice				0.96
Oil paints, any colour				0.92–0.96
Silver plate on copper		0.013	0.017	
Aluminium film 400A on Mylar			0.009	0.025
Aluminium 200A on Mylar			0.015	0.035
Nickel coating on copper		0.027	0.033	

Conduction losses on the supporting neck

Neck geometry

Thermal conductivity properties; Barron, pag. 376, Table 7.9

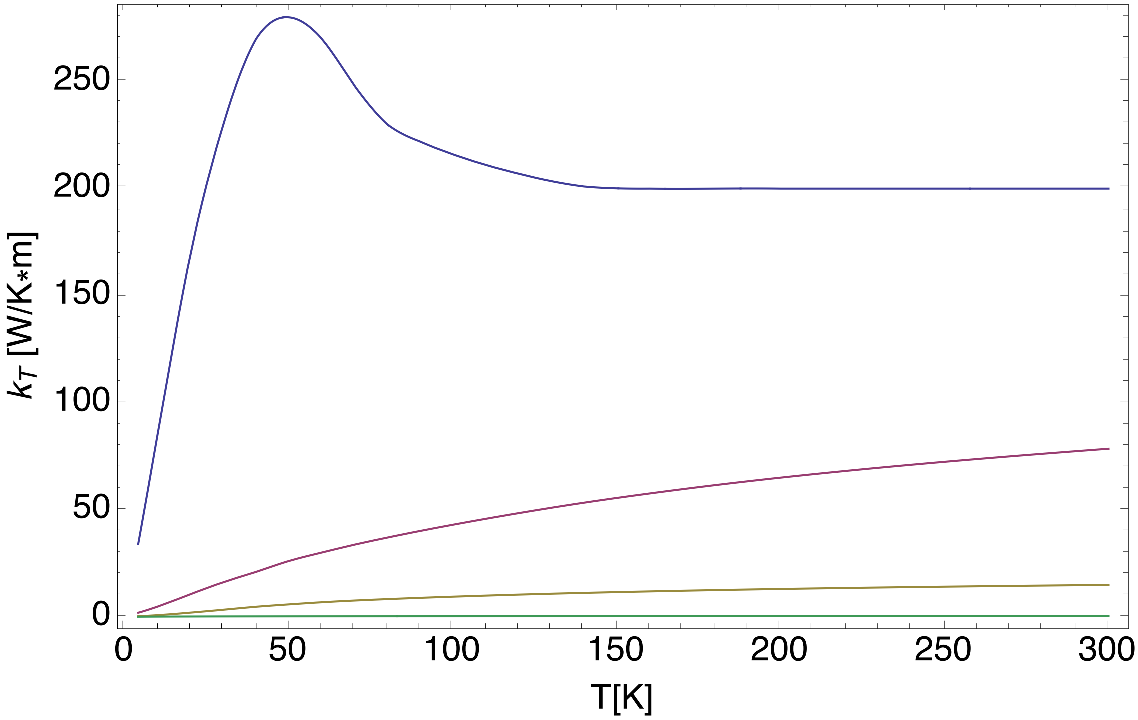
```
T = {4, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 120, 140, 160, 180, 200, 250, 300};
kTAluminum = {34, 86, 170, 230, 270, 280,
   270, 248, 230, 222, 216, 207, 201, 200, 200, 200, 200, 200};
kTBeCu = {1.9, 4.8, 10.6, 16.2, 21, 26.1, 30, 33.7, 37, 40.1,
   43, 48.4, 53.3, 57.6, 61.5, 65, 72.4, 78.5};
kTSS = {0.24, 0.77, 1.95, 3.3, 4.7, 5.8, 6.8, 7.6, 8.26, 8.86,
   9.4, 10.36, 11.17, 11.86, 12.47, 13, 14.07, 14.9};
kTTeflon = {45, 96, 141, 174, 193, 208, 219, 228, 235, 241,
    245, 251, 255, 257, 258, 259, 260, 260} * 10^-3;

Table[{{T[[i]]}, kTAluminum[[i]]}, {i, 1, Length[T]}];
kTAluminum = ListInterpolation[%];
Table[{{T[[i]]}, kTBeCu[[i]]}, {i, 1, Length[T]}];
kTBeCu = ListInterpolation[%];
Table[{{T[[i]]}, kTSS[[i]]}, {i, 1, Length[T]}];
kTSS = ListInterpolation[%];
Table[{{T[[i]]}, kTTeflon[[i]]}, {i, 1, Length[T]}];
kTTeflon = ListInterpolation[%];
```

```
Tv = Table[i, {i, 5, 305, 5}];
kTAluminumv = Table[kTAluminum[Tv[[i]]], {i, 1, Length[Tv]}];
kTAluminumIntv =
  Table[NIntegrate[kTAluminum[x], {x, 4.2, Tv[[i]]}], {i, 1, Length[Tv]}];
kTTeflonv = Table[kTTeflon[Tv[[i]]], {i, 1, Length[Tv]}];
kTTeflonIntv =
  Table[NIntegrate[kTTeflon[x], {x, 4.2, Tv[[i]]}], {i, 1, Length[Tv]}];
TableForm[Transpose[
  N[{Tv, kTAluminumv, kTAluminumIntv, kTTeflonv, kTTeflonIntv}]],
 TableHeadings → {None, {"T[K]", "Alu kT[W/m*K]", "Alu-Int kT[W/m]",
    "Teflon kT[W/m*K]", "Teflon-Int kT[W/m]"}}]
```

T[K]	Alu kT[W/m*K]	Alu-Int kT[W/m]	Teflon kT[W/m*K]	Teflon-Int kT[W/m]
5.	42.4519	31.2473	0.0552981	0.0410027
10.	86.	351.772	0.096	0.426207
15.	129.51	891.183	0.12274	0.977734
20.	170.	1641.84	0.141	1.63948
25.	202.75	2576.11	0.159125	2.39109
30.	230.	3660.17	0.174	3.22531
35.	253.125	4870.33	0.184625	4.12307
40.	270.	6181.	0.193	5.06781
45.	278.125	7554.18	0.201	6.05323
50.	280.	8951.84	0.208	7.07614
55.	277.	10346.2	0.213875	8.1312
60.	270.	11715.2	0.219	9.21364
65.	259.5	13039.8	0.22375	10.3207
70.	248.	14308.5	0.228	11.4503
75.	238.125	15523.2	0.231688	12.5997
80.	230.	16692.7	0.235	13.7666
85.	225.25	17830.	0.238188	14.9497
90.	222.	18947.7	0.241	16.1478
95.	218.797	20049.5	0.243188	17.3585
100.	216.	21136.3	0.245	18.5791
105.	213.403	22209.7	0.246731	19.8085
110.	211.05	23270.8	0.2483	21.0461
115.	208.922	24320.6	0.249719	22.2912
120.	207.	25360.3	0.251	23.5431
125.	205.141	26390.6	0.252188	24.8011
130.	203.5	27412.1	0.25325	26.0647
135.	202.109	28426.	0.254188	27.3334
140.	201.	29433.6	0.255	28.6064
145.	200.438	30437.1	0.255648	29.8831
150.	200.125	31438.4	0.256188	31.1627
155.	200.	32438.7	0.256633	32.4448
160.	200.	33438.6	0.257	33.7289
165.	199.945	34438.5	0.257305	35.0147
170.	199.938	35438.2	0.257563	36.3019
175.	199.961	36437.9	0.257789	37.5903
180.	200.	37437.8	0.258	38.8797
185.	200.039	38437.9	0.258211	40.1703
190.	200.063	39438.2	0.258438	41.4619
195.	200.055	40438.5	0.258695	42.7547
200.	200.	41438.6	0.259	44.0489
205.	200.	42438.6	0.259223	45.3445
210.	200.	43438.6	0.259429	46.6411
215.	200.	44438.6	0.259613	47.9387
220.	200.	45438.6	0.259771	49.2372
225.	200.	46438.6	0.259902	50.5364
230.	200.	47438.6	0.26	51.8361
235.	200.	48438.6	0.260063	53.1363
240.	200.	49438.6	0.260086	54.4367
245.	200.	50438.6	0.260066	55.7371
250.	200.	51438.6	0.26	57.0373
255.	200.	52438.6	0.260021	58.3373
260.	200.	53438.6	0.260034	59.6375
265.	200.	54438.6	0.26004	60.9377
270.	200.	55438.6	0.26004	62.2379
275.	200.	56438.6	0.260036	63.5381
280.	200.	57438.6	0.260029	64.8382
285.	200.	58438.6	0.26002	66.1384
290.	200.	59438.6	0.260011	67.4384
295.	200.	60438.6	0.260004	68.7385
300.	200.	61438.6	0.26	70.0385
305.	200.	62438.6	0.26	71.3385

```
Plot[{kTAluminum[x], kTBeCu[x], kTSS[x], kTTeflon[x]},
 {x, 4, 300}, FrameLabel → {"T[K]", "kT [W/K*m]"}]
```



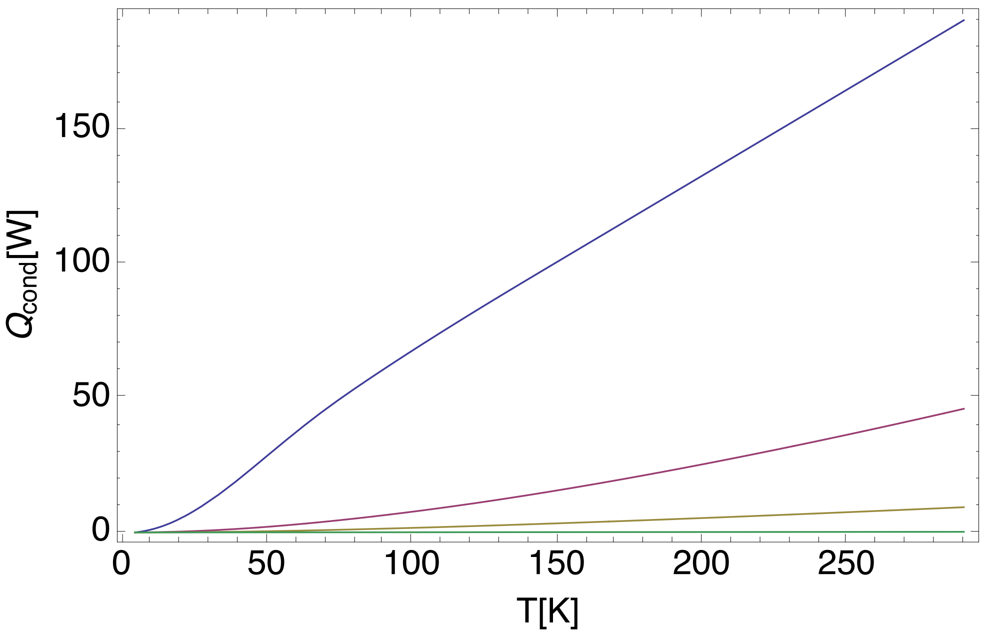
Cylinder of Lneck = 50mm, ID = 50mm, thickness = 1mm
Consider different materials, from Alu, SS, Epoxy
The end parts of the cylinder are at T1 =4.2K and T2 = 300K

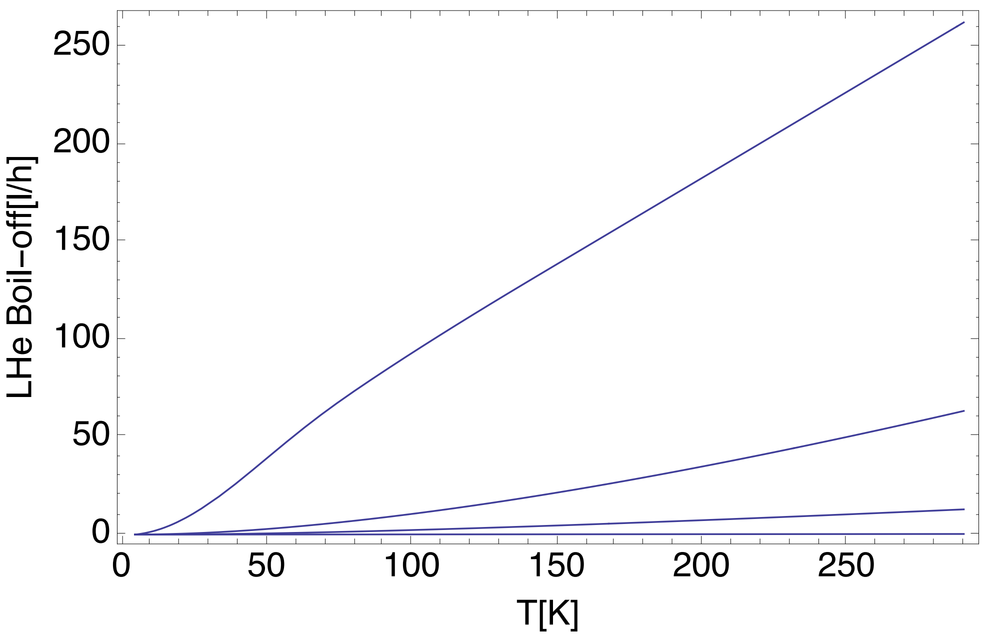
```
dNeck = 1*10^-3;
LNeck = 0.05;
ri = 50*10^-3/2; ro = ri+dNeck;
Sneck = Pi*(ro^2-ri^2);
```

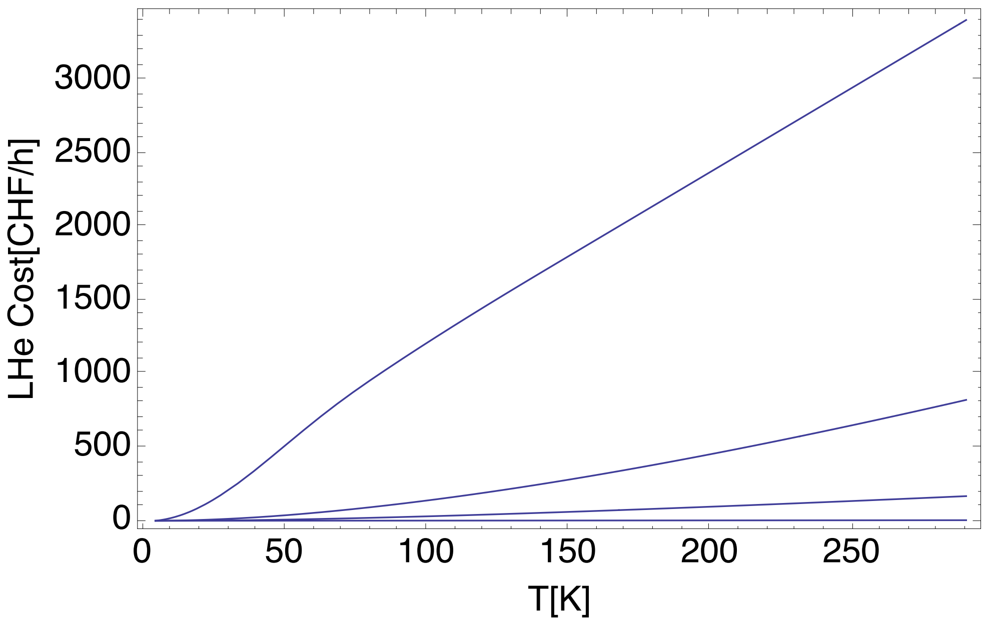
Conduction losses, boil-off rates and noil-off cost for the four different materials: Alu, BeCu, SS, teflon
All above mentioned quantities are calculated assuming the cold temperature at T1=4.2K and a variable hot temperature T2

```
QcondAlu[T1_, T2_] := Sneck/LNeck*NIntegrate[kTAluminum[x], {x, T1, T2}]
QcondBeCu[T1_, T2_] := Sneck/LNeck*NIntegrate[kTBeCu[x], {x, T1, T2}]
QcondSS[T1_, T2_] := Sneck/LNeck*NIntegrate[kTSS[x], {x, T1, T2}]
QcondTeflon[T1_, T2_] := Sneck/LNeck*NIntegrate[kTTeflon[x], {x, T1, T2}]
```

```
Plot[{QcondAlu[T1, x], QcondBeCu[T1, x], QcondSS[T1, x], QcondTeflon[T1, x]},
 {x, T1, T3}, FrameLabel -> {"T[K]", "Qcond[W]"}]
Plot[{QcondAlu[T1, x], QcondBeCu[T1, x], QcondSS[T1, x], QcondTeflon[T1, x]}/LHe/
   ρHe*3600, {x, T1, T3}, FrameLabel -> {"T[K]", "LHe Boil-off[l/h]"}]
Plot[{QcondAlu[T1, x], QcondBeCu[T1, x], QcondSS[T1, x], QcondTeflon[T1, x]}/LHe/
   ρHe*3600*CostHe, {x, T1, T3}, FrameLabel -> {"T[K]", "LHe Cost[CHF/h]"}]
```

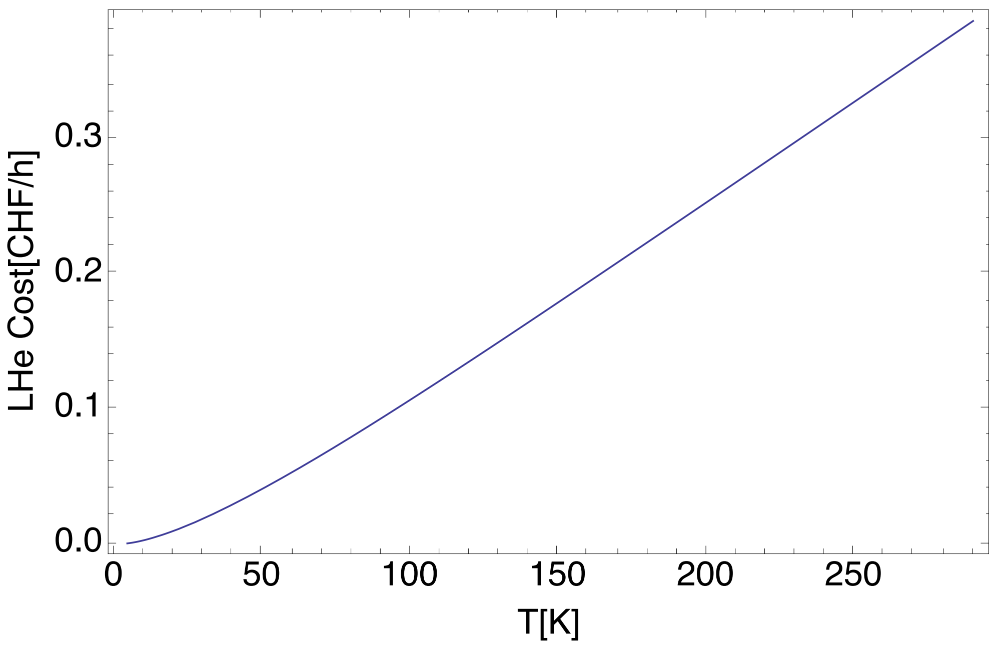

Q_{cond}[W]
150
100
50
0
0
50
100
150
200
250
T[K]


LHe Boil-off[l/h]
250
200
150
100
50
0
0
50
100
150
200
250
T[K]



Hourly boil-off cost for a Teflon neck versus hot temperature

```
Plot[{QcondTeflon[T1, x]} / LHe / ρHe * 3600 * CostHe,
 {x, T1, T3}, FrameLabel -> {"T[K]", "LHe Cost[CHF/h]"}]
```



Herschel

With regard to the last slide of the Cryogenic course, below is the simple calculation of the maximum heat load on the helium bath for satisfying the three year operation of the Herschel mission.

```
VHe = 2400. (*[l]*);
mHe = 2400 * 0.125
LossRate = mHe / (3 * 365 * 24 * 3600)
```

300.

3.17098×10^{-6}

```
Qtot = LossRate * LHe //
  Print["Herschel total heat load on helium sistem: Qtot = ", #, "[W]"] &
```

Herschel total heat load on helium sistem: Q_{tot} = 0.0662735[W]

This quantity has to be compared with radiation power density (> 1 kW/m^2) seen by Herschel when facing the Sun. The effective surface of Herschel is at least a few m^2!