Fusion and Industrial Plasma Technologies
Millimeter Wave Systems
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The goal of the problems presented here is to get a phenomenological understanding of what the quality factor of a resonant cavity is, and to show that the gyrotron instability is subject to a threshold.

I Quality factor of a resonant cavity

The quality factor Q is a dimensionless parameter that describes the ability of a cavity to retain the energy stored in it. It is defined as the ratio between the stored electromagnetic energy W_s and the energy lost per oscillation cycle P_{loss}/ω,

$$Q = \frac{\omega W_s}{P_{loss}}$$

— Write the power balance equation of the cavity and solve for the stored energy.
— Knowing that the electric field amplitude goes like the square root of the stored energy, write the time dependence of the electric field $E(t)$.
— Assuming that $E(t) = 0$ for $t < 0$, Fourier transform it and show that $\left|\tilde{E}(\omega)\right|^2$, as measured for instance by a Schottky detector has a Lorentzian shape.
— Relate the quality factor to the shape of the curve

II Gyrotron instability threshold

The power lost by a magnetized electron beam in a resonant cavity can be modelled as

$$W_{gyro} = (aE^2 - bE^4)I_b V_b$$

where a and b are given constants depending on the beam and cavity parameters in a way not relevant here, and where I_b and V_b are respectively the electron beam current and energy.
— At fixed beam energy, and getting inspiration from the previous problem, show that there is a current I_{start} below which the cavity-electron beam power balance equation has no non-zero stationary solution.
— For $I_b > I_{start}$, show that the equilibrium point is stable.