Fusion and industrial Plasma Technologies

Dimensioning of a superconducting solenoid

Exercise 1

- Requirements
- Calculate the overall current
- Suggest number of turns and operating current

Exercise 2

- Calculate the self inductance
- Calculate the hoop load
- Estimate the need of structural support

Exercise 3

- Discuss the discharge requirement in case of quench
- Discuss the hot spot temperature
- Discuss an option for graded conductor

Requirement and input data

- *Generation of 4 T inside the solenoid*

- *Bath cooling (4.2 K)*

- *Use NbTi superconductor (scaling law -> current density)*

- *Free bore of the solenoid, ø = 50mm*

- *Length of the solenoid λ = 500mm*

- *Thin (single?) layer winding*

- *NbTi composite: cu:non-cu = 2, σ_y = 300 MPa*

- *Suggested criteria for engineering margins:*

  $\Delta T = 0.5\ K$ $\sigma_{op} \leq 2/3\ \sigma_y$ $T_{hot\ spot} \leq 150\ K$

Calculate overall current

- *Apply Ampere law to find the overall current*

- *Use "long solenoid" approximation*

The "long solenoid" approximation tell us that the central field, B_c, is homogeneous and vertical inside the solenoid. The flux lines close at infinite, i.e. the field is 0 outside the solenoid.

Applying the Ampere law on the dotted path in the sketch, which include the current $I_{tot}(\Delta\lambda/\lambda)$ the two horizontal segments give 0 contribution (90° orientation of path and field). The outer segment gives also 0 contribution (because of 0 field).

$$\oint B \cdot d\ell = \mu_0 I \implies B_c \cdot \Delta\lambda = \mu_0 \frac{\Delta\lambda}{\lambda} I_{tot}$$

$$I_{tot} = \frac{B_c \cdot \lambda}{\mu_0} = 1.59 \, MA$$

λ

$\Delta\lambda$
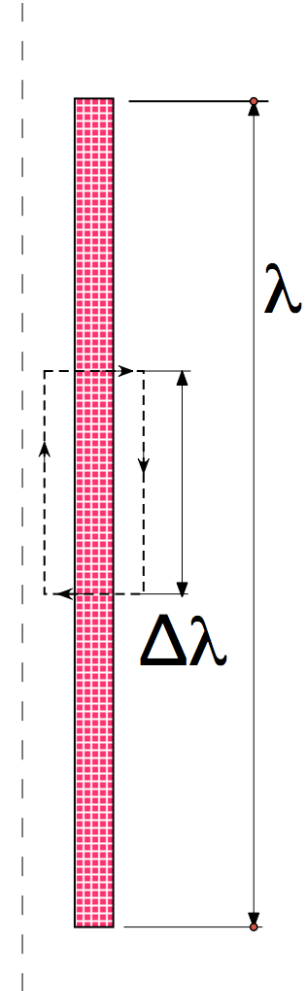
Calculate the current density at operating conditions

- *Retain approximately B_c as B_{op} for the conductor*
- *Retain $T = T_{bath} + \Delta T = 4.7$ K*
- *Calculate from scaling law J_{NbTi}*
- *Normalize J to strand area*
- *Calculate total strand area*

- *$b = 4 / 14.61 = 0.2738$*
- *$t = 4.7 / 9.03 = 0.5205$*
- *$J_{c\ NbTi}$ (4T, 4.7K) = 3043 A/mm^2*
- *$J_{c\ Strand}$ (4T, 4.7K) = 1014 A/mm^2*
- *$J_{op\ Strand}$ (4T, 4.2K) ≈ 1000 A/mm^2*
- *$A_{strand} = I_{tot}/J_{op} = 1590$ mm^2*

Number of turns and operating current

- *Discuss the implications of the selections*

- *Is a single layer realistic?*

The total current I_{tot} can be obtained by many combinations of number of turns, n, and operating current I_{op}. The criteria for a sound selection are technology and common sense. Let consider two extreme cases:

- *n=1, I_{op}= 1.59 MA The conductor should be a 500 mm x 3.18 mm slab, wrapped to a cylinder. Problems about the current injection (where are the terminals), the current leads (huge heat load in the cryostat) and the power supply (1.59 MA converter)…*

- *n=1000 000, I_{op}= 1.59 A The NbTi composite would have a diameter (non-insulated) of ≈ 45 μm (impossible to handle). The inductance would be in the range of kHy with very large voltage requirements…*

A sound solution aims at a reasonable current (avoid kA range for power supply and current leads). The sound range is between 100A and 300A, say 159A / 10 000 turns. (non-insulated diameter ≈ 0.45 mm)

A single layer is not possible with a round strand. A rectangular conductor would be necessary, to be wound on the short edge of 500mm/10 000 = 0.05mm.

Calculate the self inductance

- *Use the flux definition and the Faraday law to extract the self inductance*

$$\Phi = B \cdot \pi R^2 = \frac{\mu_0 \, n I_{op}}{\lambda} \pi R^2$$

$$V = n \dot{\Phi} = L \dot{I}_{op}$$

$$L = \mu_0 \frac{n^2 \, \pi R^2}{\lambda} = 494 \, mHy$$

$$E_{st} = \frac{L I_{op}^2}{2} = 6.24 \, kJ$$

Kamil Sedlak

- *Add electrical insulation (varnish, 20-30 μm thick) for final strand diameter.*

- *Check number of layers, i.e. adjust the number of turns (Nt) and operating current (I_{op}) for an integer number of layers.*

$A_{comp} = 0.159$ mm^2
$\varnothing_{bare} = 0.45$ mm
$\varnothing_{ins} = 0.5$ mm

- *Varnish insulation ≈ 25 μm thick*

- *1000 turns per layer -> 10 layers*

- *The thickness of the solenoid is s = 5 mm*

- *No need of adjustment!!*

Calculate the hoop load

- *Calculate the Lorentz force for the operating conditions*

- *Discuss limits and approximation*

- *Estimate the average load from the winding geometry*

$$F = I_{tot} \int B \times d\ell$$

Over a cross section, the tension in the winding is $T = I_{tot}B(R)R$.

Assuming a field distribution, linear from 4T to 0 from the inner radius to the outer radius, and an average radius of 27.5 mm, the average hoop load is 79.5 kN over the winding pack.

For the innermost single turn, the tension is $I_{op}B_{max}R_{min} = 15.9$ N

Estimate the stress

- *Estimate the peak stress is at the innermost turn.*

- *Discuss the need of an additional mechanical support (i.e. can the superconducting strand alone withstand the hoop stress, or is there a need to include an additional mechanical support?*

The peak stress is at the innermost turn, where the tension per strand is 15.9 N, is:

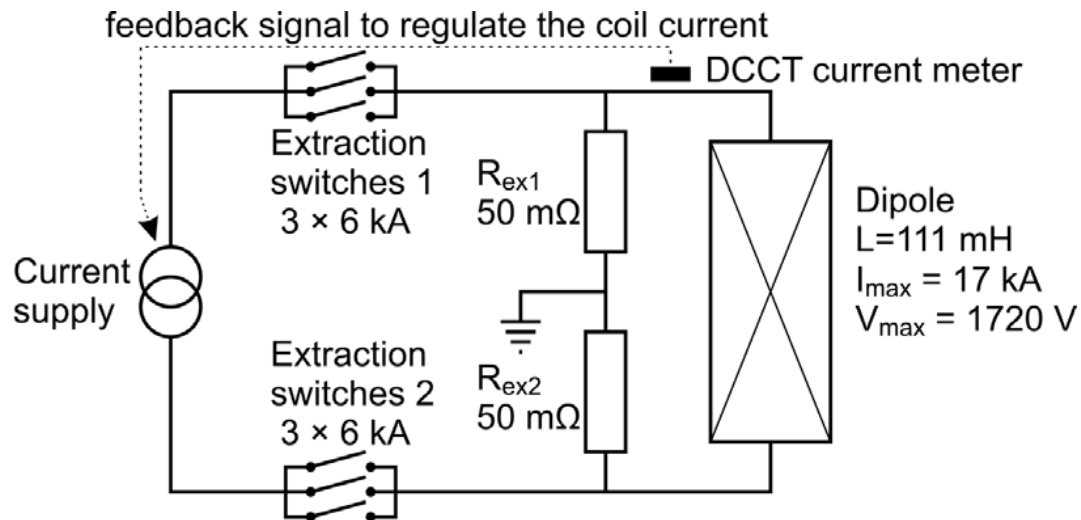$$\sigma_{max}^{strand} = \frac{15.9}{A_{comp}} = 100 MPa$$

Need of structural support?

- *The stress of the winding pack is well within the allowable, including the peak stress on a "free standing" innermost turn*

- *A detailed analysis, e.g. FE, including the electrical insulation properties (glass epoxy impregnation) and the shear stress at the insulation between layers, will provide a map of the stress distribution with actual peak lower than estimated here.*

Discuss the discharge requirement in case of quench

- *Using the scheme similar to Lecture 1 and retaining a maximum voltage at the terminal of 100 V, calculate for the "thin solenoid" the*

 - *Dump resistor*

  - *Decay time constant*

Current evolution during the fast current discharge:

$$I = I_{\mathrm{op}}\, e^{-t/\tau}$$

feedback signal to regulate the coil current

DCCT current meter

Current supply

Extraction switches 1
3 × 6 kA

R_{ex1}
50 mΩ

Extraction switches 2
3 × 6 kA

R_{ex2}
50 mΩ

Dipole
L=111 mH
I_{max} = 17 kA
V_{max} = 1720 V

Estimate the hot spot temperature

- *Calculate the dissipated ohmic energy at the hot spot, assuming a delay time for quench detection, $t_{delay} = 0.5$ s and the calculated dump time constant*

- *Under adiabatic conditions (no heat exchange from the strand to the surrounding), estimate the temperature rise at the hot spot*
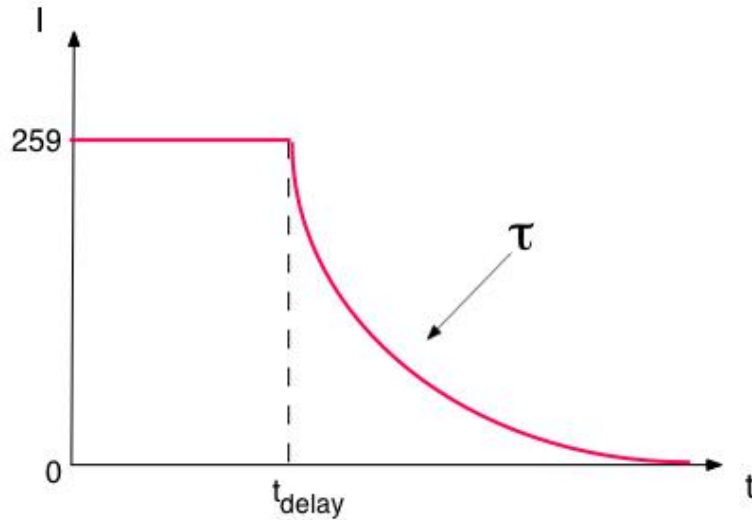
$\rho_{copper} = 1.6 \cdot 10^{-10}$ Ωm

$\rho_{NbTi} = 2.1 \cdot 10^{-8}$ Ωm *(non-superconducting mode)*

$\eta_{copper} = 8.9 \cdot 10^3$ kg/m^3

$\eta_{NbTi} = 6.5 \cdot 10^3$ kg/m^3

$Cp_{copper} = 0.0011T^3 + 0.011T$ $J/K \cdot kg$

$Cp_{NbTi} = 0.0023T^3 + 0.145T$ $J/K \cdot kg$

Discuss an option for graded conductor

- *Accounting for the layer winding and the field distribution in radial direction, propose a two-grades winding*

- *Propose a layout for the low grade conductor and discuss the implications for hot spot temperature*