



## Superconducting Magnets: Exercise 2

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## Dimensioning of a superconducting solenoid

### Exercise 1

- Requirements
- Calculate the overall current
- Suggest number of turns and operating current

### Exercise 2

- Calculate the self inductance
- Calculate the hoop load
- Estimate the need of structural support

### Exercise 3

- Discuss the discharge requirement in case of quench
- Discuss the hot spot temperature
- Discuss an option for graded conductor

# Requirement and input data

- *Generation of **4 T** inside the solenoid*
- *Bath cooling (**4.2 K**)*
- *Use NbTi superconductor (scaling law -> current density)*
- *Free bore of the solenoid,  $\phi = 50\text{mm}$*
- *Length of the solenoid  $\lambda = 500\text{mm}$*
- *Thin (single?) layer winding*
- *NbTi composite: **cu:non-cu = 2**,  $\sigma_y = 300 \text{ MPa}$*
- *Suggested criteria for engineering margins:*

$$\Delta T = 0.5 \text{ K} \quad \sigma_{op} \leq 2/3 \sigma_y \quad T_{hot \ spot} \leq 150 \text{ K}$$

# Calculate overall current

- Apply Ampere law to find the overall current
- Use “long solenoid” approximation

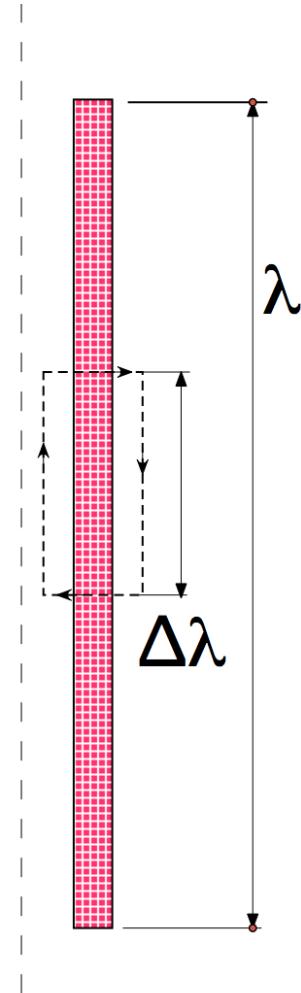
The “long solenoid” approximation tell us that the central field,  $B_c$ , is homogeneous and vertical inside the solenoid. The flux lines close at infinite, i.e. the field is 0 outside the solenoid.

Applying the Ampere law on the dotted path in the sketch, which include the current  $I_{tot}(\Delta\lambda/\lambda)$  the two horizontal segments give 0 contribution (90° orientation of path and field). The outer segment gives also 0 contribution (because of 0 field).

$$\oint B \cdot d\ell = \mu_0 I \quad \Rightarrow \quad B_c \cdot \Delta\lambda = \mu_0 \frac{\Delta\lambda}{\lambda} I_{tot}$$



$$I_{tot} = \frac{B_c \cdot \lambda}{\mu_0} = 1.59 \text{ MA}$$



# Calculate the current density at operating conditions

- Retain approximately  $B_c$  as  $B_{op}$  for the conductor
- Retain  $T = T_{bath} + \Delta T = 4.7 \text{ K}$
- Calculate from scaling law  $J_{NbTi}$
- Normalize  $J$  to strand area
- Calculate total strand area
  - $b = 4 / 14.61 = 0.2738$
  - $t = 4.7 / 9.03 = 0.5205$
  - $J_{c NbTi} (4T, 4.7K) = 3043 \text{ A/mm}^2$
  - $J_{c Strand} (4T, 4.7K) = 1014 \text{ A/mm}^2$
  - $J_{op Strand} (4T, 4.2K) \approx 1000 \text{ A/mm}^2$
  - $A_{strand} = I_{tot} / J_{op} = 1590 \text{ mm}^2$

- Discuss the implications of the selections
- Is a single layer realistic?

The total current  $I_{tot}$  can be obtained by many combinations of number of turns,  $n$ , and operating current  $I_{op}$ . The criteria for a sound selection are technology and common sense. Let consider two extreme cases:

- $n=1, I_{op}= 1.59 \text{ MA}$  The conductor should be a 500 mm x 3.18 mm slab, wrapped to a cylinder. Problems about the current injection (where are the terminals), the current leads (huge heat load in the cryostat) and the power supply (1.59 MA converter)...
- $n=1000\,000, I_{op}= 1.59 \text{ A}$  The NbTi composite would have a diameter (non-insulated) of  $\approx 45 \mu\text{m}$  (impossible to handle). The inductance would be in the range of kHy with very large voltage requirements...

A sound solution aims at a reasonable current (avoid kA range for power supply and current leads). The sound range is between 100A and 300A, say 159A / 10 000 turns. (non-insulated diameter  $\approx 0.45 \text{ mm}$ )

A single layer is not possible with a round strand. A rectangular conductor would be necessary, to be wound on the short edge of  $500\text{mm}/10\,000 = 0.05\text{mm}$ .

# Calculate the self inductance

- *Use the flux definition and the Faraday law to extract the self inductance*

# Suggest the conductor size and number of layers

- Add electrical insulation (varnish, 20-30  $\mu\text{m}$  thick) for final strand diameter.
- Check number of layers, i.e. adjust the number of turns ( $N_t$ ) and operating current ( $I_{op}$ ) for an integer number of layers.

# Calculate the hoop load

- *Calculate the Lorentz force for the operating conditions*
- *Discuss limits and approximation*
- *Estimate the average load from the winding geometry*

# Estimate the stress

- *Estimate the peak stress is at the innermost turn.*
- *Discuss the need of an additional mechanical support (i.e. can the superconducting strand alone withstand the hoop stress, or is there a need to include an additional mechanical support?*