

An aerial photograph of a river valley, likely the Rhône river near Geneva, with green hills and mountains in the background under a blue sky with light clouds.

Superconducting Magnets: Exercise 2



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Dimensioning of a superconducting solenoid

Exercise 1

- Requirements
- Calculate the overall current
- Suggest number of turns and operating current

Exercise 2

- Calculate the self inductance
- Calculate the hoop load
- Estimate the need of structural support

Exercise 3

- Discuss the discharge requirement in case of quench
- Discuss the hot spot temperature
- Discuss an option for graded conductor

Requirement and input data

- Generation of **4 T** inside the solenoid
- Bath cooling (**4.2 K**)
- Use NbTi superconductor (scaling law \rightarrow current density)
- Free bore of the solenoid, $\varnothing = \mathbf{50mm}$
- Length of the solenoid $\lambda = \mathbf{500mm}$
- Thin (single?) layer winding
- NbTi composite: **cu:non-cu = 2**, $\sigma_y = \mathbf{300 MPa}$
- Suggested criteria for engineering margins:

$$\Delta T = 0.5 K \quad \sigma_{op} \leq 2/3 \sigma_y \quad T_{hot\ spot} \leq 150 K$$

Calculate overall current

- Apply Ampere law to find the overall current
- Use “long solenoid” approximation

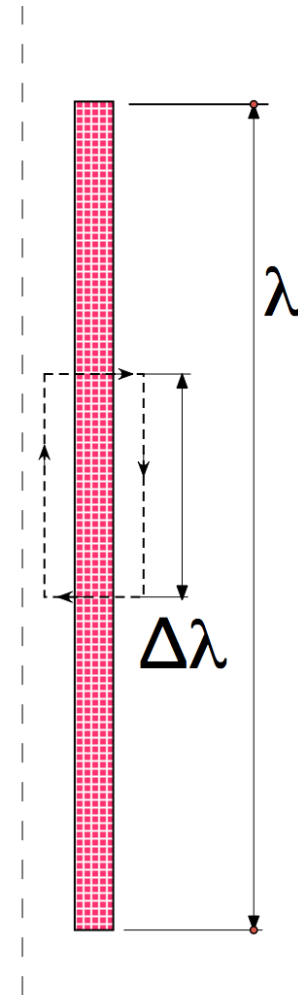
The “long solenoid” approximation tell us that the central field, B_c , is homogeneous and vertical inside the solenoid. The flux lines close at infinite, i.e. the field is 0 outside the solenoid.

Applying the Ampere law on the dotted path in the sketch, which include the current $I_{\text{tot}}(\Delta\lambda/\lambda)$ the two horizontal segments give 0 contribution (90° orientation of path and field). The outer segment gives also 0 contribution (because of 0 field).

$$\oint B \cdot d\ell = \mu_0 I \Rightarrow B_c \cdot \Delta\lambda = \mu_0 \frac{\Delta\lambda}{\lambda} I_{\text{tot}}$$



$$I_{\text{tot}} = \frac{B_c \cdot \lambda}{\mu_0} = 1.59 \text{ MA}$$



Calculate the current density at operating conditions

- Retain approximately B_c as B_{op} for the conductor
- Retain $T = T_{bath} + \Delta T = 4.7 \text{ K}$
- Calculate from scaling law J_{NbTi}
- Normalize J to strand area
- Calculate total strand area

- $b = 4 / 14.61 = 0.2738$
- $t = 4.7 / 9.03 = 0.5205$
- $J_{c \text{ NbTi}} (4T, 4.7K) = 3043 \text{ A/mm}^2$
- $J_{c \text{ Strand}} (4T, 4.7K) = 1014 \text{ A/mm}^2$
- $J_{op \text{ Strand}} (4T, 4.2K) \approx 1000 \text{ A/mm}^2$
- $A_{strand} = I_{tot} / J_{op} = 1590 \text{ mm}^2$

Number of turns and operating current

- Discuss the implications of the selections
- Is a single layer realistic?

The total current I_{tot} can be obtained by many combinations of number of turns, n , and operating current I_{op} . The criteria for a sound selection are technology and common sense. Let consider two extreme cases:

- $n=1$, $I_{\text{op}}= 1.59 \text{ MA}$ The conductor should be a 500 mm x 3.18 mm slab, wrapped to a cylinder. Problems about the current injection (where are the terminals), the current leads (huge heat load in the cryostat) and the power supply (1.59 MA converter)...
- $n=1000\ 000$, $I_{\text{op}}= 1.59 \text{ A}$ The NbTi composite would have a diameter (non-insulated) of $\approx 45 \mu\text{m}$ (impossible to handle). The inductance would be in the range of kHy with very large voltage requirements...

A sound solution aims at a reasonable current (avoid kA range for power supply and current leads). The sound range is between 100A and 300A, say **159A / 10 000 turns**. (non-insulated diameter $\approx 0.45 \text{ mm}$)

A single layer is not possible with a round strand. A rectangular conductor would be necessary, to be wound on the short edge of $500\text{mm}/10\ 000 = 0.05\text{mm}$.

Calculate the self inductance

- *Use the flux definition and the Faraday law to extract the self inductance*

Suggest the conductor size and number of layers

- *Add electrical insulation (varnish, 20-30 μm thick) for final strand diameter.*
- *Check number of layers, i.e. adjust the number of turns (Nt) and operating current (I_{op}) for an integer number of layers.*

Calculate the hoop load

- *Calculate the Lorentz force for the operating conditions*
- *Discuss limits and approximation*
- *Estimate the average load from the winding geometry*

Estimate the stress

- *Estimate the peak stress is at the innermost turn.*
- *Discuss the need of an additional mechanical support (i.e. can the superconducting strand alone withstand the hoop stress, or is there a need to include an additional mechanical support?*