Quantum Information and Quantum Computing, Problem set 12

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The goal of this problem set is to familiarize with the notion of stabilizers and stabilizer QECCs. This problem will not include any hands-on exercise on the IBM-Q platform.

Problem 1 : Correctable and non correctable errors in Shor's 9-qubit code

While the nine-qubit code can correct an arbitrary single-qubit error, it can also correct some multiple qubit errors. In what follows, X_i, Y_i, or Z_i represents the X, Y, or Z gate applied to the i-th physical qubit of the code.

1. Which of the following errors can be corrected by the nine-qubit code: X_1X_3, X_2X_7, X_5Z_6, Z_5Z_6, Y_2Z_8?
2. Suppose we perform the usual error correction procedure on the nine-qubit code after one of the above two-qubit errors has occurred. This returns us to an encoded state, but it may not be the correct encoded state. For those errors that cannot be corrected, calculate the operation that is performed on the encoded state. That is, if we start with $\alpha|0_L\rangle + \beta|1_L\rangle$, what state do we end up with?

Problem 2 : Combining stabilizer codes

Suppose we have two stabilizer codes S_1 of type $[[n_1, k_1, d_1]]$ and S_2 of type $[[n_2, k_2, d_2]]$. Suppose S_1 has generators $M_1, \ldots, M_{n_1-k_1}$ and S_2 has generators $N_1, \ldots, N_{n_2-k_2}$. Suppose we define now generators $M_i \otimes \mathbb{I}_{n_2}$ and $\mathbb{I}_{n_1} \otimes N_j$.

1. Prove that the generators so defined form an Abelian group, and therefore define an (n_1+n_2)-qubit stabilizer code.
2. Calculate k, the number of encoded qubits in the code.
3. Calculate the distance d of the code.

Problem 3 : Logical codewords of the 5-qubit stabilizer code

Consider the $[[5, 1, 3]]$ stabilizer code whose generators are

$$\begin{matrix} X & Z & Z & X & I \\ I & X & Z & Z & X \\ X & I & X & Z & Z \\ Z & X & I & X & Z \end{matrix} \tag{1}$$

1. Prove that the Pauli operator $Z_L = Z \otimes Z \otimes Z \otimes Z \otimes Z$ commutes with all generators of the code. Prove that, as a consequence, Z_L can be taken to represent the logical Z gate acting on the logical qubit encoded by the code.
2. Using the result of the previous point, find the expressions for the two logical codewords $|0_L\rangle$ and $|1_L\rangle$ in terms of the states of the 5-qubit computational basis.