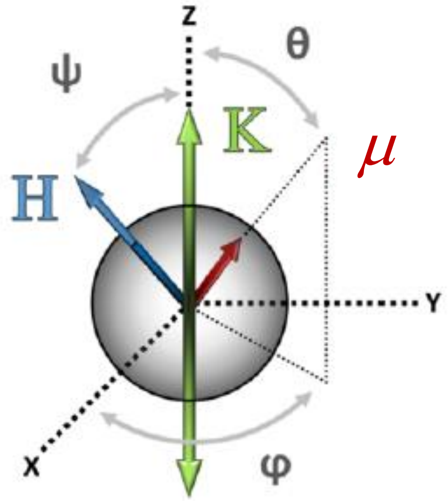
5.1 Bit magnetization reversal: B and T ≠ 0

Brown model.

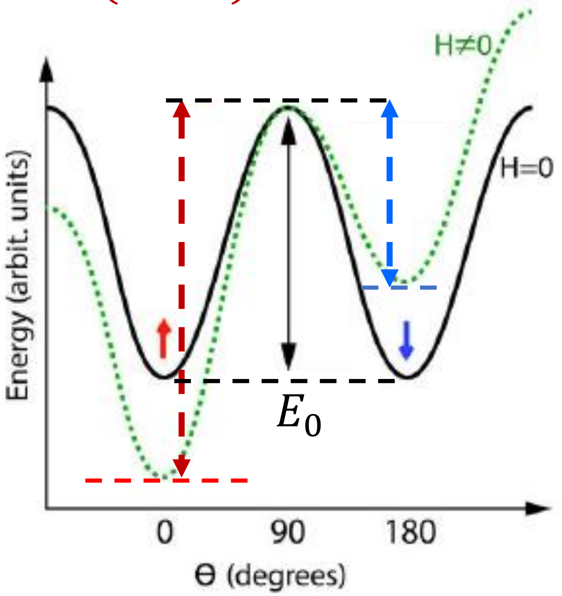
Consider a particle of magnetization μ and uniaxial magnetic anisotropy energy *K*. The magnetization relaxation time is given by $\tau = \tau_0 e^{\frac{E_{rev}}{k_B T}}$. In zero field, the barrier for magnetization reversal is the same for up and down state. In an applied field, this is not necessarily true.

In the approximation of small magnetic fields demonstrate that:

1) When the field is applied along the magnetization easy axis, state up and down need to overcome different barriers, as shown in the figure with $h = \frac{H}{H_K}$ and $H_K = \frac{2K}{\mu}$
2) When the field is applied along the magnetization hard axis, state up and down have to overcome the same barrier, $E_{rev} = K(1-h)^2$



$E_{rev,\uparrow} = K(1+h)^2$ $E_{rev,\downarrow} = K(1-h)^2$



W.F. Brown, Phys. Rev. 130, 1677 (1963) DOI: https://doi.org/10.1103/PhysRevB.104.094433

5.1 Bit magnetization reversal: B and T $\neq 0$ - Solution

1) The particle energy is given by $\varepsilon = Ksin^2\theta - \mu Hcos\theta$

The extremes of the curve can be found by equating to zero the derivative of the energy: $\frac{d\varepsilon}{d\theta} = sin\theta(2K\,cos\theta + \mu H) = 0$

The solution $sin\theta = 0$ leads to two minima $\varepsilon_1 = -\mu H$ and $\varepsilon_2 = \mu H$

The other solution $cos\theta = -\frac{\mu H}{2K}$ corresponds to the maximum $\varepsilon_m = K + \frac{\mu^2 H^2}{4K} = K\left(1 + \frac{H^2}{H_K^2}\right) = K(1 + h^2)$

The reversal barrier for magnetization up is $E_{rev,\uparrow} = \varepsilon_m - \varepsilon_1 = K(1 + h^2) + \mu H = K(1 + h^2) + 2hK = K(1 + h)^2$ and for magnetization down is $E_{rev,\downarrow} = \varepsilon_m - \varepsilon_2 = K(1 + h^2) - \mu H = K(1 + h^2) - 2hK = K(1 - h)^2$

2) The particle energy is given by $\varepsilon = Ksin^2\theta - \mu Hcos\left(\frac{\pi}{2} - \theta\right) = Ksin^2\theta - \mu Hsin\theta = K(sin^2\theta - 2h\,sin\theta)$

The extremes of the curve can be found by equating to zero the derivative of the energy: $\frac{d\varepsilon}{d\theta} = K\,cos\theta(2\,sin\theta - 2h) = 0$

The solution $cos\theta = 0$ i.e. $sin\theta = 1$ leads to the maximum $\varepsilon_m = K - 2hK$

The other solution $sin\theta = h$ corresponds to two minima $\varepsilon_1 = \varepsilon_2 = -Kh^2$

The barrier for magnetization reversal does not dipend on the direction of the magnetization and equals $E_{rev,\uparrow\downarrow} = \varepsilon_m - \varepsilon_{1,2} = K - 2hK + K\,h^2 = K(1 - h)^2$

5.2 Magnetization reversal for Fe islands on Mo(110)

Spin polarized STM is used to characterize Fe nanostructures, 1 atomic layer high, grown on Mo(110). Switching rate *vs* area of individual island is shown in the figure. What is the size of a domain wall expected on larger islands

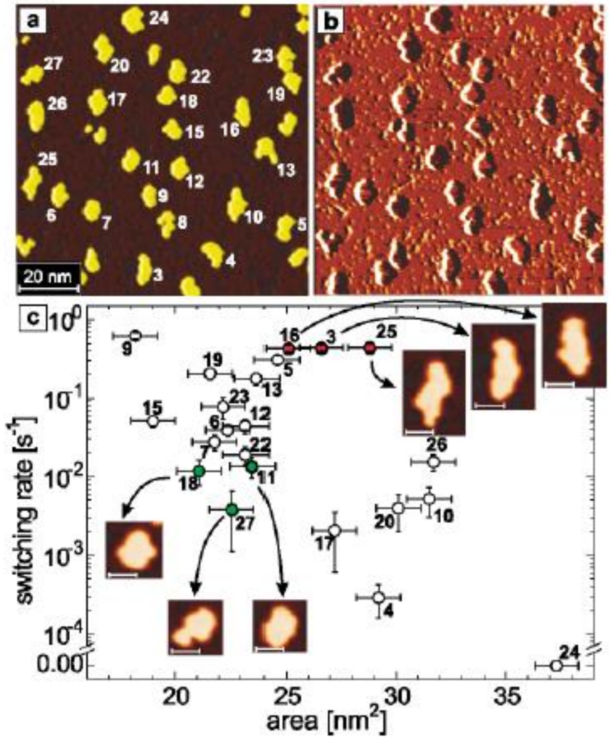


FIG. 3 (color). (a) Topography and (b) magnetic dI/dU signal of numbered Fe islands on Mo(110). (c) Plot of the switching rate versus the area of individual islands. The scatter of the switching rate points to a shape-dependent crossover from coherent rotation of compact Fe islands towards nucleation and expansion of reversed domains in elongated islands. Insets: topography of selected Fe islands (scale bar: 5 nm).

From DOI: 10.1103/PhysRevLett.92.067201

5.2 Magnetization reversal for Fe islands on Mo(110) - Solution

From the figure we realize that elongated islands, with a dimension longer than 5 nm, switch faster than small circular islands with diameter d < 5 nm. We can then deduce that $L_{cr} \approx 5\ nm$. The size of the domain wall is $\delta_{DW} = \pi\sqrt{\frac{A}{K}} = \frac{\pi}{4} L_{cr} \approx 4\ nm$

The experimental value is shown in the figure. The small difference is due to the fact that formulas are for ideal geometrical shape while reality has more complex geometris

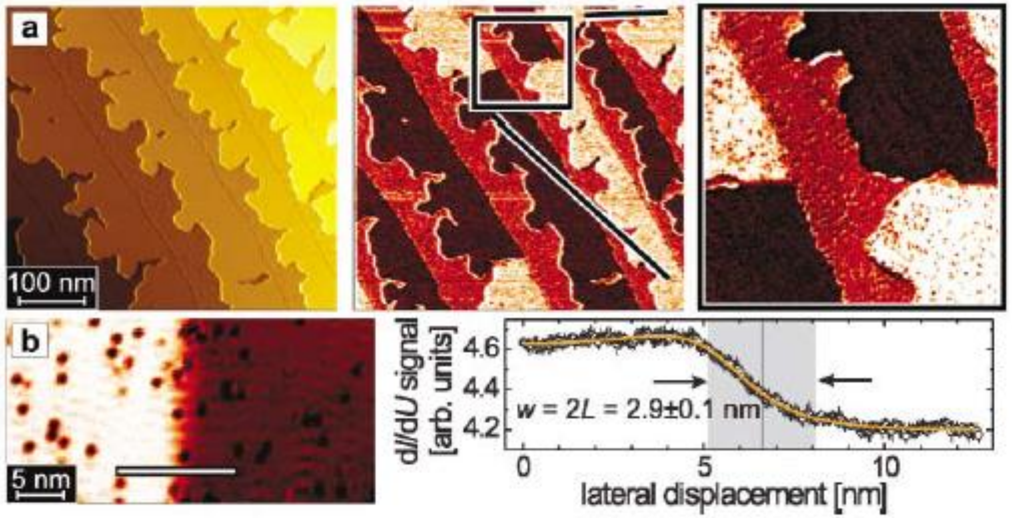


FIG. 4 (color). (a) Overview showing the topography (left panel) and magnetic dI/dU signal (middle panel) of Fe nanowires on Mo(110). Four domain walls can be recognized. Right panel: two domain walls at higher magnetification. (b) Zoom into the domain wall region (inset, rotated by 90°). The domain wall width is $w = 2.9 \pm 0.1$ nm (orange line).

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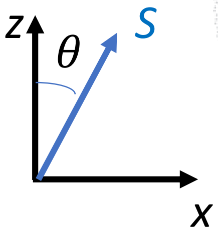
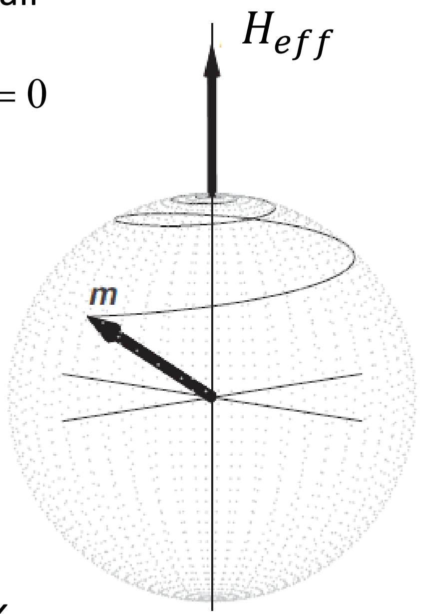
5.3 LLG equation application

With the help of the LLG equation, try to figure out the behavior of a spin in the following situations (you can test by yourself the correctness of the answer with the help of the simulator https://demonstrations.wolfram.com/PrecessionOfMagnetizationUsingTheLandauLifshitzEquation/)
In particular we are interesting to predict if the spin reverses and compare reversal speeds

1) Let consider the situation of a strong damping parameter ($\alpha = 0.1$). Spin pointing up ($\theta = 0$) and external field pointing down ($H_z < 0$): does the spin reverse?
2) Spin pointing up ($\theta = 0$) and external field with two components ($H_z < 0, H_x = 0.001$) and $\alpha = 0.1$: does the spin reverse? Does the value oh H_z has an effect on the reversal time? Does H_x has an effect on the reversal time (H_x << H_z)?
3) Spin canted ($\theta = 0.1$) and external field along z ($H_z = -1, Hx = 0.0$) and $\alpha = 0.1$: does the spin reverse? Is reversal time different with respect to the situation $\theta = 0$, $H_z = -1, Hx = 0.1$?
4) Spin pointing up ($\theta = 0$) and external field with two components ($H_z = -1, Hx = 0.1$): is the reversal faster for small or large α ?
5) Spin pointing up ($\theta = 0$) and external field with two components ($H_z = -1, Hx = 0.1$): what does the spin do if $\alpha = 0$?





5.4 Equivalence of the LLG equation expressions

Demonstrate that the two expressions for the LLG equation

1) $\frac{d\boldsymbol{m}}{dt} = -\frac{\gamma}{1+\alpha^2}(\boldsymbol{m} \wedge \boldsymbol{H}_{eff}) - \frac{\gamma}{1+\alpha^2}\frac{\alpha}{m}(\boldsymbol{m} \wedge [\boldsymbol{m} \wedge \boldsymbol{H}_{eff}])$

2) $\frac{d\boldsymbol{m}}{dt} = -\gamma(\boldsymbol{m} \wedge \boldsymbol{H}_{eff}) + \frac{\alpha}{m}\left(\boldsymbol{m} \wedge \frac{d\boldsymbol{m}}{dt}\right)$

are equivalent

(hints:
- start from expression 2
- use the vector relations in the table)

General vector relations:

$$\boldsymbol{a} \times \boldsymbol{b} = -\boldsymbol{b} \times \boldsymbol{a} \quad \text{(A.1)}$$

$$(\boldsymbol{a} + \boldsymbol{b}) \times \boldsymbol{c} = (\boldsymbol{a} \times \boldsymbol{c}) + (\boldsymbol{b} \times \boldsymbol{c}) \quad \text{(A.2)}$$

$$(\lambda\, \boldsymbol{a} \times \boldsymbol{b}) = (\boldsymbol{a} \times \lambda\, \boldsymbol{b}) = \lambda\, (\boldsymbol{a} \times \boldsymbol{b}) \quad \text{(A.3)}$$

$$\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b} \cdot (\boldsymbol{c} \times \boldsymbol{a}) = \boldsymbol{c} \cdot (\boldsymbol{a} \times \boldsymbol{b}) \quad \text{(A.4)}$$

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b}\, (\boldsymbol{a} \cdot \boldsymbol{c}) - \boldsymbol{c}\, (\boldsymbol{a} \cdot \boldsymbol{b}) \quad \text{(A.5)}$$

$$(\boldsymbol{a} \times \boldsymbol{b}) \cdot (\boldsymbol{c} \times \boldsymbol{d}) = \boldsymbol{a} \cdot [\, \boldsymbol{b} \times (\boldsymbol{c} \times \boldsymbol{d})\,] = (\boldsymbol{a} \cdot \boldsymbol{c})(\boldsymbol{b} \cdot \boldsymbol{d}) - (\boldsymbol{b} \cdot \boldsymbol{c})(\boldsymbol{a} \cdot \boldsymbol{d}) \quad \text{(A.6)}$$

$$(\boldsymbol{a} \times \boldsymbol{b}) \times (\boldsymbol{c} \times \boldsymbol{d}) = \boldsymbol{c}\, [\, (\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{d}\,] - \boldsymbol{d}\, [\, (\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{c}\,] \quad \text{(A.7)}$$

$$\boldsymbol{a} \times [\, (\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{a}\,] = \boldsymbol{a} \times [\, \boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{a})\,] = \boldsymbol{a} \times \boldsymbol{b} \quad \text{(A.8)}$$

$$\begin{aligned} \boldsymbol{e}_x \times \boldsymbol{e}_y &= \boldsymbol{e}_z \\ \boldsymbol{e}_y \times \boldsymbol{e}_z &= \boldsymbol{e}_x \\ \boldsymbol{e}_z \times \boldsymbol{e}_x &= \boldsymbol{e}_y \end{aligned} \quad \text{(A.9)}$$

We can start from 2) and replace the last part with the entire expression 2)

$$\frac{d\boldsymbol{m}}{dt} = -\gamma\left(\boldsymbol{m} \wedge \boldsymbol{H}_{eff}\right) + \frac{\alpha}{m}\,\boldsymbol{m} \wedge \left[-\gamma\left(\boldsymbol{m} \wedge \boldsymbol{H}_{eff}\right) + \frac{\alpha}{m}\left(\boldsymbol{m} \wedge \frac{d\boldsymbol{m}}{dt}\right)\right]$$

$$\frac{d\boldsymbol{m}}{dt} = -\gamma\left(\boldsymbol{m} \wedge \boldsymbol{H}_{eff}\right) + \frac{\alpha}{m}\left[-\gamma\,\boldsymbol{m} \wedge \left(\boldsymbol{m} \wedge \boldsymbol{H}_{eff}\right) + \frac{\alpha}{m}\boldsymbol{m} \wedge \left(\boldsymbol{m} \wedge \frac{d\boldsymbol{m}}{dt}\right)\right]$$

We re-write $\boldsymbol{m} \wedge \left(\boldsymbol{m} \wedge \frac{d\boldsymbol{m}}{dt}\right) = \boldsymbol{m}\left(\boldsymbol{m} \cdot \frac{d\boldsymbol{m}}{dt}\right) - \frac{d\boldsymbol{m}}{dt}(\boldsymbol{m} \cdot \boldsymbol{m}) = -\boldsymbol{m}^2\frac{d\boldsymbol{m}}{dt}$

N.B.: following 2) $\boldsymbol{m}$ and $\frac{d\boldsymbol{m}}{dt}$ are orthogonal

$$\frac{d\boldsymbol{m}}{dt} = -\gamma\left(\boldsymbol{m} \wedge \boldsymbol{H}_{eff}\right) - \gamma\frac{\alpha}{m}\boldsymbol{m} \wedge \left(\boldsymbol{m} \wedge \boldsymbol{H}_{eff}\right) - \alpha^2\frac{d\boldsymbol{m}}{dt}$$

$$\frac{d\boldsymbol{m}}{dt}(1+\alpha^2) = -\gamma\left(\boldsymbol{m} \wedge \boldsymbol{H}_{eff}\right) - \gamma\frac{\alpha}{m}\boldsymbol{m} \wedge \left(\boldsymbol{m} \wedge \boldsymbol{H}_{eff}\right)$$