Nonlinear Optics
Phys-501

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Laboratory for fundamental BioPhotonics (LBP)

Institutes of bioengineering and materials science

EPFL

Aqueous Systems

From water to neurons via membranes and semiconductors

Why: Water and Us

Liquid Water – well studied, but still unknown

> 10000 original research articles/year

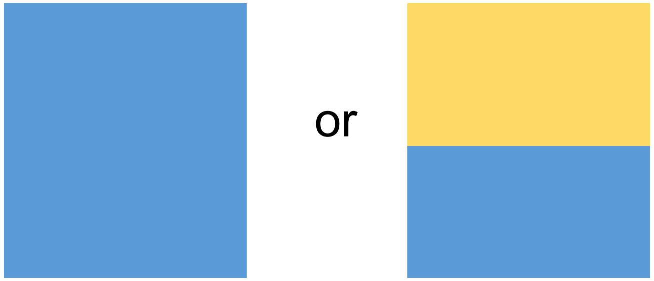
> 100 models that aim to describe water

Biology tends to consider water as a passive background (lacking actual information; but we know D_2O kills while H_2O is ok)

A biological representation of water

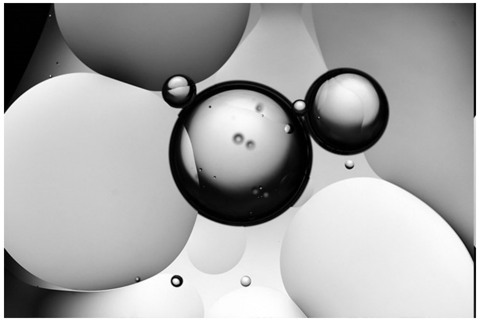
Experimental methods that can probe molecular properties of aqueous systems **are few**

Model systems for studying interfaces or solutions are overly simplistic:



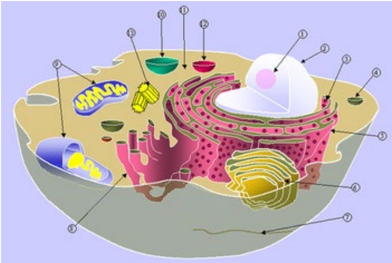
Little fundamental molecular understanding of realistic aqueous systems such as 3D nano, and micron scale systems, effects of confinement, and flow … how does surface chemistry impact materials properties ?

Droplets

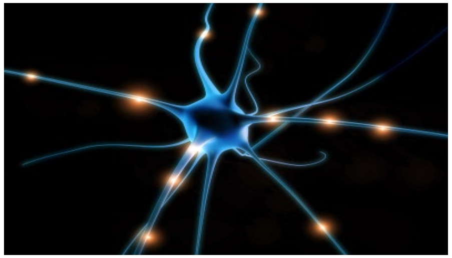


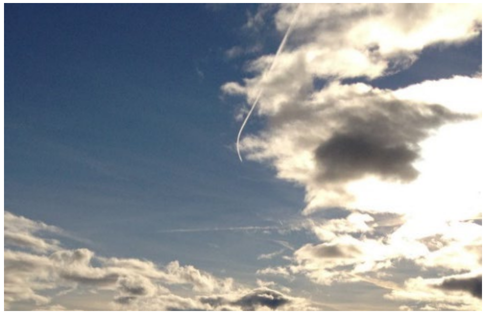
Adiposomes

Cells

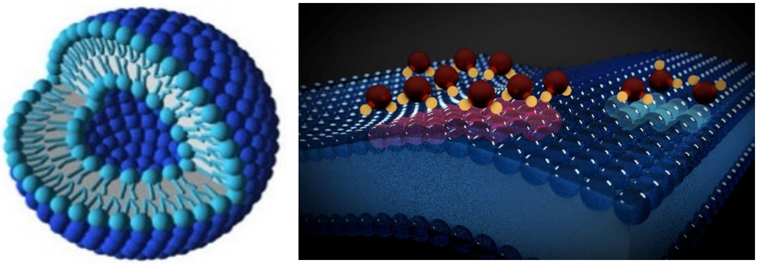


Neurons

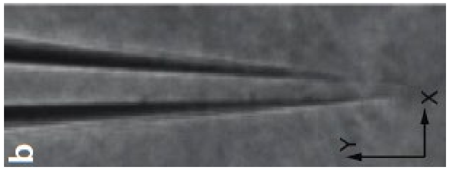




Aerosols

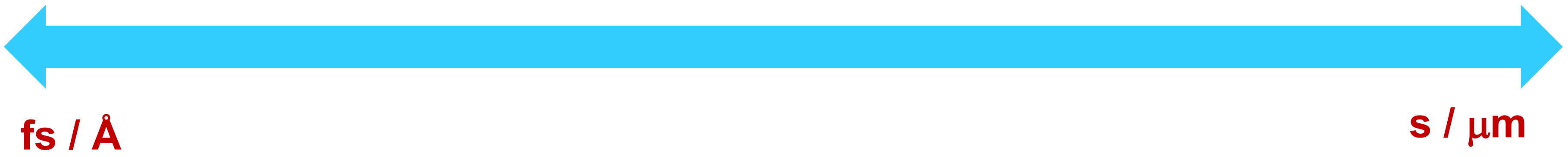


Liposomes / Membranes



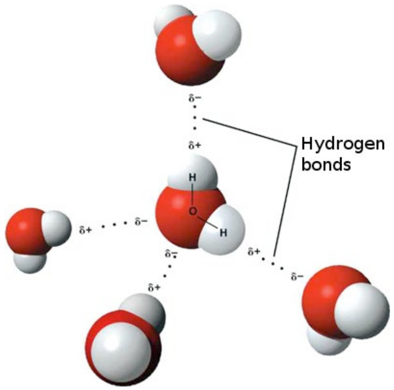
Pores

Laboratory for fundamental BioPhotonics



fs / Å

s / μm



http://www.tumblr.com/tagged/action%20potential

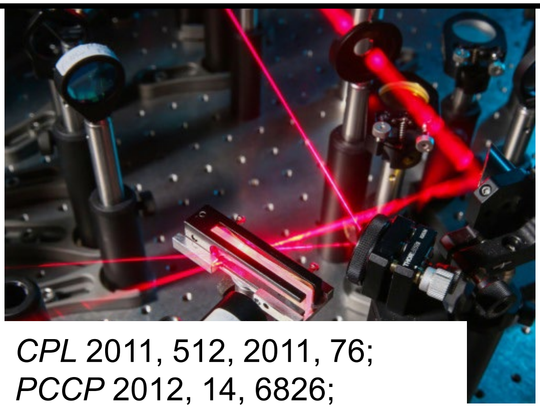
Fundamental understanding of aqueous systems

1. Develop optical tools that can access multi-length scale molecular level information at interfaces and in solution

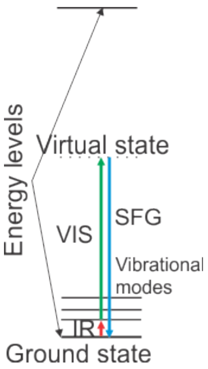
2. Investigate aqueous solutions and interfaces of 3D micron and nanoscale systems

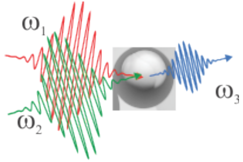
Techniques

Unique Tools



CPL 2011, 512, 2011, 76;
PCCP 2012, 14, 6826;





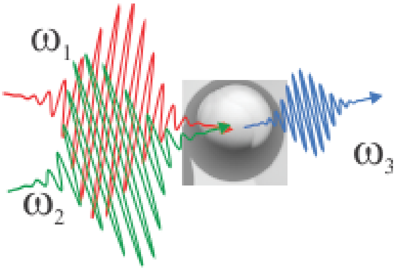
Sum Frequency Scattering

Molecular structure at interfaces of 3D systems.

- Map chemical composition, interactions, orientation and ordering of molecules

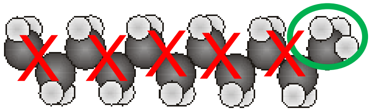
Sources for SHG and SFG

SHG / SFG is **forbidden** In centrosymmetric or isotropic media



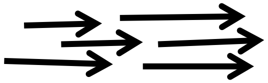
Centro-
symmetric
crystals

Isotropic medium
(= the same in
every direction)

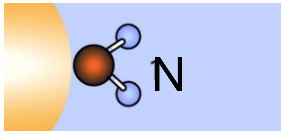


Symmetric arrangement
of molecular groups

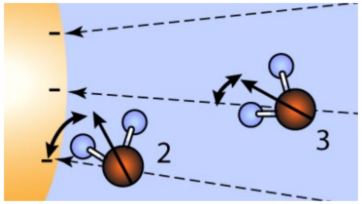
SHG / SFG is **allowed** in non-centrosymmetric environments



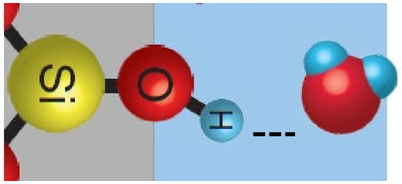
Crystals:
collagen,
microtubules,
$BaTiO_3$,…



Interface
(along z)
$I_{SH} \sim N^2 |\Gamma_s^{(2)}|^2$



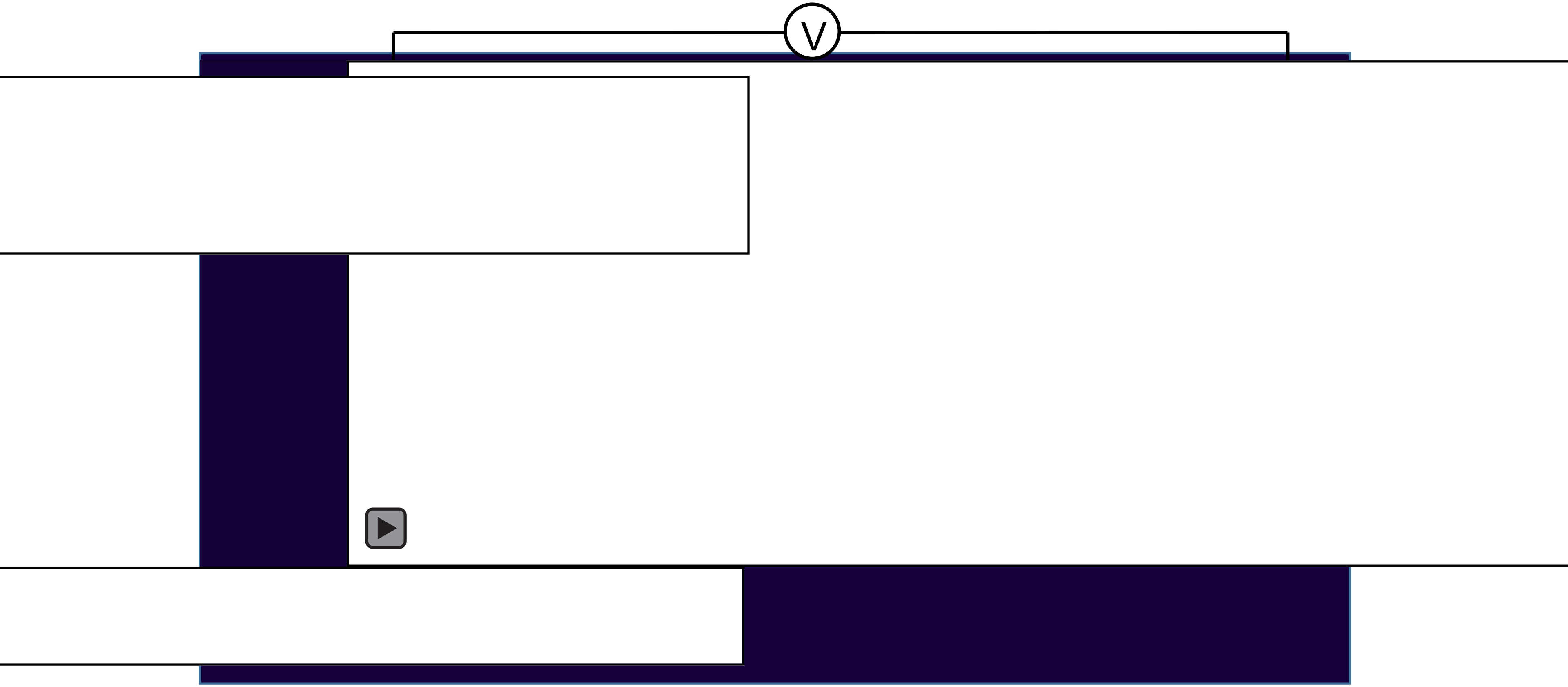
Electric field oriented water
$I_{SH} \sim N^2 |\Gamma_s^{(2)} + \Phi_0|^2$



H-bonding

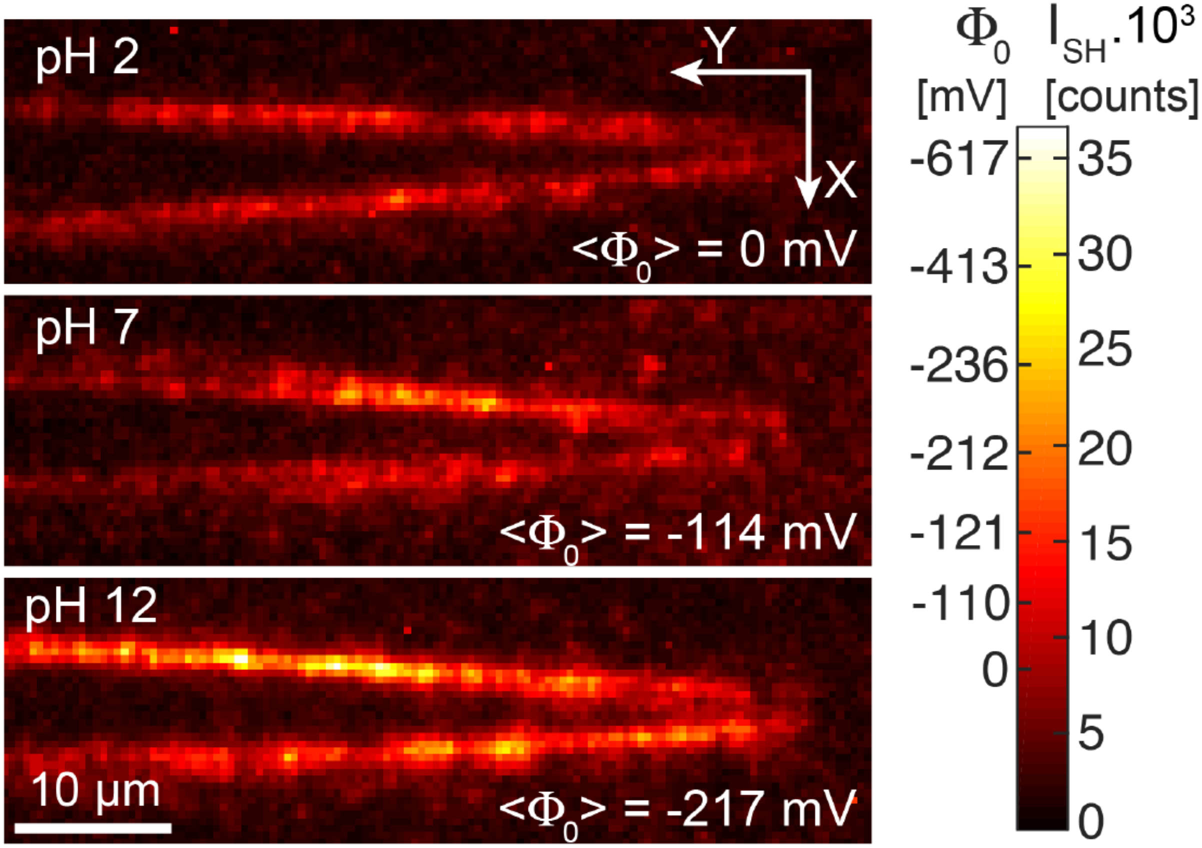
Second Harmonic Microscopy

Second Harmonic Microscopy



Science (2017) 357, 384-388

Electrostatic Potential Maps



NLO

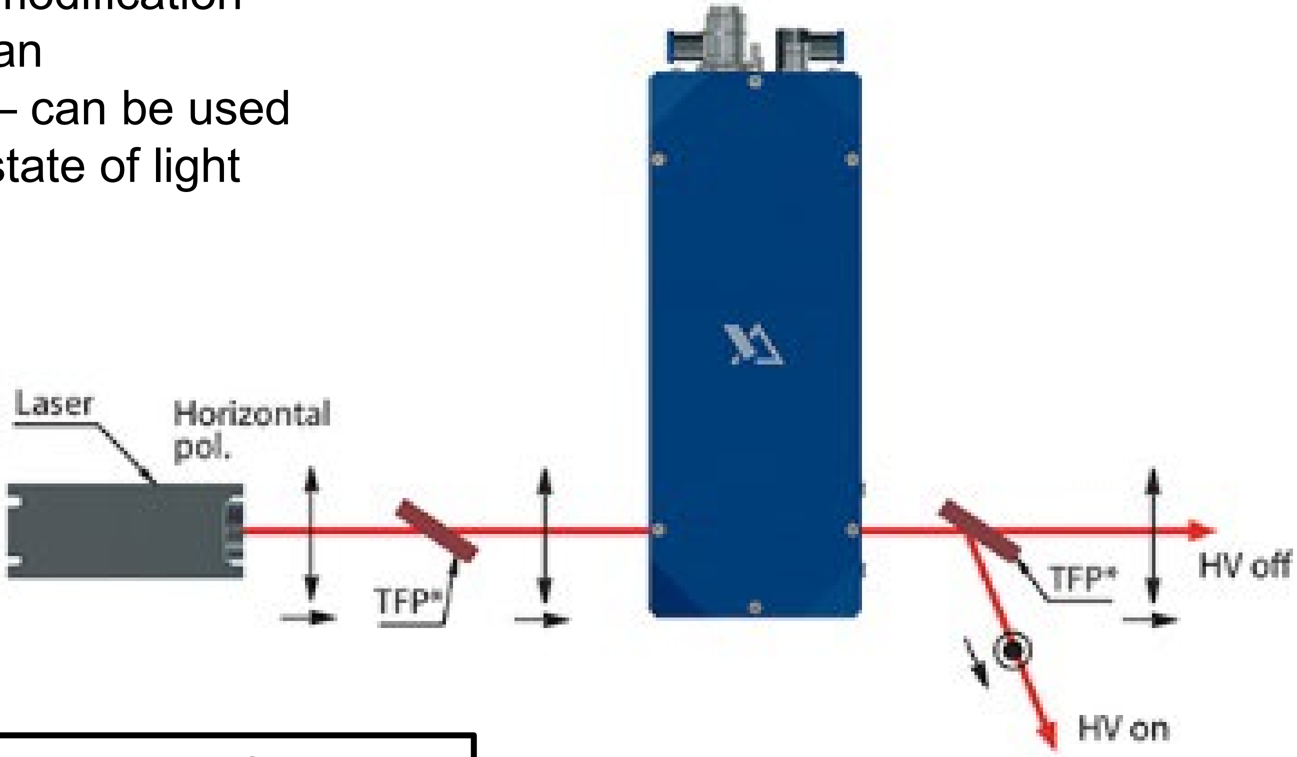
Main Reasons for Using NLO:

- Generation & Amplification of Light
- Manipulation of Light & Matter
- Material Science Tool
- Life Science Tool (e.g. brain imaging)
- … it is fun … ☺

Examples

Optical Switch

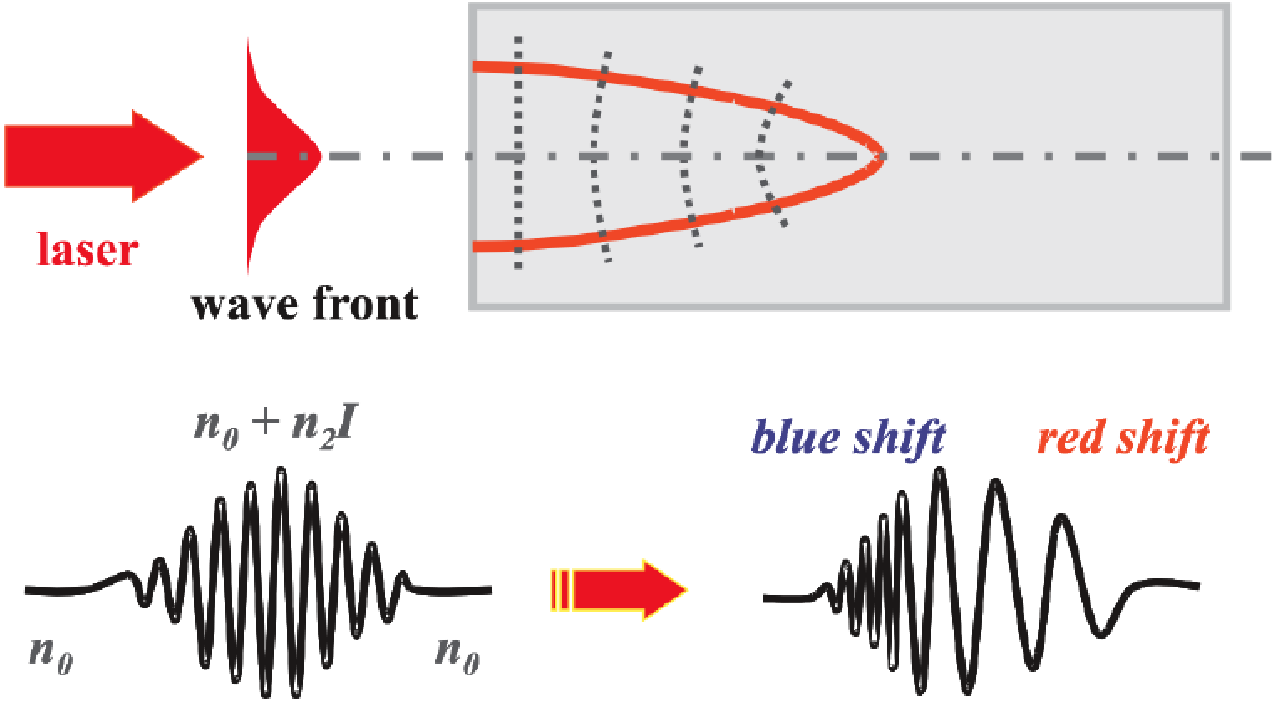
Electro-optic effect – the modification of the refractive index with an electrostatic field (voltage) – can be used to change the polarization state of light



Pulse picker – controllable selection of pulses
Used in laser systems to control the repetition rate or to inject / eject light into a cavity

Processes: Second-order nonlinear optical transitions
Relevant physical property:
Second-order nonlinear optical susceptibility – non-centrosymmetric material:
$\chi_{ijk}^{(2)}(\omega_q; \omega_m, \omega_n)$

Generating Light I



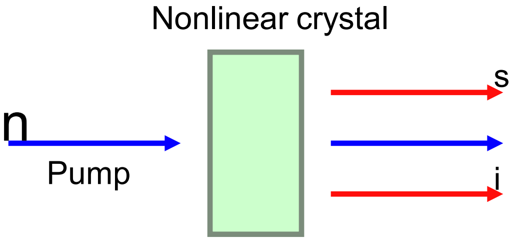
Kerr Lens Mode locking: High intensity light modifies the refractive index of a material that generates apertures and lenses; these can be used to generate femtosecond pulses

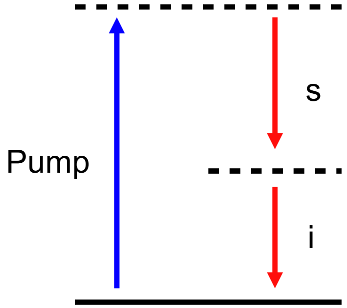
Process: Third-order nonlinear optical transitions
Relevant physical property:
Third-order nonlinear optical susceptibility: $\chi^{(3)}_{ijkl}(\omega_q;\omega_m,\omega_n,\omega_p)$

Generating light II

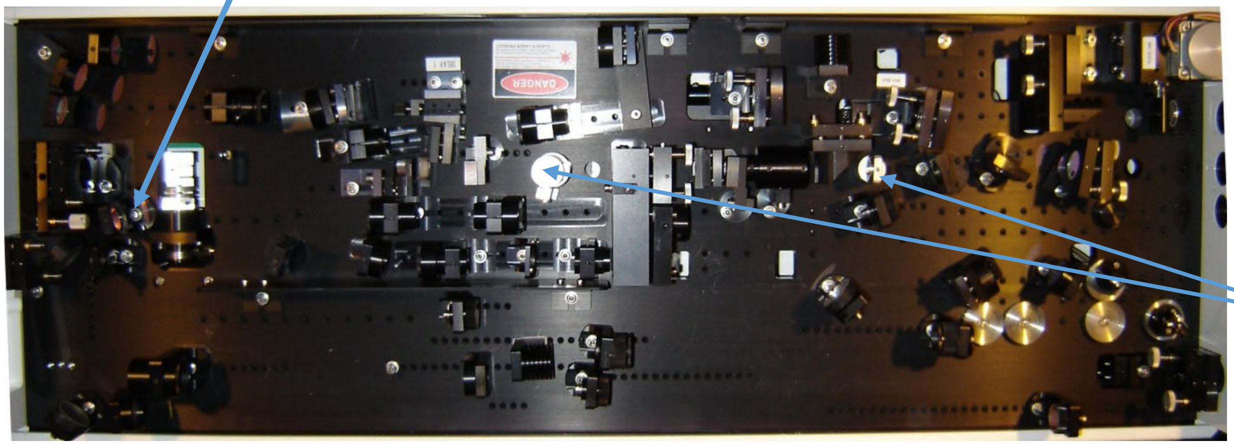
Optical Parametric Generation

(Stochastic parametric down conversion)

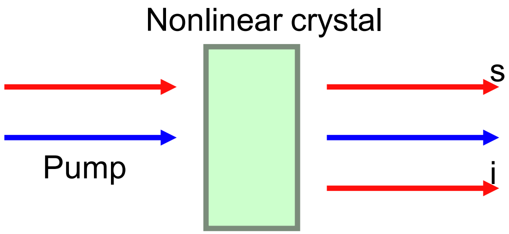




Generate a broad spectrum of pulses

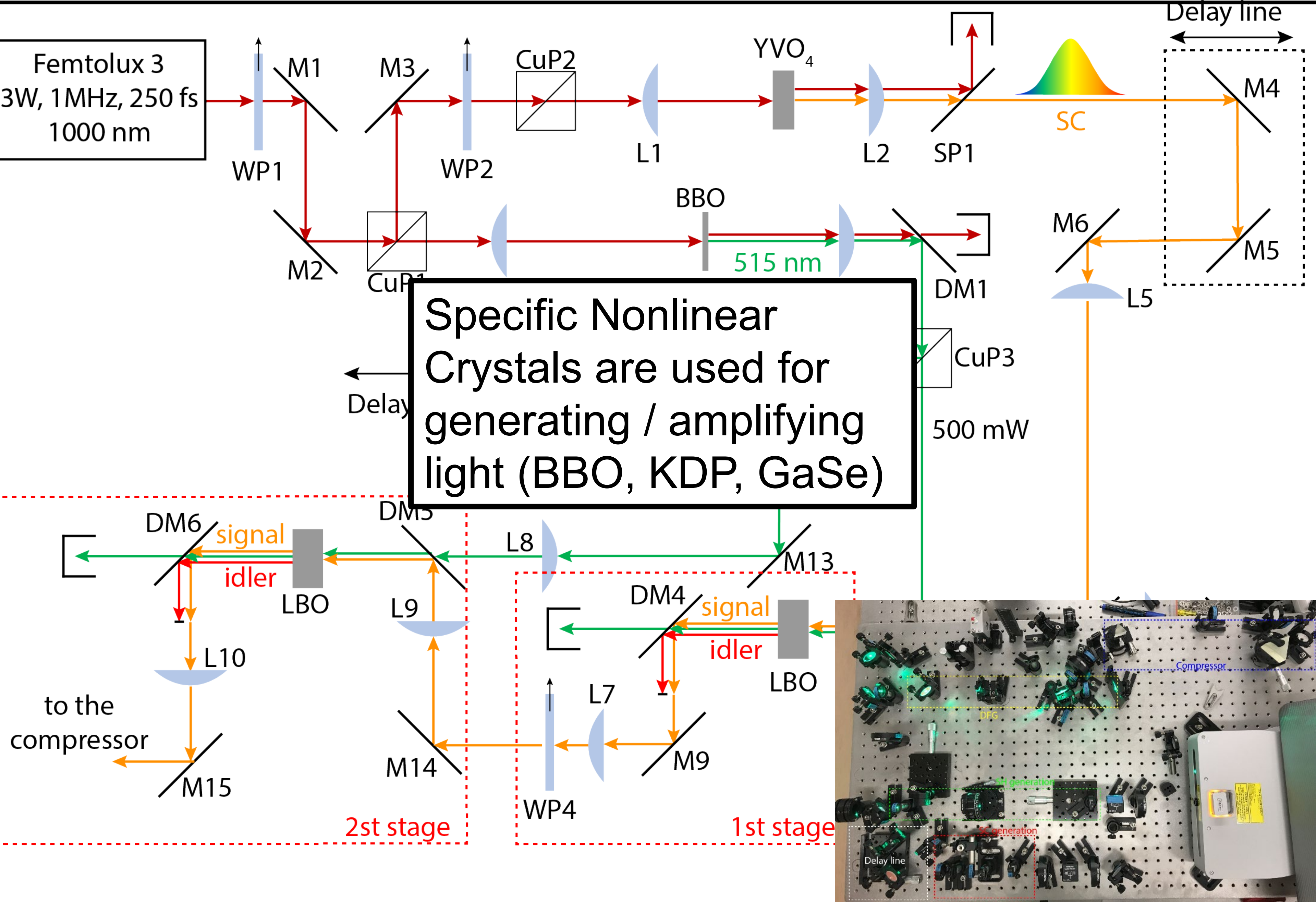


Optical Parametric Amplifier



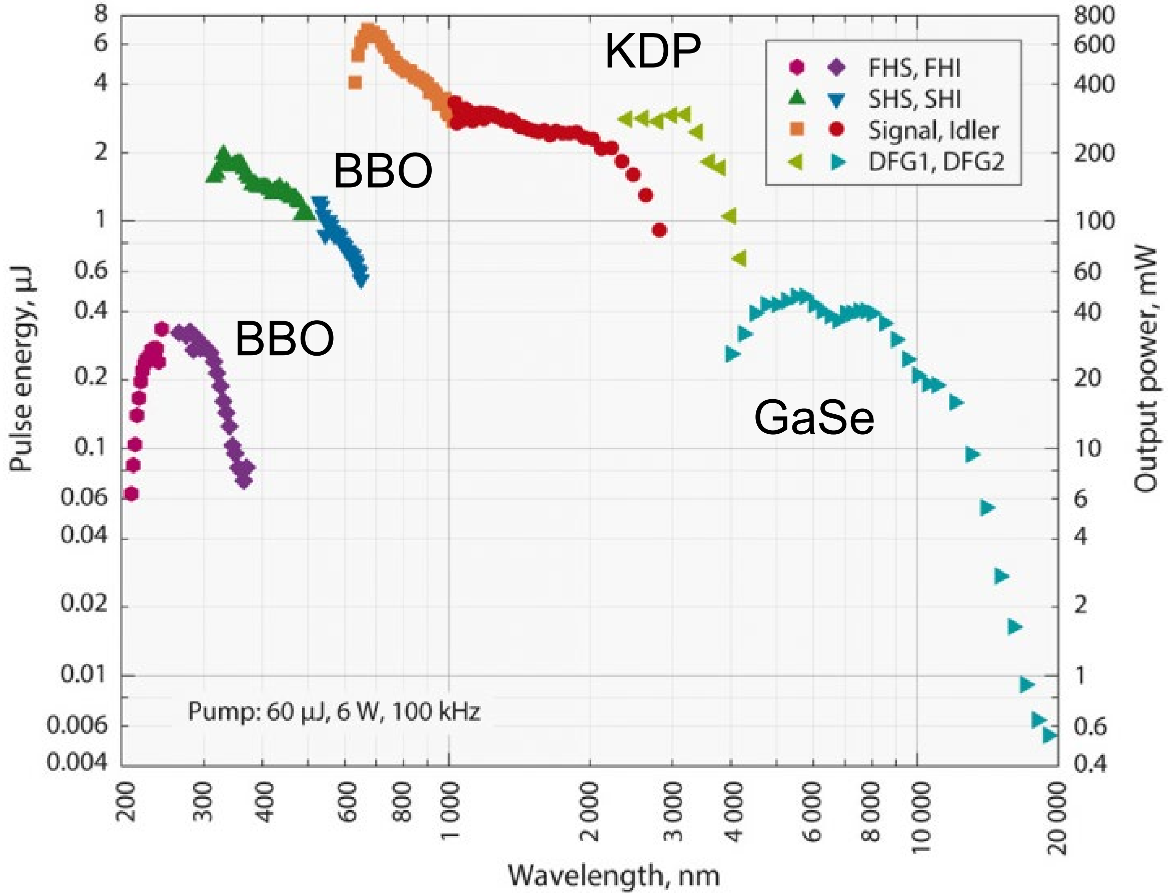
Amplify the low energy light with high intensity pulses

Generating Light II



Specific Nonlinear Crystals are used for generating / amplifying light (BBO, KDP, GaSe)

Generating light II

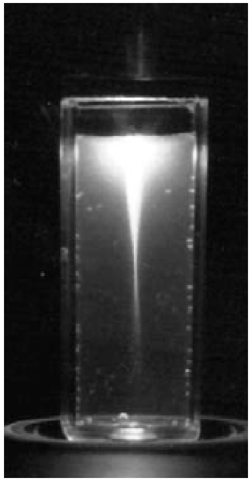


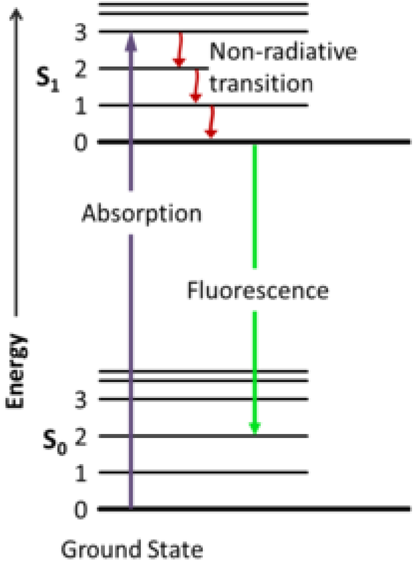
Processes: Second-order nonlinear optical transitions

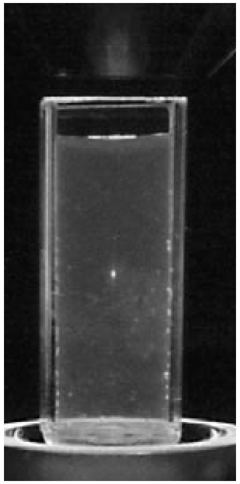
Relevant physical property:

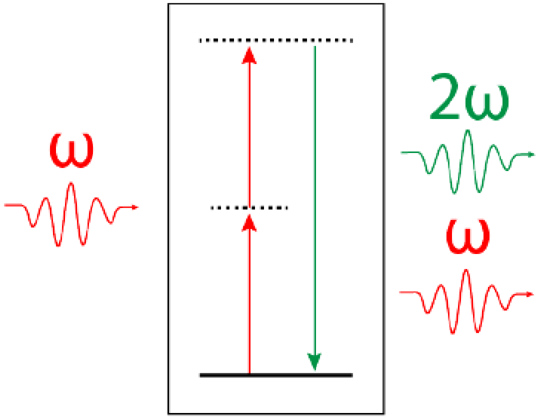
Second-order nonlinear optical susceptibility: $\chi_{ijk}^{(2)}(\omega_q;\omega_m,\omega_n)$

Imaging









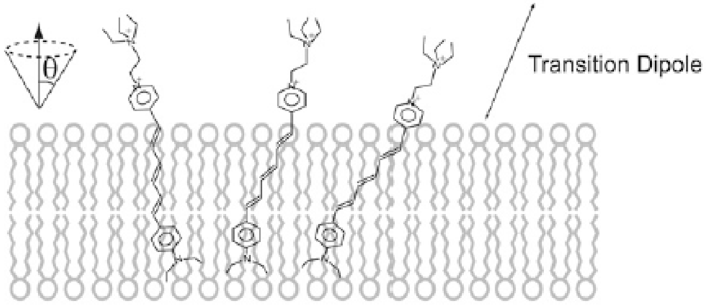
Linear (fluorescence imaging)
Low resolution
Unspecific to material structure

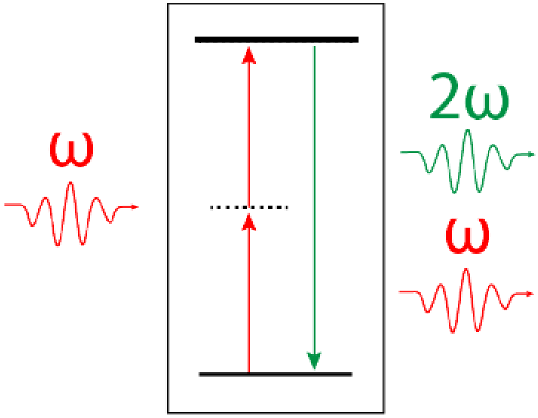
Nonlinear (second harmonic, 2-photon fluorescence imaging)
High resolution
Specific to material structure

Processes: Second-order nonlinear optical transitions
Relevant physical property:
Second-order nonlinear optical susceptibility: $\chi^{(2)}_{ijk}(\omega_q;\omega_m,\omega_n)$

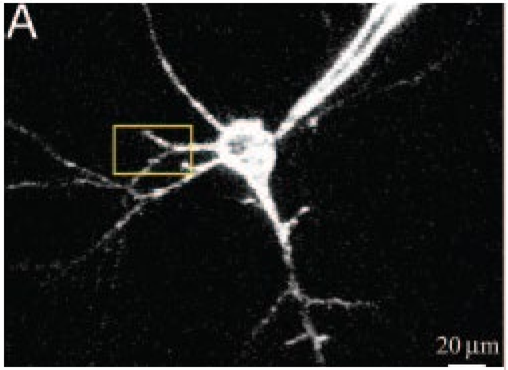
SH Imaging

Voltage sensitive fluorophores can be used in SH imaging

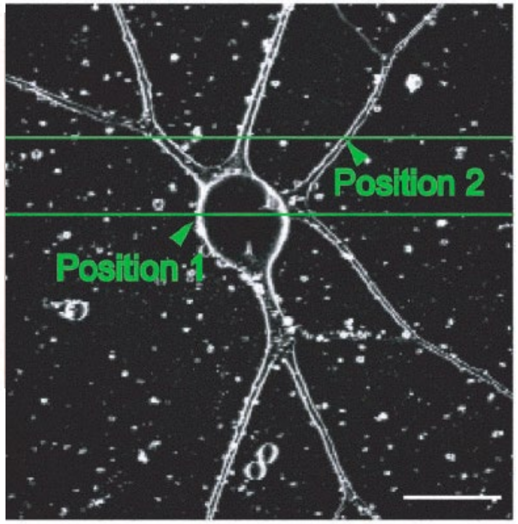


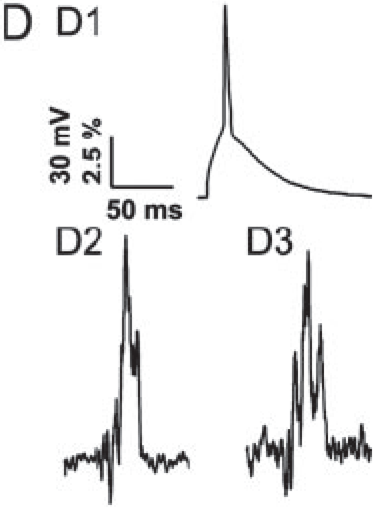


Time and spatially resolved propagation
of action potential – ‘seeing thoughts’



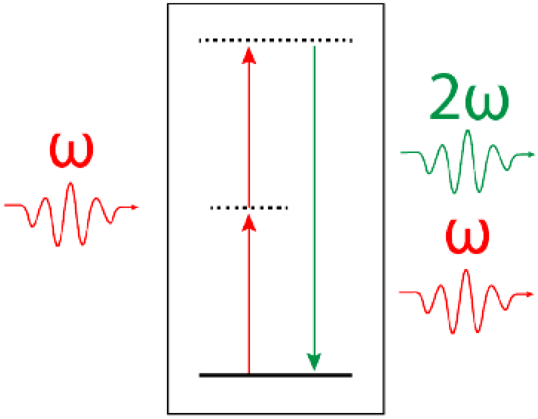
Eisenthal / Yuste: PNAS ***103, 786*** (2006); *PNAS* ***103,***18779, (2006); *Biophys. J., 2007.* ***93: p. L26-8.***

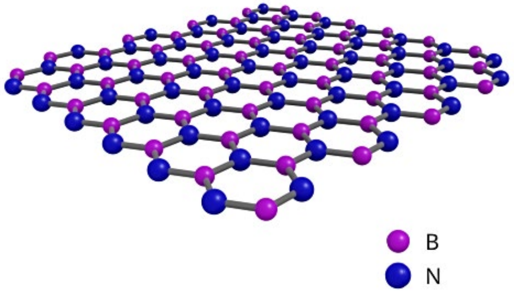


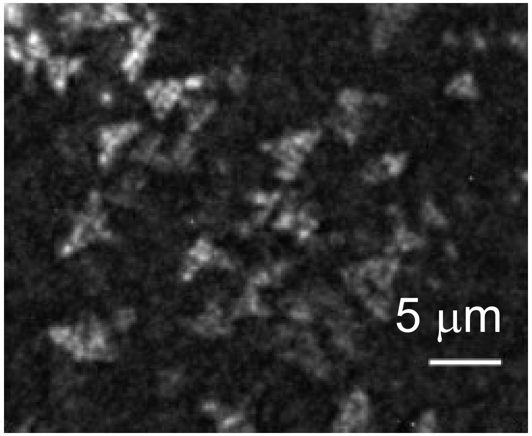


SH Imaging

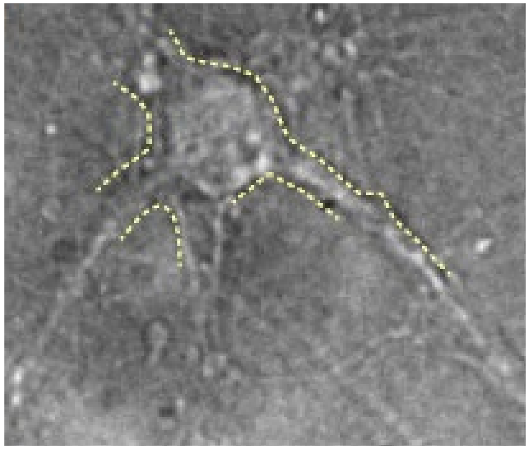
Image non-centrosymmetric structures in materials / cells and tissue to understand how they work / or to detect cancer metastasis / other medical conditions

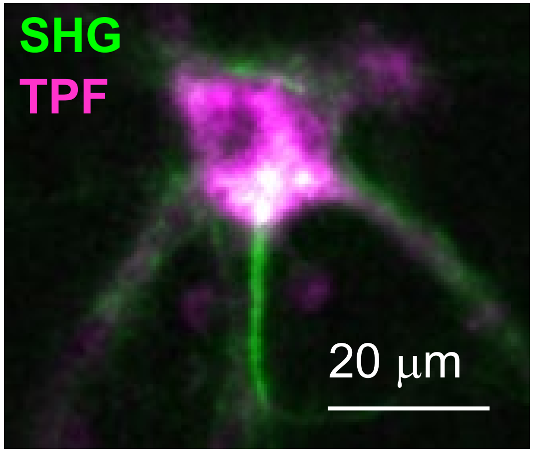


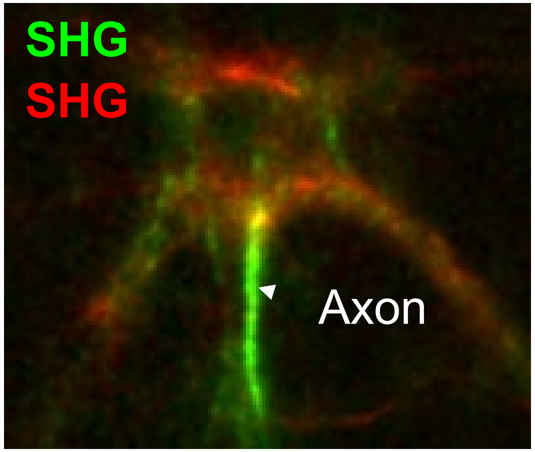


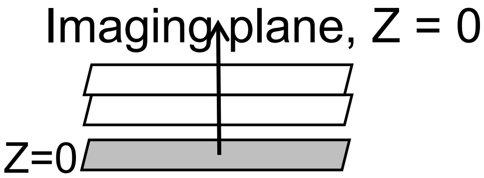


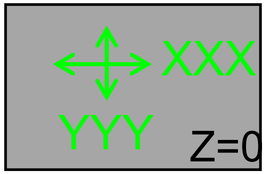
organelles in living neurons

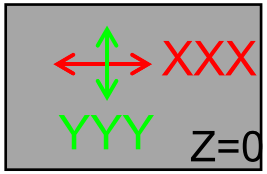












Impact on Society

Impact of nonlinear optics using Nobel Prizes as a metric

2022: Aspect, Clauser, Zeilinger: for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science

2018: Morau, Strickland, Ashkin: chirped pulse amplification – enables generation of high intensity femtosecond pulses & optical tweezers

2014: Hell & Moerner: Imaging (super resolution / STED)

2012: Boyle, Smith and Kao: development of CCD sensor & fibers for optical communication

2005: Hansch, Hall, Glauber: Laser based precision spectroscopy

1997: Zewail: ultrafast spectroscopy on gasses

1981: Bloembergen: development of laser based spectroscopy

1964: Prokorov & Townes: development of the laser & principle behind it
….

The NLO course

Procedural remarks

Course Procedure

Follow the course
Do exercises

Grade:
Based on answers to an exam that will be given after the course – it is an ***open book exam –*** *we will have books in the exam room, no notes*

Exam date: TBD

Teaching Assistants: Zhi Li & Nelson Correia

EXAM

Grading is based upon an exam in end of this semester.

At approximately lecture 7/8 there will be a possibility to do a voluntary intermediate test. The result of this test can be used for the exam.

As follows:
A question / or multiple smaller questions will be asked during the test.

The result of test can be used to replace the result of the question that has the worst score during the exam.

Course Material

MOODLE:
https://moodle.epfl.ch/course/view.php?id=16006

Own notes
Slides
Exercises
R. W. Boyd, Nonlinear Optics, Ed. 3

Course Material

Topics	Material
1. Introduction / overview of nonlinear optical phenomena	Chapter 1.1-1.6
2. Wave description of nonlinear optical processes	Chapter 2.1-2.5
3. The intensity dependence of the refractive index	Chapter 4.1-4.3, part 4.4-4.5
4. Electro-optic and photorefractive effects	Chapter 11.1 – 11.3
5. Optically induced damage	Chapter 12
	+ slides & notes on board

4 ECTS

Location: CM1121 Wed 12.15 – 4.00 PM

Topics & Learning Objectives

Provide the foundations to understand the physics behind nonlinear optical processes.

- Start from basic principles and build up concepts needed to describe NLO interactions (**E**, **D**, **P**, $\chi^{(n)}$)
- Use / understanding of tensors
- What is **polarization** and how can we define **the response** of a material to a set of intense beams / pulses?
- How do material properties (energy levels; spatial structure) relate to the emitted field / NLO processes?
- Given a simple model of an atom, how does a NLO response arise?

Topics & Learning Objectives

- For the simplest of geometries, how can we describe the generation of new frequencies starting from the equations that relate waves to matter (Maxwells' Equations, ME) ?
- What is phase matching and how can it be used?
- How can an intense pulse modify the optical and electrical properties of a material? Are there different mechanisms?
- Under which conditions do these effects lead to optical damage?
- Connect the fundamental processes with applications

Topics & Learning Objectives

At the end of the course you will be able to:

- Read nonlinear optical papers without skipping the eqs

- Understand the foundation of how devices work that use NLO interactions

- Perform analysis needed to calculate NLO effects (includes deeper insights in math and physics)

- Have knowledge of the few most widely used processes

Note that this course aims for **the foundations** and not provide a corollary of applications.

Relevant BSc background

Analysis / Linear Algebra:
Euclidean vector space, partial derivatives, differential, Jacobian, Hessian, Taylor expansion, gradient, chain rule, implicit function theorem, Lagrange multipliers, multiple integrals, ordinary differential equation, Matrices, Vectors, differential operators, surface and curvelinear integrations, Fourier Analysis,…

Materials: from chemistry to properties

- Atomistic aspects, chemical bonds, periodic table of elements
- Structure of materials
- Electrical properties: energy bands; dielectrics, origin of polarization, relative dielectric constant and capacitance.
- Optical properties: link between energy and wavelength of a photon.
- Optical properties of metals, ceramics and polymers in terms of reflection, transmission and absorption. Refractive indices and Beer-Lambert law

Relevant BSc background

Physics III / IV
Electromagnetism
- Maxwell's equations
- Electrostatics
- Polarization and magnetization of matter
- Radiation
- Propagation of waves
- Electromagnetic waves
- Reflection, refraction, interference, and diffraction

Optical Engineering:
Geometric optics
Wave optics, interference
Maxwell optics and polarization
Fourier optics and diffraction
Waveguides, optical fibers and cavities
Photons and light-matter interaction
Lasers and photodetectors

MT Master Courses NLO connects with

Imaging Optics:

Introduction to 0ptical imaging systems such as camera objectives and microscopes. Discussion of imaging formation. Principles of design of imaging optics with geometrical optics and analysis with raytracing. Presentation of different applications in photography and microscopy.

Detection Optics:

Students analyse the fundamental characteristics of optical detectors. Thermal and photoemissive devices as well as photodiodes and infrared sensors are studied. CCD and CMOS cameras are analysed in detail. Single photon detection is explained.

Lecture 1

Motivation
Course Coordinates
Course Procedure
Course Material
Learning Objectives
Connection to other MT courses
Exam / Grading

Topic 1: Introduction & the nonlinear optical susceptibility

Lecture 1+2: The NLO susceptibility

Materials & Energy
Light
Light & Matter
Concept of polarization
Scalars, vectors, matrices, tensors
Time domain definition of P
Definition of material response
The NLO susceptibility – a start
Molecular vs. Macroscopic polarization (units)

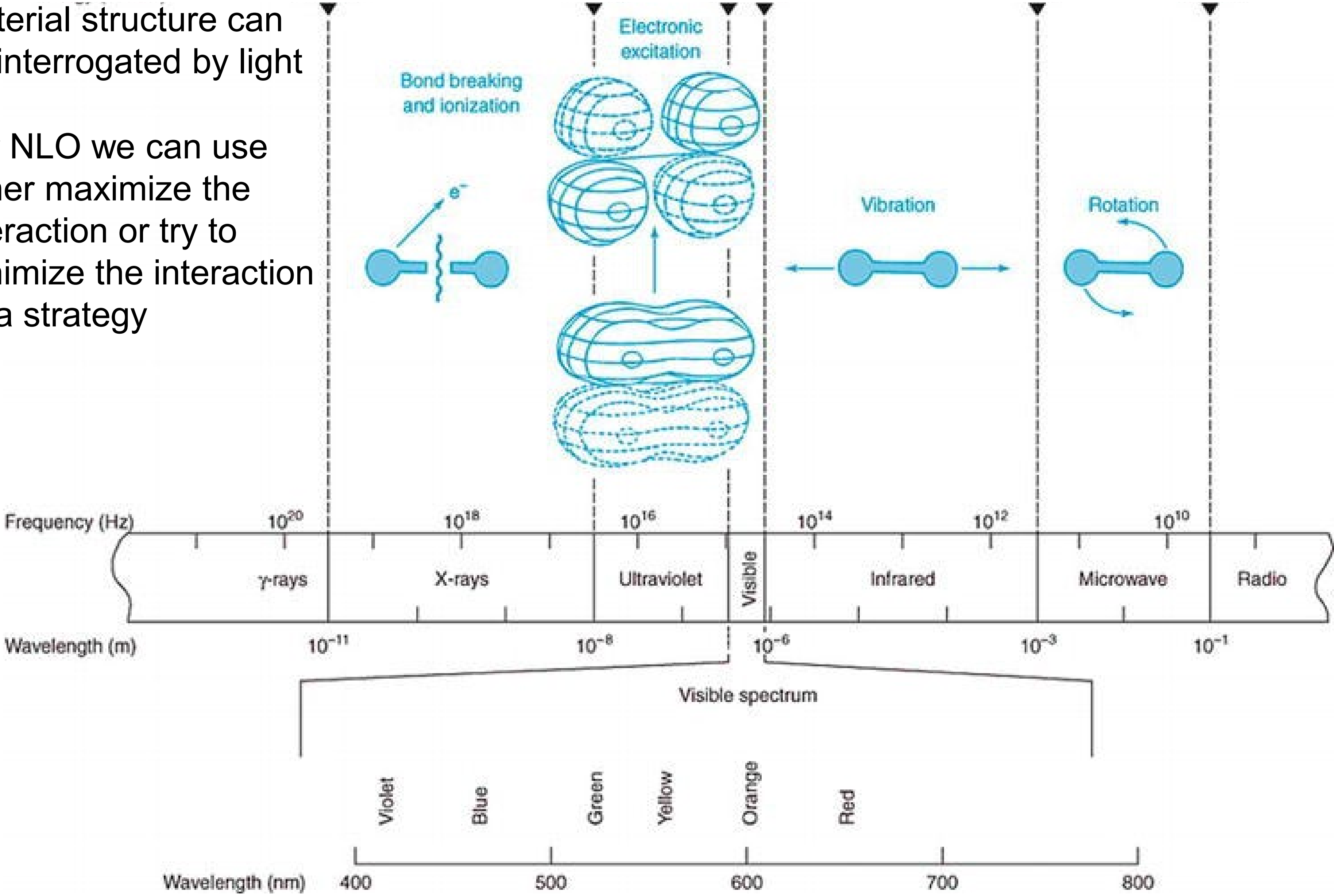
Material: CH 1.1 -1.3
Exercises lecture 1 + 2: Additional exercise 0; exercise 2,3,1 (in that order)

Materials & Energy

General Structure

Material structure can be interrogated by light

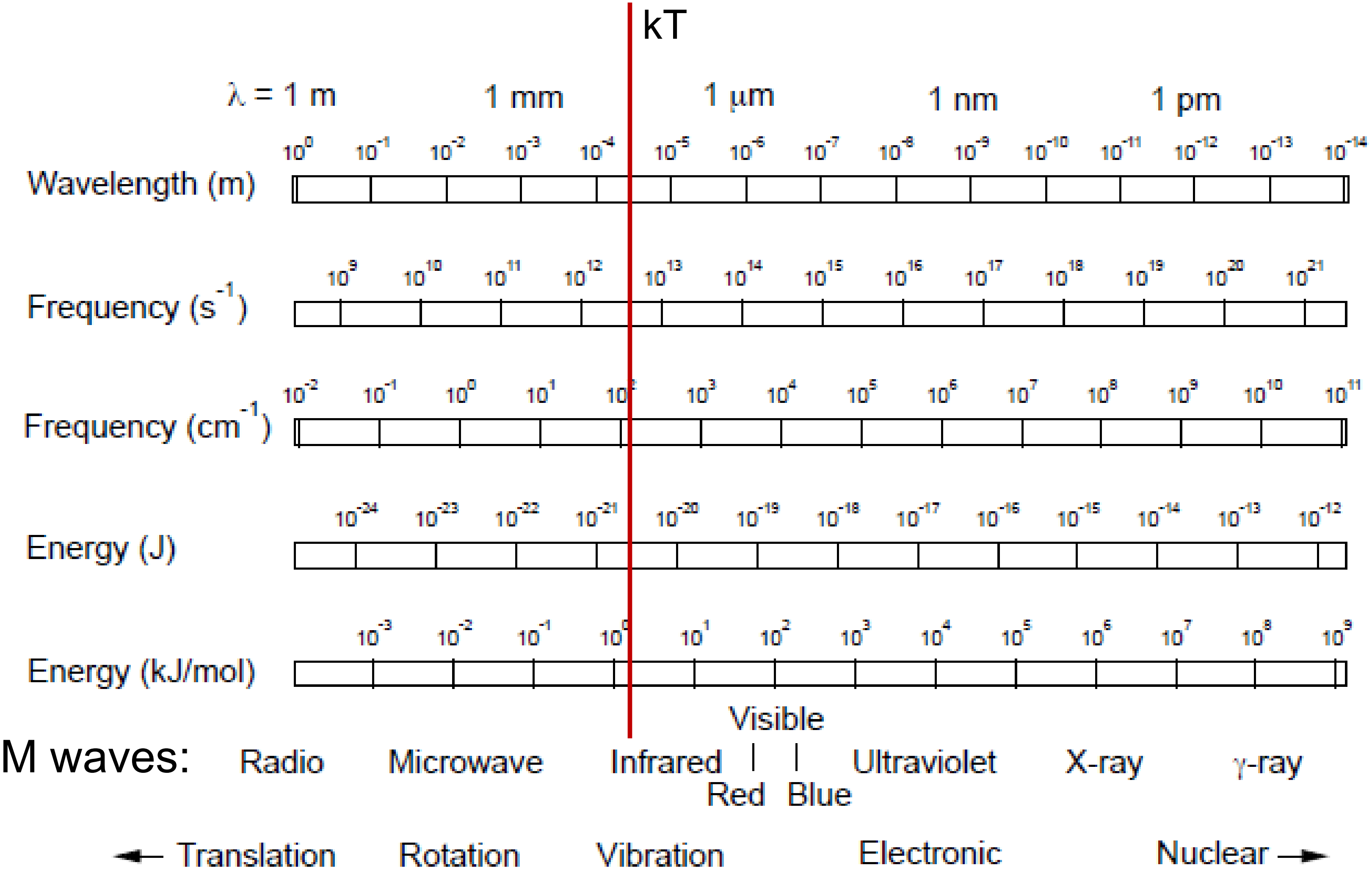
For NLO we can use either maximize the interaction or try to minimize the interaction as a strategy



Energy & Units

At room temperature kT= 4.11 10^{-21} J; 0.025 eV; 208 cm^{-1}

Per mole: 2.47 kJ/mol

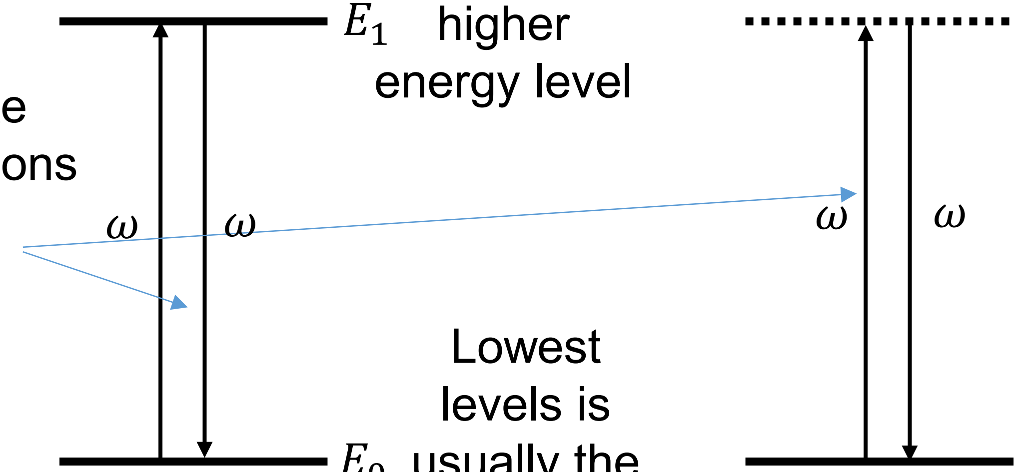


E-M waves:

Type of energy levels

Energy Level Diagrams

Materials are represented by their energy levels



Arrows indicate photon transitions (absorption & emission)

higher energy level

Dotted lines are virtual energy states: there is no occupied energy level here

Lowest levels is usually the Ground State (G. S)

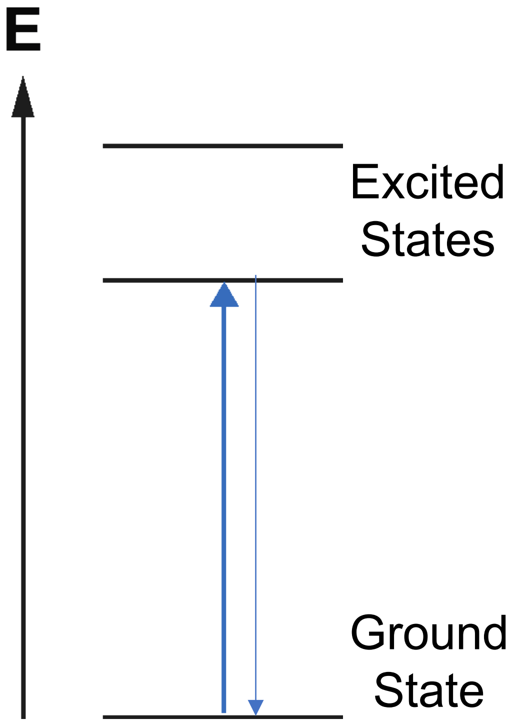
Solid lines are energy states (wave functions / orbitals / bands)

Photon energy $(h\omega)$
matches the energy level difference:
resonant interaction

$$E_1 - E_0 = h\omega$$

Transitions: Electronic

UV-Vis Light

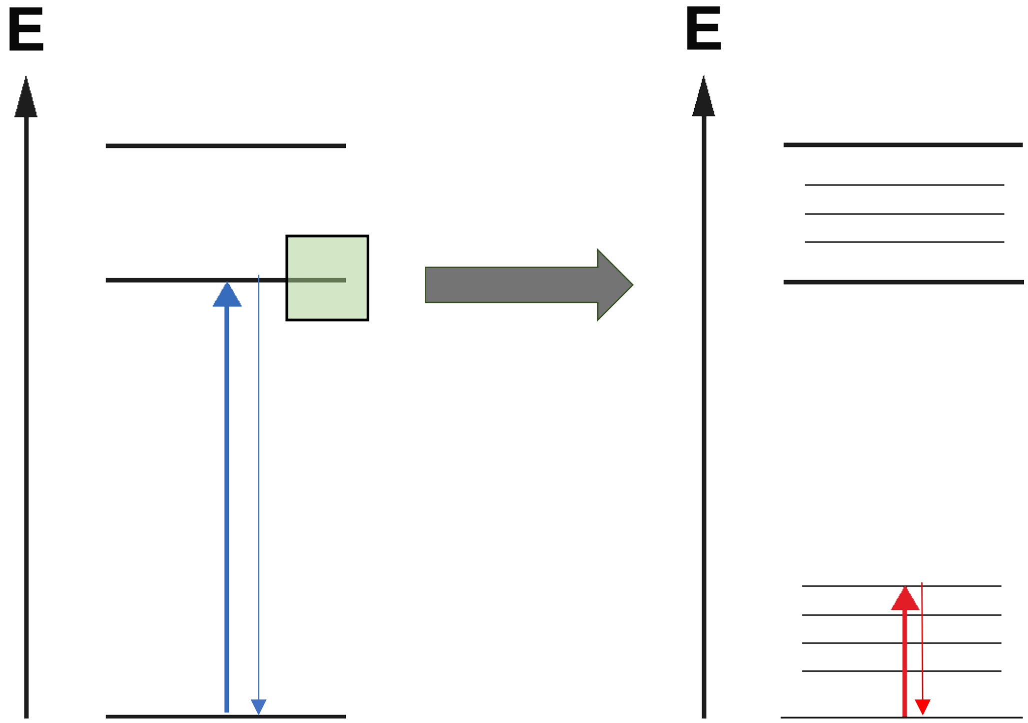


Quantized levels
Store E
as orbital motion

Transitions: Vibrational

UV-Vis Light

IR Light

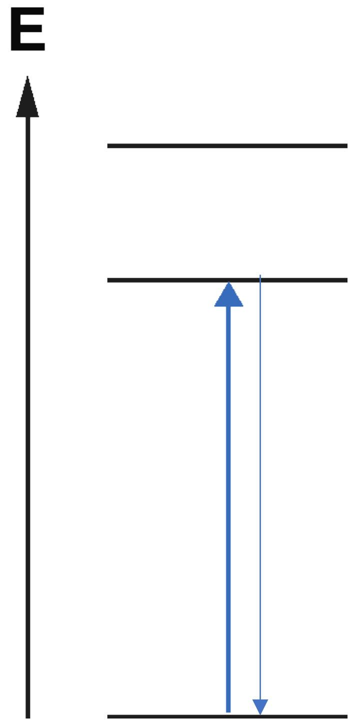


Quantized levels
Store E
as orbital motion

Quantized levels
Store E as
vibrational motion

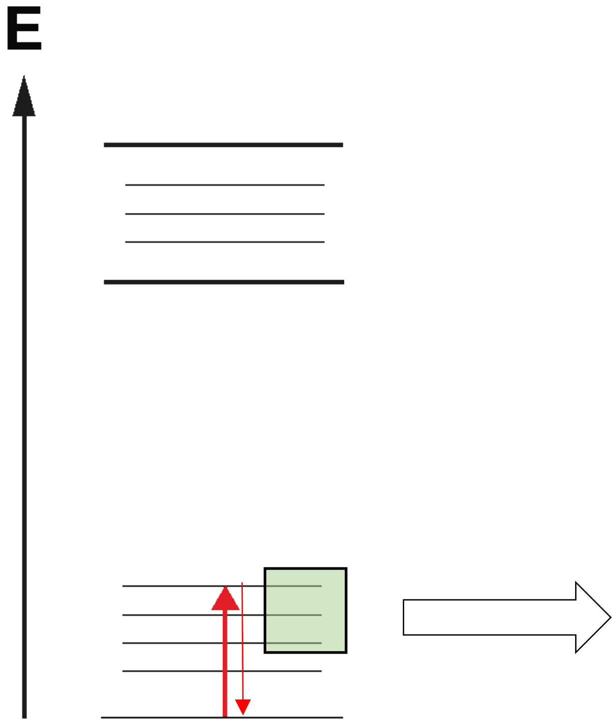
Transitions: Rotational

UV-Vis Light



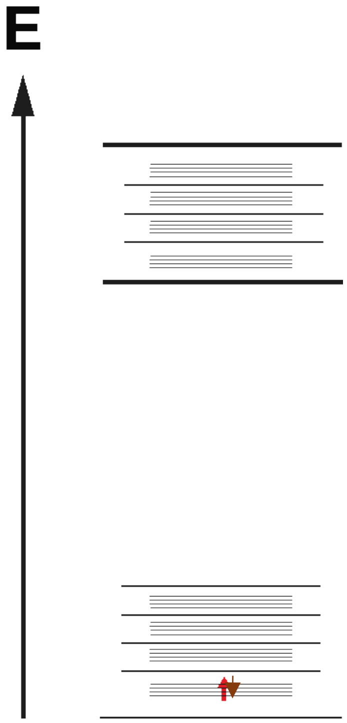
Quantized levels
Store E
as orbital motion

IR Light



Quantized levels
Store E as
vibrational motion

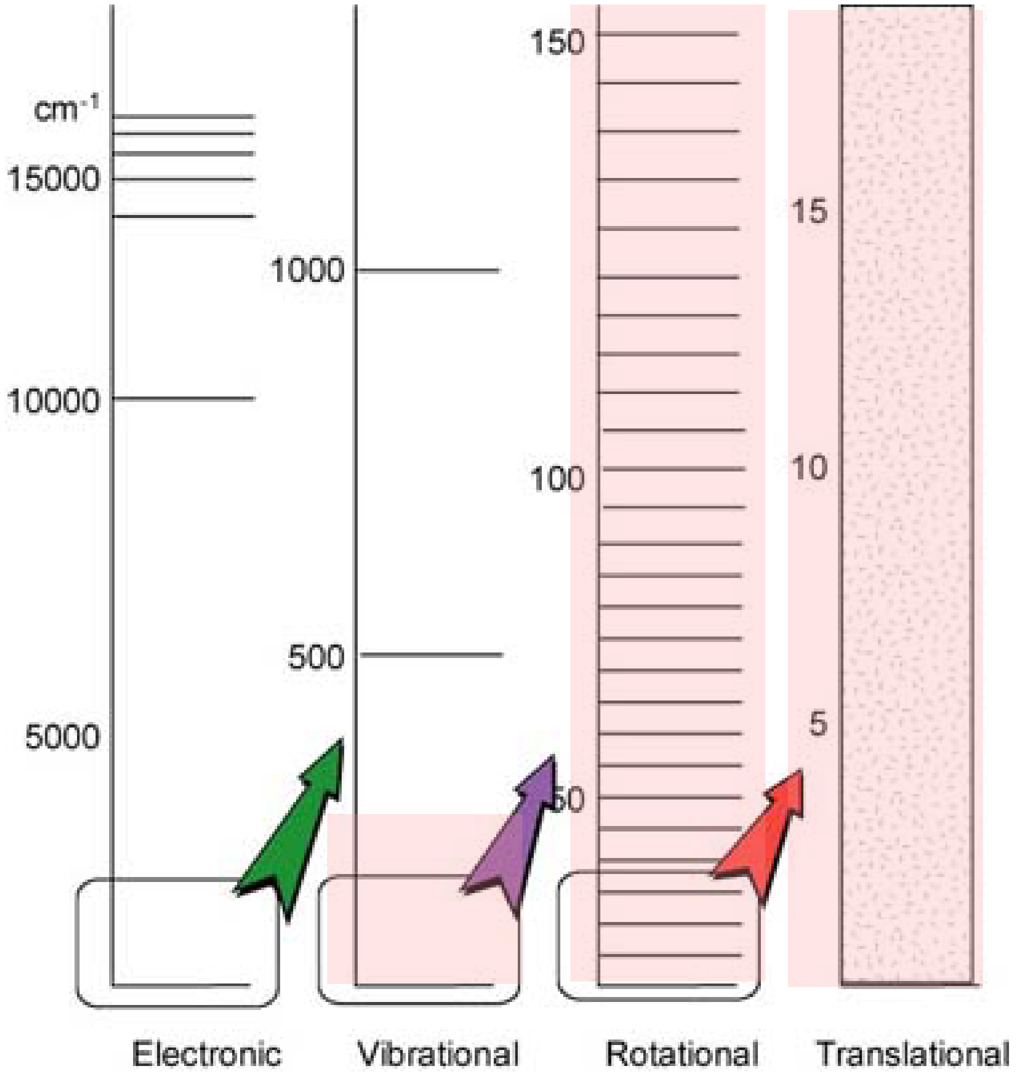
Microwave



Quantized levels
Store E as
rotational motion

Energies & States

There are different types of transitions that represent different energies



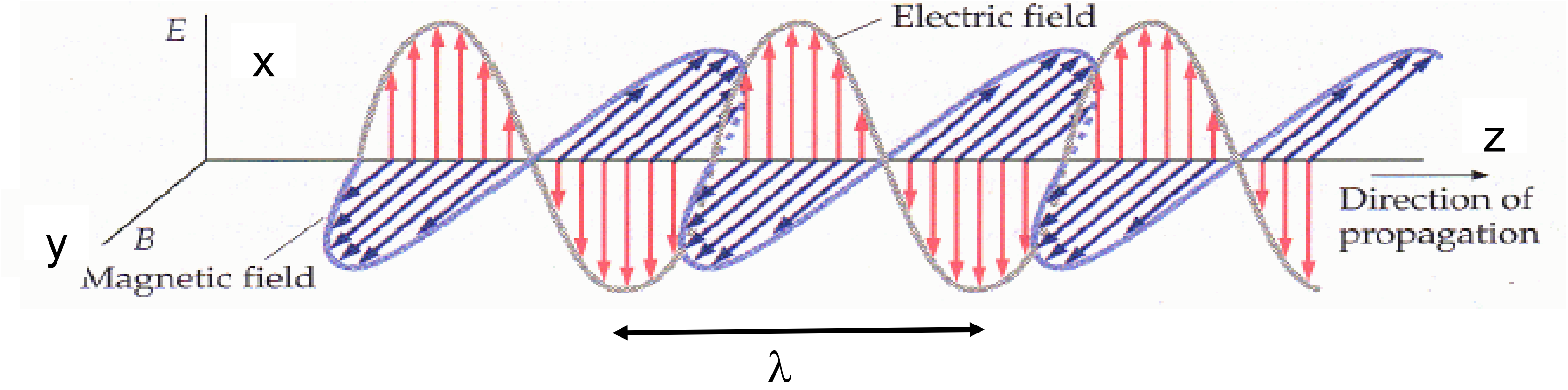
- Smaller energies probe finer structure.
- Core electrons report on atomic species.
- Valence electrons report on bonds
- Vibrational modes report on specific molecular groups or 3D structure

Thermal energy at room temperature:
1 kT = 208 cm^{-1} at 293 K

Light

Light in a medium

Linear polarized light:



Light is an oscillating electric and magnetic field:

Traveling wave: $$\mathbf{E} = \mathbf{E_0} e^{-i(\mathbf{kz}-\omega t)}$$

Amplitude
& Direction of oscillation
(polarization)

Frequency

Wave vector (here: $\boldsymbol{k} = k\boldsymbol{z}$) = direction of propagation

$$k = \frac{cn(\omega)}{\lambda}$$

→ refractive index of material, depends on frequency

→ wavelength

More properties of (pulsed) light

Traveling plane wave, vector: $\boldsymbol{E} = \boldsymbol{E_0} e^{-i(\boldsymbol{kz}-\omega t)}$

Field with multiple frequencies
(a pulsed beam) $\boldsymbol{E}(\boldsymbol{r}, t) = \sum_n \boldsymbol{E}_n(\boldsymbol{r}, t) = \sum_n \boldsymbol{E}_n(\boldsymbol{r}) e^{-i\omega_n t}$

Intensity: $I = \frac{cn\varepsilon_0}{2} |E|^2$ [J/m²s] =Power / area

Fluence: $F = \frac{E_{pulse}}{A}$ [J/m²]

Momentum of a photon: $h\boldsymbol{k} = hk\widehat{\boldsymbol{k}}$

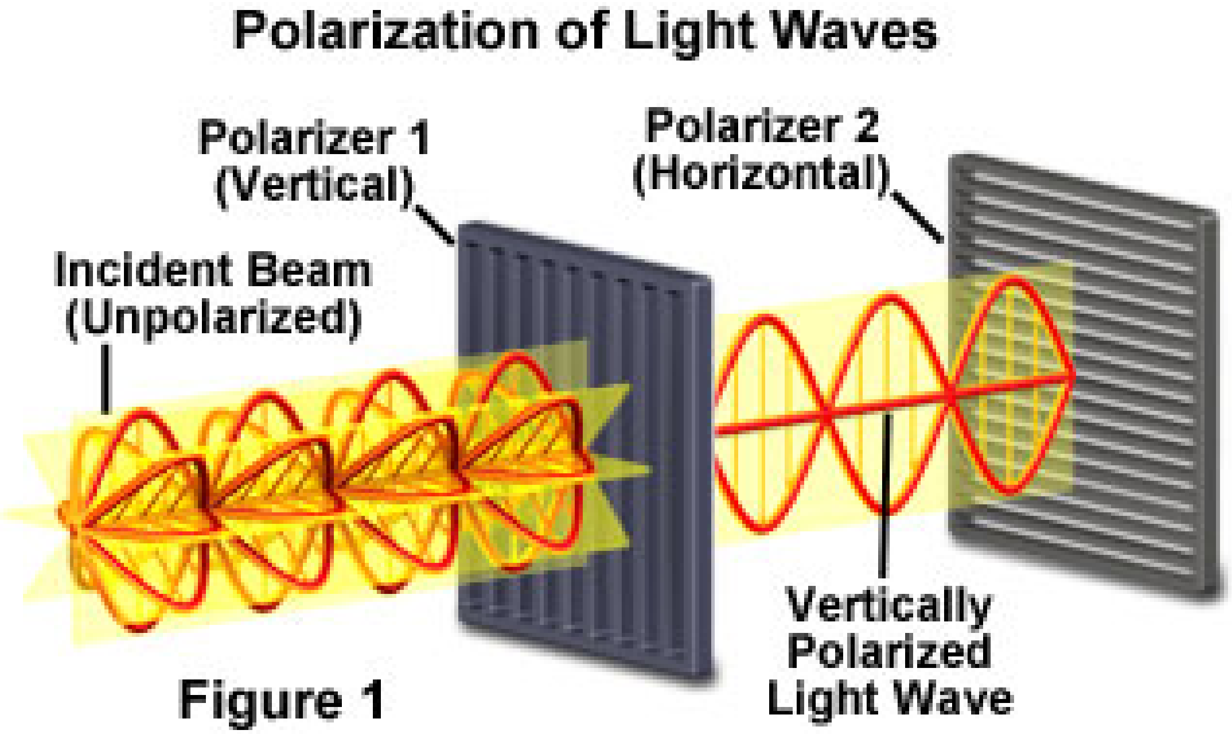
Energy of one photon: $h\frac{c}{\lambda} = h\upsilon = h2\pi\omega$

Light

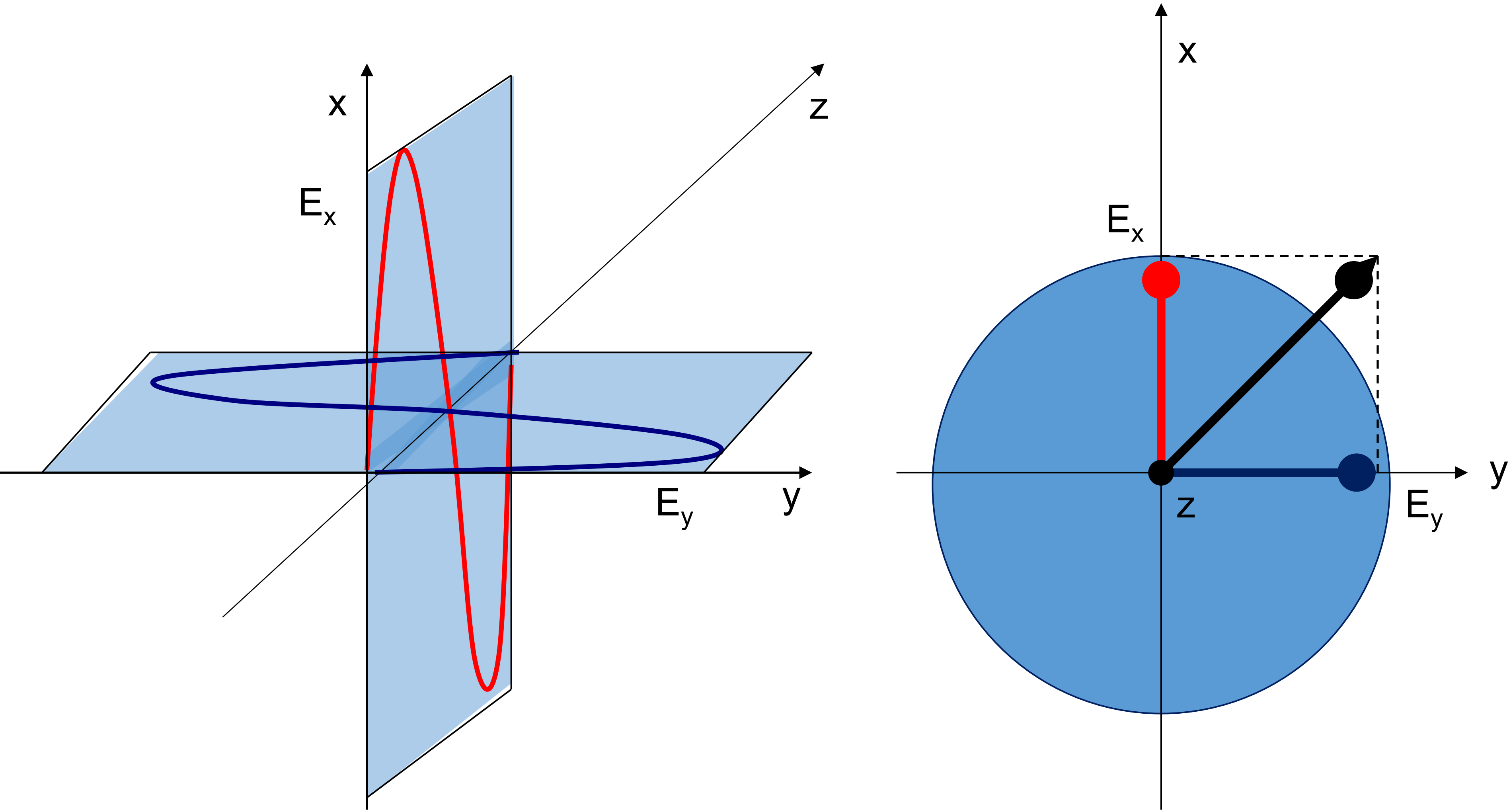
The electromagnetic field oscillates in a particular direction:

This is the polarization direction of the light ($\mathbf{E_0}$=$\mathrm{E}_0\mathbf{u}$, magnitude and direction)

The polarization direction can be modified, light can be linearly, circularly or elliptically polarized… We will mostly use linearly polarized light in this course

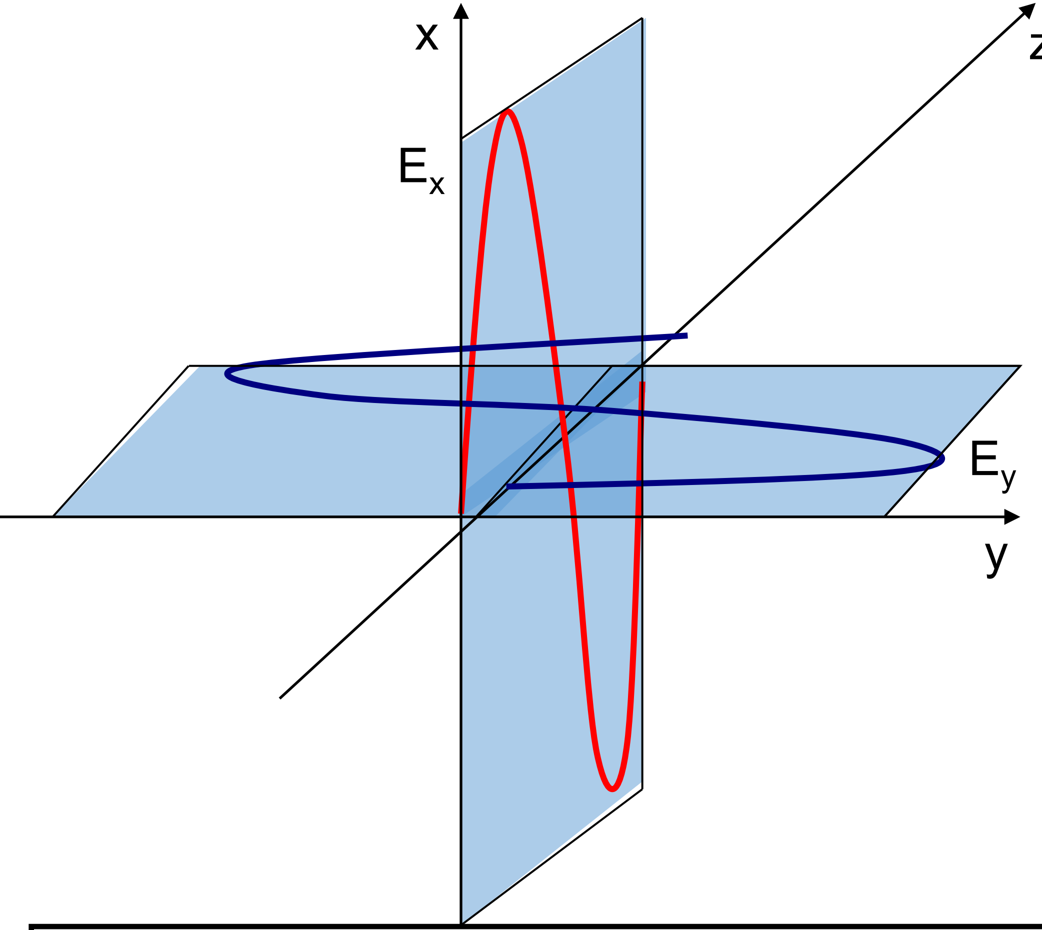


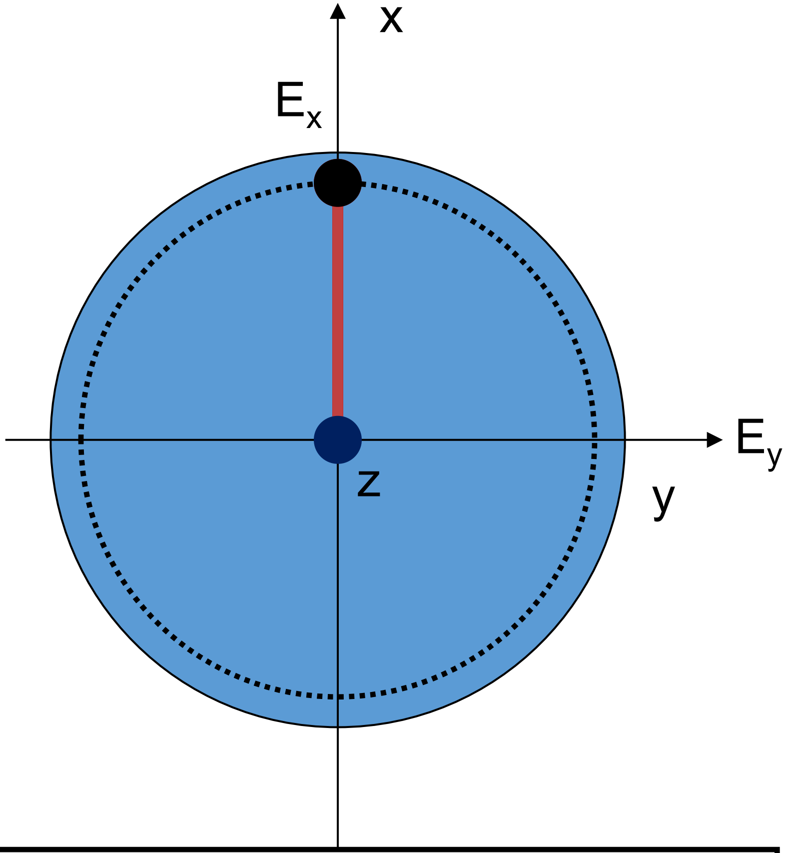
Linear Polarized Light



The electric field of linearly polarized light ($E_x + E_y$) consists of two perpendicular, equal in amplitude, linear components (E_x and E_y) that have no phase difference.

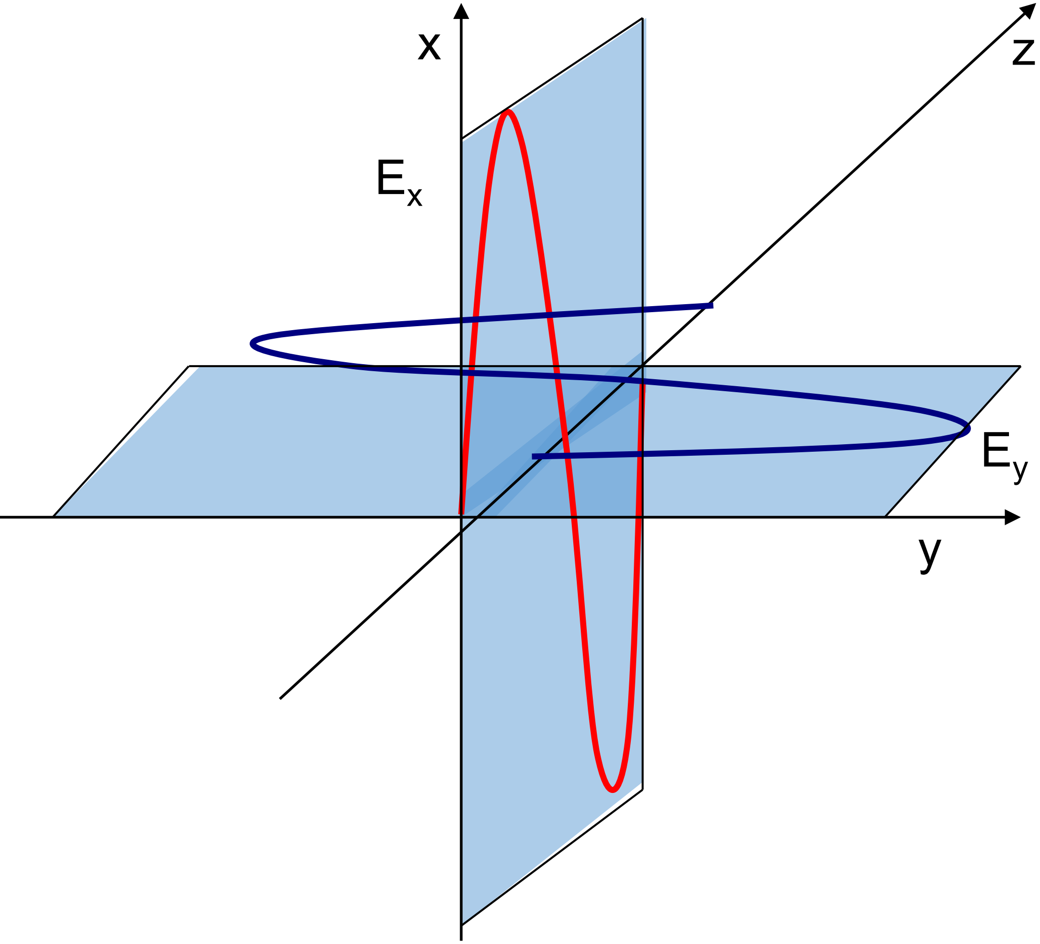
Circular Polarized Light

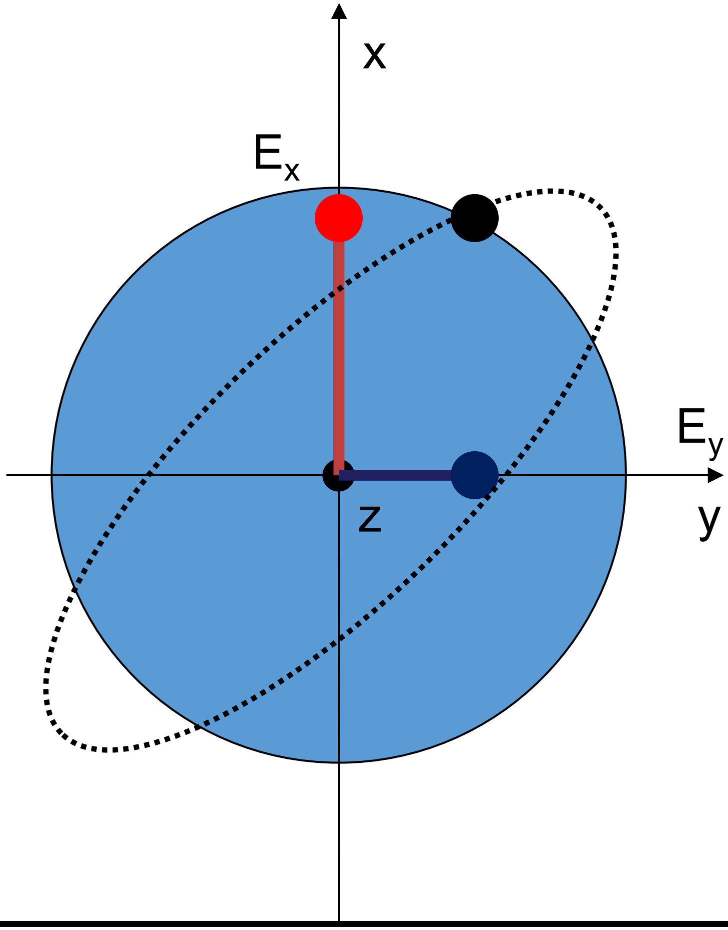




The electric field of circular polarized light (E_x+ E_y) consists of two linear components (E_x, E_y) that are perpendicular to each other, equal in amplitude, but have a phase difference of π/2. The resulting electric field rotates in a circle around the direction of propagation. The rotation direction, can be left- or right-hand circularly polarized light.

Elliptically Polarized Light





The electric field of elliptically polarized light (E_x+ E_y) consists of two linear components (E_x, E_y) which are either of equal amplitude with a phase difference other than π/2 (90°) OR they consist of two linear components (E_x, E_y) which have a different amplitude and a phase difference of π/2 (90°).

Polarized Light

Transverse electro-magnetic wave, electric field component:

$\boldsymbol{E} = \boldsymbol{E_0} e^{-i(kz-\omega t)}$ with $\boldsymbol{z} \cdot \boldsymbol{E}_0 = 0$

$\boldsymbol{B} = \sqrt{\epsilon\mu}\, \boldsymbol{z} \times \boldsymbol{E}$

Plane of oscillation (polarization) is perpendicular to propagation direction; **E** and **B** are orthogonal

In general we have for the polarization, written as a vector in (x,y,z) coordinates:

$\boldsymbol{E}_0 = (E_x, E_y e^{i\varphi}, 0)$ Elliptically polarized light; fields in x and y direction propagate with a different phase & amplitude

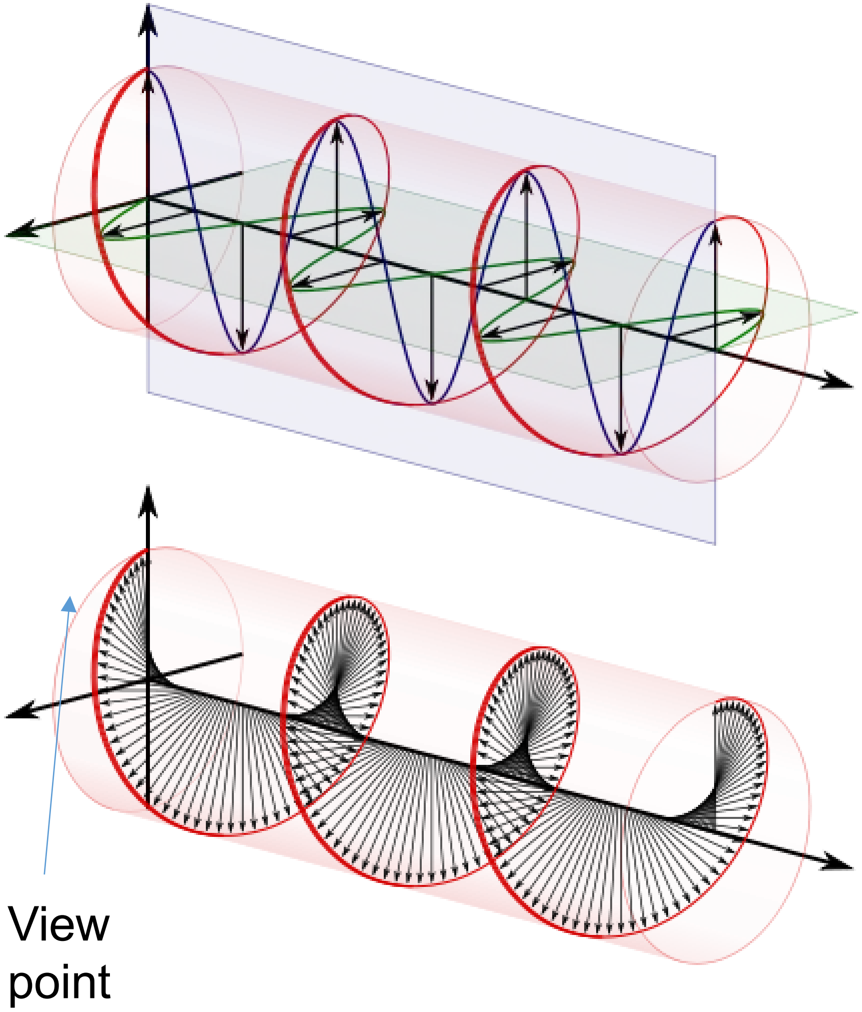
$\boldsymbol{E}_0 = (E_x, E_x e^{i\varphi}, 0)$ Elliptically polarized light; fields in x and y direction propagate with a different phase but equal amplitude

$\boldsymbol{E}_0 = (E_x, E_x e^{i\frac{\pi}{2}}, 0)$ Circular polarized light; fields in x and y direction propagate with a 90^o phase difference (with equal amplitude)

$\boldsymbol{E}_0 = (E_x, E_y, 0)$ Linear polarized light; fields in x and y direction propagate with a 0^o phase difference

Polarized Light

Elliptical and circular polarized can rotate either clockwise or counterclockwise as viewed along the direction of propagation



$$\boldsymbol{E} = \boldsymbol{E}_0 e^{-i(\boldsymbol{k}\boldsymbol{z}-\omega t)}$$

Clockwise rotation = right handed polarized light (graphs)

$$\boldsymbol{E}_0 = (E_x, E_x e^{i\frac{\pi}{2}}, 0)$$

$$\boldsymbol{E}_0 = (E_x, E_x i, 0)$$

$$\boldsymbol{E}_0 = E_x\,(\hat{x} + i\hat{y})$$

Counter-clockwise rotation = left handed polarized light

$$\boldsymbol{E}_0 = (E_x, E_x e^{-i\frac{\pi}{2}}, 0)$$

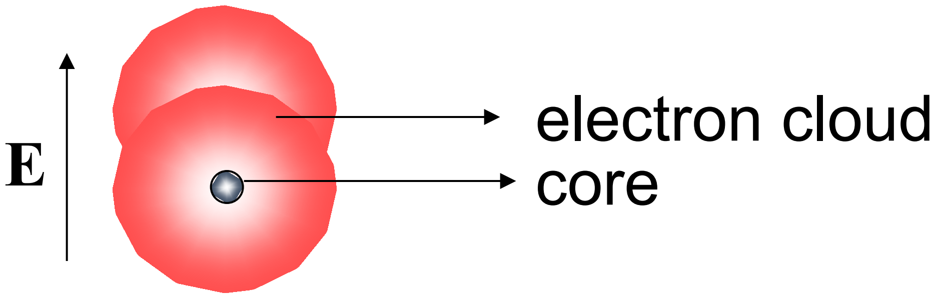
$$\boldsymbol{E}_0 = (E_x, -E_x i, 0)$$

$$\boldsymbol{E}_0 = E_x\,(\hat{x} - i\hat{y})$$

Light & Matter

Light in Matter

Interaction of electric field (**E**) with medium:



Medium consist of charged electrons and nuclei

- **E** field moves electron could – small displacement: harmonic oscillator (mas on spring)
- Oscillating charges emit electric field (which emits magnetic field)
- Original and generated e-m fields interfere

$\mathbf{P} = \varepsilon_0 \chi^{(1)}\mathbf{E}$ is material contribution to displacement field in medium: $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$

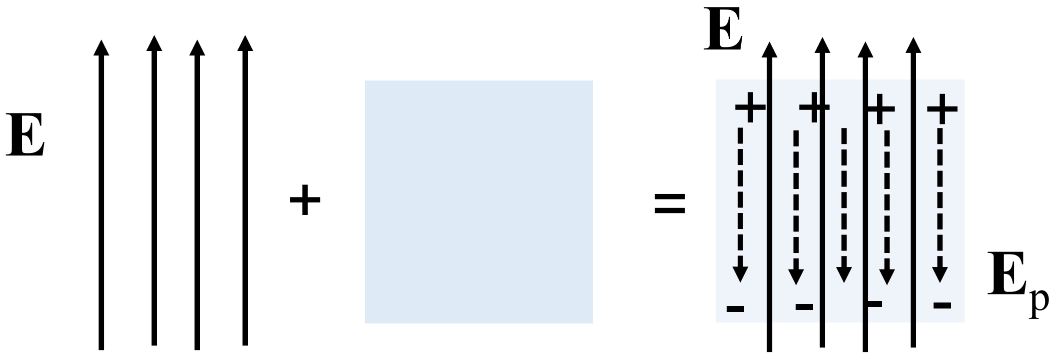
$\varepsilon = 1 + \chi^{(1)}$, ε is the dielectric permittivity

$n^2 = 1 + \chi^{(1)}$, n is the refractive index

Fields in a medium

Polarizability is the ability of a medium to respond to and reduce the magnitude of an external electromagnetic field (**E**).

Interaction of electric field (**E**) with medium:



Medium consist of charged electrons and nuclei which will be displaced by external **E** field

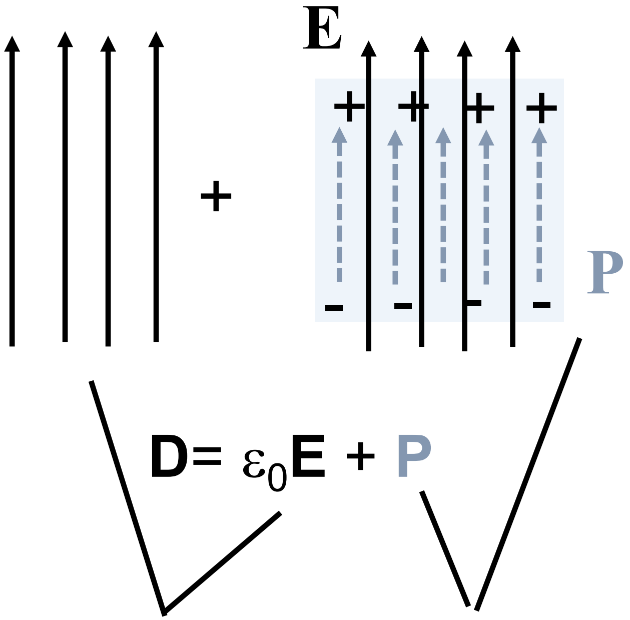
E field distorts atoms & molecules: separates/displaces charges & induce dipoles

These charge displacements create a response field ($\mathbf{E}_p$)

These two field interfere: $\mathbf{E}_{eff}=\mathbf{E}+\mathbf{E}_p$

Linear Macroscopic Polarizability (**P**)

Polarizability (**P**) is the field that is present in the medium



Displacement field (**D**) in a medium =

+

D is a field, but it is *not* an electric field. Unit: [C/m^2]. It is a charge density redistribution [(Cm)/m^3]

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

Displacement field in vacuum

Polarization (of the medium)

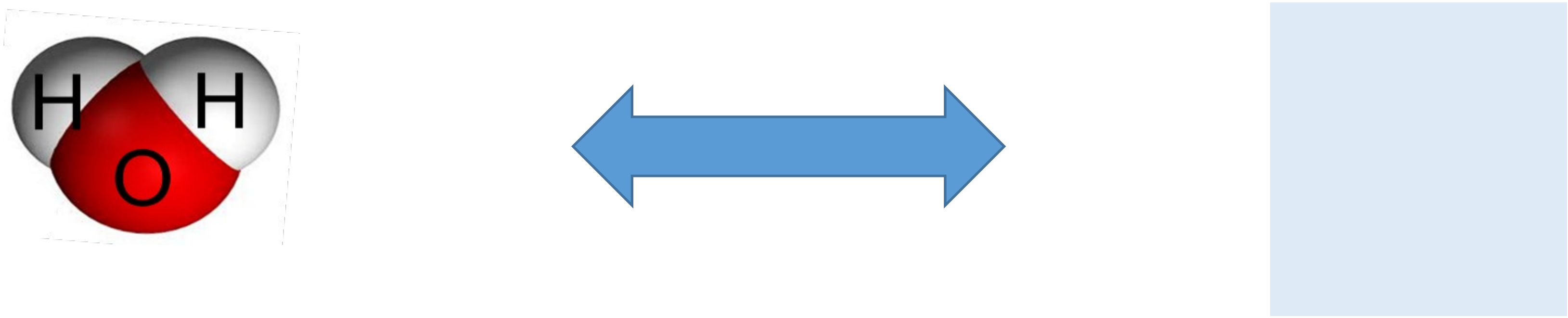
$\mathbf{P} = \varepsilon_0 \chi^{(1)} \mathbf{E}$ is the materials' contribution to **D** in medium:

$\varepsilon = 1 + \chi^{(1)}$, ε is the dielectric permittivity, frequency dependent

$n^2 = 1 + \chi^{(1)}$, n is the refractive index

A few more remarks about dipoles, quadrupoles, …. And units

Molecular vs Macroscopic polarization



Molecular polarization:
u or **p*** [Cm] (permanent dipole)

Molecular induced dipole:
$\mathbf{u}_{ind}$ or $\mathbf{p}_{ind}$

p: used for optics/spectroscopy

Macroscopic polarization **P**

$$\boldsymbol{P} = \sum_{all\ molecules} \boldsymbol{p}_i = N\langle \boldsymbol{p} \rangle$$

Number of molecules per unit volume
contributing to the polarization

$$\chi^{(1)} = \frac{N}{\varepsilon_0} \langle \alpha \rangle_{orientations}$$

Polarization can be induced by:
1 - Separating charge within a molecule (with or without a charge or dipole). This is electronically induced polarization.
2 – Rotating dipoles. This is dipolar induced polarization.

Higher Order Macroscopic Polarizability (**P**)

Polarizability (**P**) is the field that is present in the medium

P = $\varepsilon_0\, \chi^{(1)}$**E** is material contribution to **D** in medium

But at high electromagnetic field strength the electron displacement is no longer very small and there can also be higher order terms in E:

$$\boldsymbol{P} = \boldsymbol{P}^{(1)}(\boldsymbol{E}) + \boldsymbol{P}^{(2)}(\boldsymbol{E}^2) + \boldsymbol{P}^{(3)}(\boldsymbol{E}^3) + \ldots$$

With terms:

$$P_i^{(1)}(\omega) = \epsilon_0 \chi_{ij}^{(1)}(\omega;\omega) E_j(\omega)$$

$$P_i^{(2)}(2\omega) = \epsilon_0 \chi_{ijk}^{(2)}(2\omega;\omega,\omega) E_j(\omega) E_k(\omega) \qquad 2\omega = \omega + \omega$$

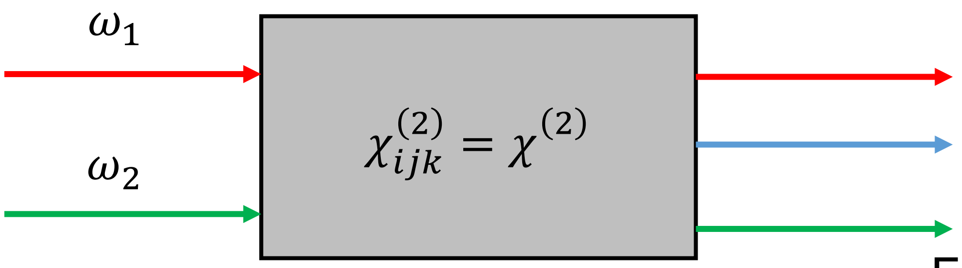
$$P_i^{(3)}(3\omega) = \epsilon_0 \chi_{ijkl}^{(3)}(3\omega;\omega,\omega,\omega) E_j(\omega) E_k(\omega) E_l(\omega) \qquad 3\omega = \omega + \omega + \omega$$

photons out   photons in

These are examples of different frequencies that can be generated; these are called second harmonic and third harmonic generation.
Any other combination is also possible (sum frequency generation)

SFG* and SHG*

Sum Frequency Generation:



ω_1

ω_2

$\chi_{ijk}^{(2)} = \chi^{(2)}$

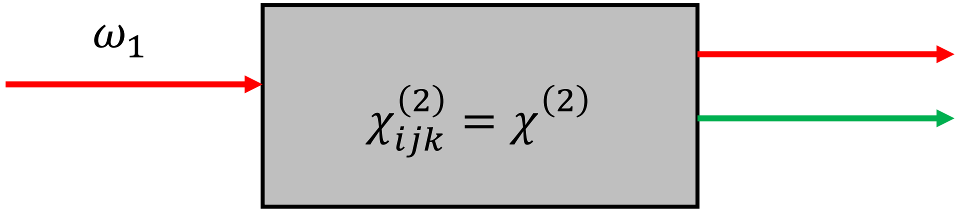
$\omega_0 = \omega_1 + \omega_2$

From Maxwell Equations:

$$P_i^{(2)}(\omega_0) = \epsilon_0\, \chi_{ijk}^{(2)}(\omega_0;\omega_1,\omega_2)\, E_j(\omega_1)E_k(\omega_2) \qquad I_i(\omega_0) \sim \left|P_i^{(2)}(\omega_0)\right|^2$$

Direction of E-field (polarization: x,y,z direction)

Second Harmonic Generation:



ω_1

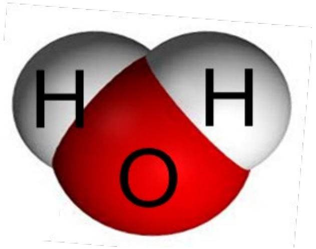
$\chi_{ijk}^{(2)} = \chi^{(2)}$

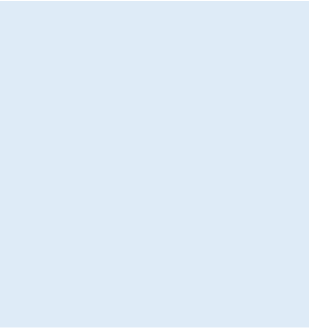
$\omega_2 = 2\omega_1$

$$P_i^{(2)}(\omega_2) = \epsilon_0\, \chi_{ijk}^{(2)}(\omega_2;\omega_1,\omega_1)\, E_j(\omega_1)E_k(\omega_1) \qquad I_i(\omega_0) \sim \left|P_i^{(2)}(\omega_0)\right|^2$$

*Summation convention: double indices should be summed over

Molecular vs Macroscopic polarization





Molecular polarization:
μ or **p** [Cm] (permanent dipole)

μ: used quantum mechanically;
p: used for optics/spectroscopy

Macroscopic polarization **P**

$$\boldsymbol{P} = \sum_{all\ molecules} \boldsymbol{p}_i = N\langle \boldsymbol{p} \rangle$$

Number of molecules per unit surface contributing to the polarization

For second order processes:

$$p_a^{(2)} = \beta_{abc}^{(2)} E_b E_c$$

$$P_i^{(2)}(2\omega) = \epsilon_0 \chi_{ijk}^{(2)} E_j(\omega) E_k(\omega)$$

$$\boldsymbol{\chi}_s^{(2)} = \frac{N}{\epsilon_0} \langle \boldsymbol{\beta}^{(2)} \rangle$$

All molecular orientations

Molecular coordinates

Macroscopic material coordinates

Coupling between light and matter

The Maxwell equations describe the coupling between light and matter

Incoming and generated fields

electric fields are produced by changing magnetic fields (Faraday)

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\boldsymbol{\nabla} \cdot \boxed{\boldsymbol{D}} = \rho$$

electric field diverges from electric charge (Coulomb)

circulating magnetic fields are produced by changing electric fields and by electric currents (Ampere)

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \boxed{\frac{\partial \boldsymbol{D}}{\partial t}}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$$

No magnetic monopolaes

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$$

$$\boldsymbol{P} = \boldsymbol{P}^{(1)}(\boldsymbol{E}) + \boldsymbol{P}^{(2)}(\boldsymbol{E}^2) + \boldsymbol{P}^{(3)}(\boldsymbol{E}^3) + \ldots$$

$$P_i^{(3)}(3\omega) = \epsilon_0 \chi_{ijkl}^{(3)}(3\omega; \omega, \omega, \omega) E_j(\omega) E_k(\omega) E_l(\omega)$$

Polarization in material that is generated at different frequencies radiates waves at those frequencies and couple with incoming fields

Some notes on: Scalars, Vectors, Matrices, Tensors

What are tensors
Contractions and tensor products
Kronecker delta function
Levi-Cevita epsilon tensor symbol and its uses

Key concepts Lecture 1

Light, Matter recap
Interactions in a medium
Displacement field
Macroscopic vs molecular polarization
Primer on tensors

Book Material: CH 1.1 -1.3

Exercises: 0 (additional exercises), 2,3,1

Chapter 1

Light – Matter Interactions: E, D, P

Time Domain picture

Definition of NLO susceptibility

Properties of the NLO susceptibility

- -Reality of fields
- -Permutation Symmetry
- -Lossless media
- -Kleinman Symmetry
- -Spatial symmetry

Anharmonic Oscillator Model

- -static
- -dynamic