Magnetism in materials

Week 07

1. In first approximation, the Pauli paramagnetic susceptibility is temperature-independent and is given by

$$\chi_{P0} = \frac{3n\mu_0\mu_B^2}{2k_BT_F} \tag{1}$$

where n is the electron density and T_F the Fermi temperature. Show that for a metal at $T \ll T_F$ the first temperature-dependent correction to the Pauli magnetic susceptibility is

$$\chi_P = \chi_{P0}\left(1 - \frac{\pi^2T^2}{12T_F^2}\right) \tag{2}$$

and estimate the size of this correction for a typical metal at room temperature.

Hint Use the low-temperature Sommerfeld expansion

$$\int_{-\infty}^{\infty} \frac{H(E)}{\exp\left(\frac{E-\mu}{k_BT}\right)+1}dE = \int_{-\infty}^{\mu} H(E)dE + \frac{\pi^2}{6}(k_BT)^2H'(\mu) + O([k_BT]^4) \tag{3}$$

and the fact that for small T the chemical potential is related to the Fermi energy by

$$\mu \approx E_F\left(1 - \frac{\pi^2T^2}{12T_F^2}\right) \tag{4}$$

2. When the temperature increases the chemical potential has to decrease to conserve the total number of electrons, so it becomes more relevant the contribution from the high energy tale $(E-\mu) \gg k_BT$ where the Fermi-Dirac distribution is well approximated by $f(E) \approx e^{-(E-\mu)/k_BT}$. Show that in this regime the chemical potential can be approximated as

$$\mu = k_BT\left[\frac{3}{2}\ln\left(\frac{T_F}{T}\right) + \ln\left(\frac{4}{3\sqrt{\pi}}\right)\right] \tag{5}$$

Show that in this regime the Pauli susceptibility is well approximated by

$$\chi_P = \frac{n\mu_0\mu_b^2}{k_BT} \tag{6}$$

Hint Impose that the total number of electrons is conserved.
Use the integral $\int_0^\infty \sqrt{x}e^{-x}dx = \frac{\sqrt{\pi}}{2}$

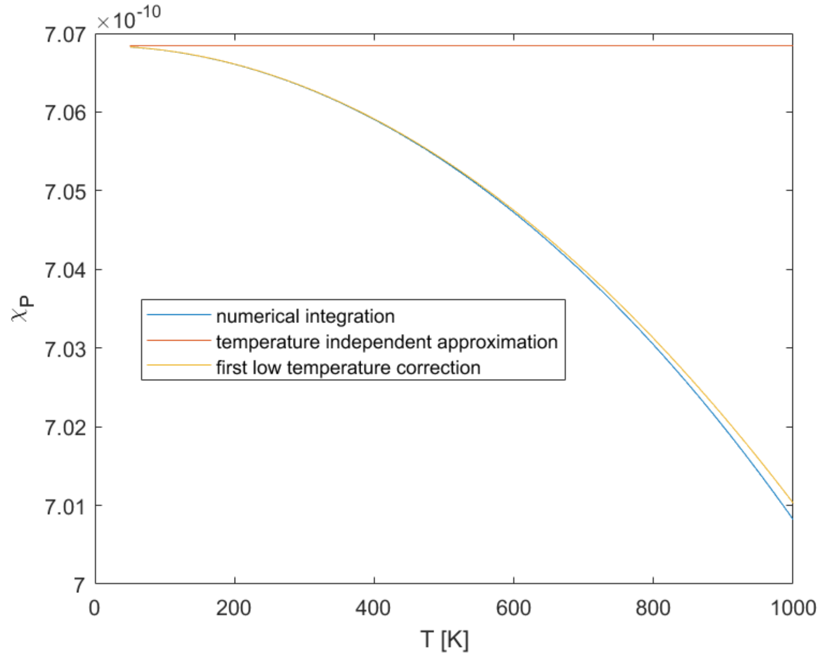

Figure 1: Pauli paramagnetic susceptibility at $T \ll T_F$ for a metal with $T_F = 10^4\,\mathrm{K}$. The blue curve is computed via numerical integration of the Fermi-Dirac distribution, the orange and yellow curves show respectively the temperature independent χ_{P0} and temperature dependent $\chi_{P0}\left(1 - \frac{\pi^2 T^2}{12 T_F^2}\right)$.

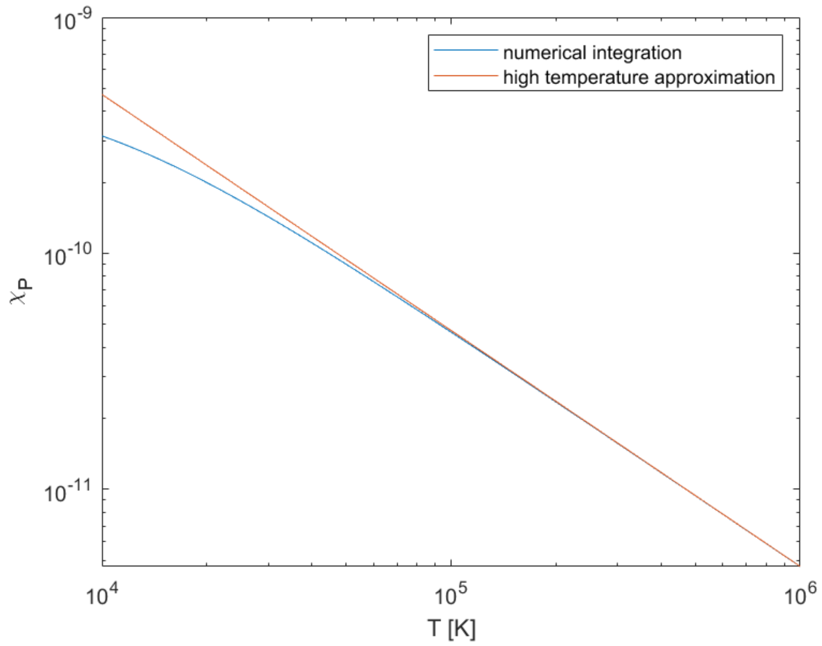

Figure 2: Pauli paramagnetic susceptibility at $T \geq T_F$ for a metal with $T_F = 10^4\,\mathrm{K}$. The blue curve is computed via numerical integration of the Fermi-Dirac distribution, the orange one shows $\chi_P = \frac{n\mu_0\mu_b^2}{k_B T}$.