MAGNETISM IN MATERIALS

Lecture 1: Introduction

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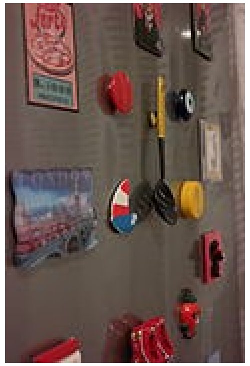
EPFL

- If something is magnetic, how do we know it is magnetic?
- What are magnetic effects that are recognized as magnetic?
- If it behaves as magnetite, it is a magnet!



[How to fight microplastic pollution with magnets - BBC Future]

image source: wikimedia.org

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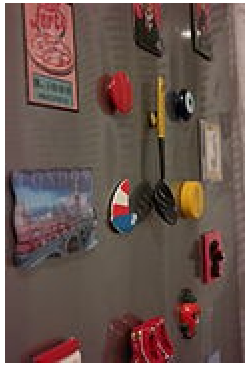

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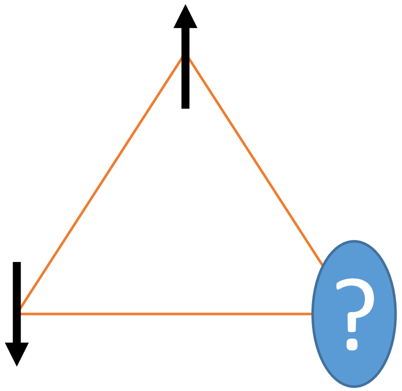
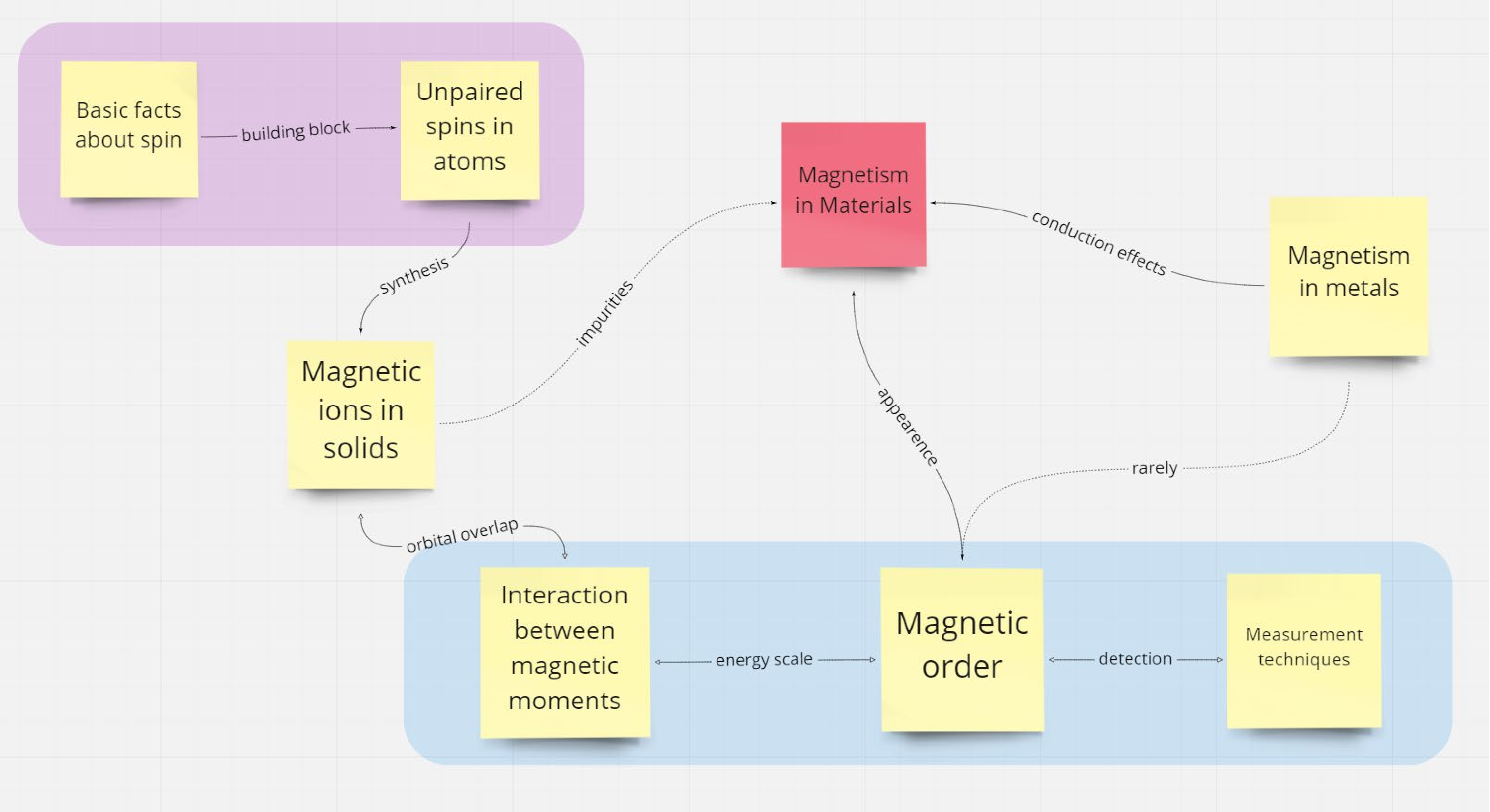


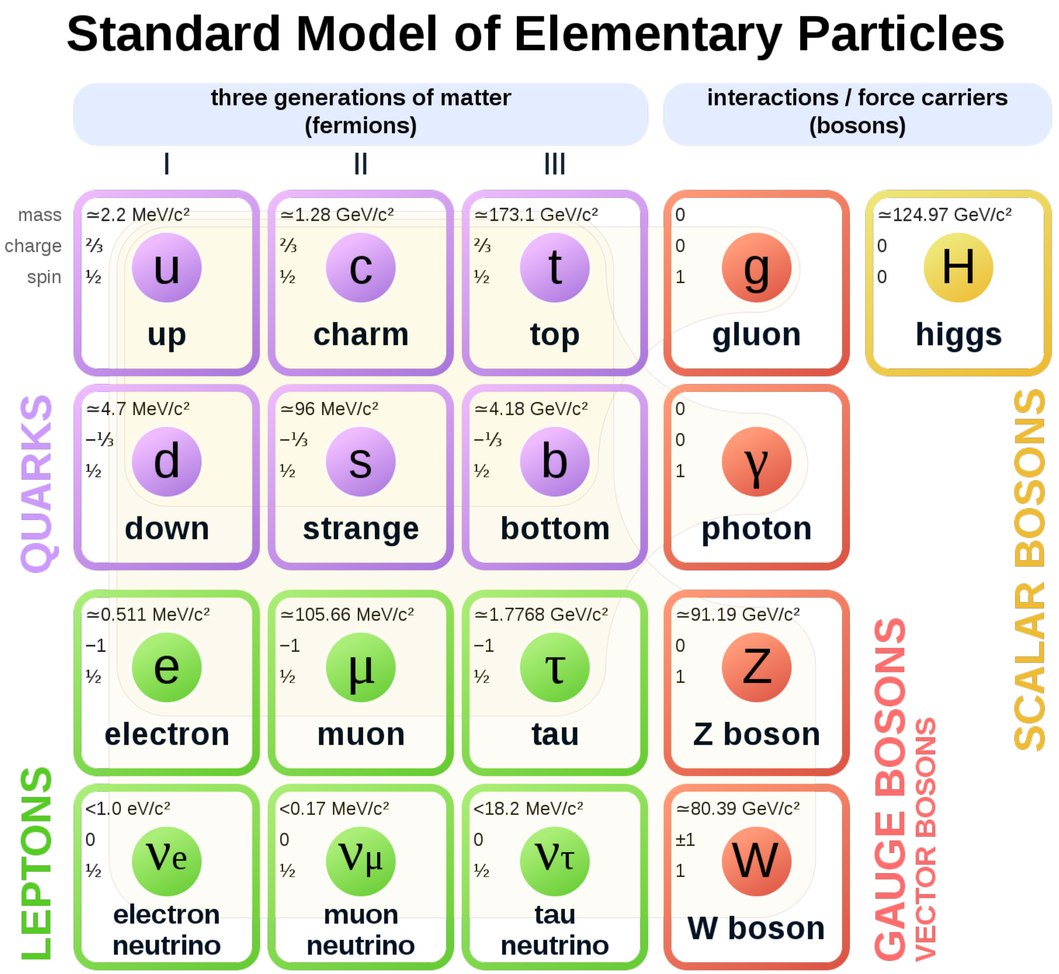
- condensed matter physics: 10^{23} magnetic moments -> emergent phenomena
- interactions (NN, NNN, dipole)
- geometry
- spintronics

image source: wikimedia.org


Basic facts about spin
building block
Unpaired spins in atoms
synthesis
Magnetic ions in solids
impurities
Magnetism in Materials
conduction effects
Magnetism in metals
appearence
rarely
orbital overlap
Interaction between magnetic moments
energy scale
Magnetic order
detection
Measurement techniques

- what does it mean a single spin?

❖ what does it mean a single spin?

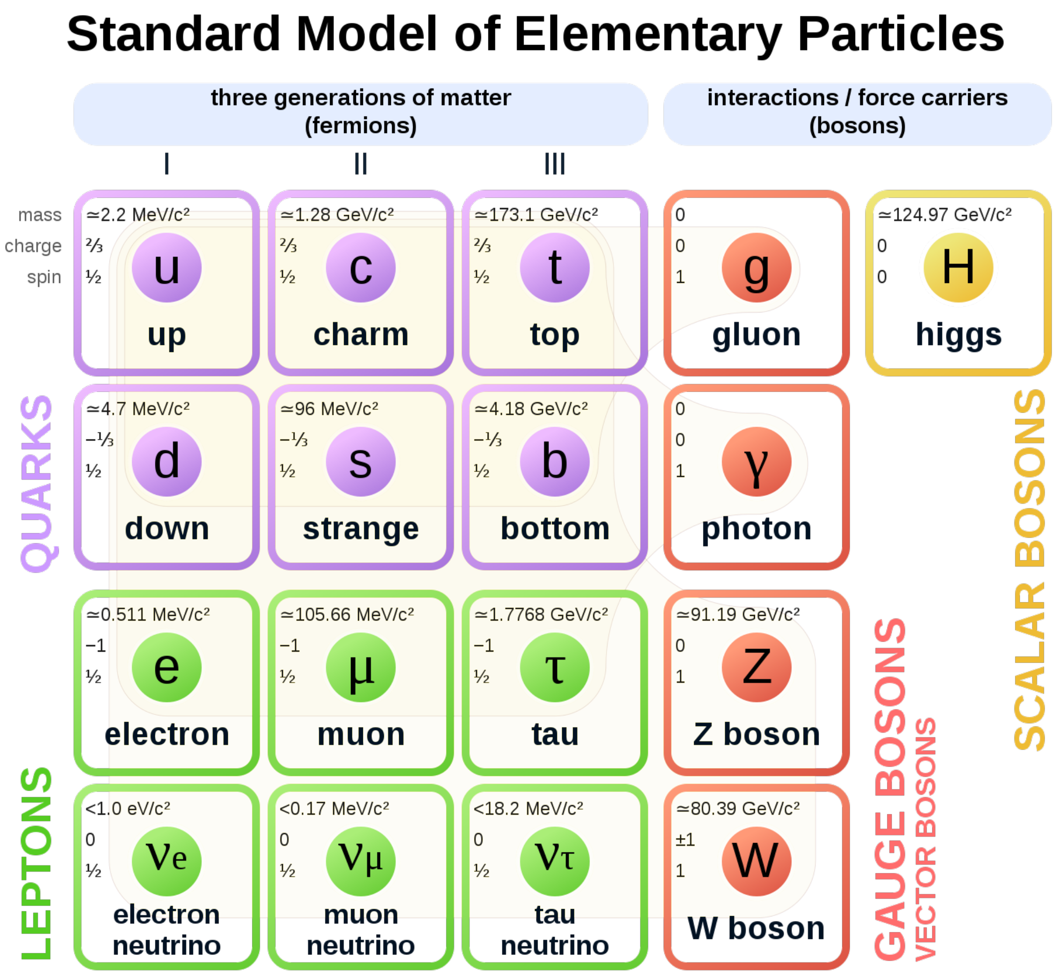


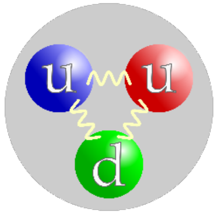
see this video: [(109) Electrons DO NOT Spin - YouTube](), especially from 8:50

image source: wikimedia.org

❖ what does it mean a single spin?

proton (and neutron) spin crisis!

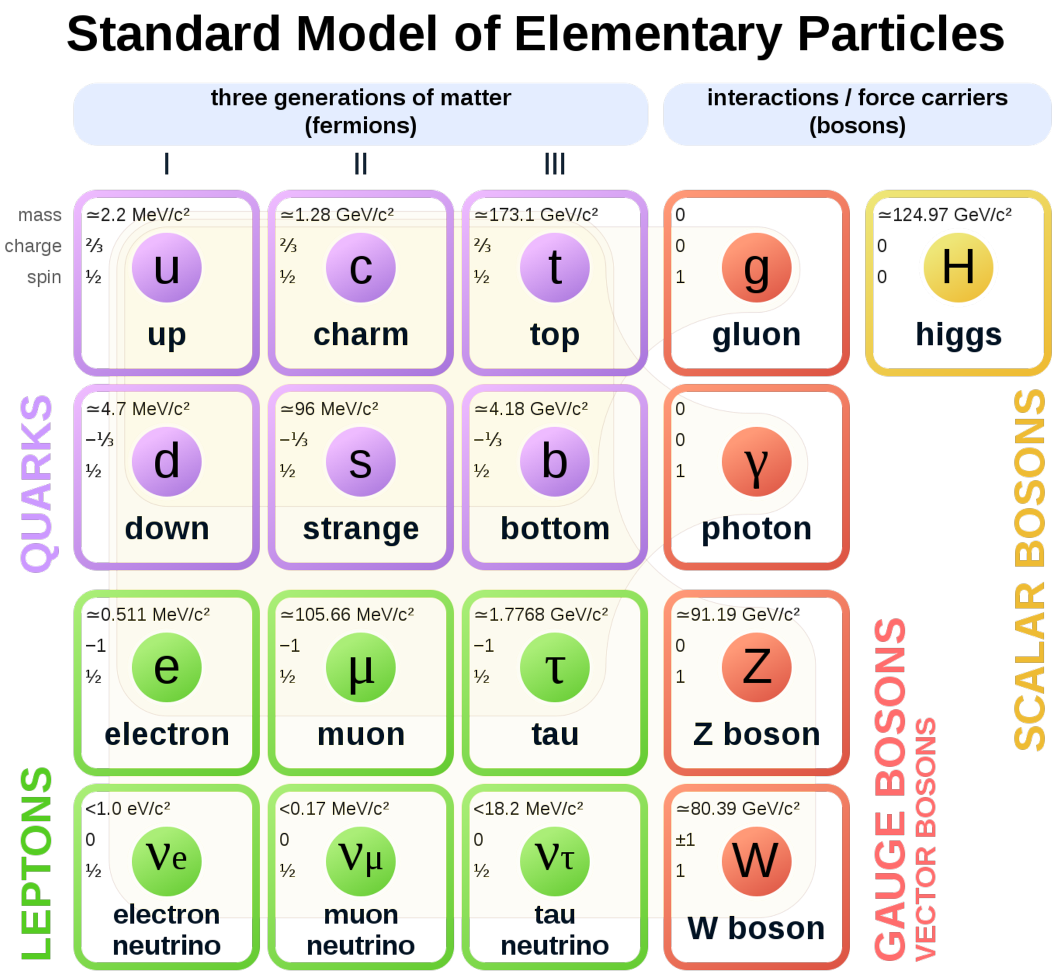




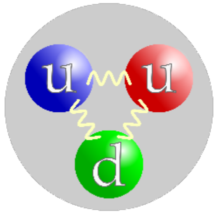
$$(S_{proton} = ½) >> \Sigma\ (S_{quark} = ½)$$

image source: wikimedia.org

- what does it mean a single spin?



proton (and neutron) spin crisis!



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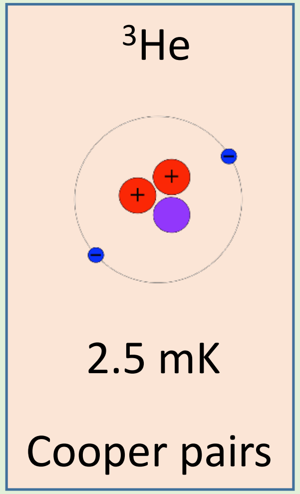

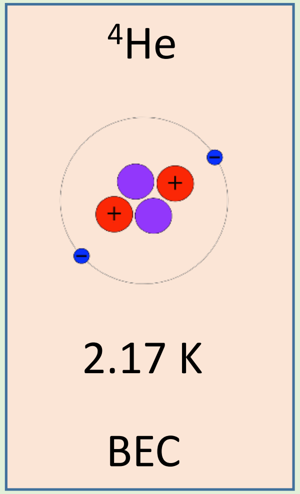

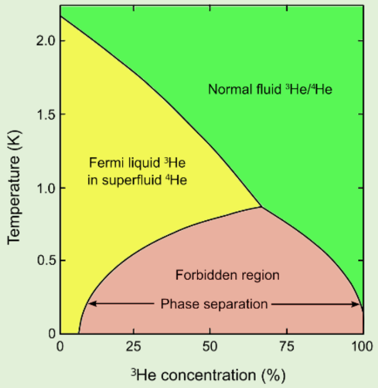

dilution refrigerator, T_{min} = 2.7 mK (continuously)

[quantum computing]

image source: wikimedia.org

Schrödinger equation

$$E = \frac{p^2}{2m} + U$$

$$\hat{E} \rightarrow i\hbar\frac{\partial}{\partial t}$$

$$\hat{p} \rightarrow -i\hbar\vec{\nabla}$$

$$\left(-\frac{\hbar}{2m}\vec{\nabla}^2 + U\right)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

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1st order in time
2nd order in space

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1st order in time
2nd order in space

Dirac equation

$$(i\gamma^\mu\partial_\mu - m)\Psi = 0$$

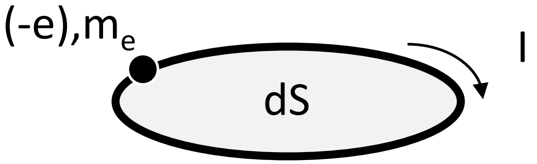
$$\gamma^0 = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

4 solutions

+m↑	**+m↓**	electron
-m↑	**-m↓**	positron

elementary magnetic dipole (no monopoles...yet)



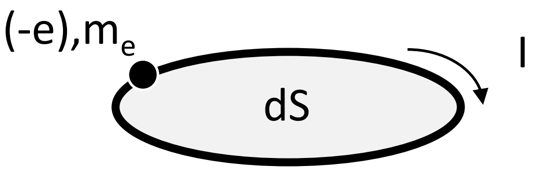
$$d\mu = IdS \text{ [Am}^2\text{]}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{\mu} = \gamma \vec{L}$$

gyromagnetic ratio

elementary magnetic dipole (no monopoles...yet)



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↓

gyromagnetic ratio

<u>electric dipole in electric field</u>

$$E = -\vec{p}_{dip}\vec{\mathcal{E}}$$

<u>magnetic dipole in magnetic field</u>

$$E = -\vec{\mu}\vec{B}$$

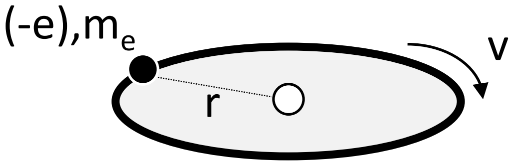
$$\frac{d\vec{L}}{dt} = \vec{T} = \vec{\mu} \times \vec{B} \longrightarrow$$

precession: $\frac{d\vec{\mu}}{dt} = \gamma\vec{\mu} \times \vec{B}$

<u>Larmor frequency</u>

$$\omega_L = \gamma B$$

elementary magnetic dipole (no monopoles...yet)



$$\mu = IS = \frac{(-e)}{\tau} r^2\pi = \frac{(-e)}{2r\pi/v} r^2\pi = \frac{(-e)m_e vr}{2m_e} = \frac{(-e)\hbar}{2m_e} = -\mu_B$$

$$\mu_B = 9.274 \times 10^{-24} \ \ \mathrm{Am}^2$$

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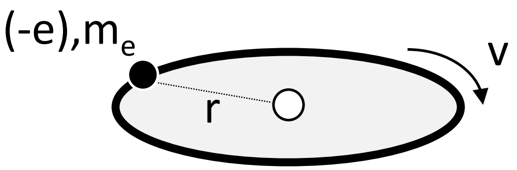

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$$\omega_L = \frac{e}{2m_e} B$$

$$\gamma = \frac{e}{2m_e} = 8.7935 \times 10^{10} s^{-1} T^{-1}$$

NIST: $\gamma_e = 1.761 \times 10^{11} s^{-1} T^{-1}$ $\frac{\gamma_e}{\gamma} \approx 2$ *g*-factor

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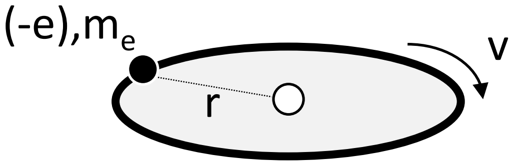

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$$H = \frac{1}{2m} (\vec{p}\vec{\sigma})^2 \longrightarrow \frac{1}{2m} \left(\vec{p}\vec{\sigma} - q\vec{A}\vec{\sigma}\right)^2 + q\varphi \boldsymbol{I} \longrightarrow \left[\frac{1}{2m}\left(\vec{p} - q\vec{A}\right)^2 + q\varphi\right] \boldsymbol{I} - \frac{q}{m} \vec{B}\vec{S}$$

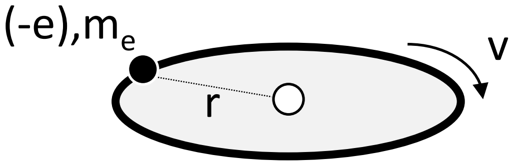
$$E = -g\mu_B \vec{B}\vec{S}$$

$$\vec{p} \to \vec{p} - q\vec{A}$$

$$\vec{\nabla} \times \vec{A} = \vec{B}$$

$$\boldsymbol{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

elementary magnetic dipole (no monopoles...yet)



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$$g = 2 + \frac{\alpha}{\pi} + \cdots = 2.0023 \ldots \qquad \alpha = \frac{e^2}{2\varepsilon_0 hc} \approx \frac{1}{137}$$

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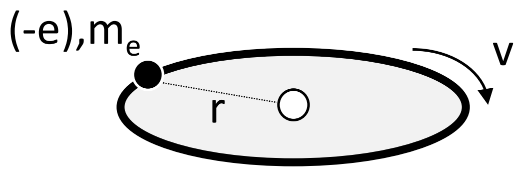

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Muon g-2 experiment

NIST: $\gamma_e = 1.761 \times 10^{11} s^{-1} T^{-1}$ $\quad \frac{\gamma_e}{\gamma} \approx 2$ $\quad$ g-factor

$$H = \frac{1}{2m}(\vec{p}\vec{\sigma})^2 \longrightarrow \frac{1}{2m}\left(\vec{p}\vec{\sigma} - q\vec{A}\vec{\sigma}\right)^2 + q\varphi \boldsymbol{I} \longrightarrow \left[\frac{1}{2m}\left(\vec{p} - q\vec{A}\right)^2 + q\varphi\right]\boldsymbol{I} - \frac{q}{m}\vec{B}\vec{S}$$

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$$E = -g\mu_B m_s B \qquad m_s = \pm\frac{1}{2}$$

$$\hat{\boldsymbol{S}} = \frac{1}{2}\hat{\boldsymbol{\sigma}}$$

$$\hat{S}_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad |\uparrow_x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad |\downarrow_x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\hat{S}_y = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad |\uparrow_y\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} \qquad |\downarrow_y\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$\hat{S}_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad |\uparrow_z\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |\downarrow_z\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\hat{\boldsymbol{S}}^2 = \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2$$

$$\hat{\boldsymbol{S}}^2|\psi\rangle = \left(\hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2\right)|\psi\rangle = \frac{3}{4}|\psi\rangle$$

$\hat{\boldsymbol{S}}^2 = s(s+1)$ 2s + 1 projections

$\hat{\boldsymbol{L}}^2 = l(l+1)$ 2l + 1 projections

$$\left.\begin{aligned} &\left[\hat{S}_\alpha, \hat{S}_\beta\right] = i\epsilon_{\alpha\beta\gamma}\hat{S}_\gamma \\ &\left[\hat{\boldsymbol{S}}^2, \hat{S}_\alpha\right] = 0 \end{aligned}\right\}$$

you can know the total spin and one component
but not two components simultaneously

(x,p_x) & (t,E)

$$\hat{S}_+ = \hat{S}_x + i\hat{S}_y = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\hat{S}_- = \hat{S}_x - i\hat{S}_y = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\hat{S}_+ |\downarrow_z\rangle = |\uparrow_z\rangle$$

$$\hat{S}_- |\uparrow_z\rangle = |\downarrow_z\rangle$$

not observables because $\hat{S}_+^\dagger \neq \hat{S}_+$

$$[\hat{S}_+, \hat{S}_-] = 2\hat{S}_z$$

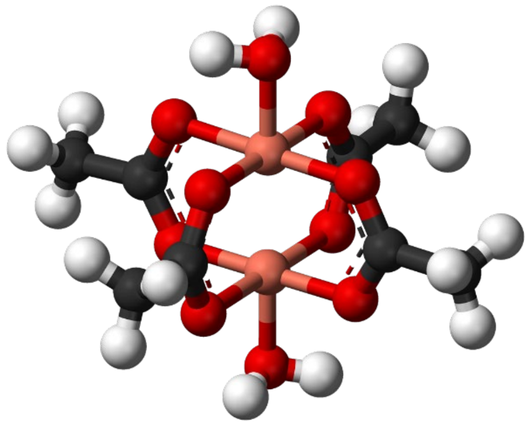
$$[\hat{S}_z, \hat{S}_\pm] = \pm\hat{S}_\pm$$

$$[\hat{\boldsymbol{S}}^2, \hat{S}_\pm] = 0$$

$$\hat{S}_+\hat{S}_- + \hat{S}_-\hat{S}_+ = 2\left(\hat{S}_x^2 + \hat{S}_y^2\right) \rightarrow \hat{\boldsymbol{S}}^2 = \frac{1}{2}\left(\hat{S}_+\hat{S}_- + \hat{S}_-\hat{S}_+\right) + \hat{S}_z^2$$

$$\mathcal{H} = J\boldsymbol{S}^a\boldsymbol{S}^b$$

- hyperfine interaction between electron and nuclear moments
- exchange interaction in a dimer



$Cu(OAc)_2$

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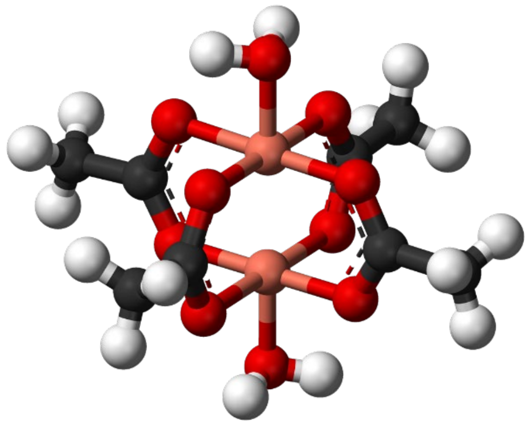

$Cu(OAc)_2$

$\boldsymbol{S}^{tot} = \boldsymbol{S}^a + \boldsymbol{S}^b\,|^2$

$(\boldsymbol{S}^{tot})^2 = (\boldsymbol{S}^a)^2 + (\boldsymbol{S}^b)^2 + 2\boldsymbol{S}^a\boldsymbol{S}^b \rightarrow \mathcal{H} = \frac{1}{2}J\left[(\boldsymbol{S}^{tot})^2 - (\boldsymbol{S}^a)^2 - (\boldsymbol{S}^b)^2\right]$

	s^{tot}	$s^{tot}(s^{tot}+1)$	E	m_s
$(\uparrow)(\downarrow)$	0	0	-3J/4	0
$(\uparrow)(\uparrow)$	1	2	J/4	-1,0,1

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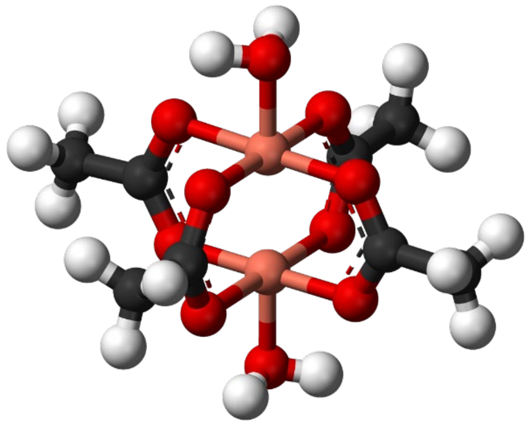

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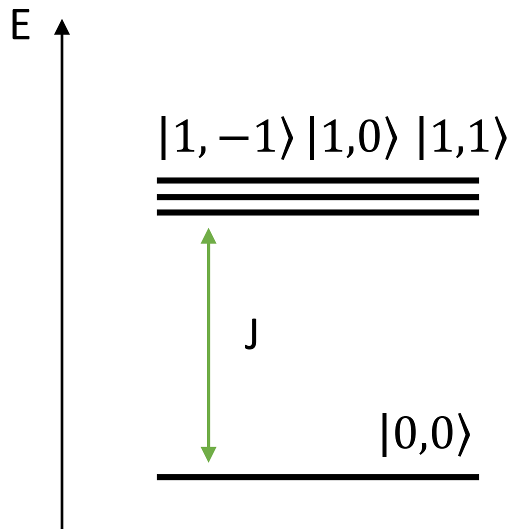

$$\boldsymbol{S}^{tot} = \boldsymbol{S}^a + \boldsymbol{S}^b\,|^2$$

$$(\boldsymbol{S}^{tot})^2 = (\boldsymbol{S}^a)^2 + \left(\boldsymbol{S}^b\right)^2 + 2\boldsymbol{S}^a\boldsymbol{S}^b \rightarrow \mathcal{H} = \frac{1}{2}J\left[(\boldsymbol{S}^{tot})^2 - (\boldsymbol{S}^a)^2 - \left(\boldsymbol{S}^b\right)^2\right]$$

	s^{tot}	$s^{tot}(s^{tot}+1)$	E	m_s
$(\uparrow)(\downarrow)$	0	0	-3J/4	0
$(\uparrow)(\uparrow)$	1	2	J/4	-1,0,1