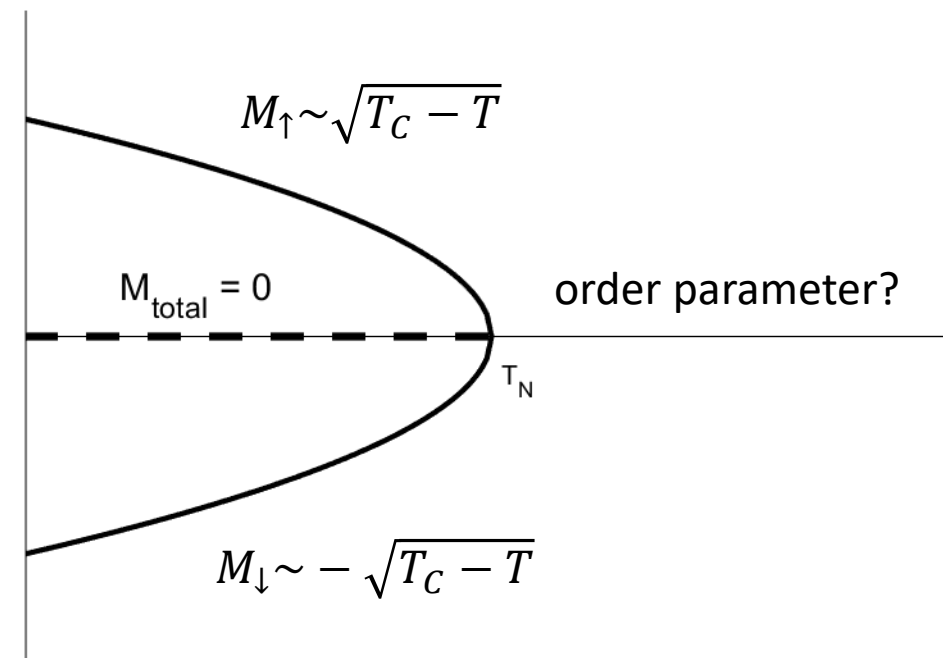
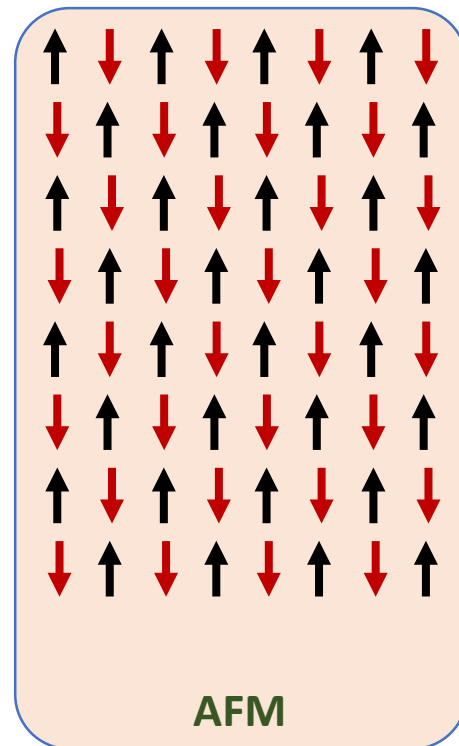
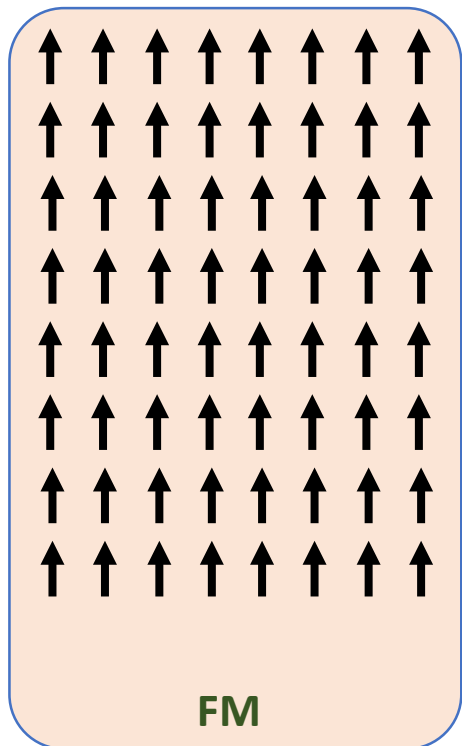


MAGNETISM IN MATERIALS

Lecture 6: Magnetic Ordering (cont.)

- ❖ antiferromagnetism
- ❖ ferrimagnetism
- ❖ incommensurate order
- ❖ spin-glass



$$B_{mf} = \lambda M$$

$$M = g\mu_B S B_S(x)$$

$$x = \frac{g\mu_B S (B_{out} + \lambda M)}{k_B T}$$

$$B_{\uparrow} = -\lambda M_{\downarrow} \quad B_{\downarrow} = -\lambda M_{\uparrow}$$

$$M_{\uparrow} = g\mu_B S B_S(x)$$

$$x = \frac{g\mu_B S (B_{out} + \lambda M_{\downarrow})}{k_B T}$$

- ❖ paramagnet → Curie law
- ❖ ferromagnet → Curie-Weiss law

$$\frac{C}{T} = \frac{\mu_0 M}{B + \lambda M} \quad \chi = \frac{C}{T - T_C}$$

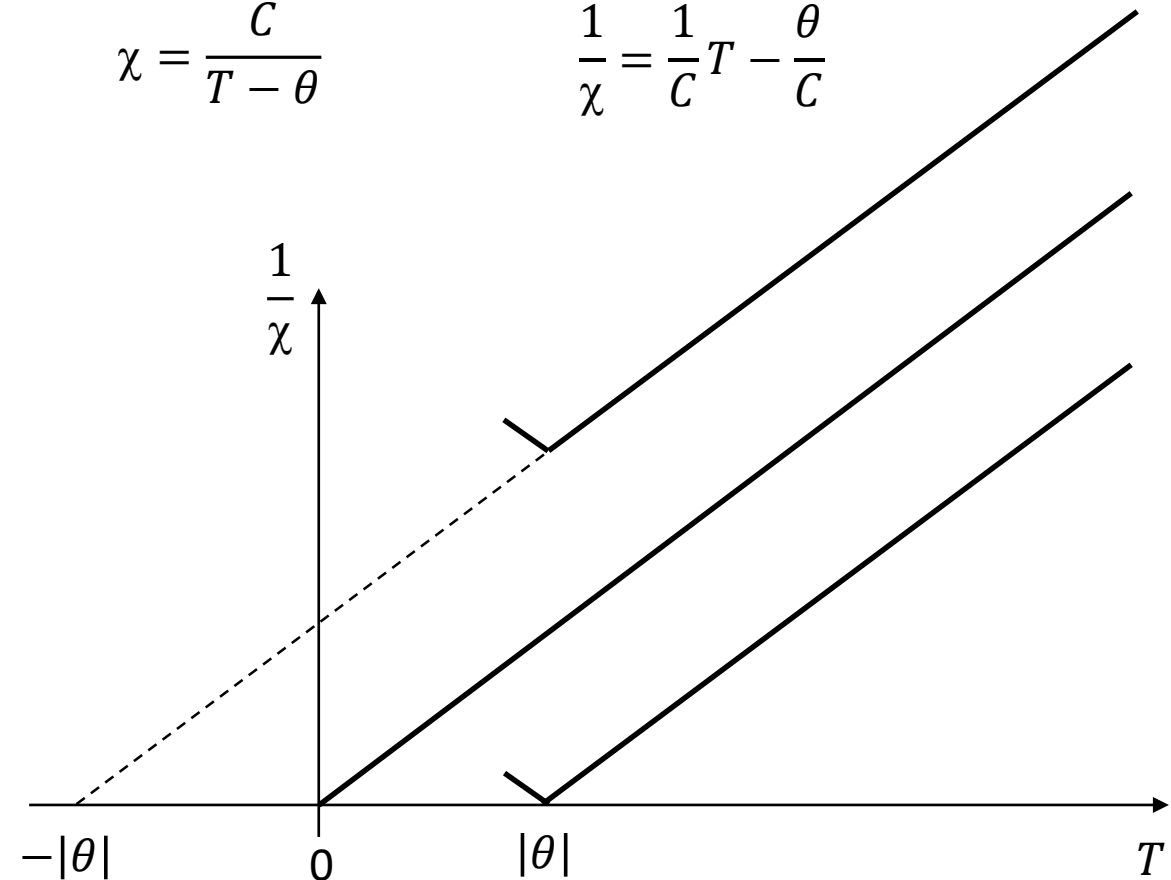
- ❖ antiferromagnet → Curie-Weiss law

$$\frac{C}{T} = \frac{\mu_0 M}{B - \lambda M} \quad \chi = \frac{C}{T + T_N}$$

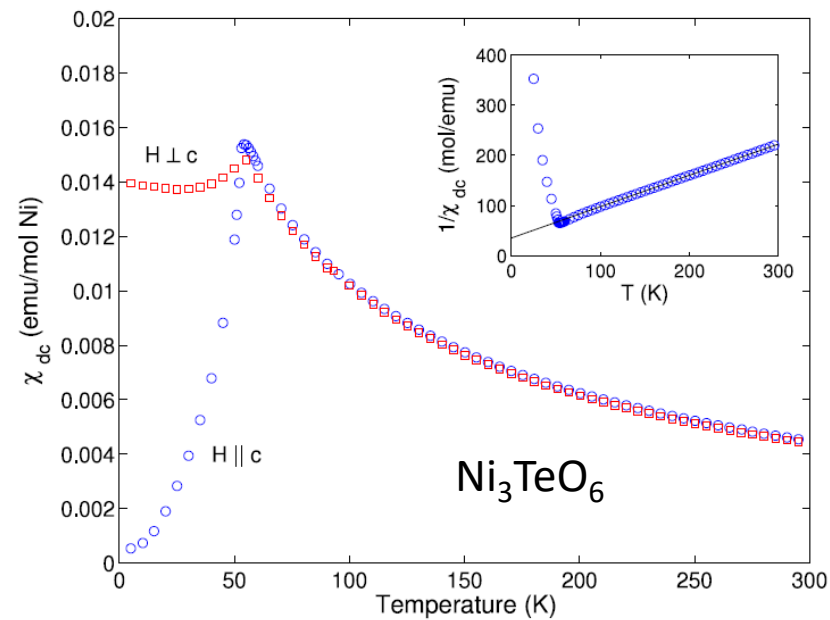
C : Curie constant
 T_C : Curie temperature/critical temperature
 T_N : Néel temperature
 θ : Weiss temperature

- ❖ general approach

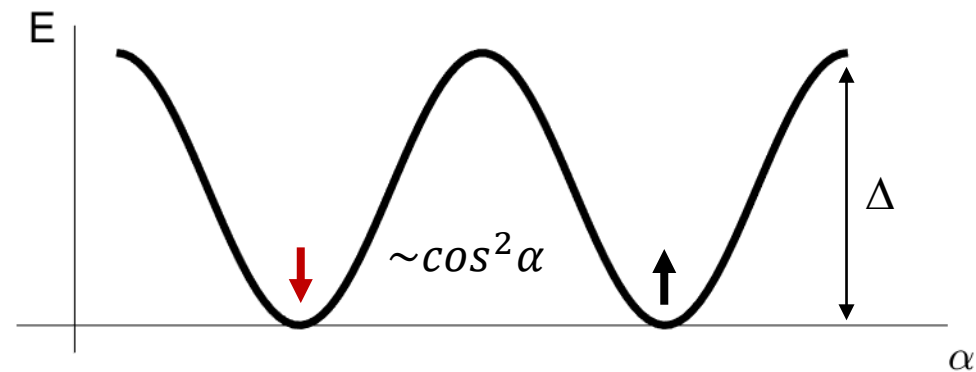
$$\chi = \frac{C}{T - \theta} \quad \frac{1}{\chi} = \frac{1}{C}T - \frac{\theta}{C}$$



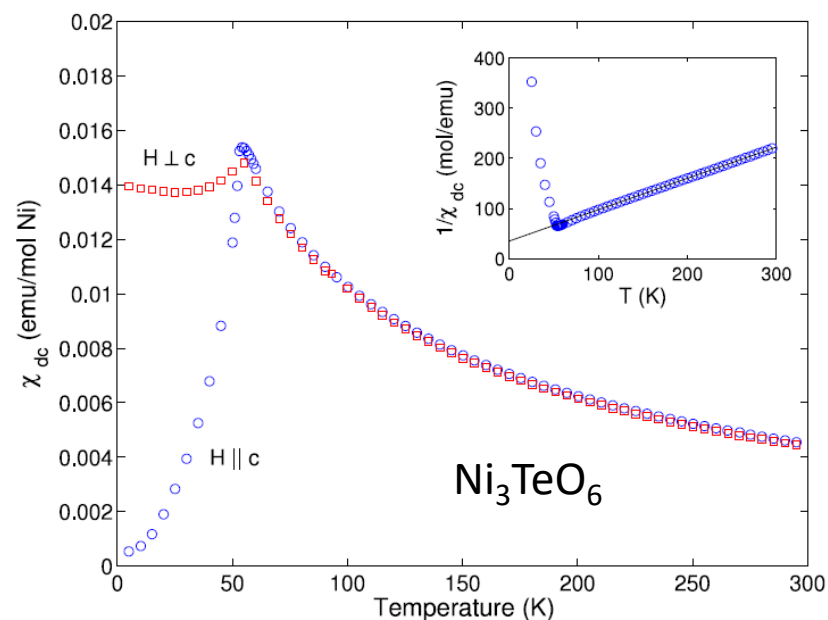
- ❖ very anisotropic magnetic response (susceptibility) below T_N



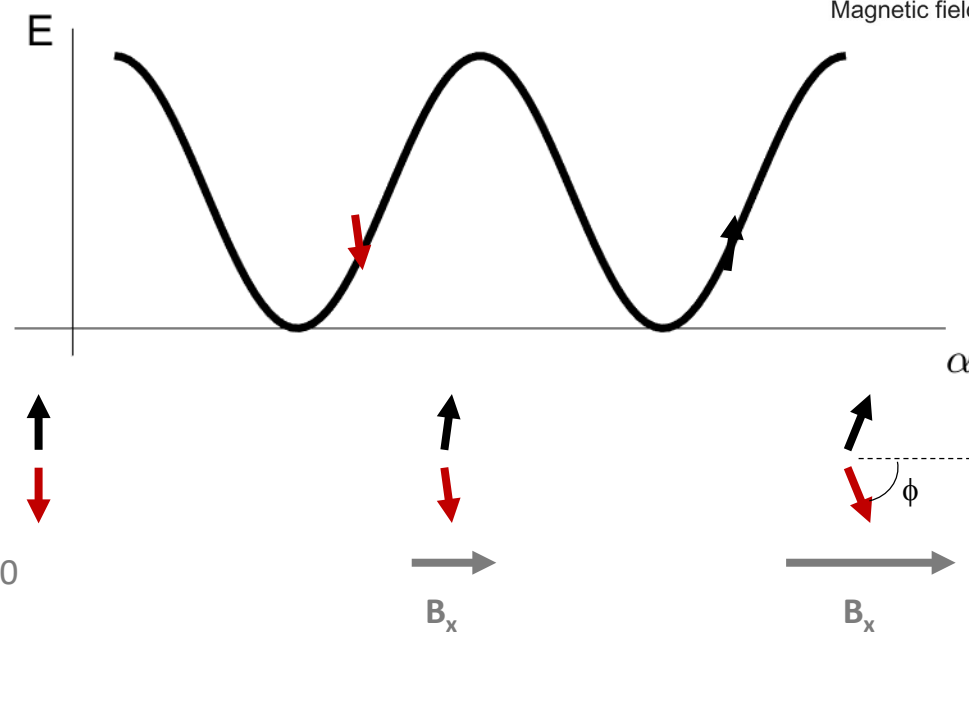
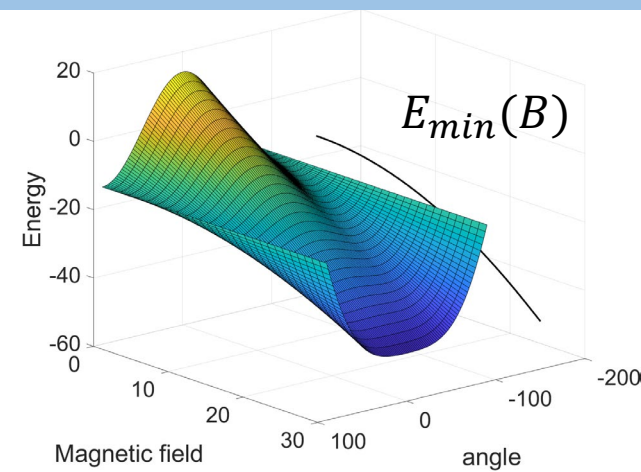
- ❖ anisotropy energy
- ❖ in the simplest form: $E = \Delta \cos^2 \alpha$
- ❖ origin in SOC



- ❖ very anisotropic magnetic response (susceptibility) below T_N

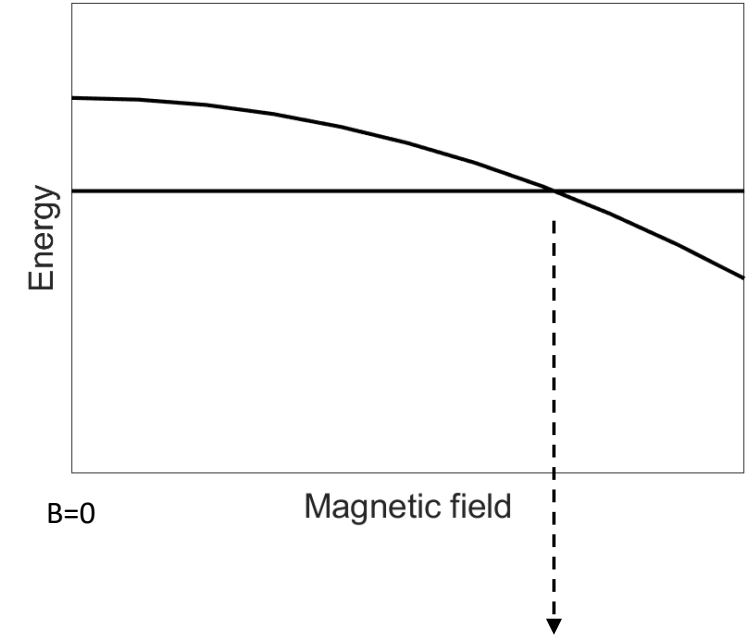
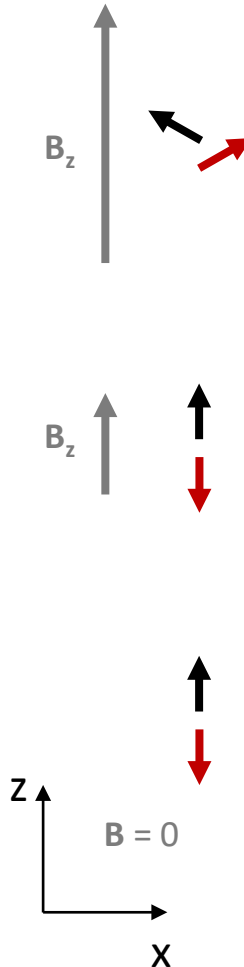
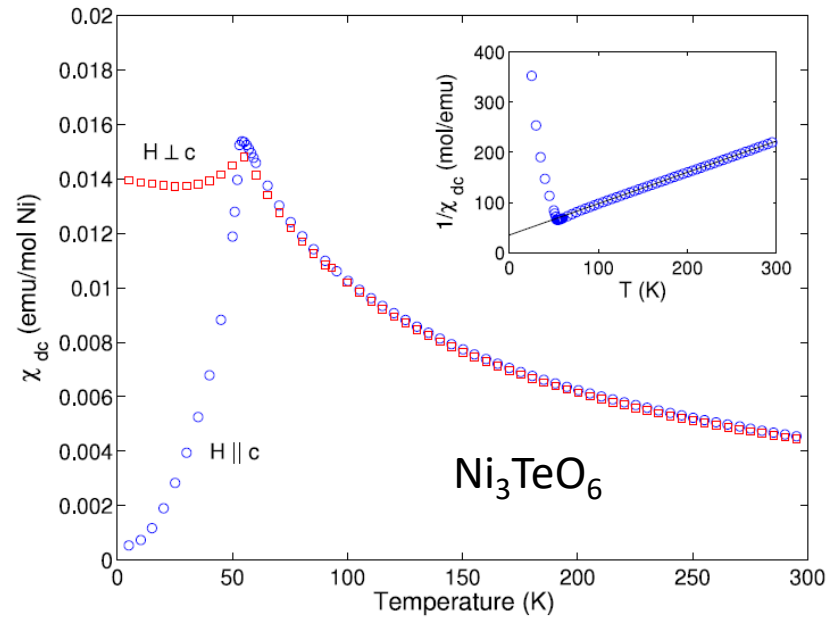


- ❖ anisotropy energy
- ❖ in the simplest form: $E = \Delta \cos^2 \alpha$
- ❖ origin in SOC



$$\hat{\mathcal{H}} = J\mathbf{S}_1\mathbf{S}_2 + g\mu_B\mathbf{B}(\mathbf{S}_1 + \mathbf{S}_2) \rightarrow E = C(J)M^2 \cos 2\varphi - 2BM \cos \varphi + 2\Delta \cos^2 \varphi$$

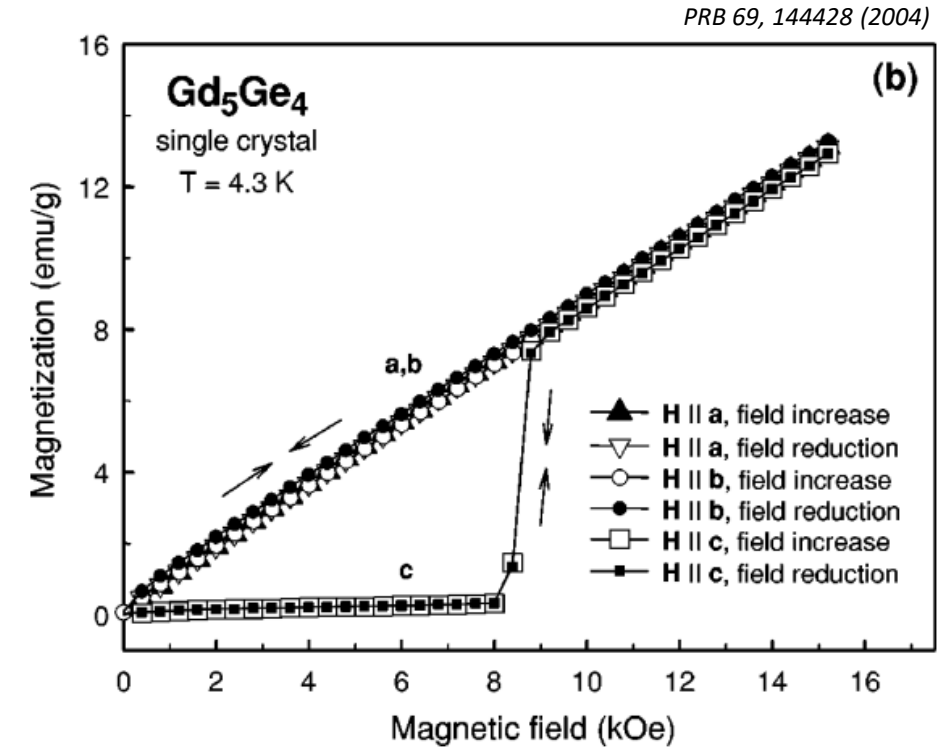
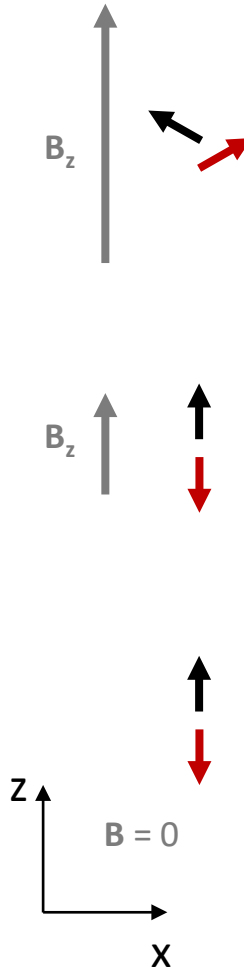
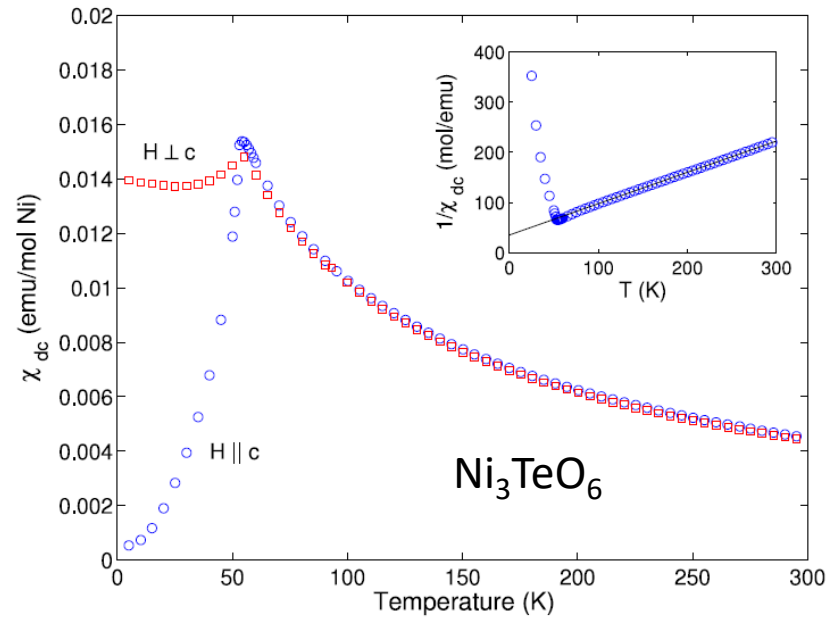
❖ very anisotropic magnetic response (susceptibility) below T_N



spin-flop: $B \approx \sqrt{C(J)\Delta}$

$$E = C(J)M^2 \cos(\theta + \gamma) - BM \cos \theta - BM \cos \gamma + \Delta \sin^2 \theta + \Delta \sin^2 \gamma$$

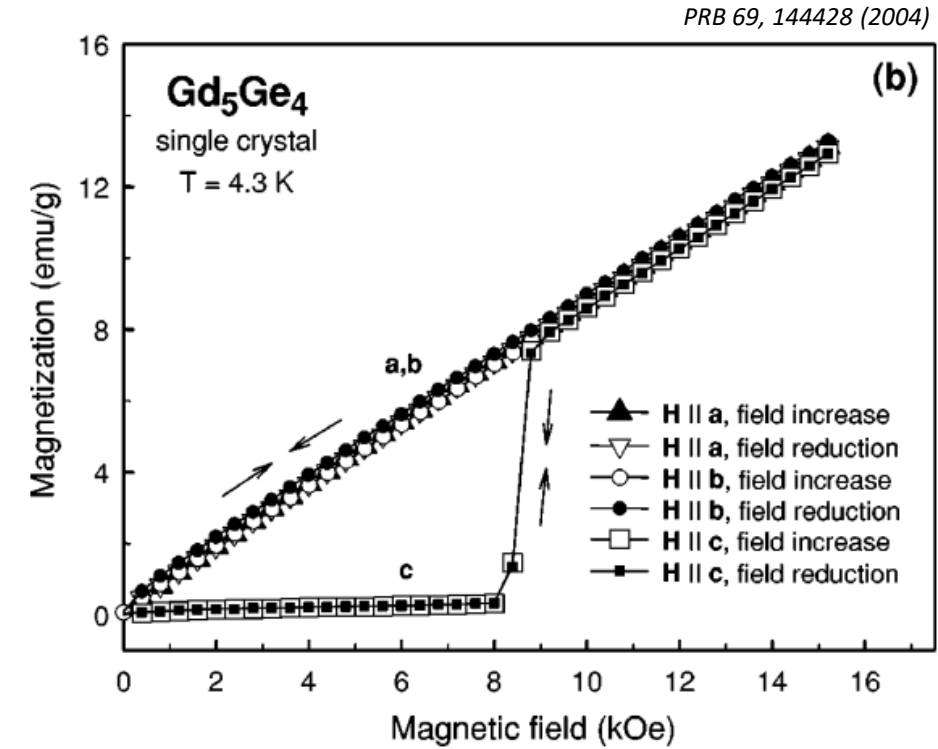
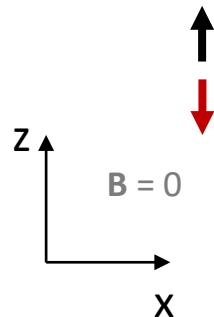
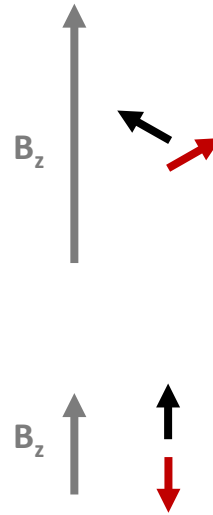
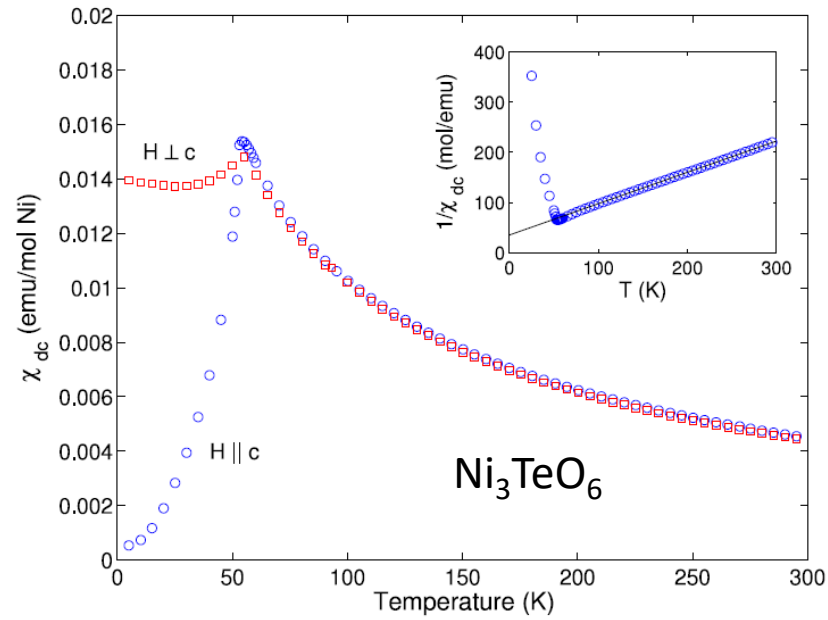
❖ very anisotropic magnetic response (susceptibility) below T_N



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❖ very anisotropic magnetic response (susceptibility) below T_N

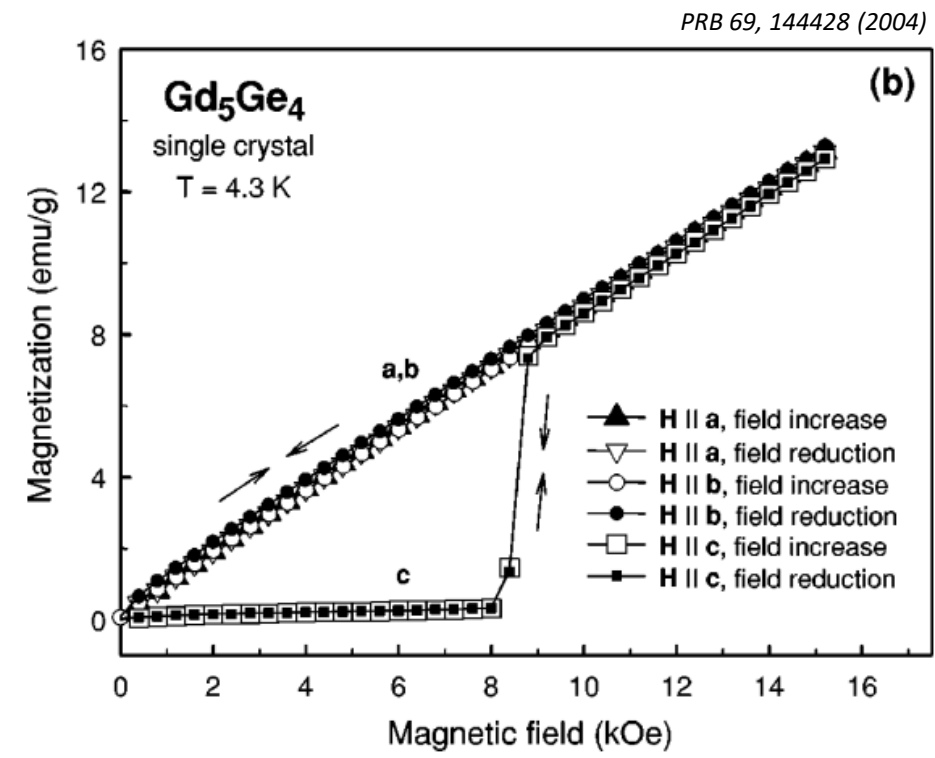
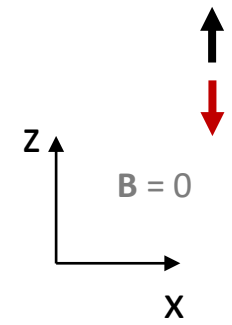
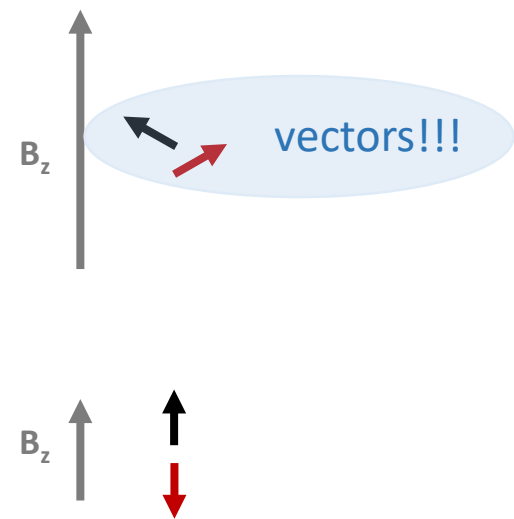
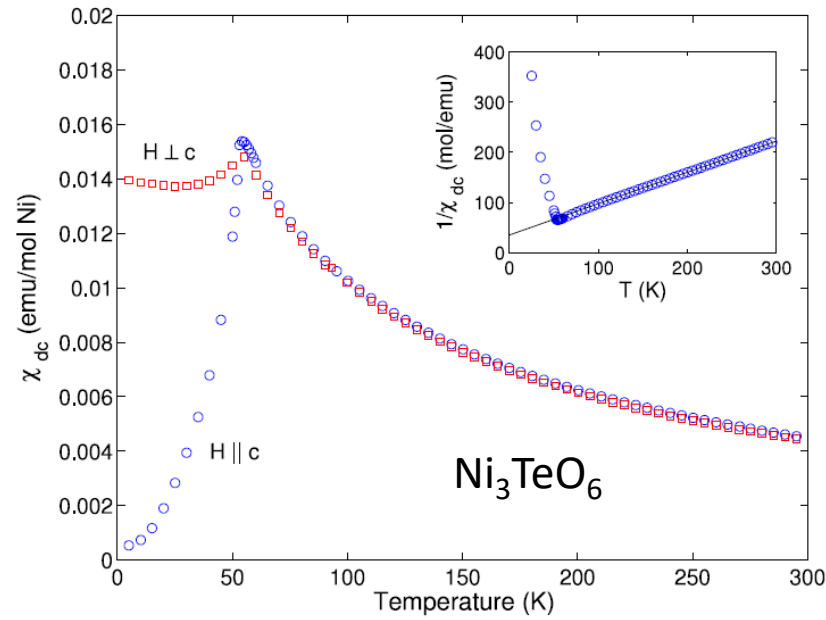


spin-flop: $B \approx \sqrt{C(J)\Delta}$

spin-flip: $B = C(J)$

$$E = C(J)M^2 \cos(\theta + \gamma) - BM \cos \theta - BM \cos \gamma + \Delta \sin^2 \theta + \Delta \sin^2 \gamma$$

❖ very anisotropic magnetic response (susceptibility) below T_N

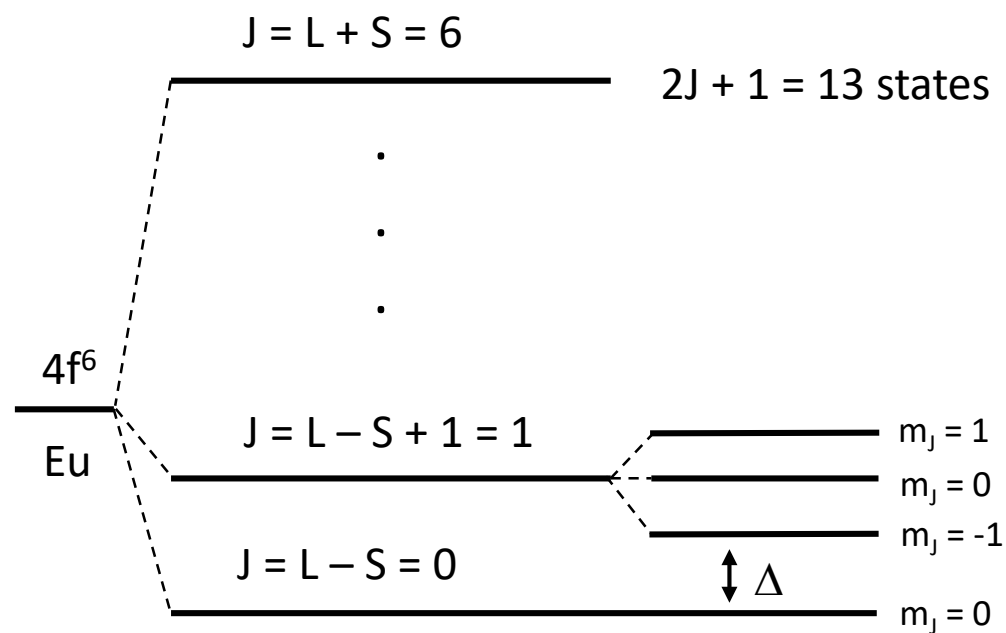


spin-flop: $B \approx \sqrt{C(J)\Delta}$

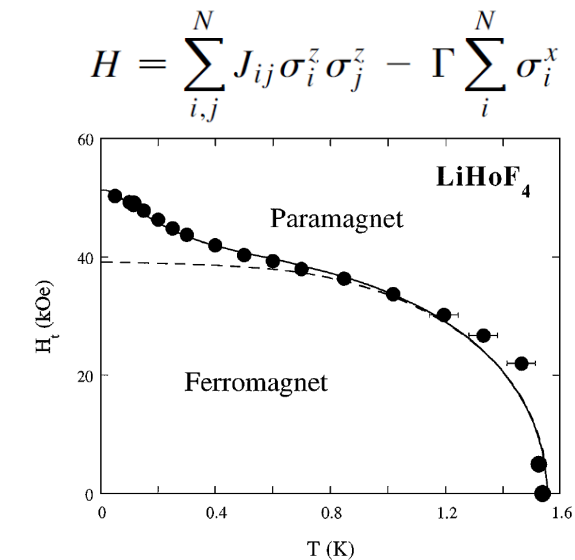
spin-flip: $B = C(J)$

$$E = C(J)M^2 \cos(\theta + \gamma) - BM \cos \theta - BM \cos \gamma + \Delta \sin^2 \theta + \Delta \sin^2 \gamma$$

- ❖ $RE = \text{Ho}$ (c -axis anisotropy) a ferromagnet with $T_c = 1.5\text{K}$
- ❖ $RE = \text{Er}$ (ab -plane anisotropy) an antiferromagnet with $T_c = 0.38\text{K}$
- ❖ transverse field induced QPT

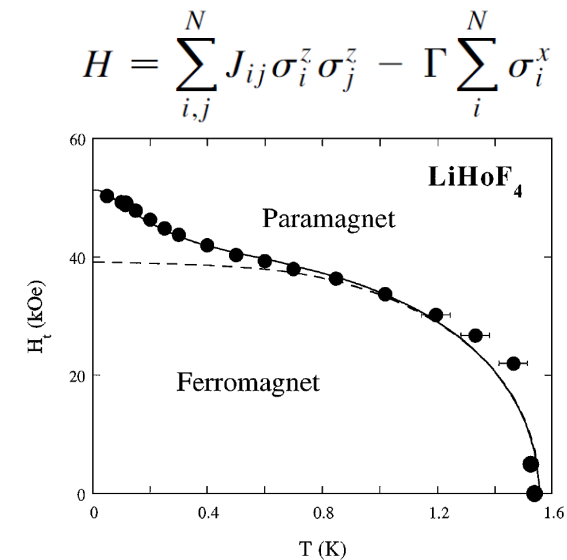
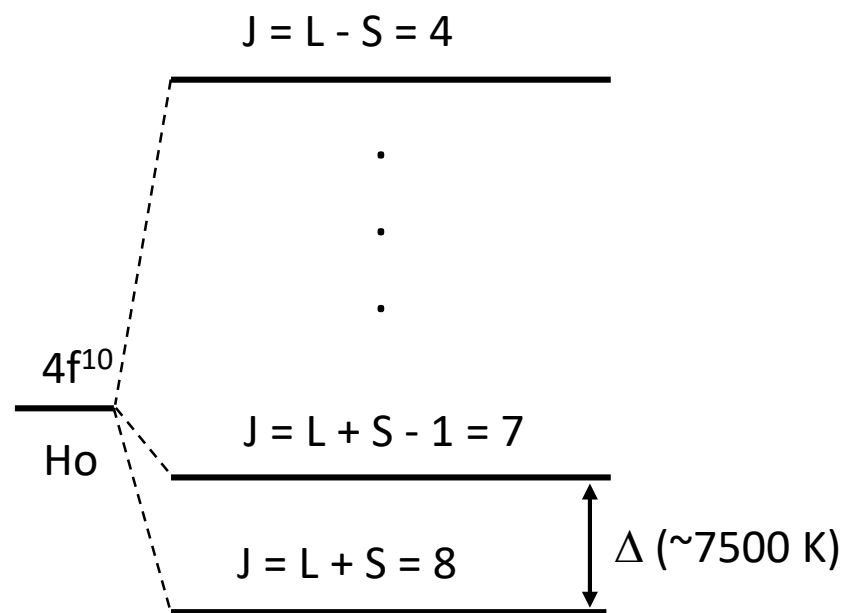


when $k_B T \sim \Delta$ ($\sim 500\text{ K}$), $\mu > 0$



PRL 77, 940 (1996)

- ❖ $RE = \text{Ho}$ (c -axis anisotropy) a ferromagnet with $T_c = 1.5\text{K}$
- ❖ $RE = \text{Er}$ (ab -plane anisotropy) an antiferromagnet with $T_c = 0.38\text{K}$
- ❖ transverse field induced QPT



PRL 77, 940 (1996)

-
- Diagram illustrating the energy levels for the $4f^{10}$ configuration, showing the splitting of the 5H_7 state (labeled Ho) into three levels:
- Top level: $J = L - S = 4$
 - Middle level: $J = L + S - 1 = 7$
 - Bottom level: $J = L + S = 8$
- The energy difference between the middle and bottom levels is indicated by a double-headed arrow and labeled Δ (~ 7500 K). A magnifying glass highlights this energy difference.

Figure 1 displays the energy level diagrams for Tb^{3+} , Ho^{3+} , and Er^{3+} ions. The y-axis represents energy in Kelvin (K), ranging from 0 to 600. The x-axis represents the magnetic field in Tesla (T), ranging from 0 to 14. The diagrams show the splitting of the ground state into sub-levels (A, B, E) and the resulting energy differences. The ground state splitting is indicated by $g_{||}$ and $g_{\perp}$ values.

Tb³⁺ ($7F_{J=6}$)

Sub-levels: A, B, E

Energy levels (K):

- A: $M_J = 4, 0, -4$ (split into $\sim A_+$ and $\sim A_-$)
- B: $M_J = 6, 2, -2, -6$ (split into $\sim B_+$ and $\sim B_-$)
- E: $M_J = 5, 1, 3$

Ground state splitting: $g_{||} = 17.7$, $g_{\perp} = 0.0$

Ho³⁺ ($5I_{J=8}$)

Sub-levels: A, B, E

Energy levels (K):

- A: $M_J = 8, 4, 0, -4, -8$ (split into $\sim A_+$ and $\sim A_-$)
- B: $M_J = 6, 2, -2, -6$ (split into $\sim B_+$ and $\sim B_-$)
- E: $M_J = 7, 3, 1, 5$

Ground state splitting: $g_{||} = 13.4$, $g_{\perp} = 0.0$

Er³⁺ ($4I_{J=15/2}$)

Sub-levels: E, E'

Energy levels (K):

- E: $M_J = 15/2, 9/2, 3/2, 1/2, -1/2, -3/2, -5/2, -7/2$
- E': $M_J = 13/2, 7/2, 5/2, 3/2, 1/2, -1/2, -3/2, -5/2$

Ground state splitting: $g_{||} = 3.1$, $g_{\perp} = 8.2$

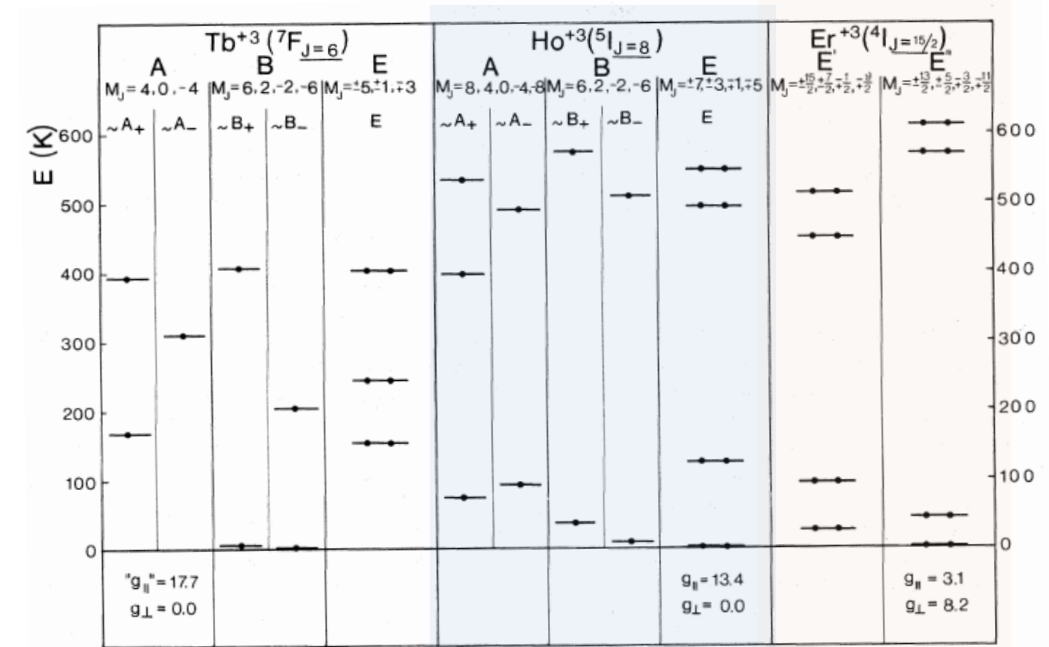
- ❖ $RE = \text{Ho}$ (c -axis anisotropy) a ferromagnet with $T_c = 1.5\text{K}$
- ❖ $RE = \text{Er}$ (ab -plane anisotropy) an antiferromagnet with $T_c = 0.38\text{K}$
- ❖ transverse field induced QPT

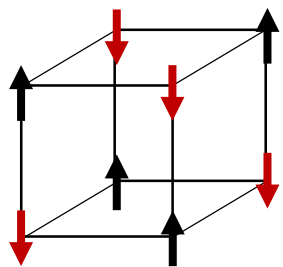
TABLE VII. Energies and wave functions for the crystal levels of Ho in LiHoF_4 transforming according to the A , E , and B representations of the S_4 point group. For the doubly degenerate levels (E representations) $g_{\parallel}$ are included, $g_{\perp} = 0$.

A						
E (K)	$e^{i1.7^\circ} 8\rangle$	$e^{-i8.2^\circ} 4\rangle$	$ 0\rangle$	$e^{i8.2^\circ} -4\rangle$	$e^{-i1.7^\circ} -8\rangle$	
535	0.397	-0.437	0.551	-0.437	0.397	$\sim A_+$
492	0.530	-0.468	0.000	0.468	-0.530	$\sim A_-$
398	-0.386	0.158	0.807	0.158	-0.386	$\sim A_+$
92	0.468	0.530	0.000	-0.530	-0.468	$\sim A_-$
74	0.440	0.533	0.212	0.533	0.440	$\sim A_+$
E						
E (K)	$ \pm 7\rangle$	$e^{\mp i9.2^\circ} \pm 3\rangle$	$e^{\mp i88.1^\circ} \mp 1\rangle$	$e^{\mp i92.5^\circ} \mp 5\rangle$	$ g_{\parallel} $	
546	0.562	-0.717	-0.371	0.183	8.6	
497	0.281	-0.305	0.788	-0.456	2.1	
124	0.070	0.023	0.488	0.870	10.0	
0	0.775	0.627	-0.060	-0.045	<u>13.4</u>	
B						
E (K)	$e^{i30.4^\circ} 6\rangle$	$e^{i22.7^\circ} 2\rangle$	$e^{-i22.7^\circ} -2\rangle$	$e^{-i30.4^\circ} -6\rangle$		
575	0.374	-0.600	-0.600	0.374	$\sim B_+$	
511	-0.423	0.566	-0.566	0.423	$\sim B_-$	
39	0.600	0.374	0.374	0.600	$\sim B_+$	
9	-0.566	-0.423	0.423	0.566	$\sim B_-$	

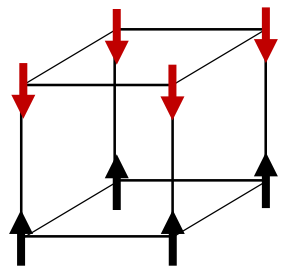
Kramers theorem:

levels with half-integer spins
are at least doubly degenerate

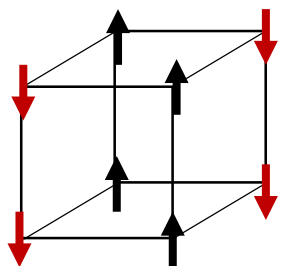




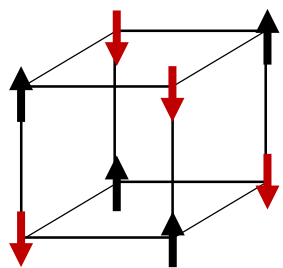
G-type AFM



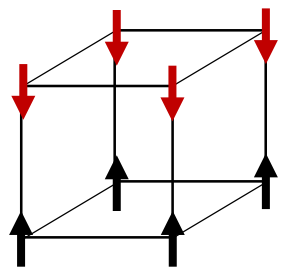
A-type AFM (FM planes)



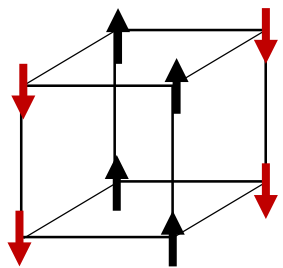
C-type AFM (FM chains)



G-type AFM



A-type AFM (FM planes)



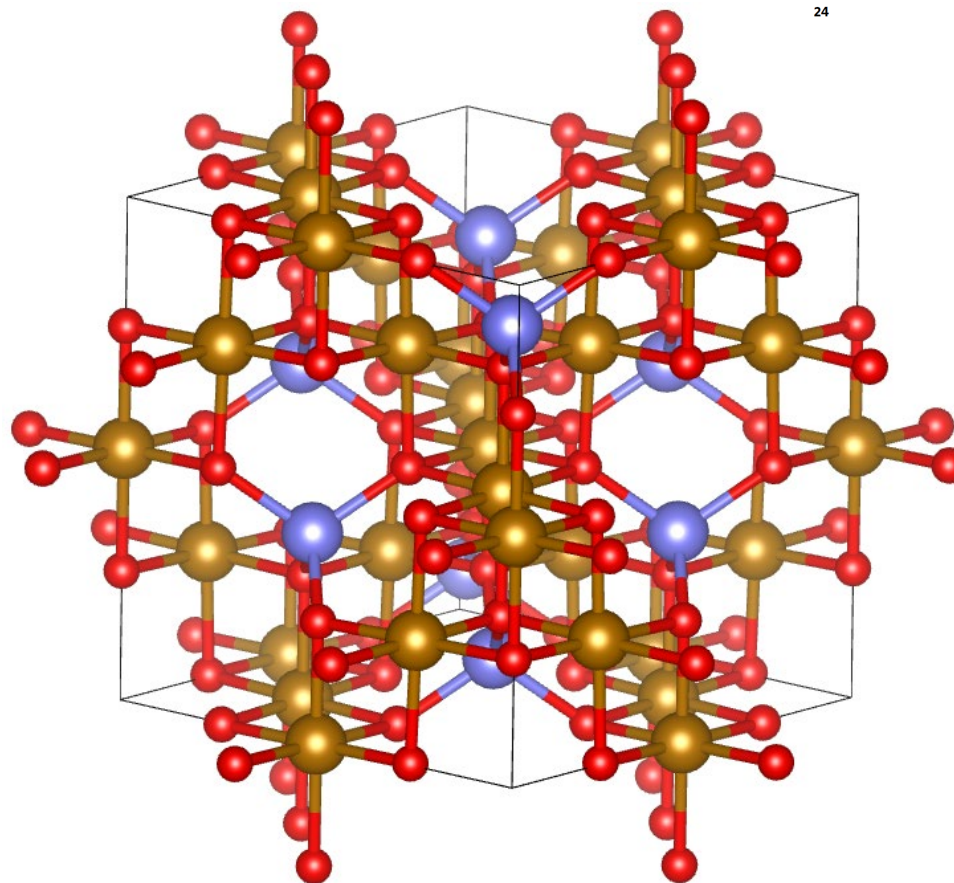
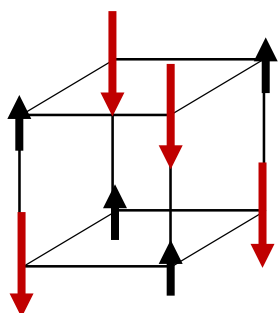
C-type AFM (FM chains)

KCuF₃

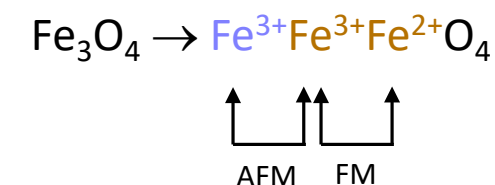
x^2-y^2 (x^2-z^2 , y^2-z^2)

$3z^2-r^2$ ($3y^2-r^2$, $3x^2-r^2$)

- ❖ FiM = unequal sublattices
- ❖ more often multiple sites with a resultant FM moment
- ❖ sometimes with a compensation point



magnetite



REVIEW ARTICLE

<https://doi.org/10.1038/s41563-021-01139-4>

nature
materials

Check for updates

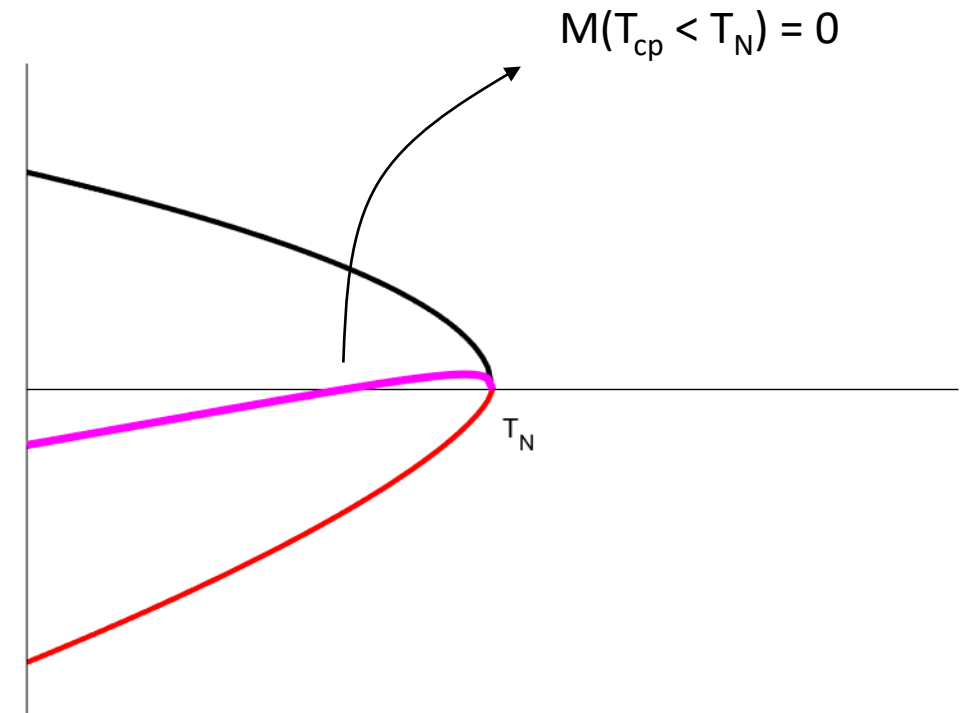
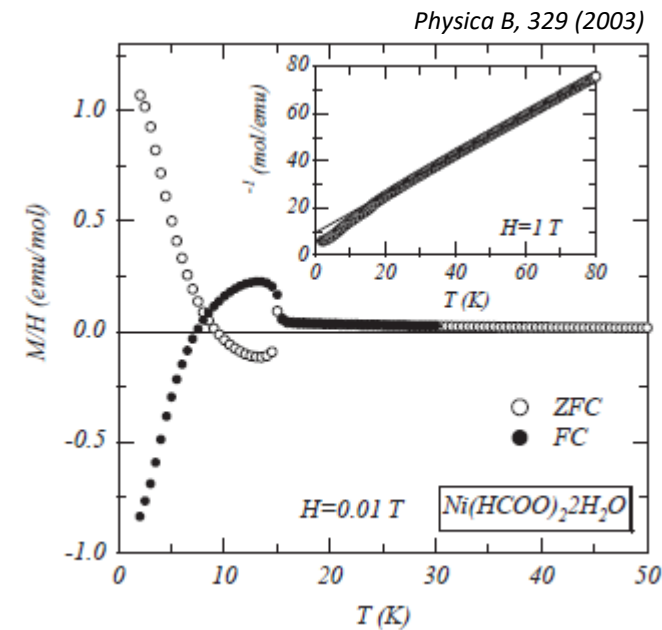
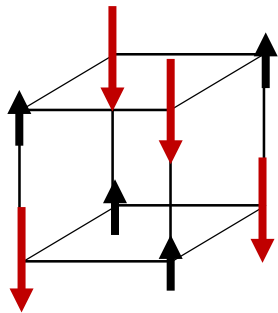
Ferrimagnetic spintronics

Se Kwon Kim¹, Geoffrey S. D. Beach², Kyung-Jin Lee^{1,3,4}, Teruo Ono^{5,6}, Theo Rasing^{7,8} and Hyunsoo Yang⁹

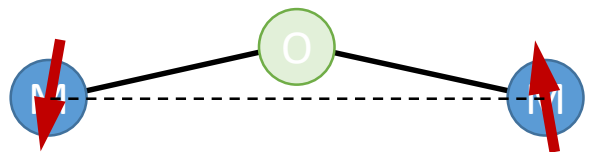
24

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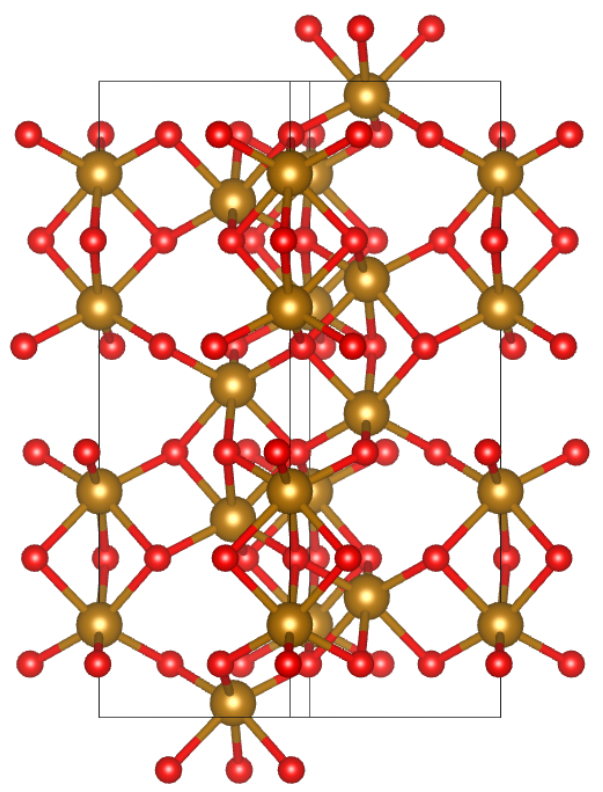
- ❖ FiM = unequal sublattices
- ❖ more often multiple sites with a resultant FM moment
- ❖ sometimes with a compensation point



❖ AFM - canting

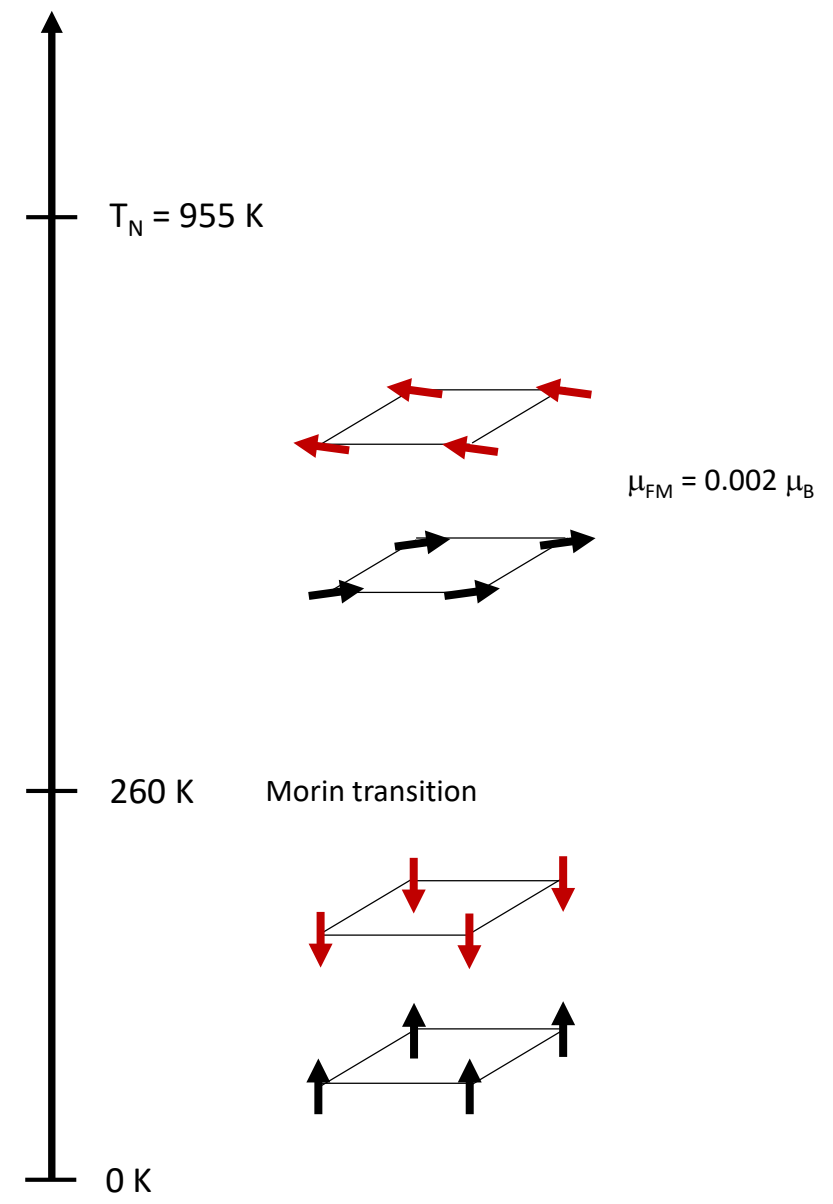


$$\hat{\mathcal{H}} = -J\mathbf{S}^a\mathbf{S}^b + D(\mathbf{S}^a \times \mathbf{S}^b)$$



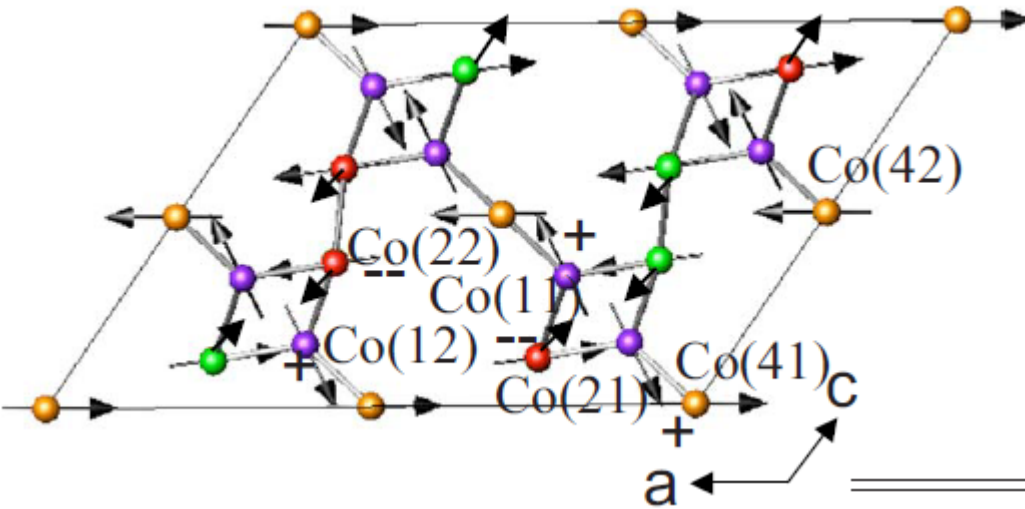
hematite

Fe₂O₃ (rust!)



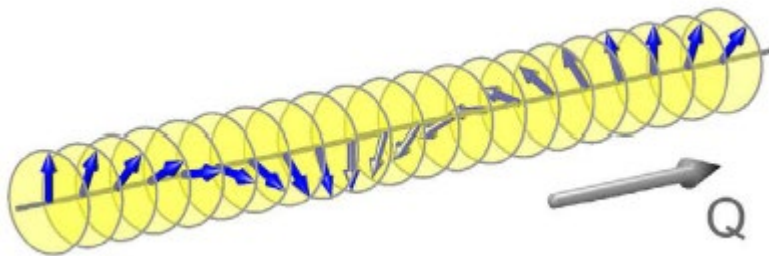
❖ sometimes it gets very complicated

Co7(TeO3)4Br6



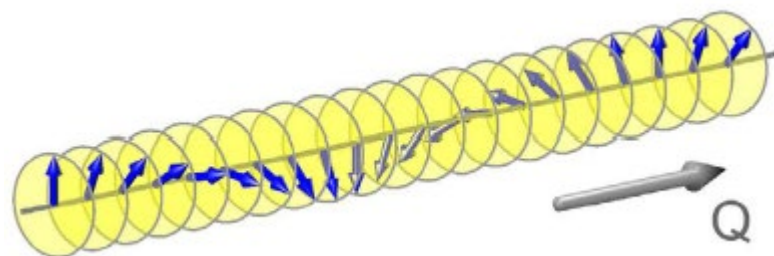
Ions	M_x	M_y	M_z	μ_B/Co	α_{li}
Co(11)	2.64(9)	1.86(6)	2.77(9)	4.25(7)	0
Co(12)	-2.64(9)	1.86(6)	-2.77(9)	4.25(7)	125(2)
Co(21)	-3.08(7)	-1.1(1)	3.08(7)	4.4(1)	95(2)
Co(22)	3.08(7)	-1.1(1)	-3.08(7)	4.4(1)	97(2)
Co(31)	-0.43(8)	-3.1(1)	2.22(8)	3.8(1)	93(2)
Co(32)	0.43(8)	-3.1(1)	-2.22(8)	3.8(1)	131(2)
Co(41)	-3.51(8)	1.33(9)	2.48(8)	4.5(1)	90(2)
Co(42)	3.51(8)	1.33(9)	-2.48(8)	4.5(1)	75(2)

❖ FM + DMI → helical order



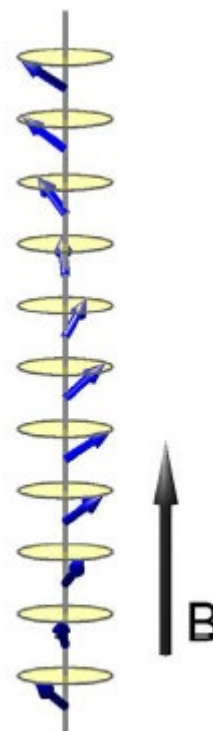
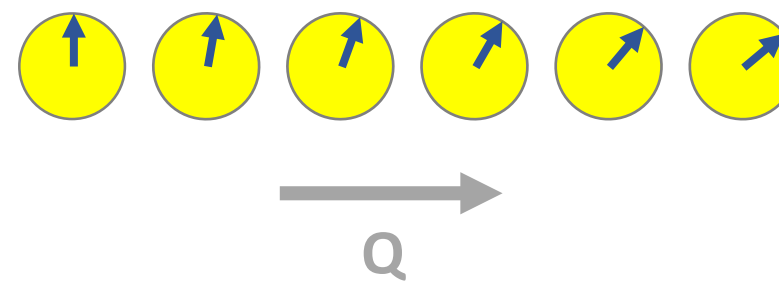
$$\lambda = 2\pi/Q \gg \text{unit cell}$$

❖ FM + DMI → helical order

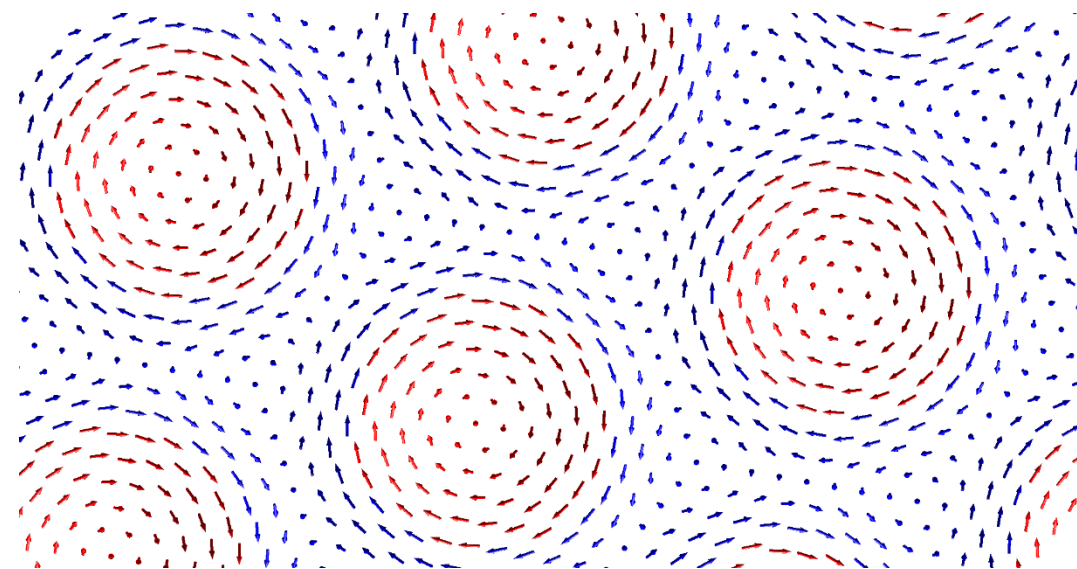
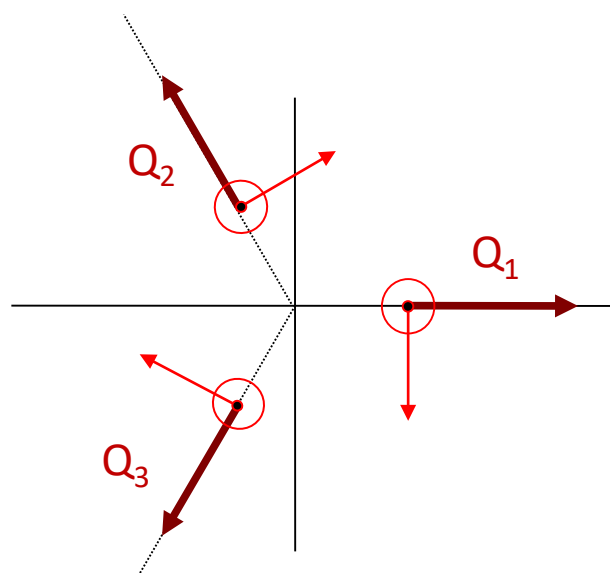
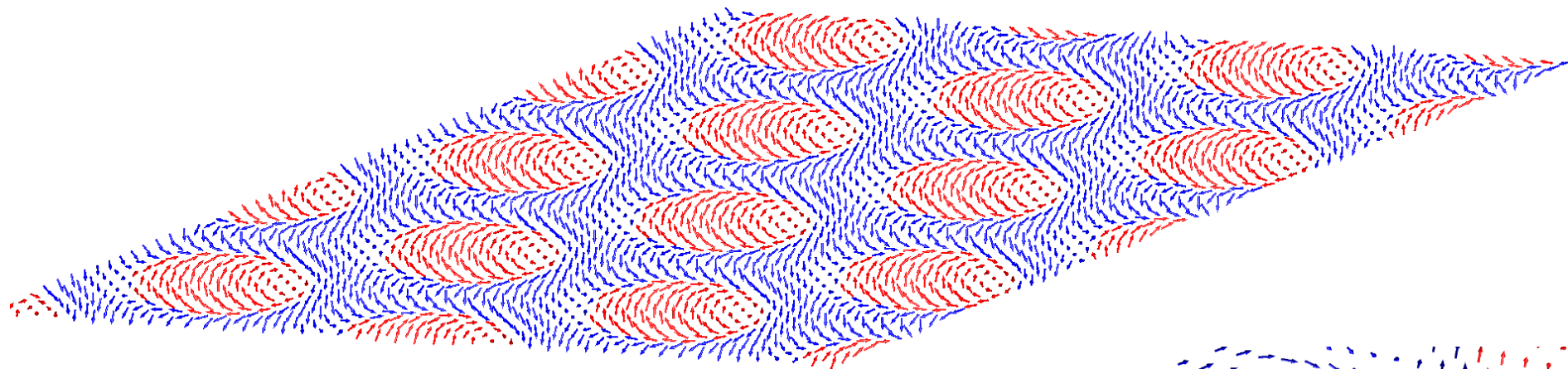


$$\lambda = 2\pi/Q \gg \text{unit cell}$$

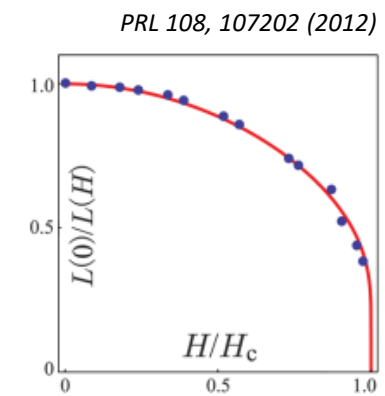
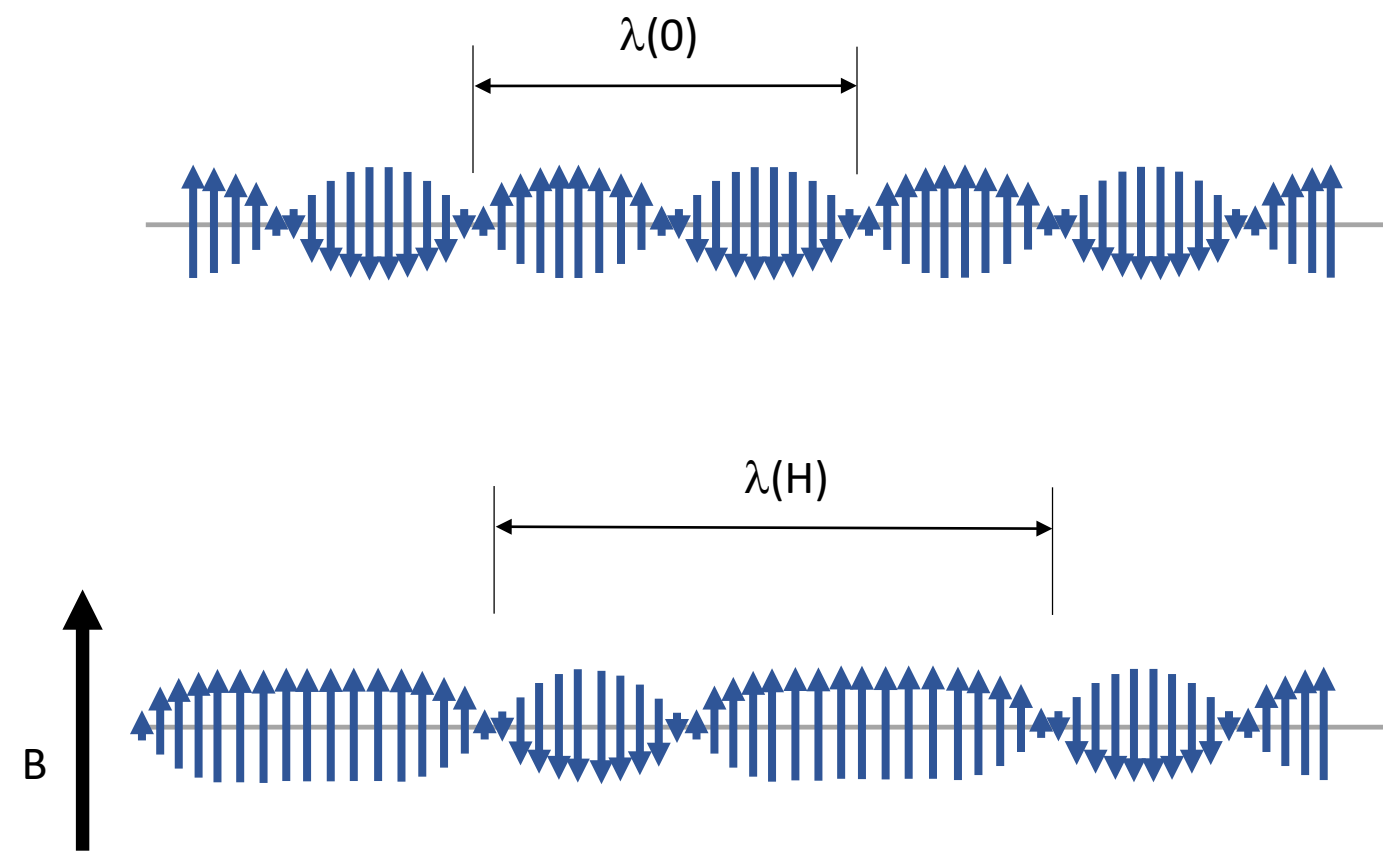
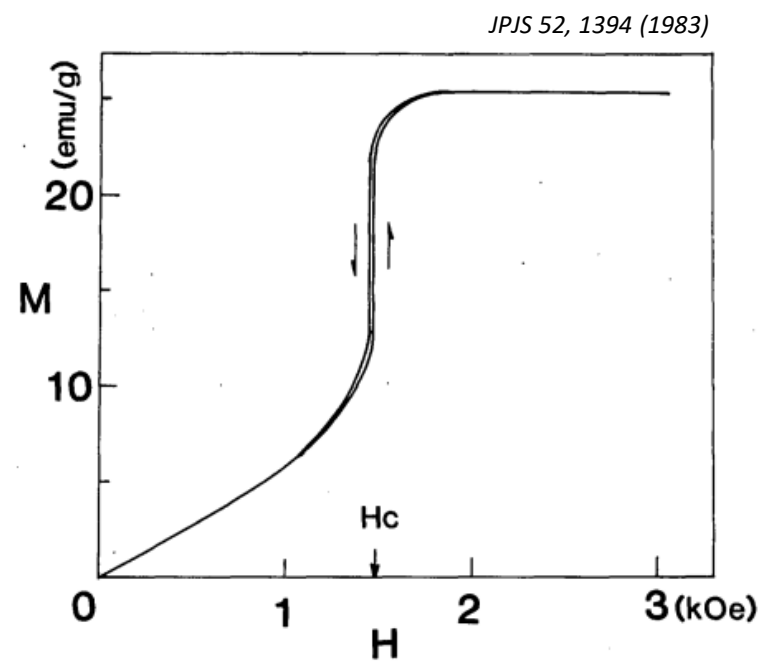
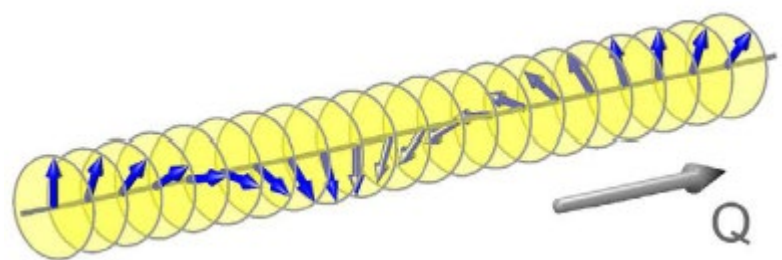
spiral order



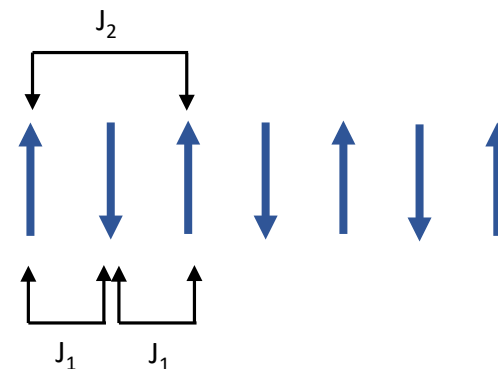
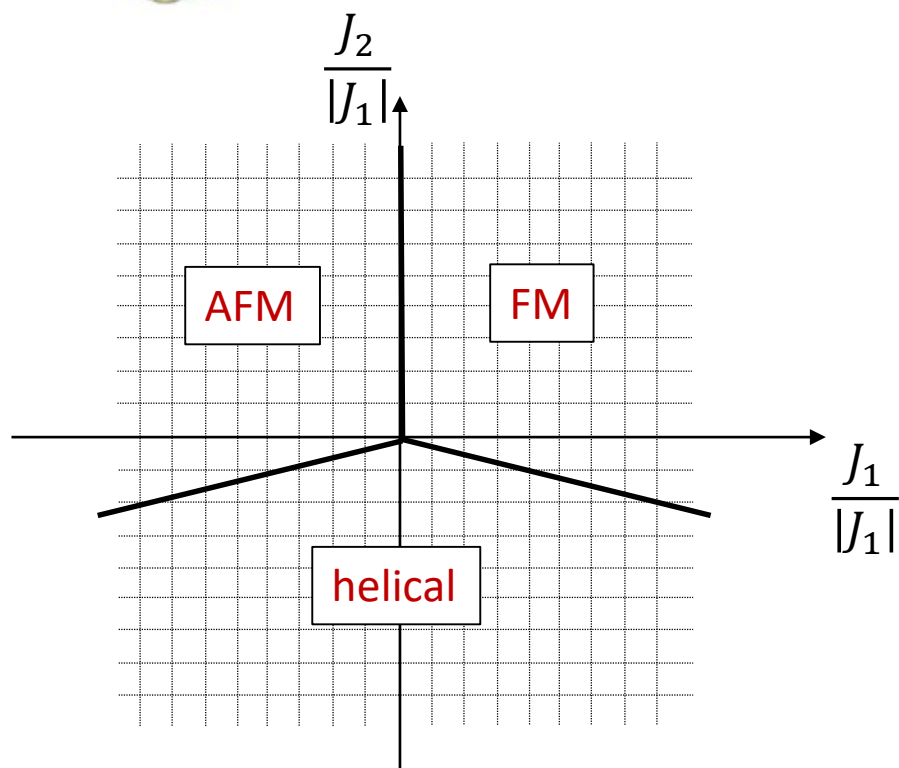
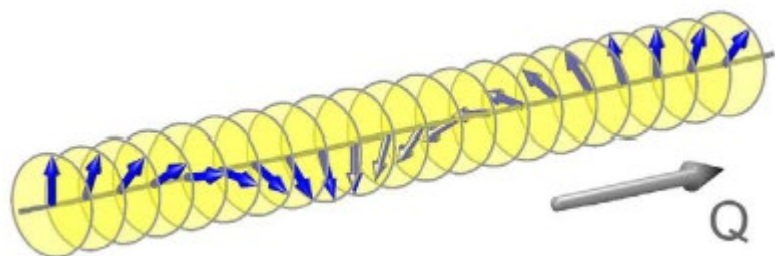
conical order (Q weakly coupled to lattice)

Skyrmion lattice

❖ soliton lattice ($\text{Cr}_{1/3}\text{NbS}_2$)



❖ helical order from AFM



$$\mathcal{H} = -J_1 \mathbf{S}_i \mathbf{S}_{i+1} - J_2 \mathbf{S}_i \mathbf{S}_{i+2}$$

$$E = -J_1 \cos \theta - J_2 \cos 2\theta$$

$$\frac{\partial E}{\partial \theta} = (J_1 + 4J_2 \cos \theta) \sin \theta = 0$$

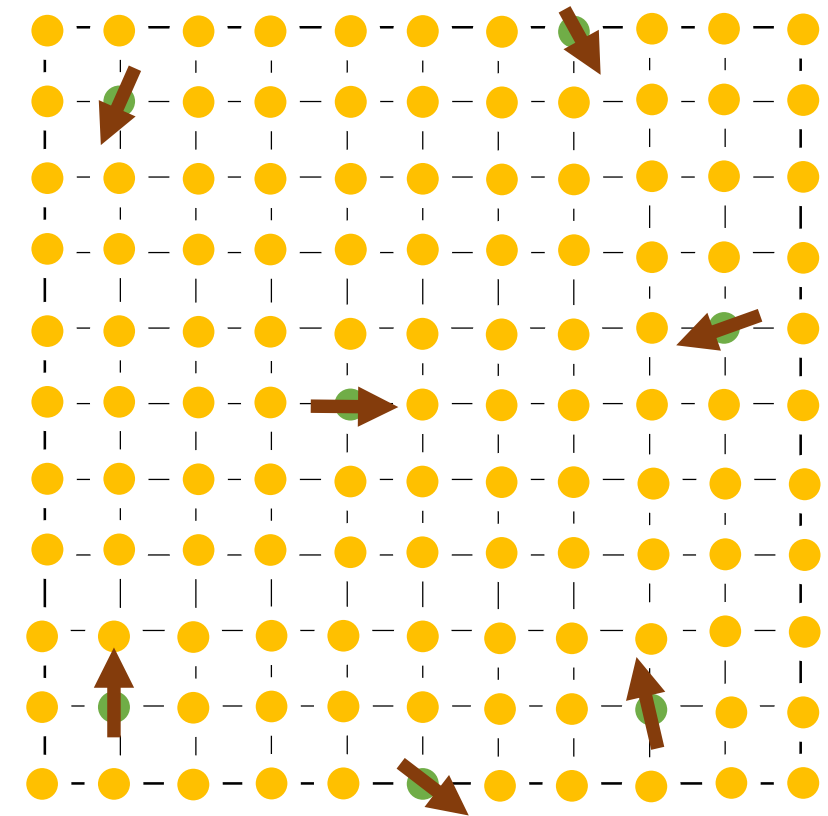
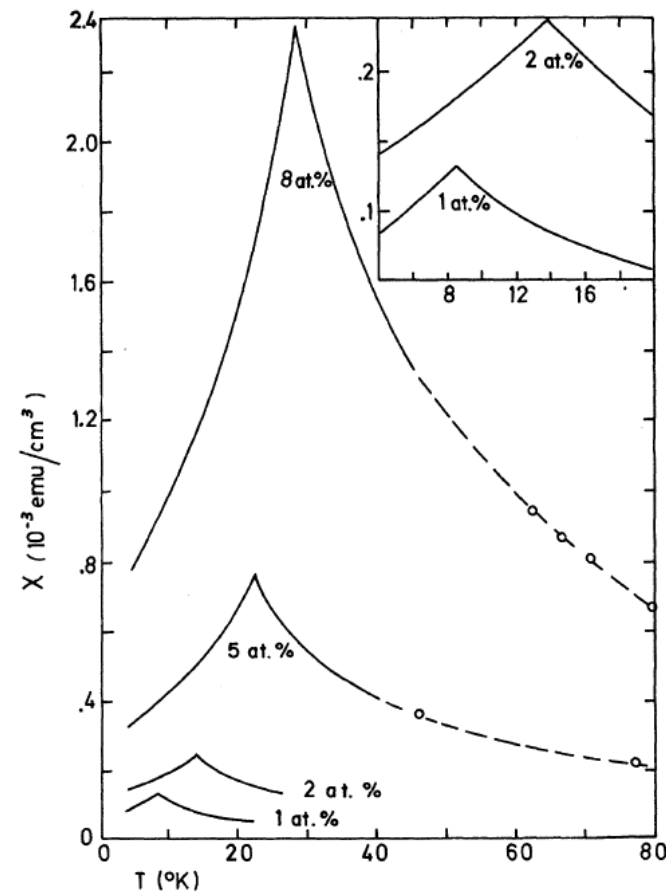
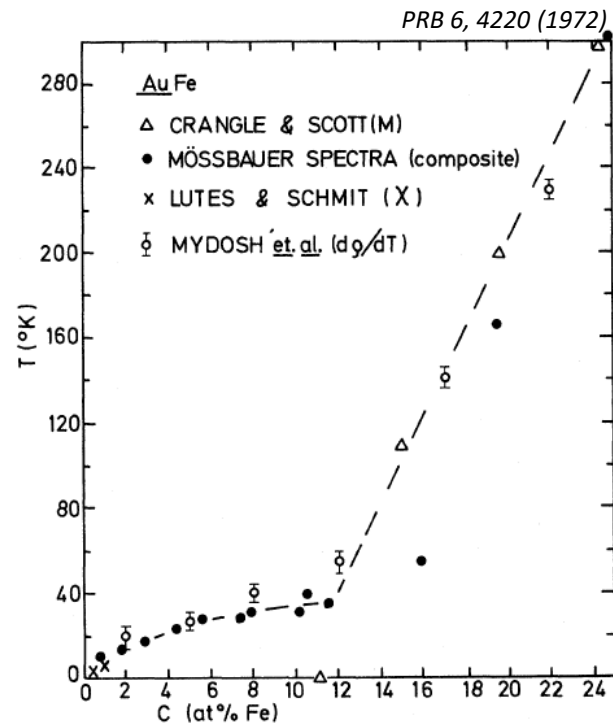
$\theta = 0$: FM
 $\theta = \pi$: AFM

$$\cos \theta = -\frac{J_1}{4J_2}$$

$$\downarrow J_2 < 0$$

$$\cos \theta = \frac{J_1}{4|J_2|} < 1$$

- ❖ often claimed, rarely proven
- ❖ started in '70s, not-so-good analogy with amorphous glasses
- ❖ a follow-up of a Kondo problem (interaction between localized and conduction electrons in a very dilute limit)

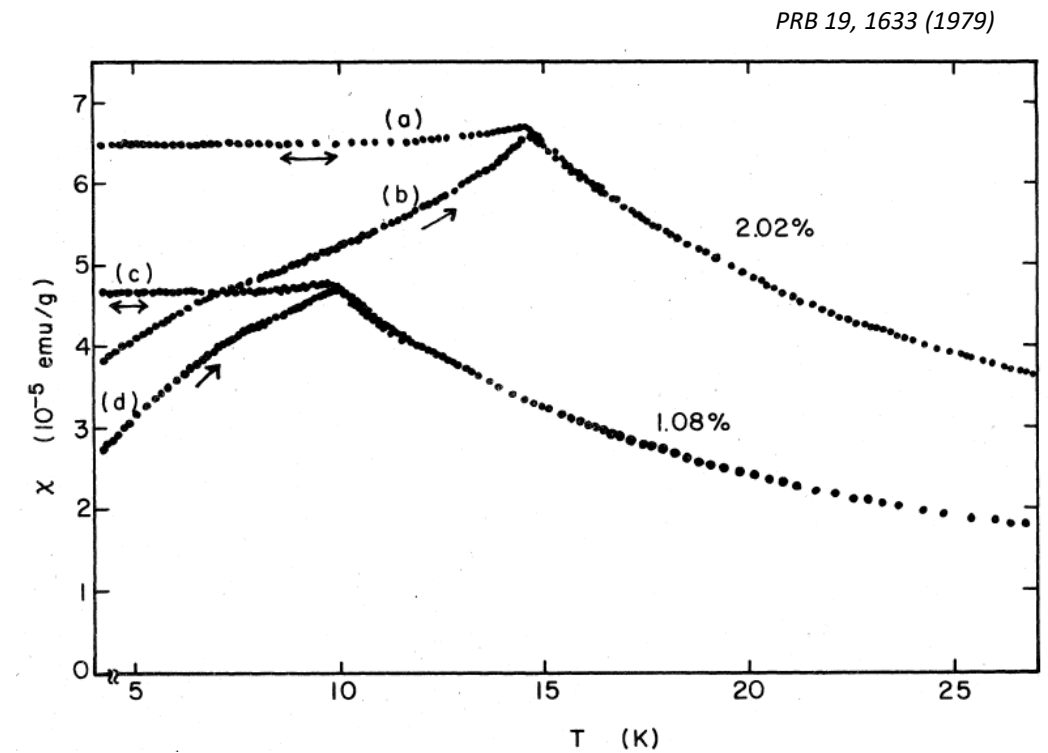


- 1) frequency dependence of T_g
- 2) FC/ZFC protocol
- 3) specific heat
- 4) aging, memory and rejuvenation effects



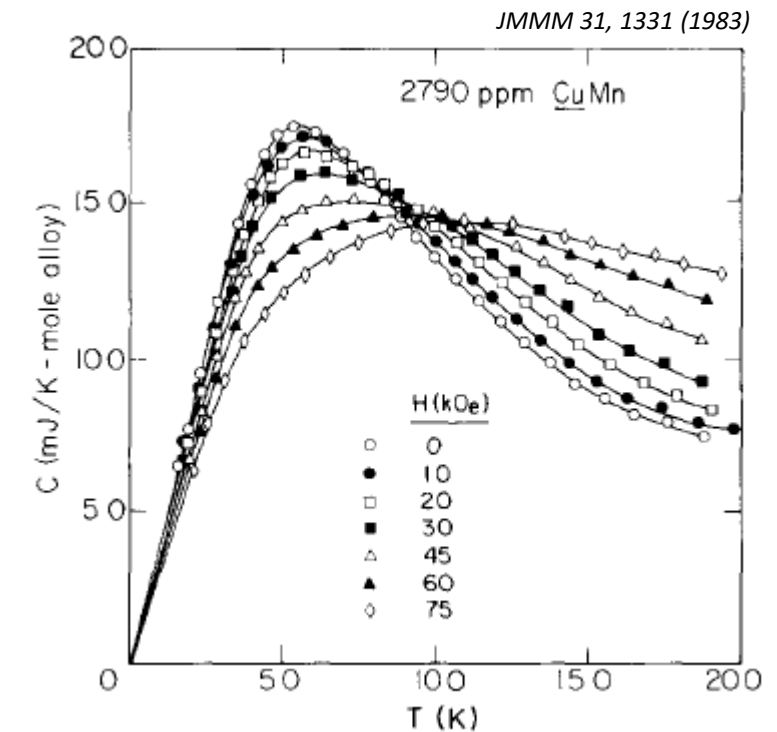
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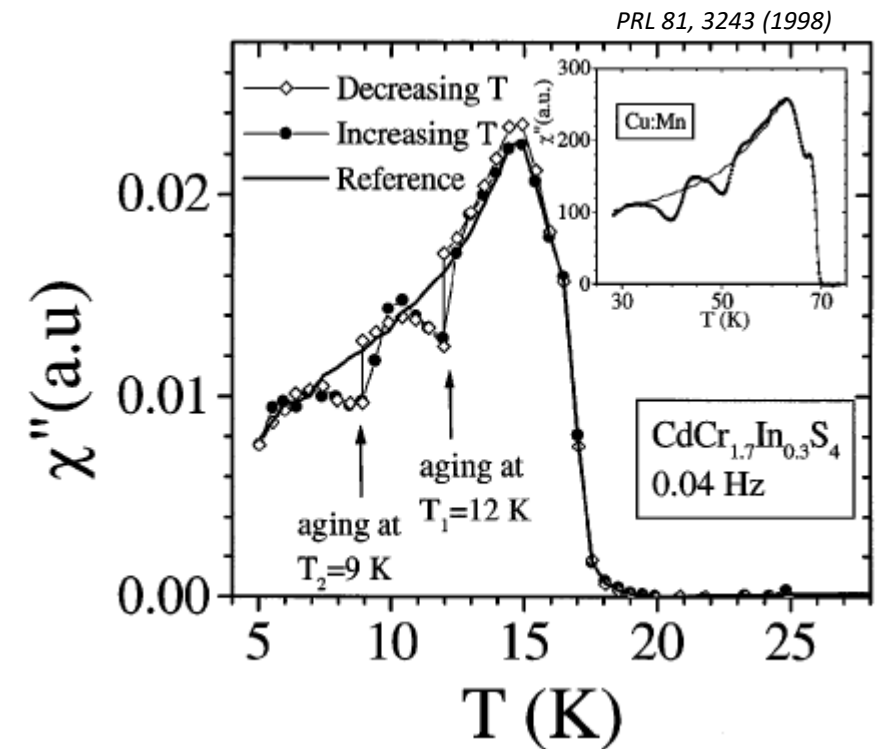
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Infinite Number of Order Parameters for Spin-Glasses

G. Parisi

Servizio Documentazione, Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali di Frascati, Frascati, Italy

(Received 22 June 1979)

This Letter shows that in the mean-field approximation spin-glasses must be described by an infinite number of order parameters in the framework of the replica theory.

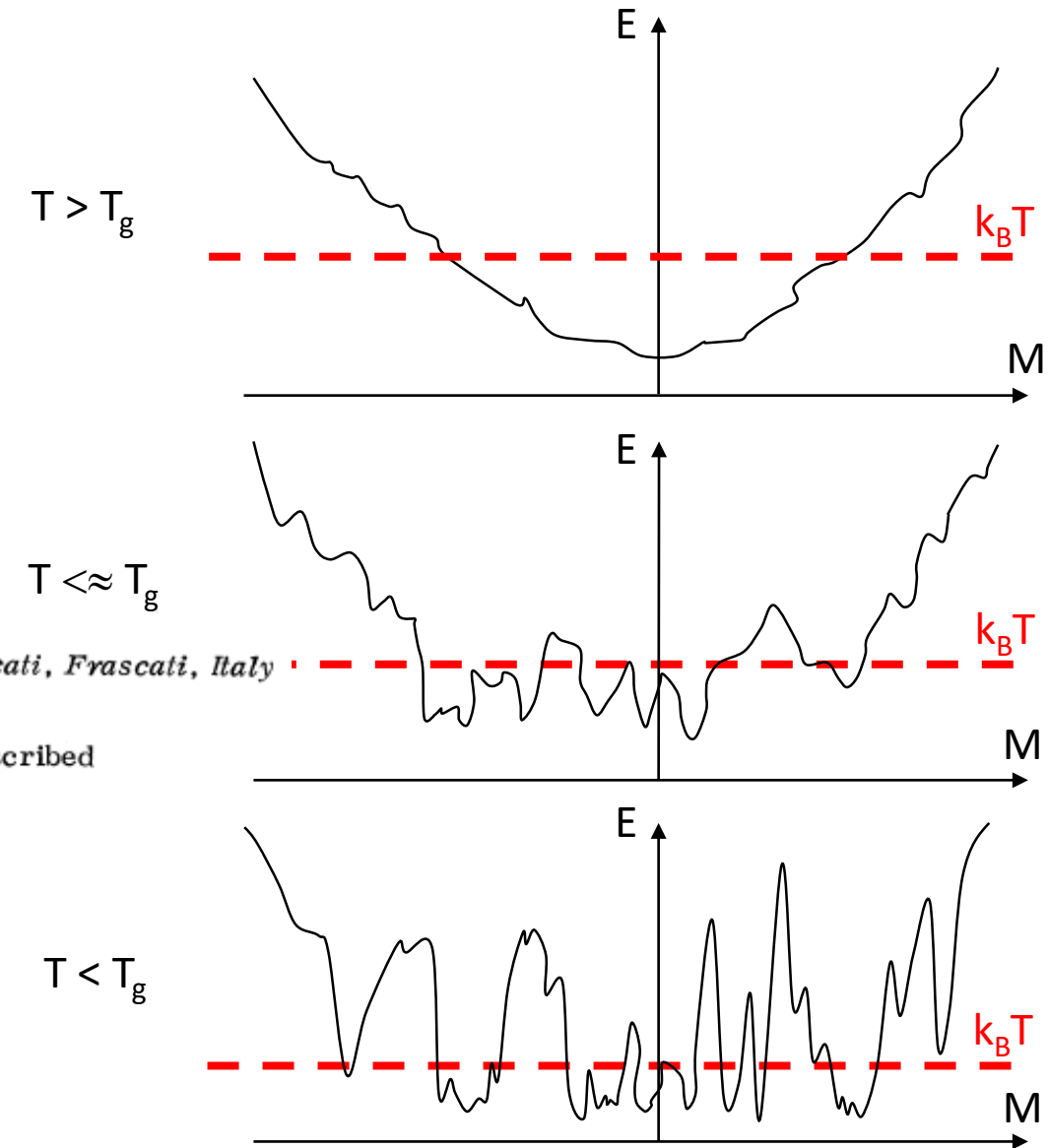
Order Parameter for Spin-Glasses

Giorgio Parisi

Università di Roma II, Tor Vergata, Rome, Italy, and Laboratori Nazionali, Istituto Nazionale di Fisica Nucleare, Frascati, Italy

(Received 1 February 1983)

An order parameter for spin-glasses is defined in a clear physical way: It is a function on the interval 0–1 and it is related to the probability distribution of the overlap of the magnetization in different states of the system. It is shown to coincide with the order parameter introduced by use of the broken replica-symmetry approach.



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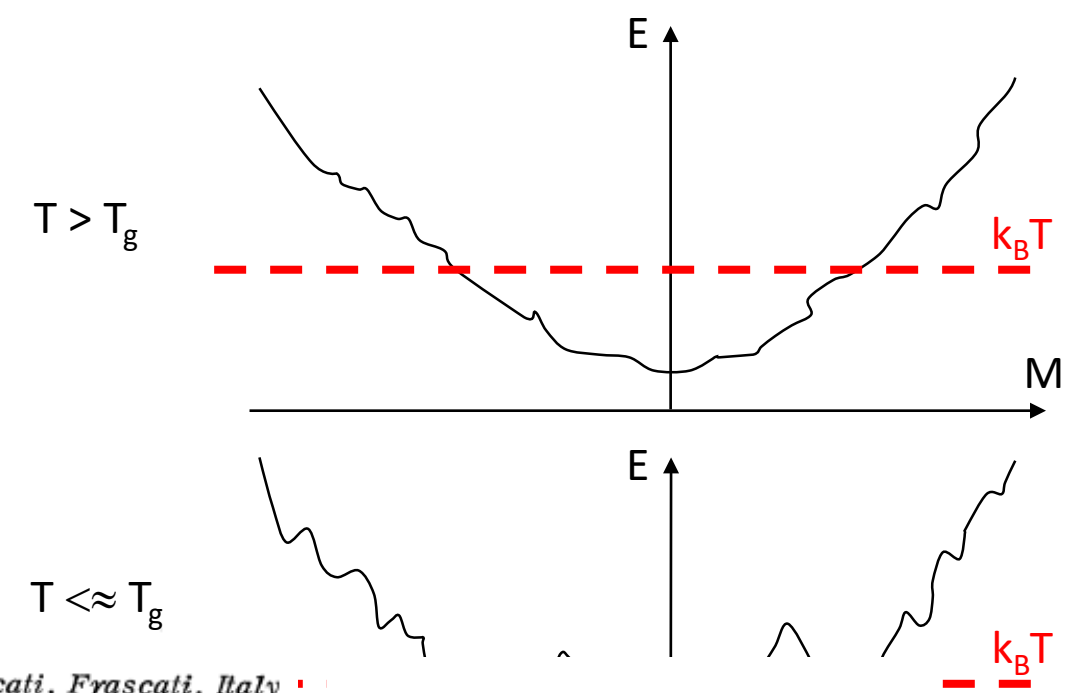
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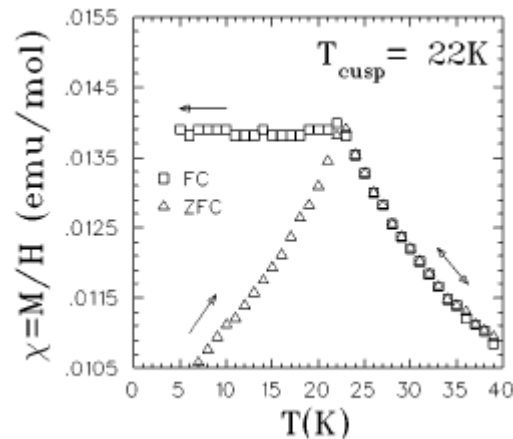
Static Critical Behavior of the Spin-Freezing Transition in the Geometrically Frustrated Pyrochlore Antiferromagnet Y₂Mo₂O₇

M. J. P. Gingras,^{1,*} C. V. Stager,² N. P. Raju,³ B. D. Gaulin,² and J. E. Greedan³
¹TRIUMF, Theory Group, 4004 Wesbrook Mall, Vancouver, British Columbia, V6T-2A3, Canada

²Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, L8S 4M1, Canada

³Department of Chemistry, McMaster University, Hamilton, Ontario, L8S 4M1, Canada
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Some frustrated pyrochlore antiferromagnets, such as Y₂Mo₂O₇, show a spin-freezing transition and magnetic irreversibilities below a temperature T_f similar to what is observed in *randomly* frustrated spin-glasses. We present results of nonlinear dc magnetization measurements on Y₂Mo₂O₇ that provide strong evidence that there is an underlying thermodynamic phase transition at T_f , which is characterized by critical exponents $\gamma \approx 2.8$ and $\beta \approx 0.8$. These values are typical of those found in random spin-glasses, despite the fact that the level of random disorder in Y₂Mo₂O₇ is immeasurably small.



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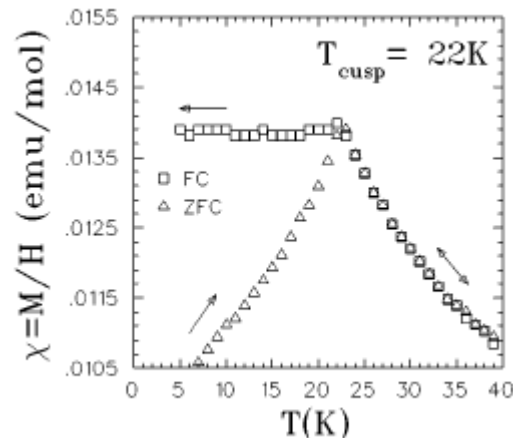
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Self-induced spin glass state in elemental and crystalline neodymium

Umut Kamber¹, Anders Bergman², Andreas Eich¹, Diana Iuşan², Manuel Steinbrecher¹, Nadine Hauptmann¹, Lars Nordström², Mikhail I. Katsnelson¹, Daniel Wegner^{1,*}, Olle Eriksson^{2,3}, Alexander A. Khajetoorians^{1,*}

Spin glasses are a highly complex magnetic state of matter intricately linked to spin frustration and structural disorder. They exhibit no long-range order and exude aging phenomena, distinguishing them from quantum spin liquids. We report a previously unknown type of spin glass state, the spin-Q glass, observable in bulk-like crystalline metallic neodymium thick films. Using spin-polarized scanning tunneling microscopy combined with ab initio calculations and atomistic spin-dynamics simulations, we visualized the variations in atomic-scale noncollinear order and its response to magnetic field and temperature. We quantified the aging phenomena relating the glassiness to crystalline symmetry and the energy landscape. This result not only resolves the long-standing debate of the magnetism of neodymium, but also suggests that glassiness may arise in other magnetic solids lacking extrinsic disorder.

Kamber *et al.*, *Science* **368**, eaay6757 (2020)
