MAGNETISM IN MATERIALS

Lecture 5: Magnetic Ordering

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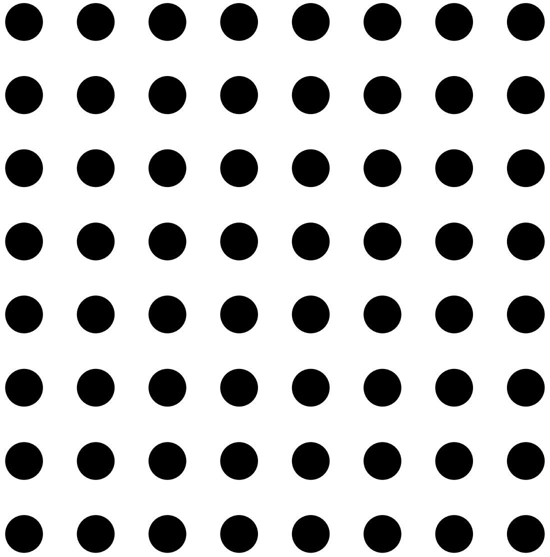
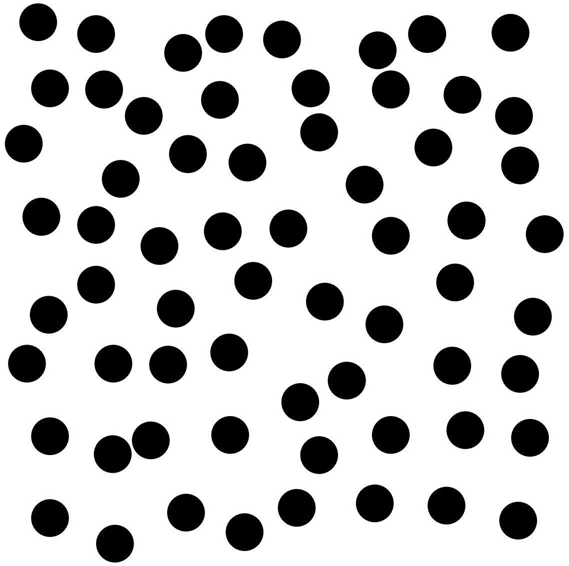
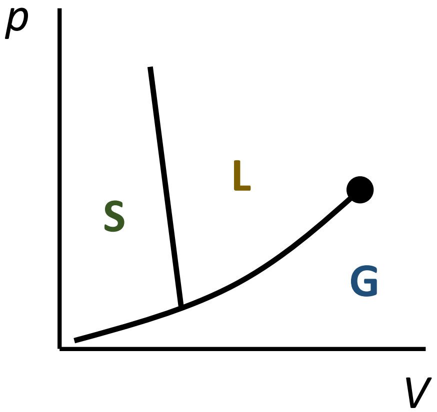
EPFL

- what is order?
- ferromagnetism
- Weiss (mean-field) model
- critical behavior
- symmetry breaking
- excitations

- analogy with crystalline matter: what defines a solid, liquid, gas?
- regular patterns? correlations? density?







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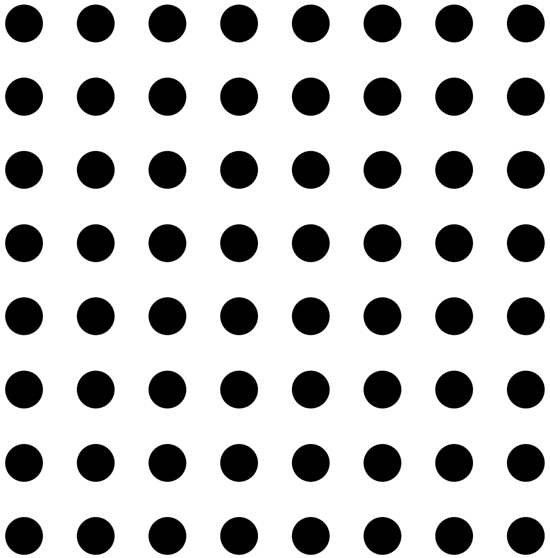

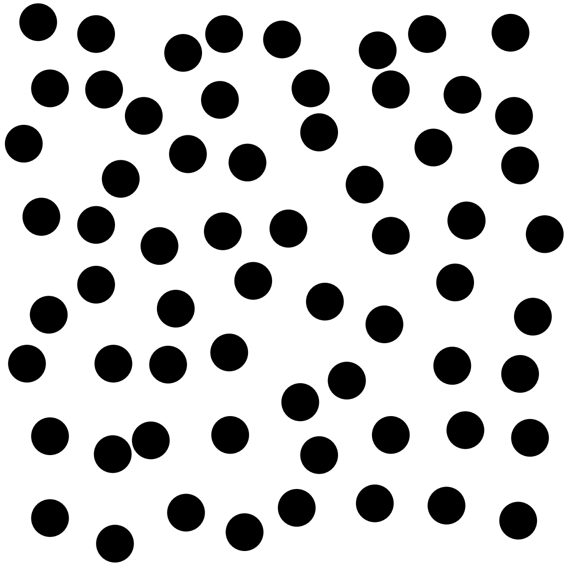

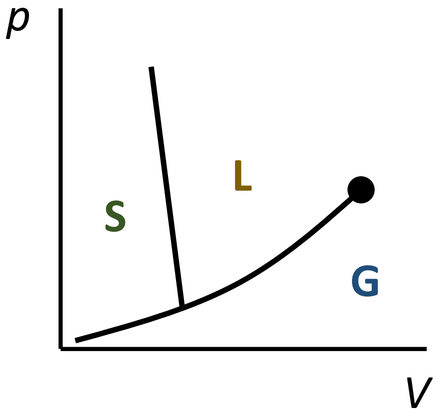

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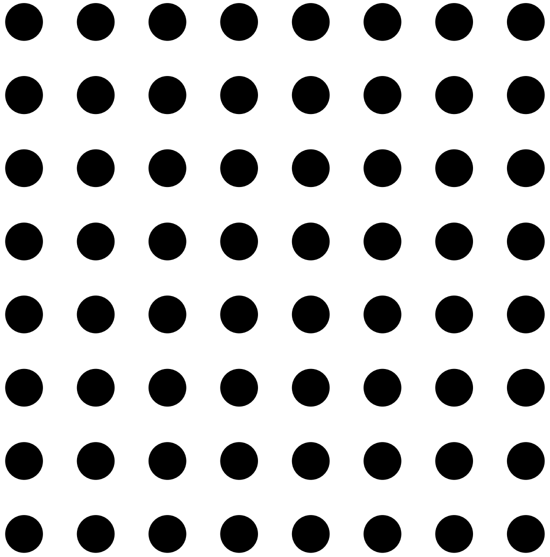
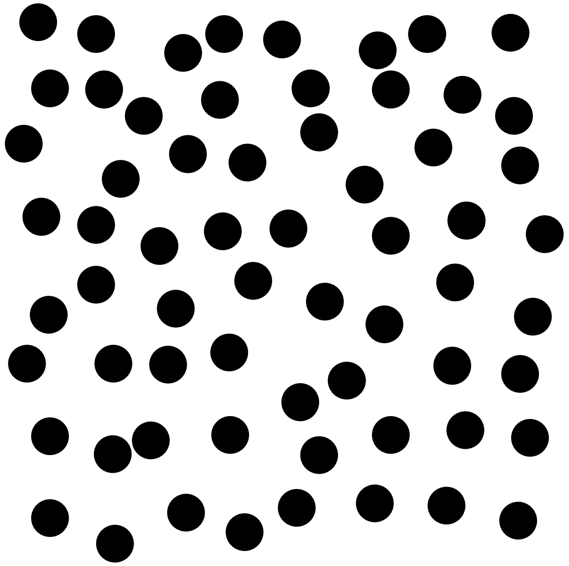




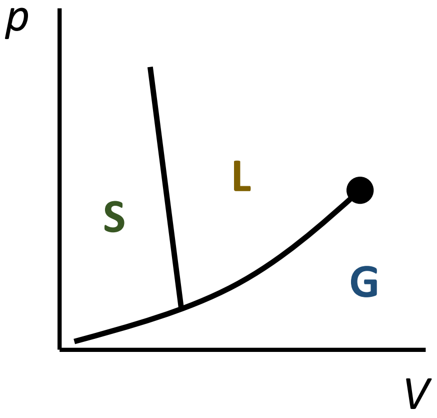
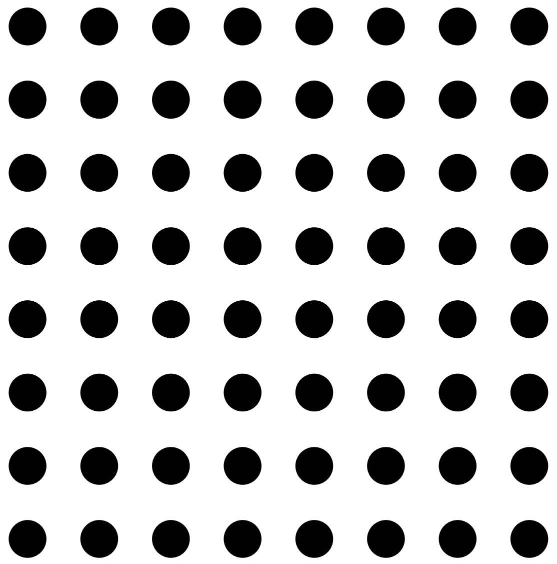
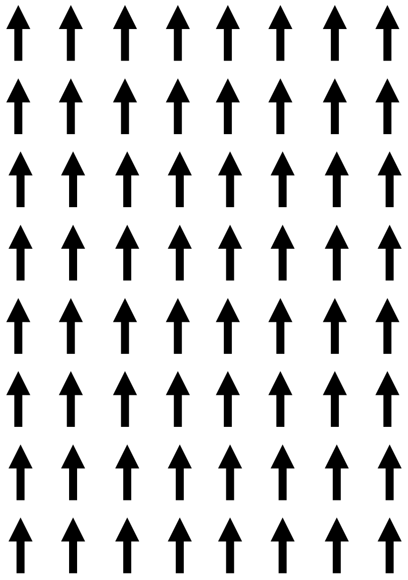
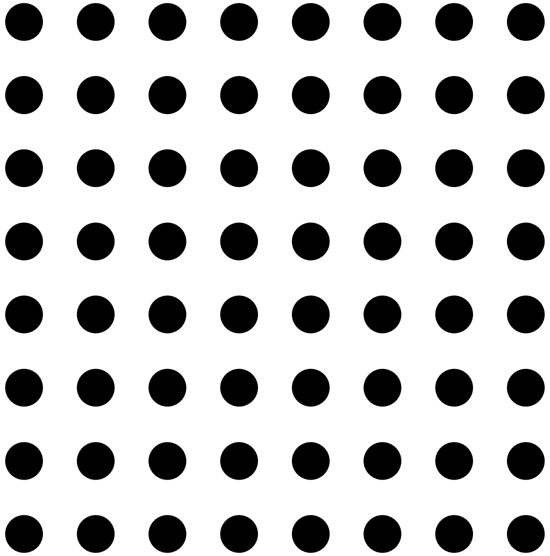
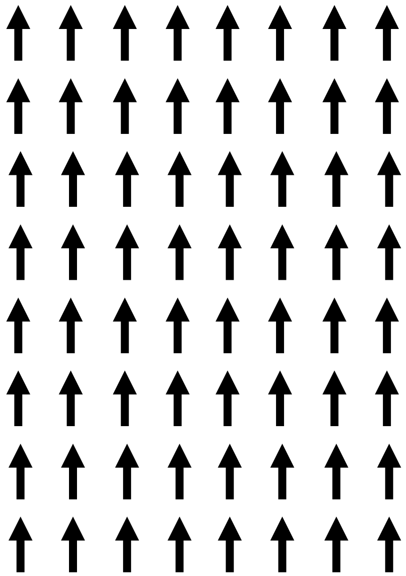


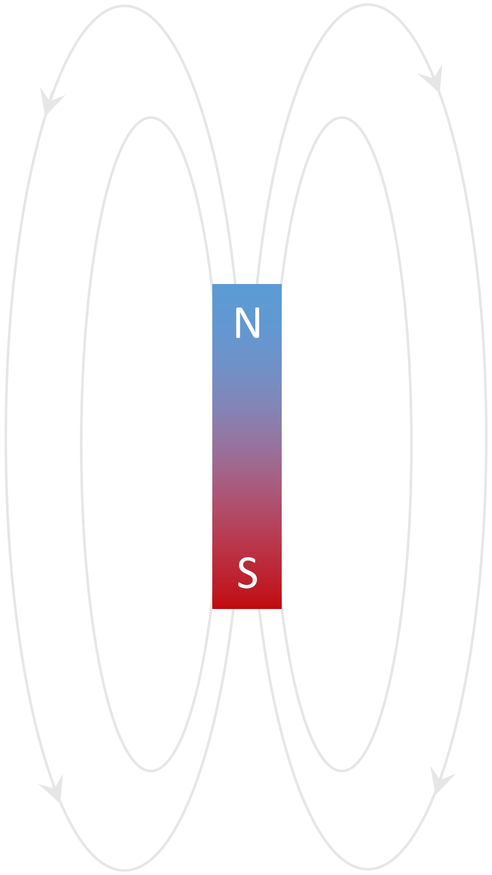
image source: wikimedia.org

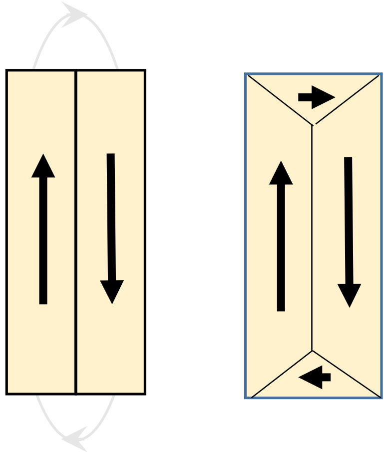




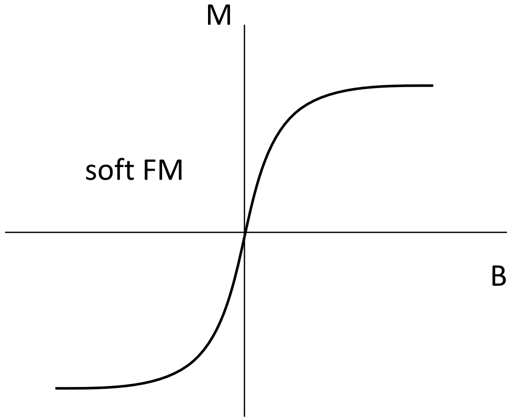


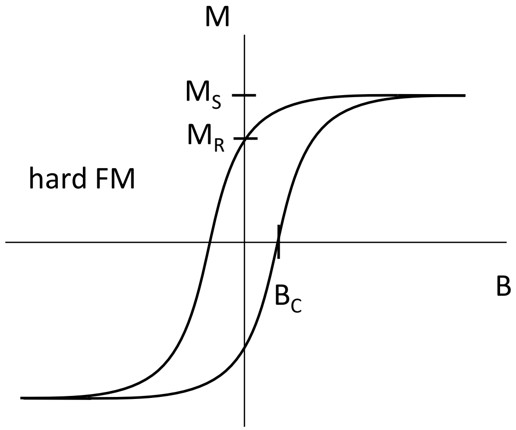




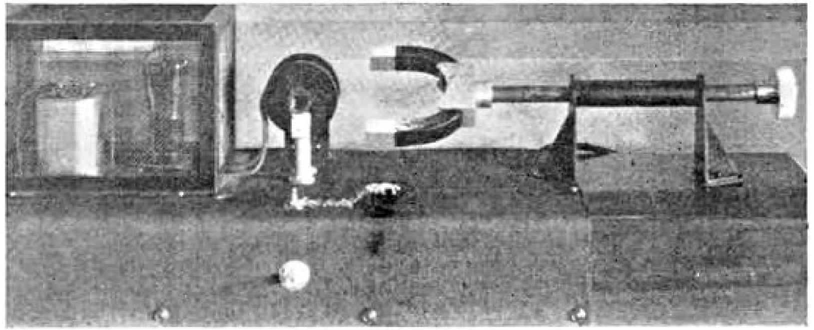


Bloch walls
(Neel for thin films)





Barkhausen effect



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- mean-field approach (Weiss model)

$$\hat{\mathcal{H}} = -\sum_{i,j} J_{ij}\boldsymbol{S}_i\boldsymbol{S}_j + g\mu_B\boldsymbol{B}_{out}\sum_i \boldsymbol{S}_i$$

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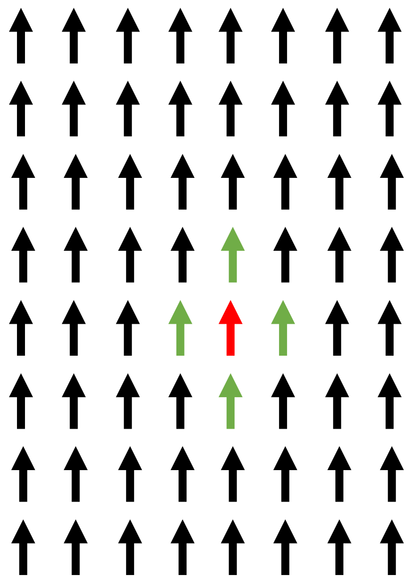

$$\boldsymbol{B}_{mf} = -\frac{2}{g\mu_B}\sum_j J_{ij}\boldsymbol{S}_j$$

$$\boldsymbol{B}_{mf} \sim \sum_j \boldsymbol{S}_j = \boldsymbol{M}$$

$$\boldsymbol{B}_{mf} = \lambda\boldsymbol{M}$$

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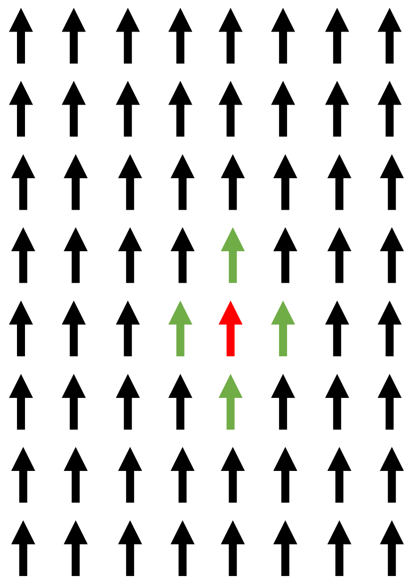
$$\hat{\mathcal{H}} = g\mu_B(\boldsymbol{B}_{out} + \boldsymbol{B}_{mf})\sum_i \boldsymbol{S}_i$$

'paramagnetic' Hamiltonian

$$M = g\mu_B S B_S(x)$$

$$x = \frac{g\mu_B S(B_{out}+\lambda M)}{k_B T}$$



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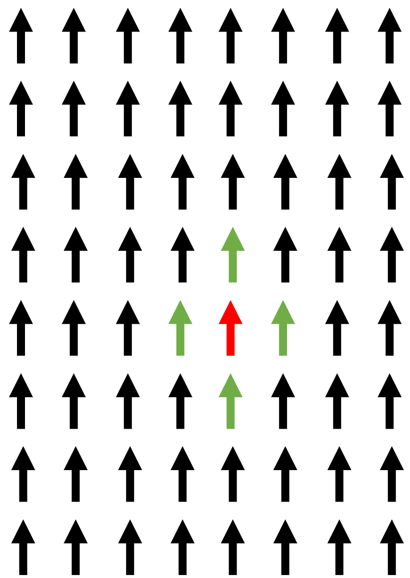

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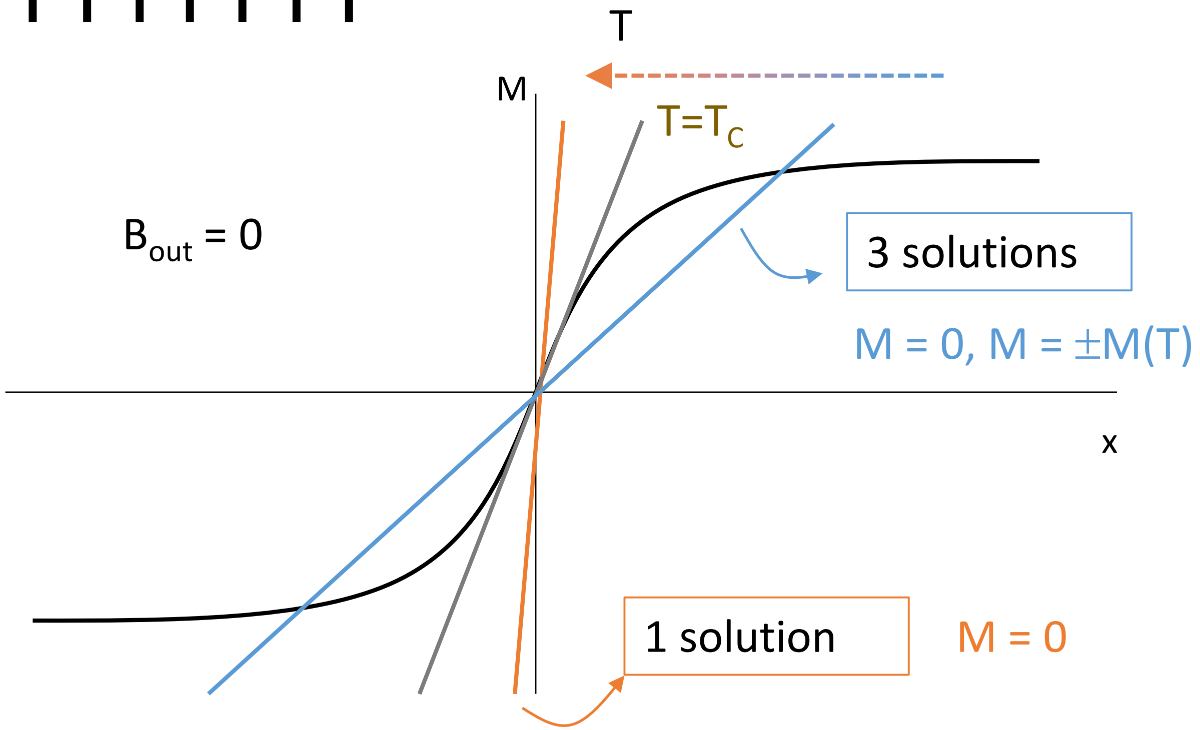


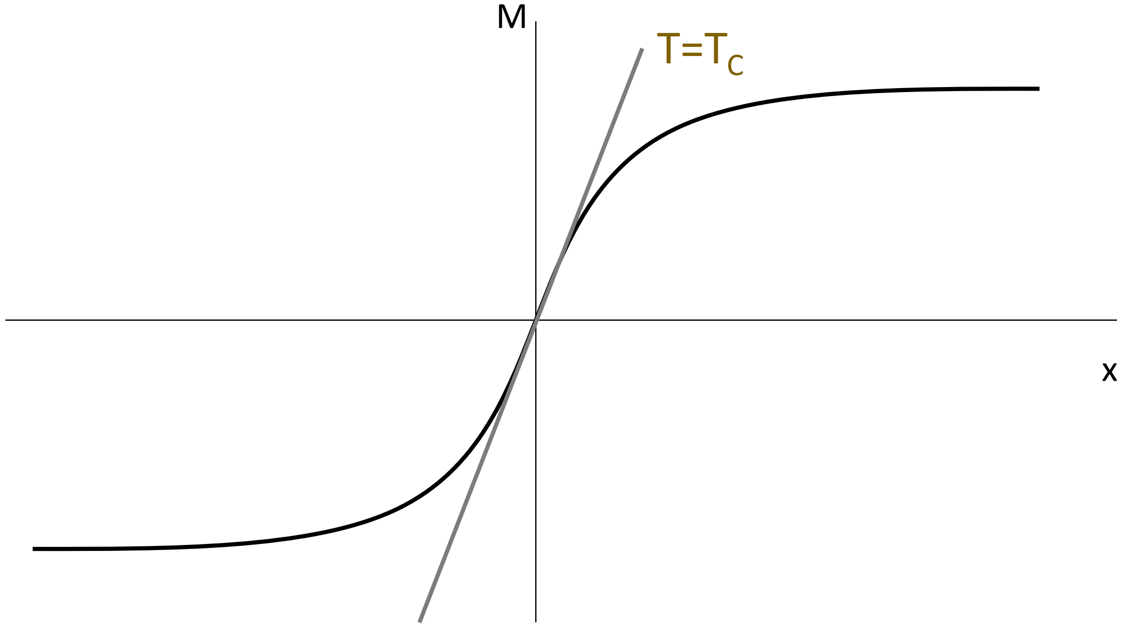
$x \ll 1$

$$B_S(x) \approx \frac{S+1}{3S}x$$

$$T_C = \frac{g\mu_B(S+1)\lambda M_S}{3k_B}$$

S = 1/2, T_C = 1000 K

B_{mf} = 1500 T



- paramagnet → Curie law

$$\chi = \frac{C}{T} = \frac{\mu_0 M}{B} \qquad \frac{C}{T} = \frac{\mu_0 M}{B + \lambda M}$$

$$\chi = \frac{C}{T - T_C}$$ Curie-Weiss law

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- dependence on magnetic field below T_C

$$B_S(x) \approx \frac{S+1}{3S} x - kx^3 \rightarrow M \sim B^{\frac{1}{3}}$$

for B ≠ 0 there is no transition!!!

- paramagnet → Curie law

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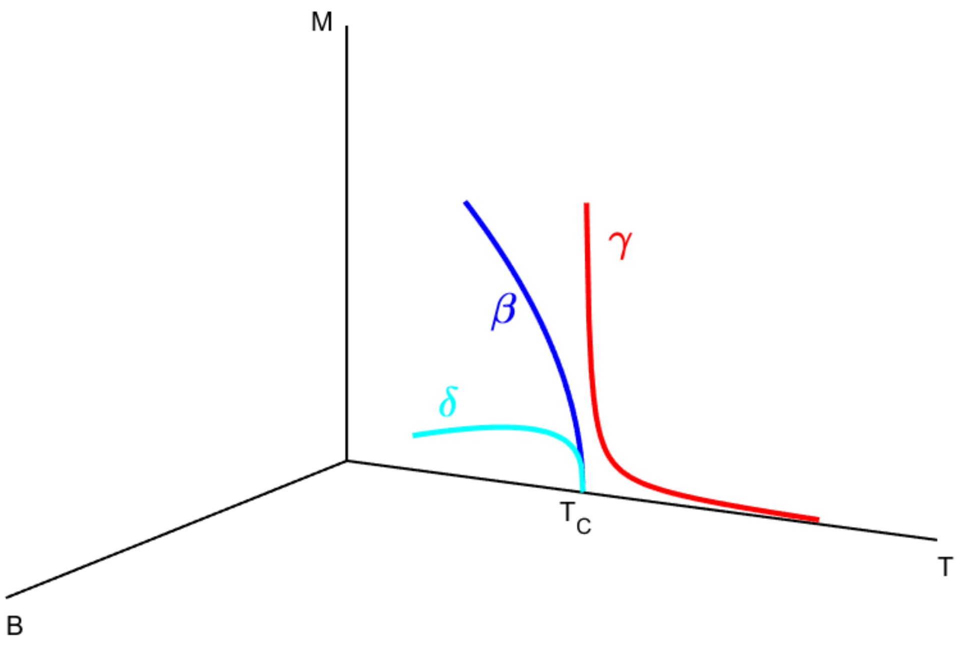

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critical exponent	observable	mean-field	Heisenberg	XY	Ising (3D)	Ising (2D)
β	$M \sim \lvert\varepsilon\rvert^{\beta}$	1/2	0.37	0.35	0.33	1/8
δ	$M \sim B^{\frac{1}{\delta}}$	3	4.8	4.8	4.8	15
γ	$\chi \sim \varepsilon^{-\gamma}$	1	1.39	1.32	1.24	7/4
α	$C_P \sim \varepsilon^{-\alpha}$	0	-0.12	-0.015	0.11	0
ν	$\xi \sim \varepsilon^{-\nu}$	1/2	0.71	0.67	0.63	1
η	$\langle\varphi(0)\varphi(r)\rangle \sim r^{-d+2-\eta}$	0	0.035	0.038	0.036	1/4

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- dependence on magnetic field below T_C

$$B_S(x) \approx \frac{S+1}{3S} x - kx^3 \rightarrow M \sim B^{\frac{1}{3}}$$

- mean-field disregards fluctuations and correlations
- calculated by various methods
- not independent (Widom relations) $\gamma = \beta(\delta - 1)$
- not necessarily identical above and below the transition

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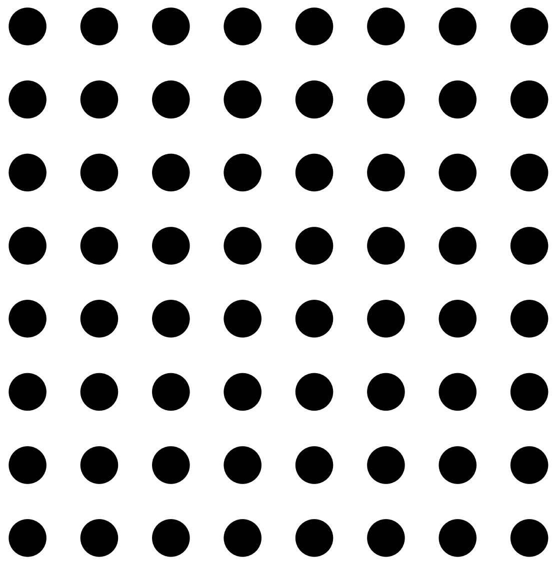
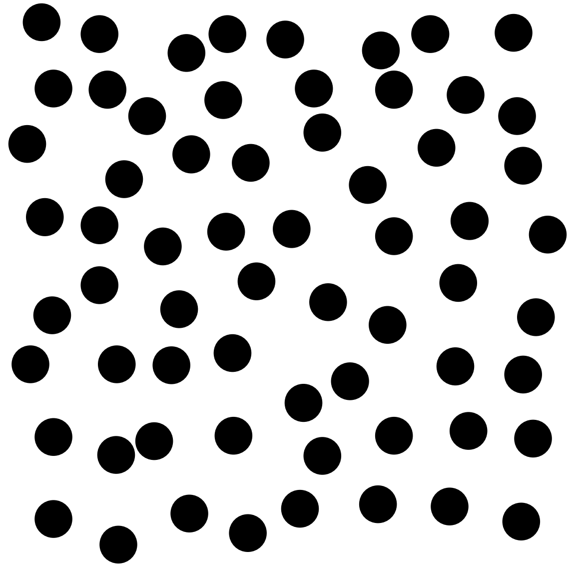
$$F = E - TS$$

disorder

order





translation, rotation

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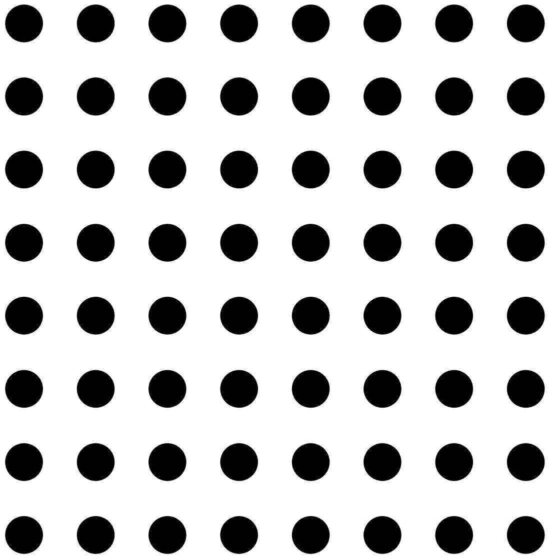
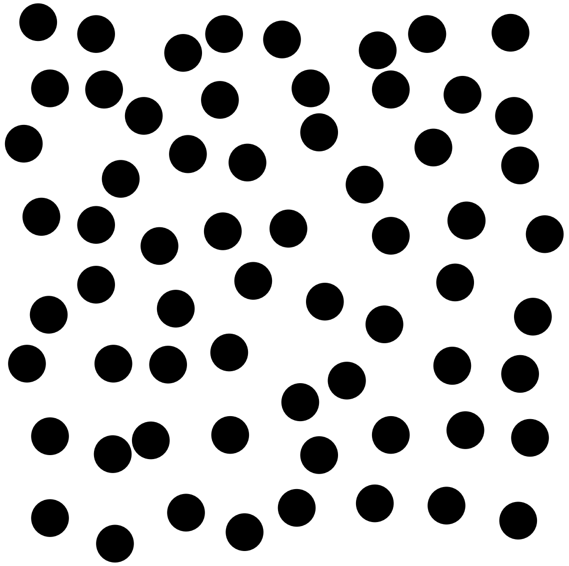
disorder

order



translation, rotation



The New York Times

Opinion

OP-ED CONTRIBUTOR

Why the Higgs Boson Matters

By Steven Weinberg

July 13, 2012

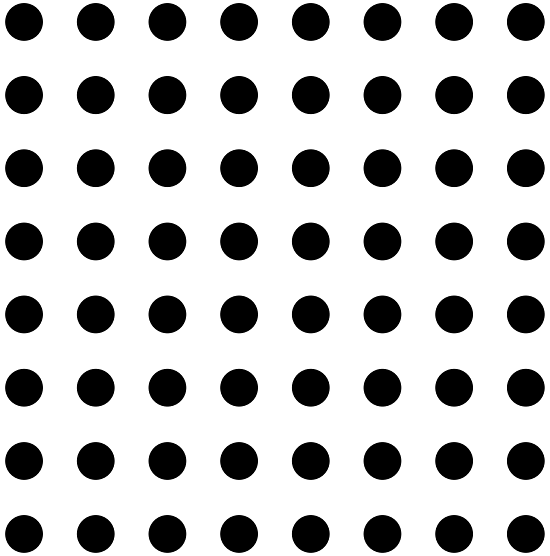
Give this article

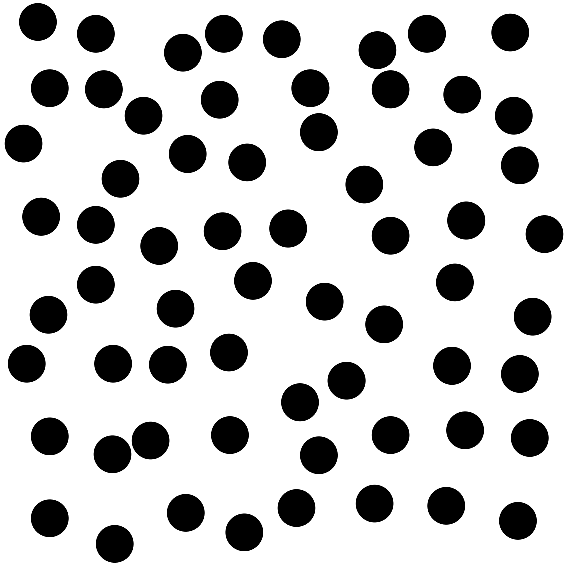
Daniel Haskett

$$F = E - TS$$

disorder

order





translation, rotation

We have a well-tested theory of elementary particles and the forces that they exert on each other, known as the Standard Model. A central feature of the Standard Model is a symmetry between two of these forces: the electromagnetic force, and the less familiar weak nuclear force, which provides the first step in the chain of reactions that gives the sun its energy.

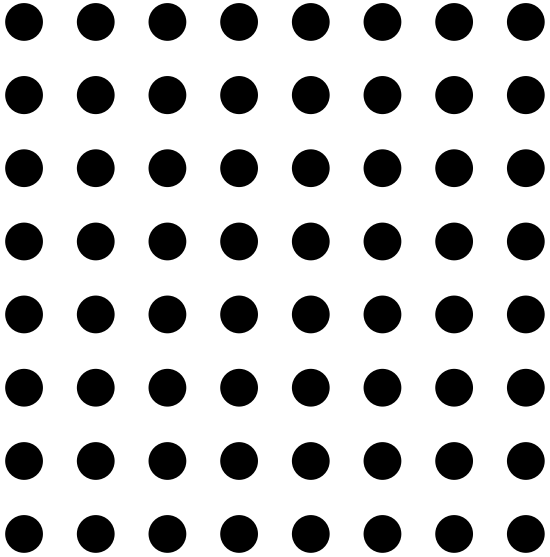
MAGNETIC ORDERING

symmetry breaking

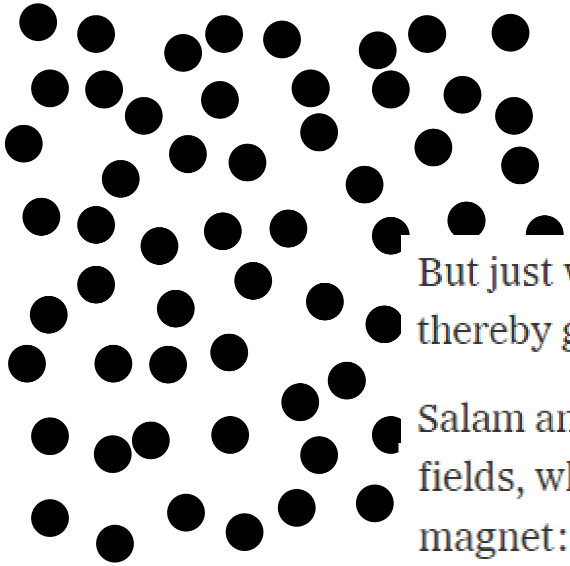
$$F = E - TS$$

disorder

order



translation, rotation



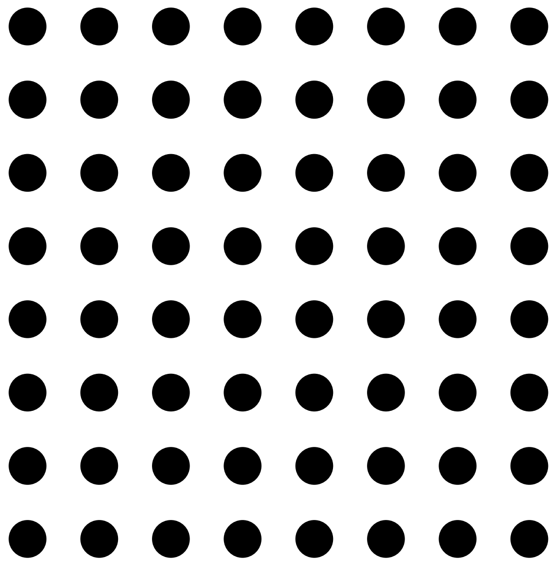
But just what is it that breaks the electroweak symmetry and thereby gives elementary particles their masses?

Salam and I assumed that the culprit is what are called scalar fields, which pervade all space. This is like what happens in a magnet: Even though the equations describing iron atoms don’t distinguish one direction in space from another, any magnetic field produced by the atoms will point in just one way. The symmetry-breaking fields in the Standard Model do not mark out directions in space — instead, they distinguish the weak from the electromagnetic forces, and give elementary particles their masses. Just as a magnetic field appears in iron when it cools and solidifies, these scalar fields appeared as the early universe expanded and cooled.

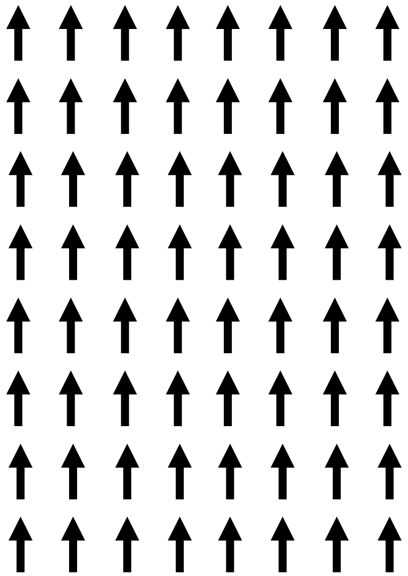
$$F = E - TS$$

disorder

order



~~translation~~, ~~rotation~~



translation, ~~rotation~~

?

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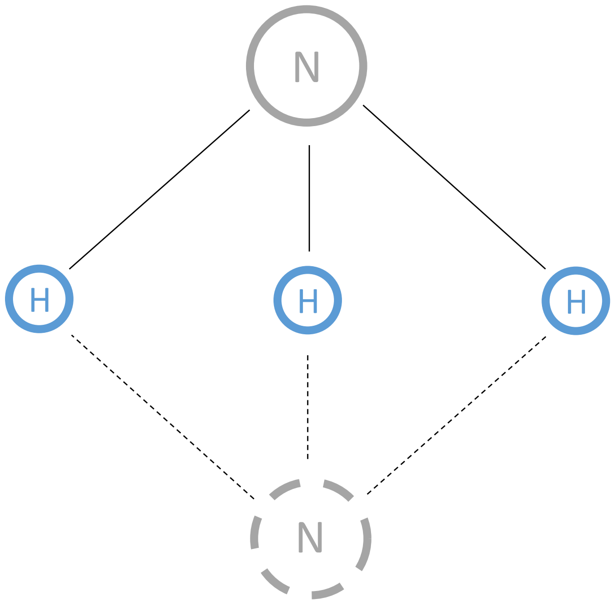


$F = E - TS$

disorder

order

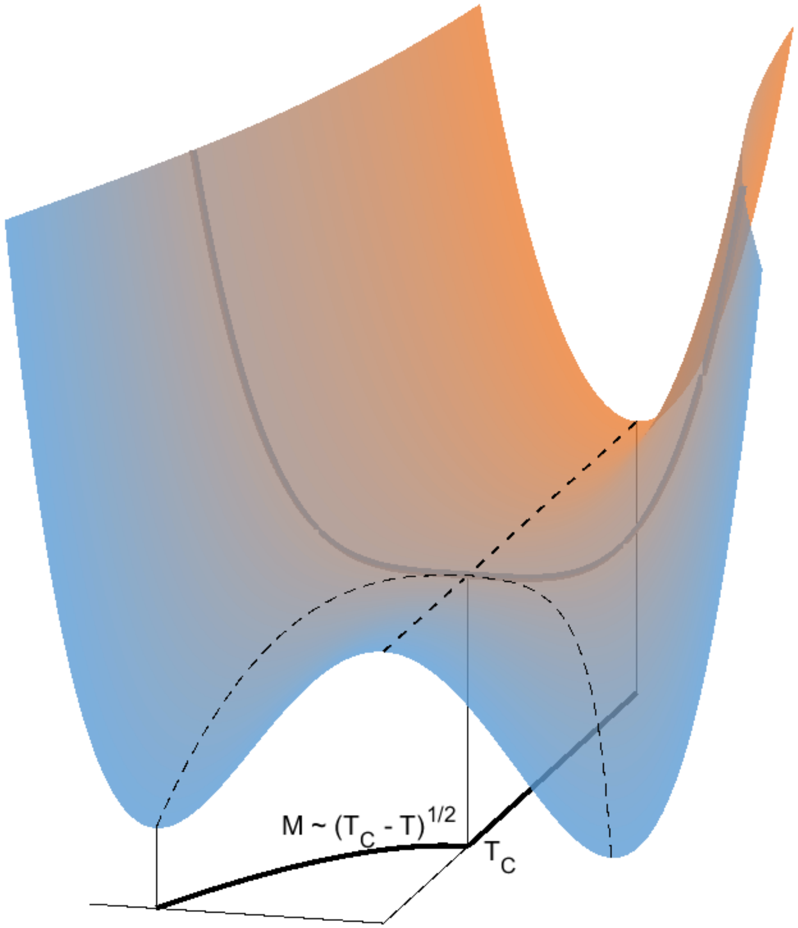
❖ Landau model of second-order phase transition

$$F(M) = F_0 + a_0(T - T_C)M^2 + bM^4$$

$$\frac{\partial F}{\partial M} = 0 \rightarrow M = 0$$

$$\rightarrow M = \pm K\sqrt{T_C - T}$$

(only close to T_C)



$$F = E - TS$$

disorder

order

$$\hat{\mathcal{H}} = -\sum_{i,j} J_{ij}\boldsymbol{S}_i\boldsymbol{S}_j$$

$$\frac{d\langle \boldsymbol{S}_i\rangle}{dt} = \frac{1}{i\hbar}\langle[\boldsymbol{S}_i, \hat{\mathcal{H}}]\rangle \xrightarrow[\text{dispersion}]{\text{magnon}} \hbar\omega = 4JS(1 - \cos qa)$$

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number of magnons: $N = \int_0^\infty \frac{g(\omega)d\omega}{e^{\hbar\omega/k_BT}-1}$

3D: $g(q)dq \sim q^2 dq$

$\Downarrow$ $T <<$

$g(\omega)d\omega \sim \omega^{1/2}d\omega$

Bloch law

$$\frac{M(0)-M(T)}{M(0)} \sim T^{3/2}$$

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NO

1D 2D

Mermin-Wagner-Berezinskii

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number of magnons:

$$N = \int_0^\infty \frac{g(\omega) d\omega}{e^{\hbar\omega / k_B T} - 1}$$

NO

1D YES 2D

Mermin-Wagner-Berezinskii

3D: $g(q)dq \sim q^2 dq$

$T <<$

$$g(\omega)d\omega \sim \omega^{1/2} d\omega$$

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