BASICS OF MRI: GRADIENTS AND KSPACE

Cristina Cudalbu
CIBM MRI EPFL AIT

PHYS 473

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OUTLINE

- Localization - Magnetic field gradients – gradient coils
- Spatial encoding
  - Slice selection
  - Frequency encoding
  - Phase encoding
- How to encode a 3D image
- Some notions about k-space
  - Discrete Fourier transform
  - What is where in k-space
  - How to fill the k-space

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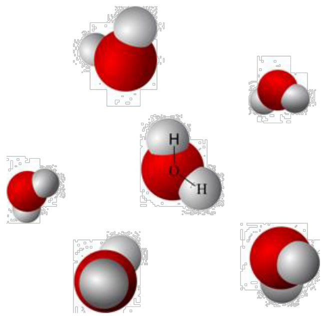
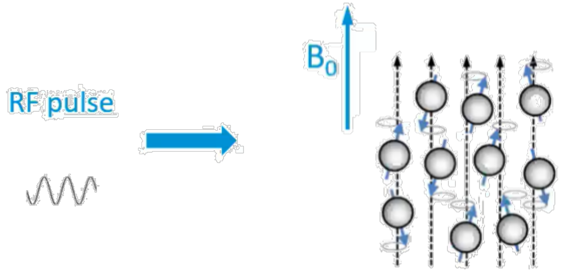
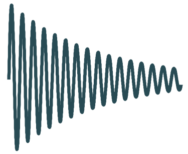
UNTIL NOW

Most MRI applications measure 1H nuclei from water

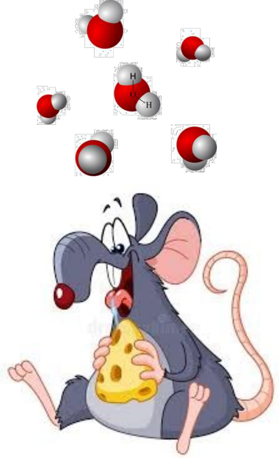


- **Static magnetic field (B_0)**
- **RF pulses**
- **FID – relaxation**
  - M_{xy} decays, M_z grows (T_2 and T_1 relaxation)

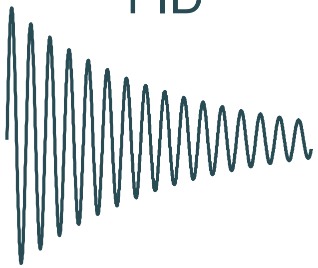




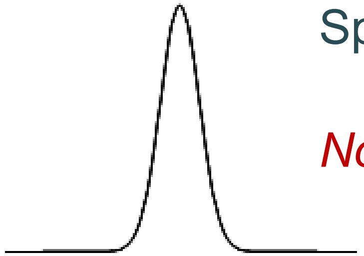
- **No spatial information - RF coils measure signal from entire body**



FID



FT →



Spectrum

No spatial information

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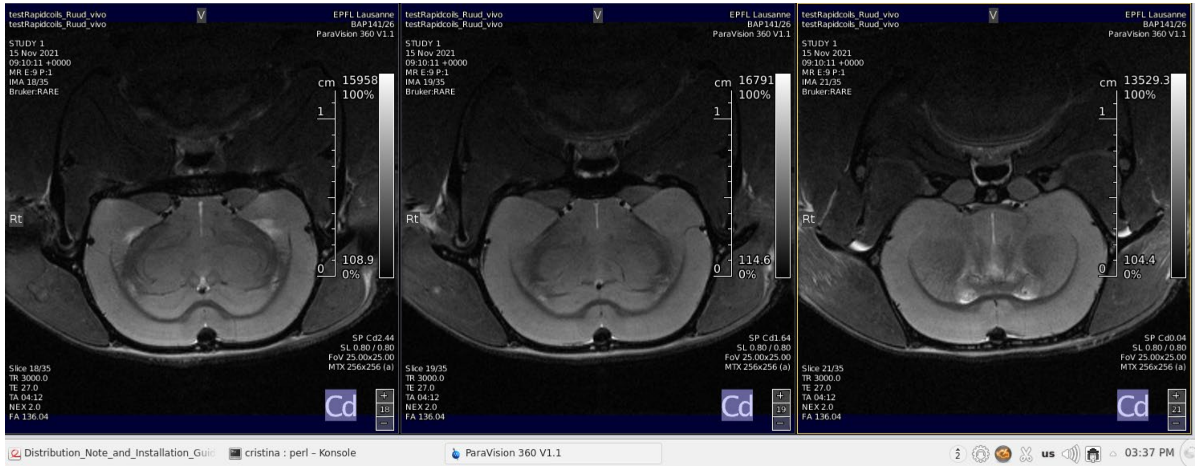
?



MRI – Magnetic Resonance Imaging

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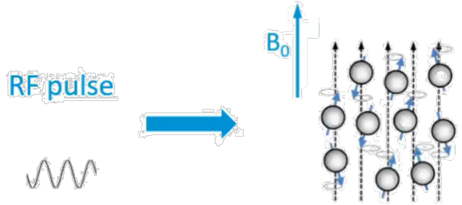
MAGNETIC RESONANCE IMAGING

Equipment



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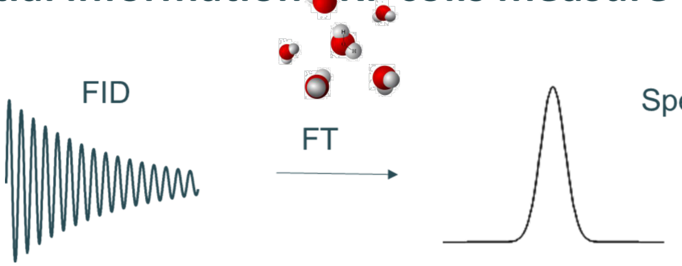
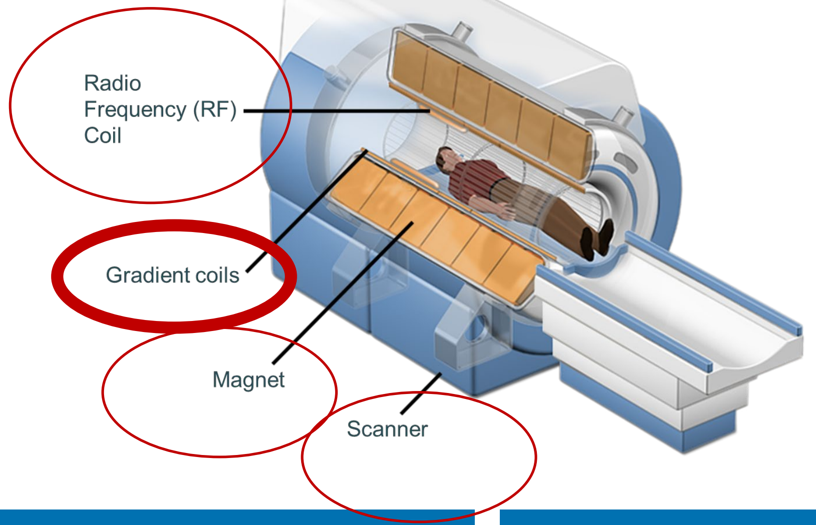


Precession Frequency $\omega_L = \gamma B_0$ Static Magnetic Field

MRI Scanner Cutaway



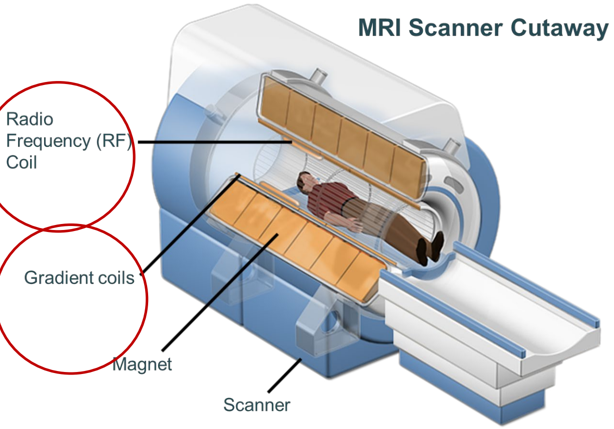
+ Magnetic field / Resonance frequency → position dependent

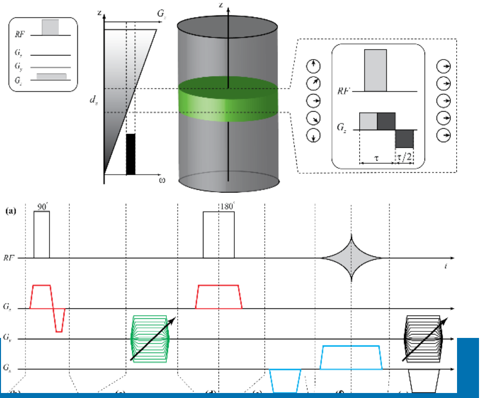
?

MRI – Magnetic Resonance Imaging

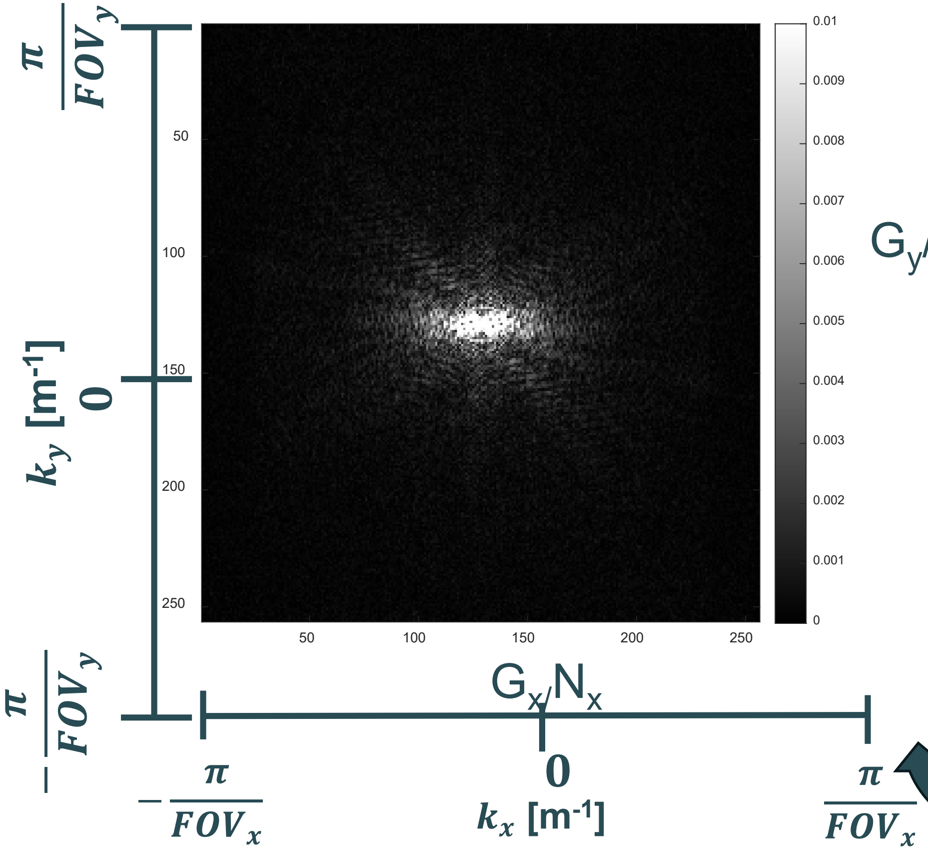
https://nationalmaglab.org

Object Space



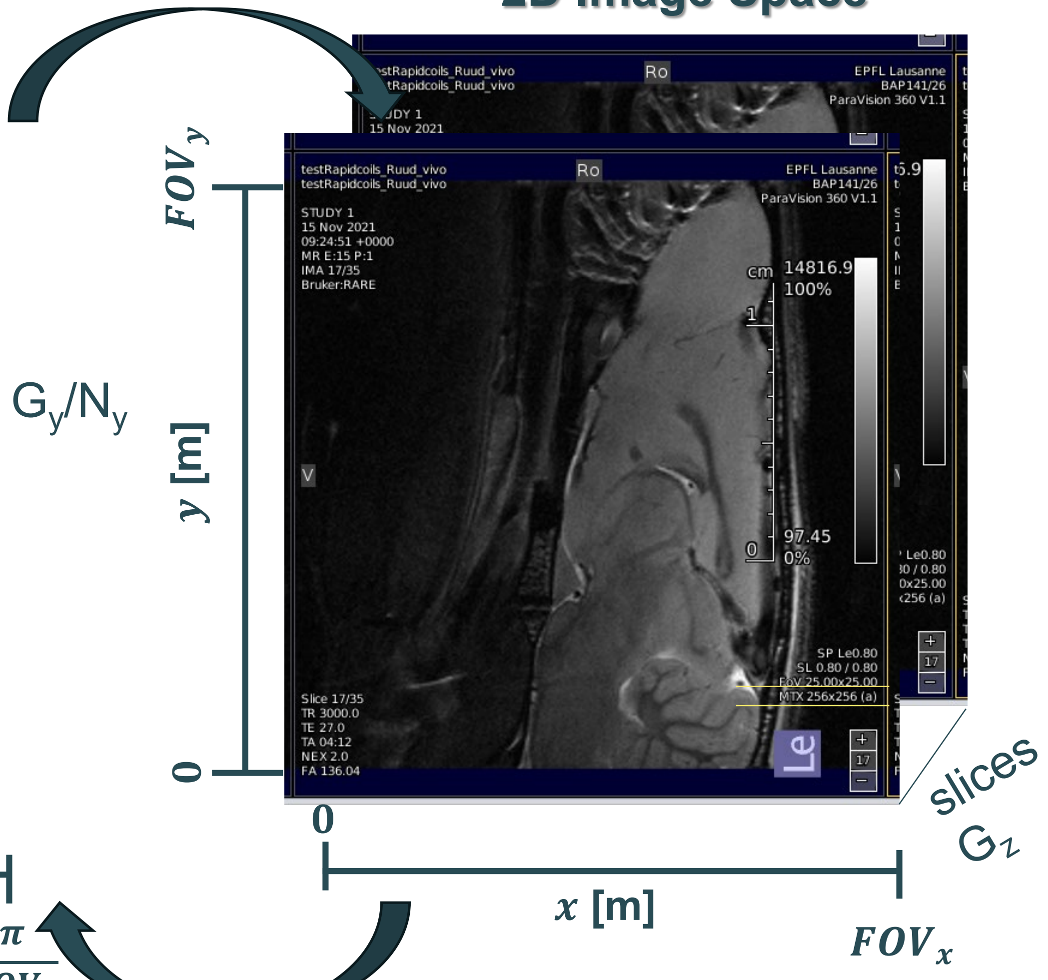


2D Kspace



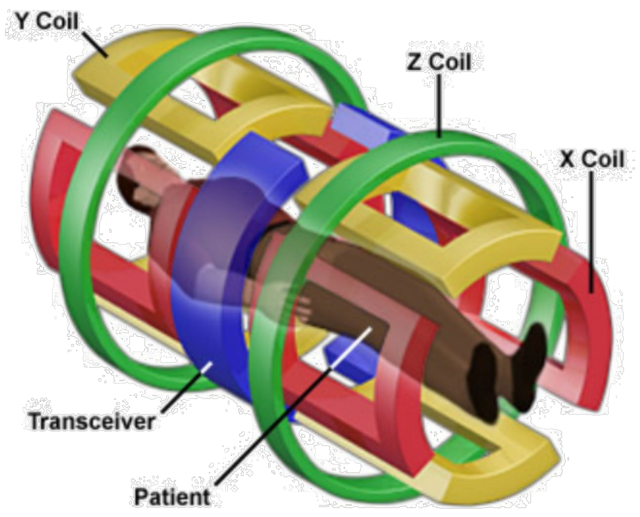
IFT x,y

2D Image Space



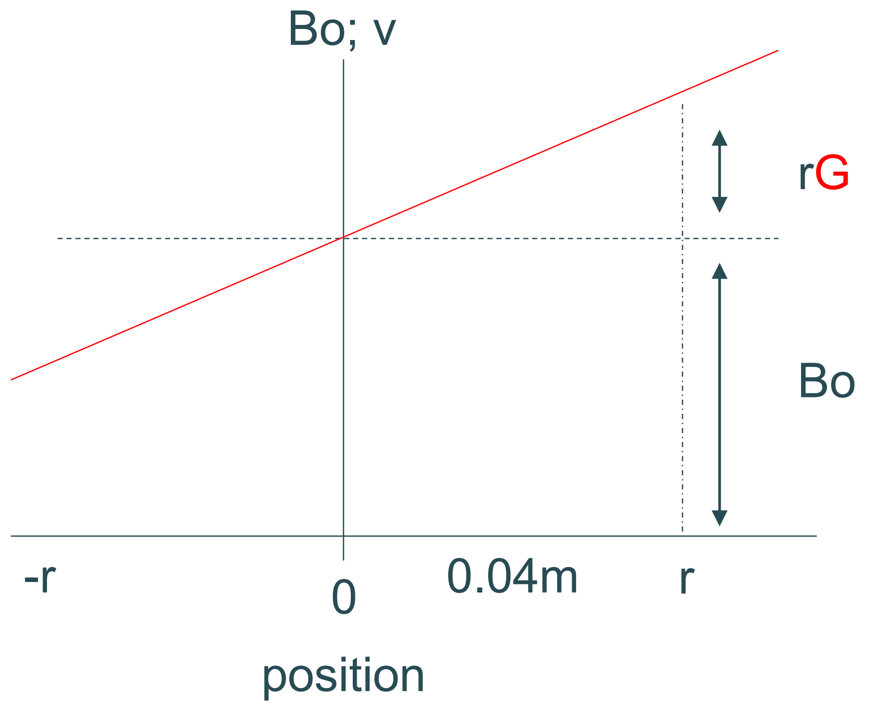
FT x,y

MAGNETIC FIELD GRADIENTS – GRADIENT COILS



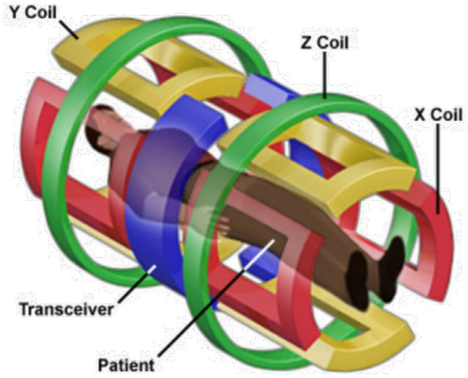
HOW TO ENCODE SPATIAL POSITION ?

Magnetic field gradients - differentiate the signal across space



Magnetic field B along z
varies spatially with x, y, and/or z:

$$\vec{G} \equiv \frac{dB_z}{d\vec{r}}$$



$$\nu = \gamma B_0$$

External magnetic field B_0 = 3.0 Tesla (T) = 30,000 Gauss (G)

Gyromagnetic ratio $\gamma(^1H)$ = 42.57 MHz/T = 4257 Hz/G

Larmor frequency ν = 127.7 MHz

$$\nu = \gamma B_0 + \gamma r G$$

Position r = 0.04 meters (m) = 4 cm

Gradient G = 0.05 T/m = 5 G/cm = 21.285 kHz/cm

Additional frequency offset = 85.14 kHz

$$\phi \text{ (radians)} = 2\pi\nu t$$

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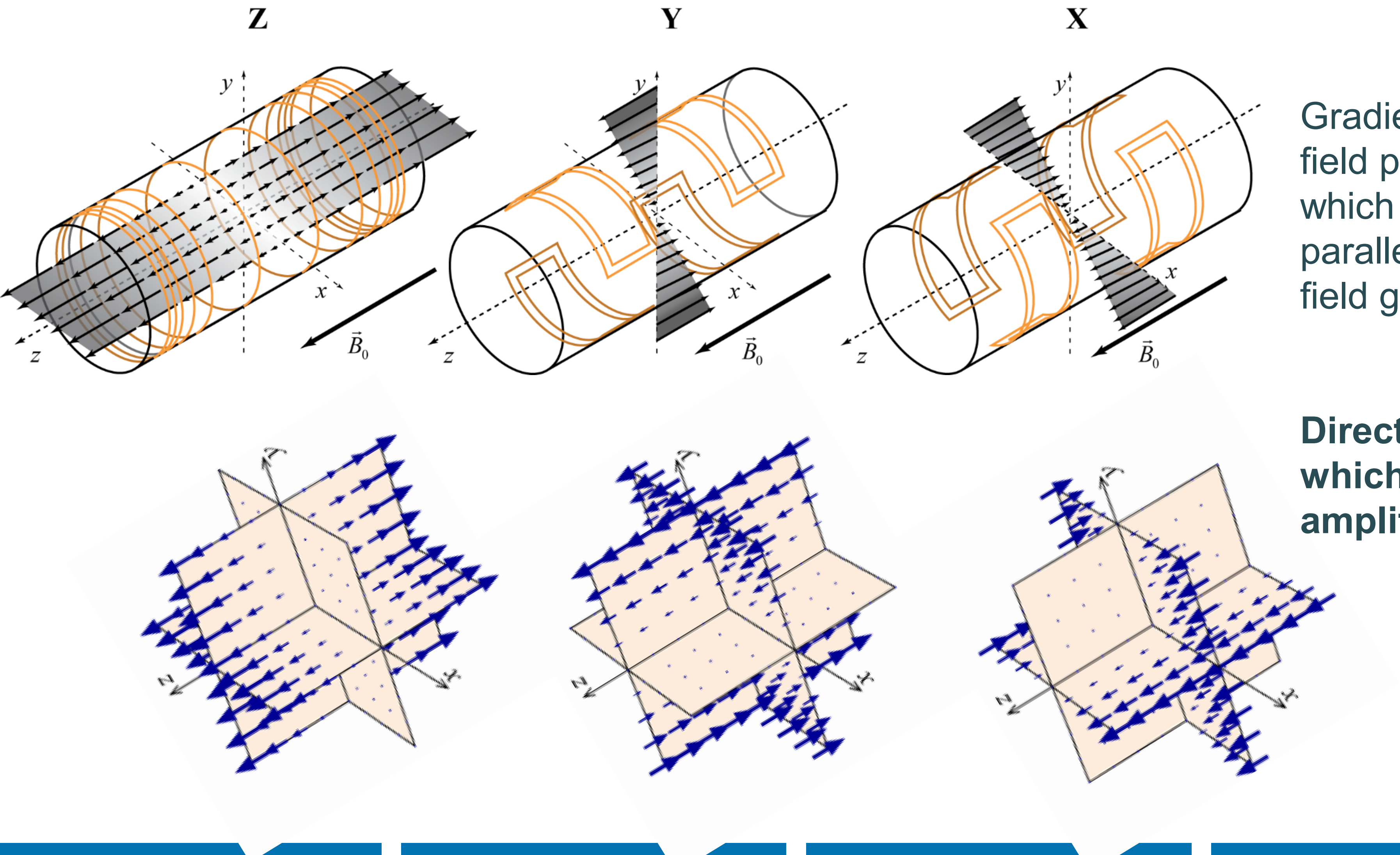
GRADIENT COILS

- Gradient coil generate a magnetic field along the main B_0 field varying linearly along one axis (x, y or z):
  - $\vec{G}_x = x \cdot G_x \cdot \vec{e}_z$
  - $\vec{G}_y = y \cdot G_y \cdot \vec{e}_z$
  - $\vec{G}_z = z \cdot G_z \cdot \vec{e}_z$
- By applying a (magnetic) gradient, we modify the resonance frequency locally:
  - $\omega = \gamma \cdot B_0 + \gamma \cdot x \cdot G_x = \omega_0 + \underbrace{\gamma \cdot x \cdot G_x}_{\delta\omega}$

Or more generally if multiple gradients are applied at the same time

- $\omega = \omega_0 + \gamma \cdot (x \cdot G_x + y \cdot G_y + z \cdot G_z)$

GRADIENT COILS



Gradient coils generate magnetic field field parallel to B_0 – added to main B_0 which is on Z direction (arrows are parallel to B_0) = direction of the magnetic field generated by the gradient

Direction of gradient = direction in which the magnetic field varies = amplitude is varying along the 3 axes

GRADIENT COILS

- Amplitude G/cm or mT/m (10.000G=1T)
- Linearity (homogeneity)
- Rise time (0 to max amplitude, 120 µs) – slew rate=G_{max}/T_{rise}
- Change current in B_0 – Lorentz force – acoustic noise in MRI

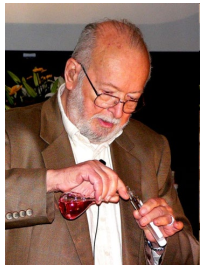
SPATIAL ENCODING

SPATIAL ENCODING

- MR relies on four different methods to spatially encode the NMR signal
- Gradient based methods:
  - Frequency Encoding (73’ Lauterbur, 1973 Mansfield)
  - Slice selection (74’ Mansfield)
  - Phase encoding (76’ Ernst)
- Geometrical method:
  - RF-coil sensitivity profile (97’ Sodickson, 99’ Pruessman, 01’ Larkman,…)

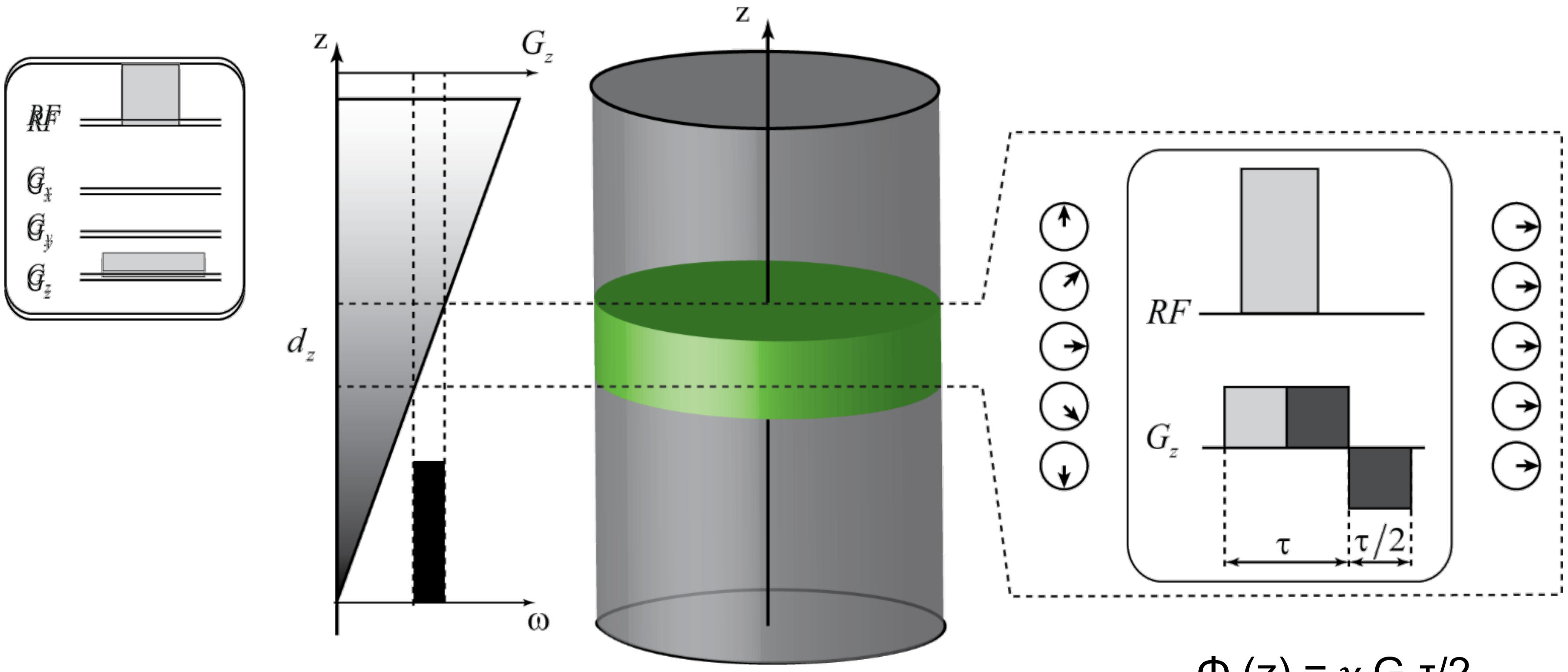
Lauterbur and Mansfield shared the 2003 Nobel Prize in Medicine

Peter Mansfield



Paul Lauterbur

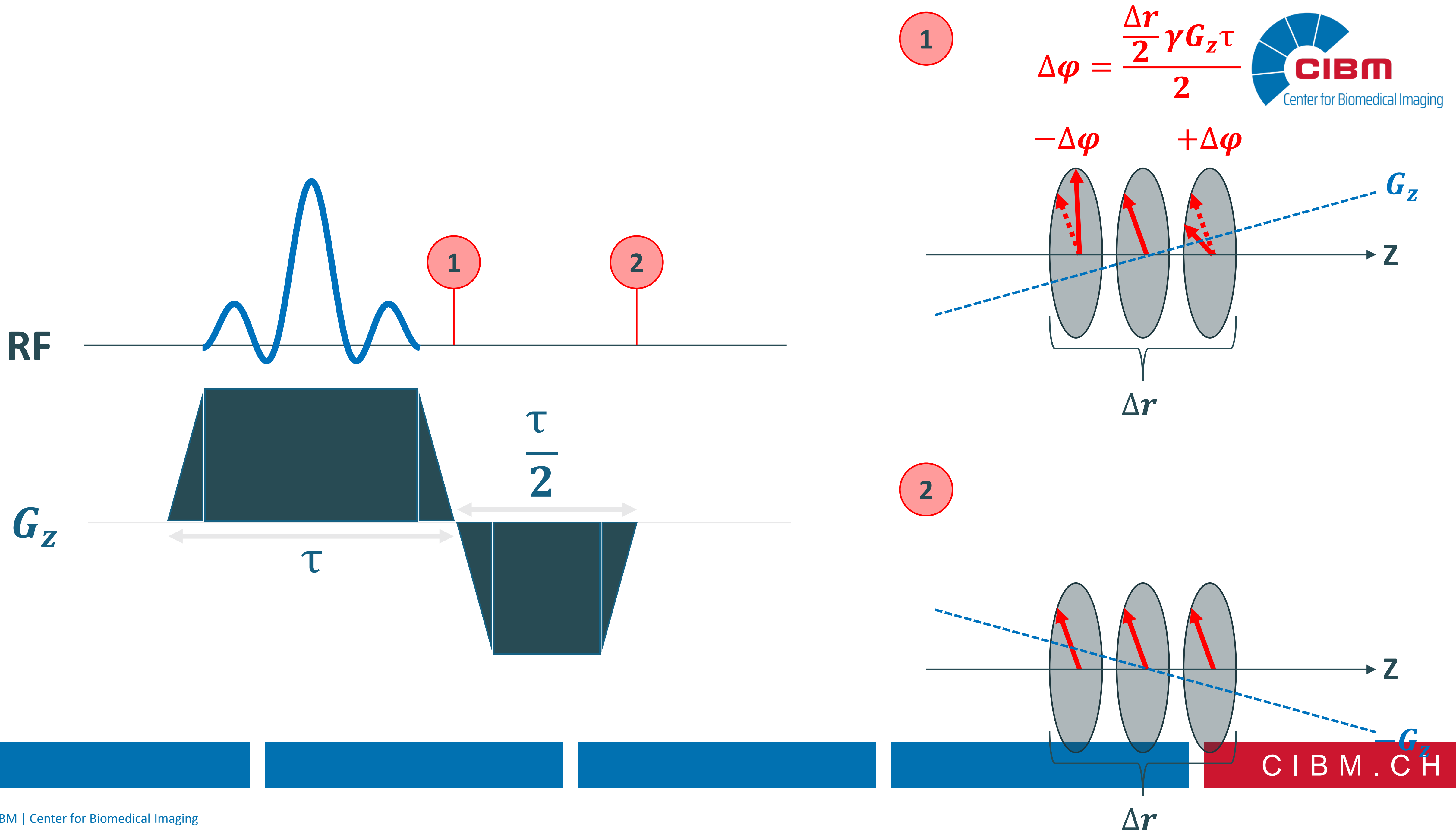
SPATIAL ENCODING – SLICE SELECTION



$$S \propto \iint_{slice} \rho(x,x)dx\,dy$$

$$\delta z = \frac{BW_{RF}}{\gamma G_z}$$

$$\Phi\ (z) = \gamma\ G_z\tau/2$$



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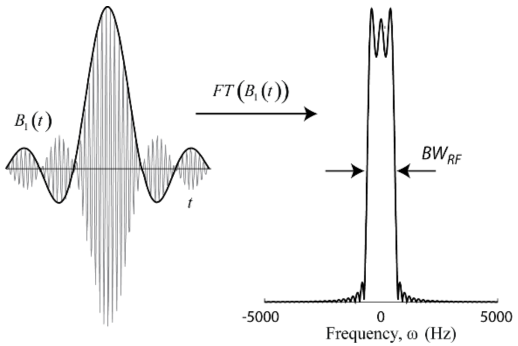
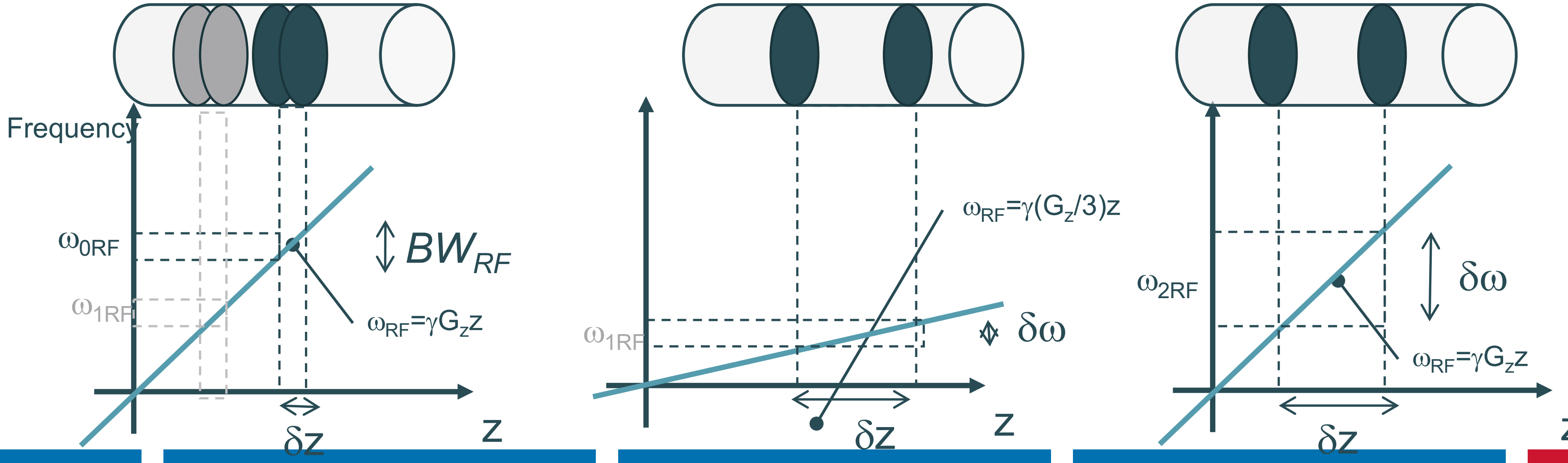
SLICE SELECTION

- Gradient is applied during an RF-pulse, which is characterized by its excitation bandwidth (BW_{RF})



$$\delta z = \frac{BW_{RF}}{\gamma G_z}$$

- The slice thickness can be changed either by adjusting the pulse length or the gradient amplitude



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EXAMPLE

$$\delta z = \frac{BW_{RF}}{\gamma G_z}$$

- 2mm slice
- 5mm out of the isocenter
- BW_{RF} = 1000 Hz
- G_z = ?

- fixed G_z and a constant RF pulse,
- the slice position can only be adjusted by changing the transmitter frequency

$\nu_{RF} = \nu_0$ + slice pos (cm) x G_z (kHz/cm) = ν_0 +2.5kHz

$$G_z = \frac{1.0}{0.2}\ kHzcm^{-1} = 5kHzcm^{-1} = 1.17Gcm^{-1} = 11.7mTm^{-1}$$

$$\gamma = 4257.6HzG^{-1}$$

While elimination of the gyromagnetic ratio makes the variables nucleus-specific, the use of uniform units (kHz and cm) greatly simplify calculations and avoids common mistakes.

- RF pulse frequency ν_{RF} (kHz),
- magnetic field strength $(\gamma/2\pi)B_0$ (kHz), and
- gradient strength $(\gamma/2\pi)G$ (kHz cm^{-1})
- γ (rad s^{-1} T^{-1}) = 267.522 ; $\gamma/2\pi$ (MHz T^{-1}) = 42.57

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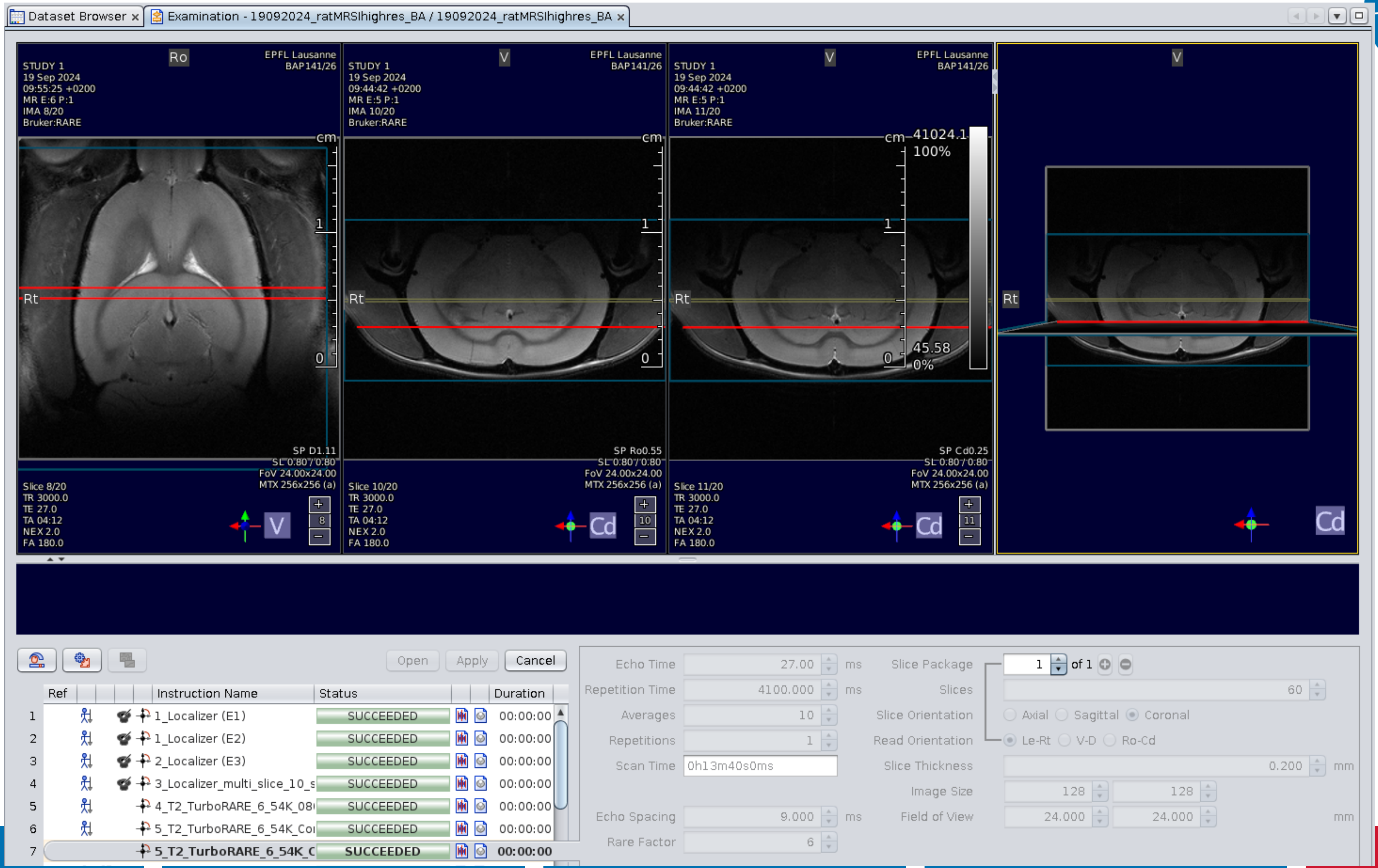
ORIENTATIONS



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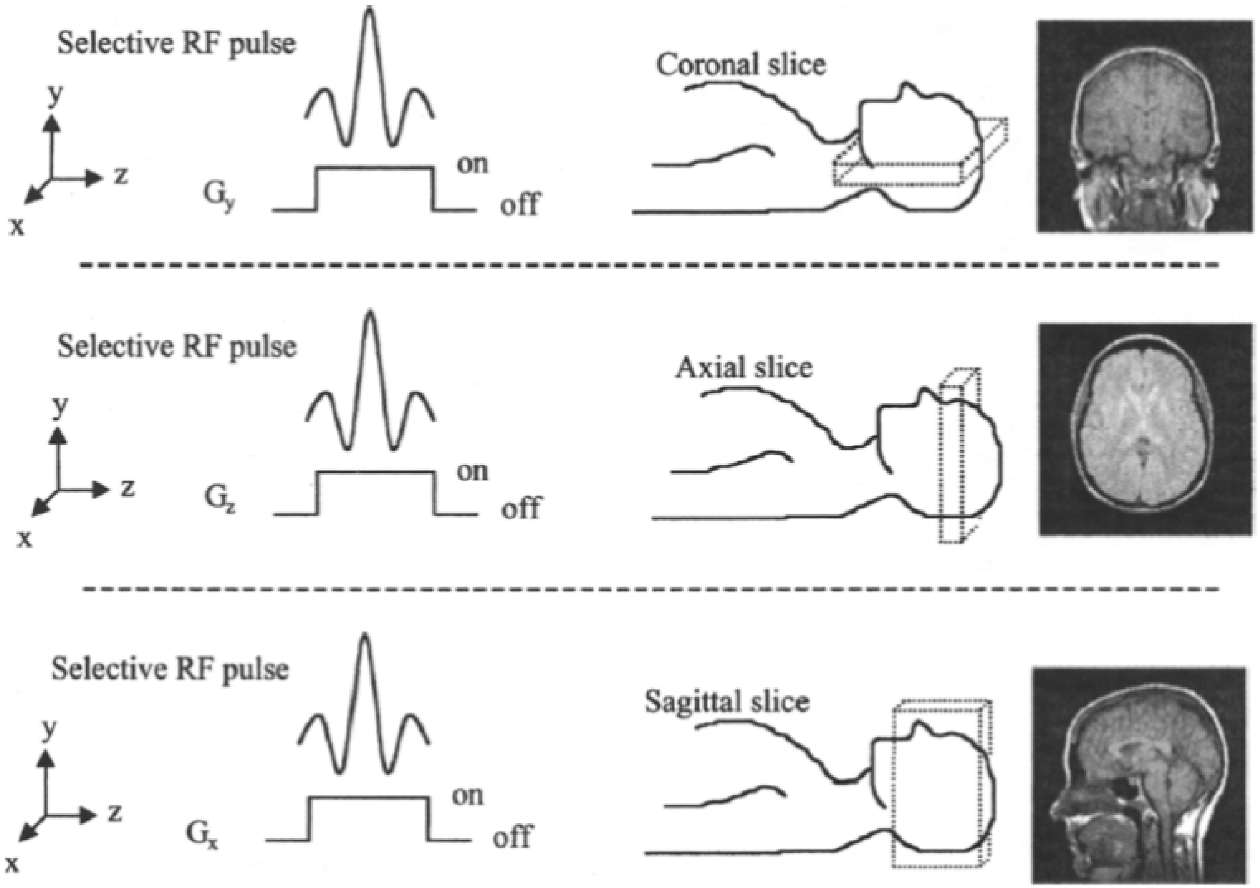

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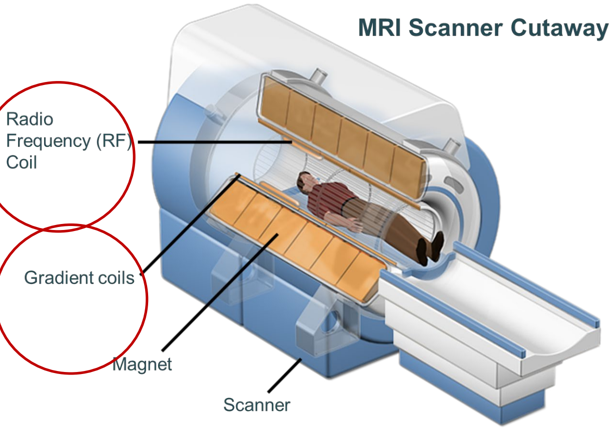
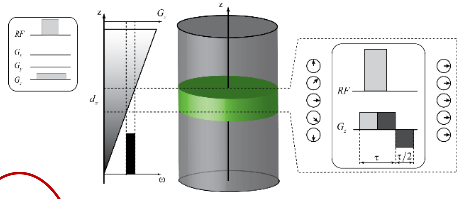
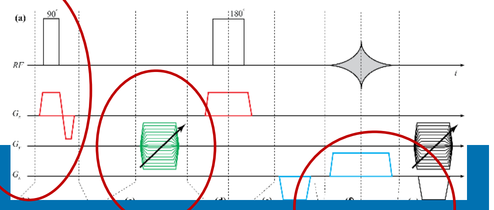
HOW DO WE FILL THE 2D IMAGE ?

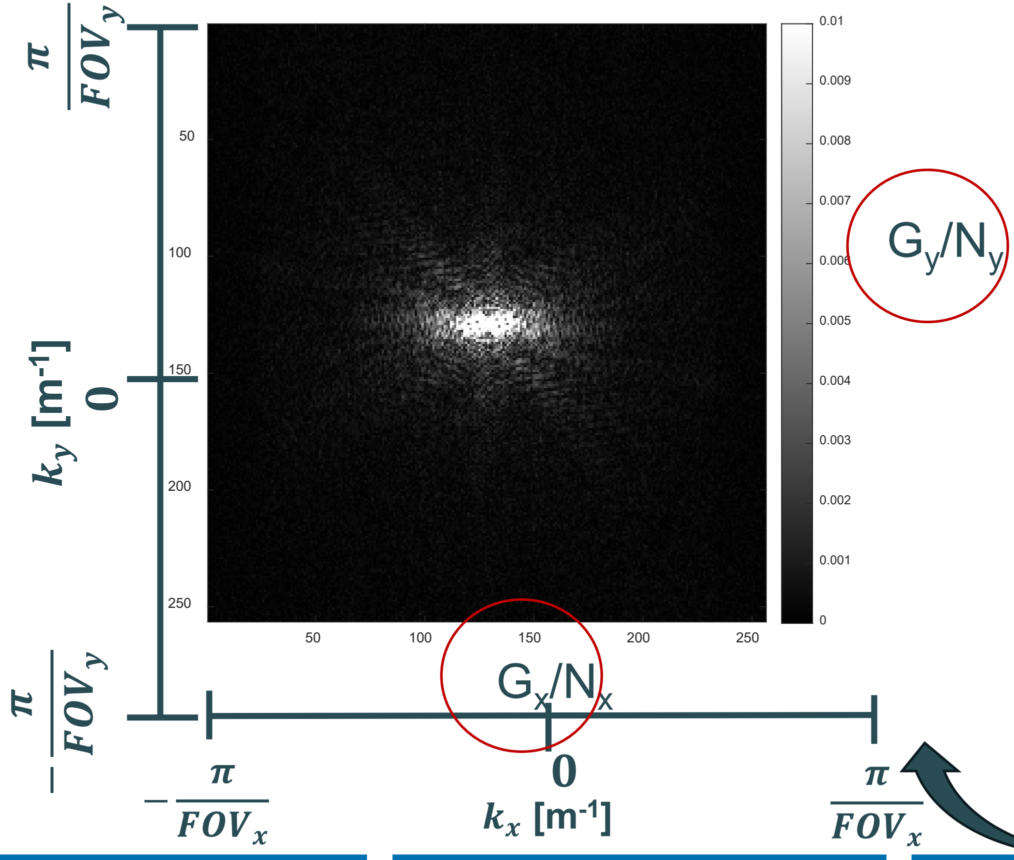
Object Space







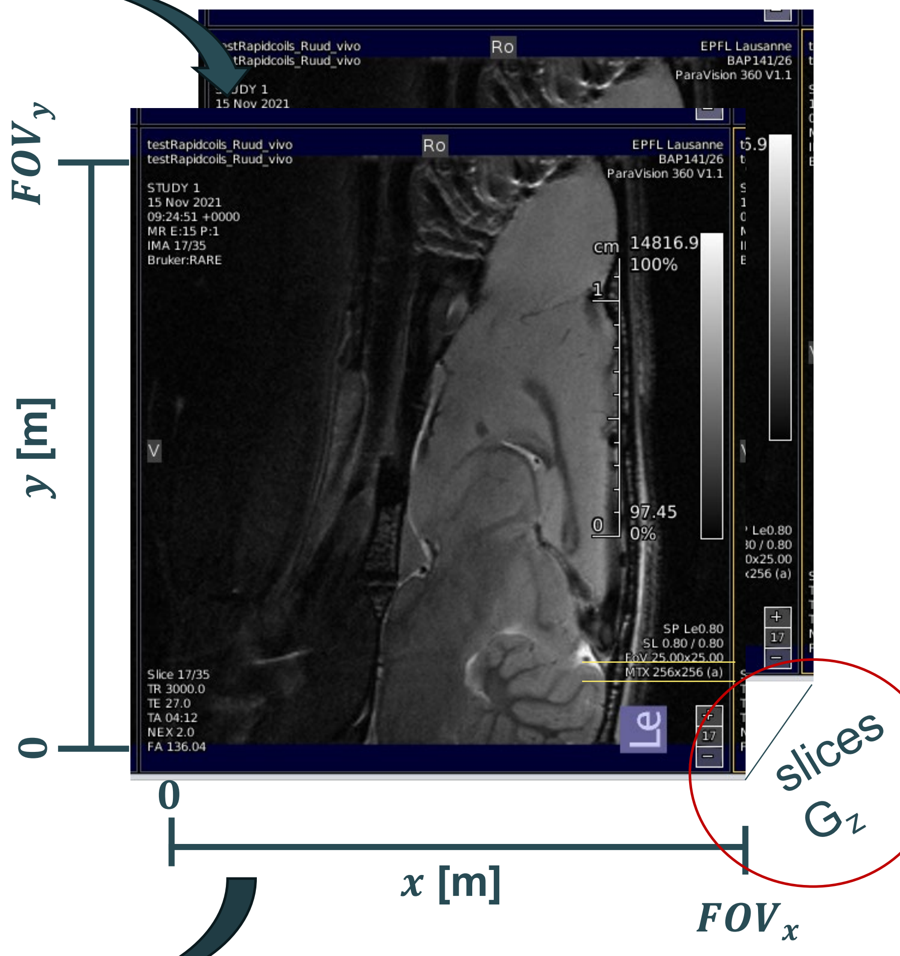
2D Kspace



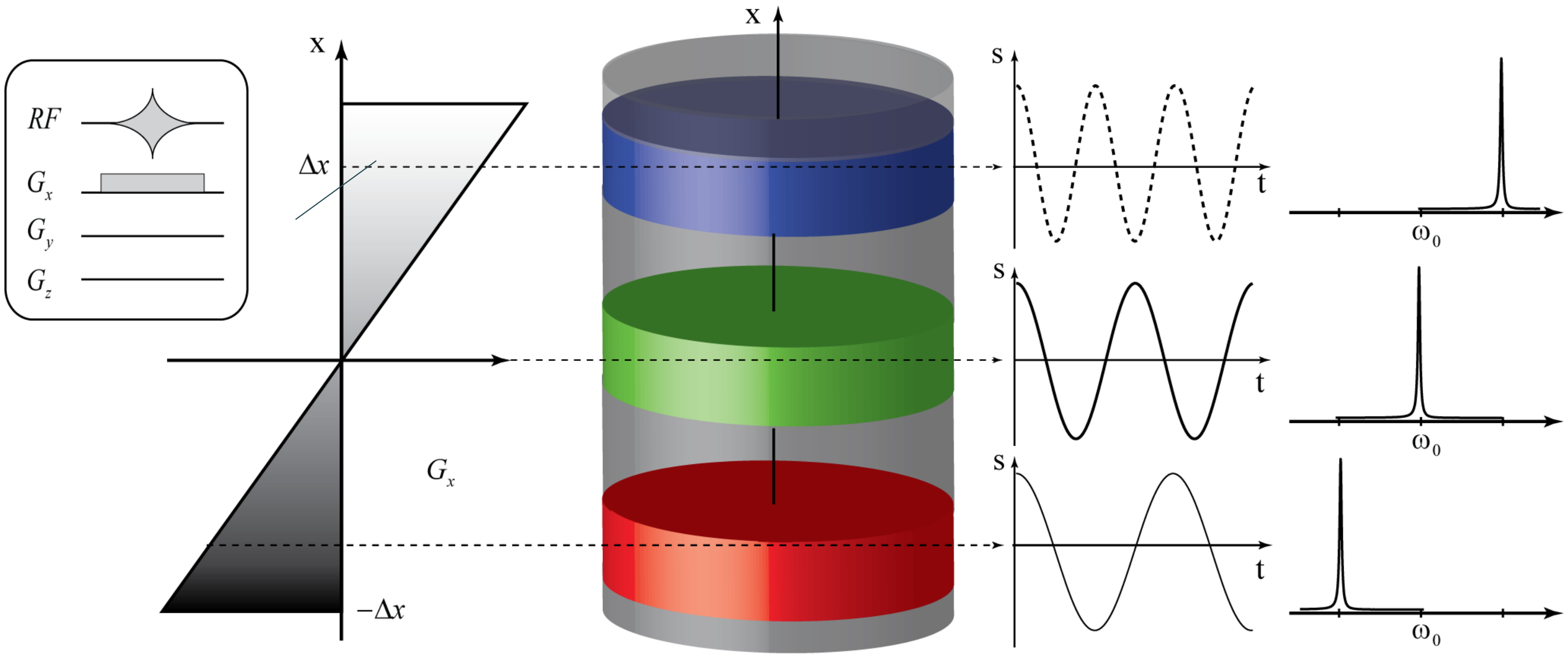
IFT x,y

FT x,y

2D Image Space



SPATIAL ENCODING – FREQUENCY ENCODING



$$\vec{G}_x = x \cdot G \cdot \vec{e}_z$$

$$\omega = \gamma \cdot x \cdot G_x$$

$$S(Gx, t) \propto \iint_{slice} \rho(x, y) e^{-j\omega t} dx\, dy = \iint_{slice} \rho(x, y) e^{-j\gamma G_x x t} dx\, dy$$

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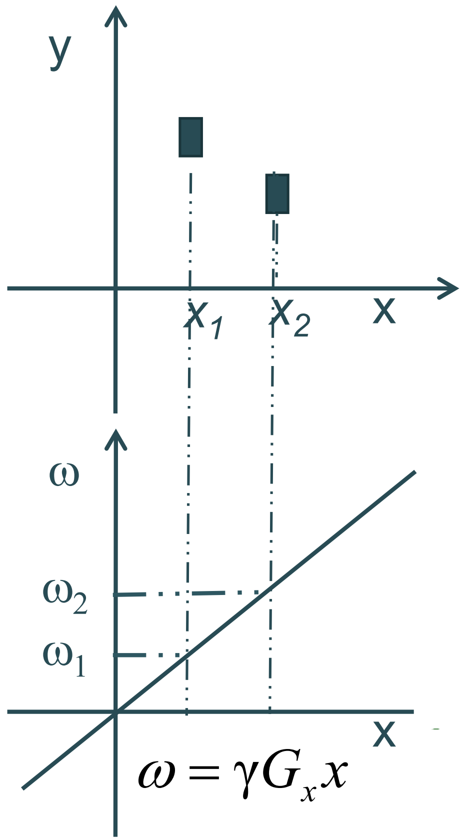
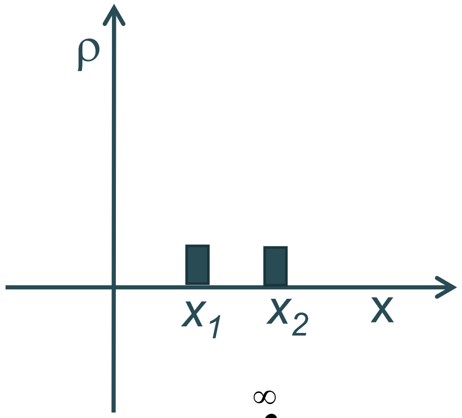
FREQUENCY ENCODING

- Consider the simple 2-dimensional object, O(x,y)

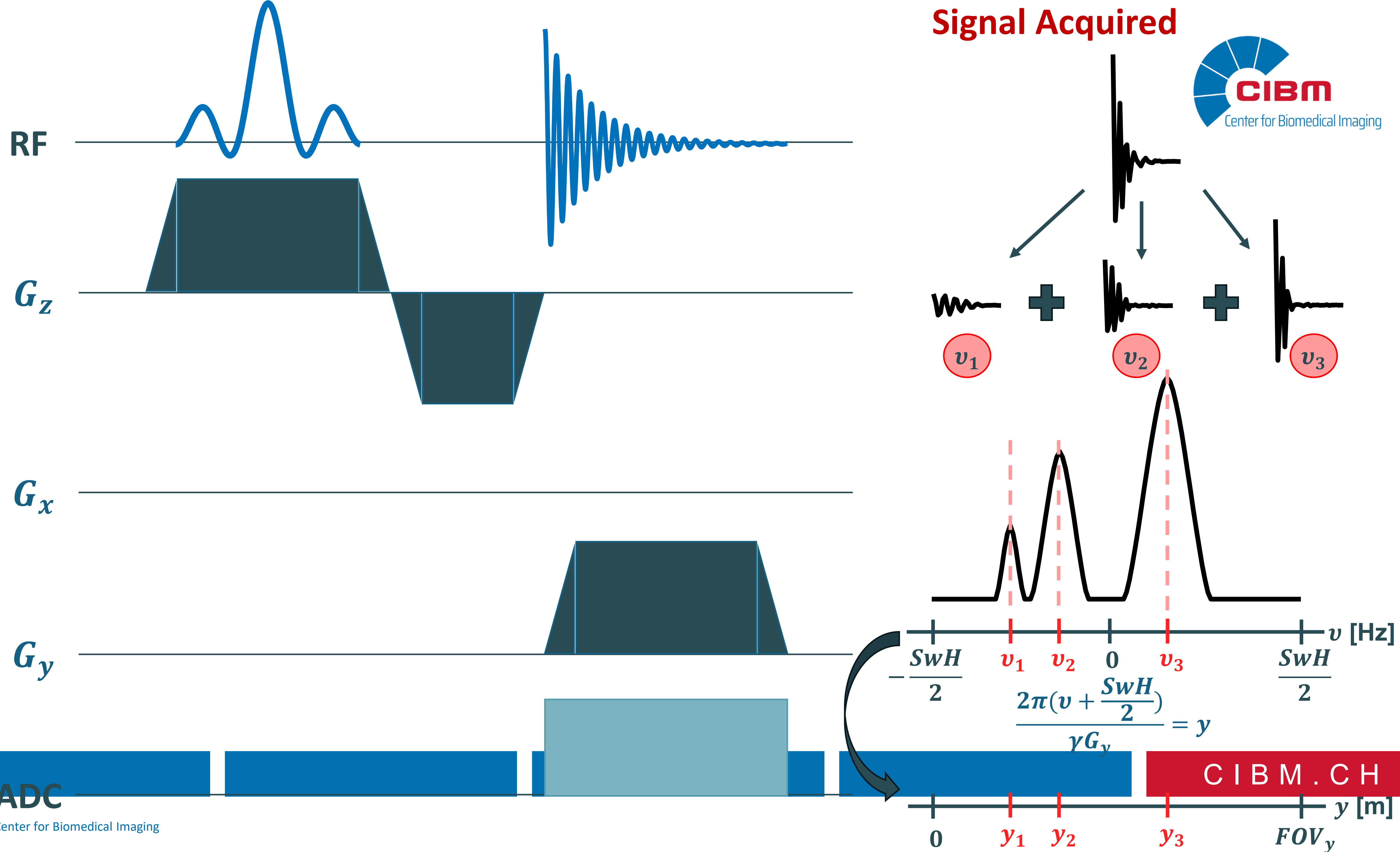




$$\rho(x = \frac{\omega}{\gamma G_x}) = \int_{-\infty}^{\infty} S(t) e^{i\omega t} dt$$

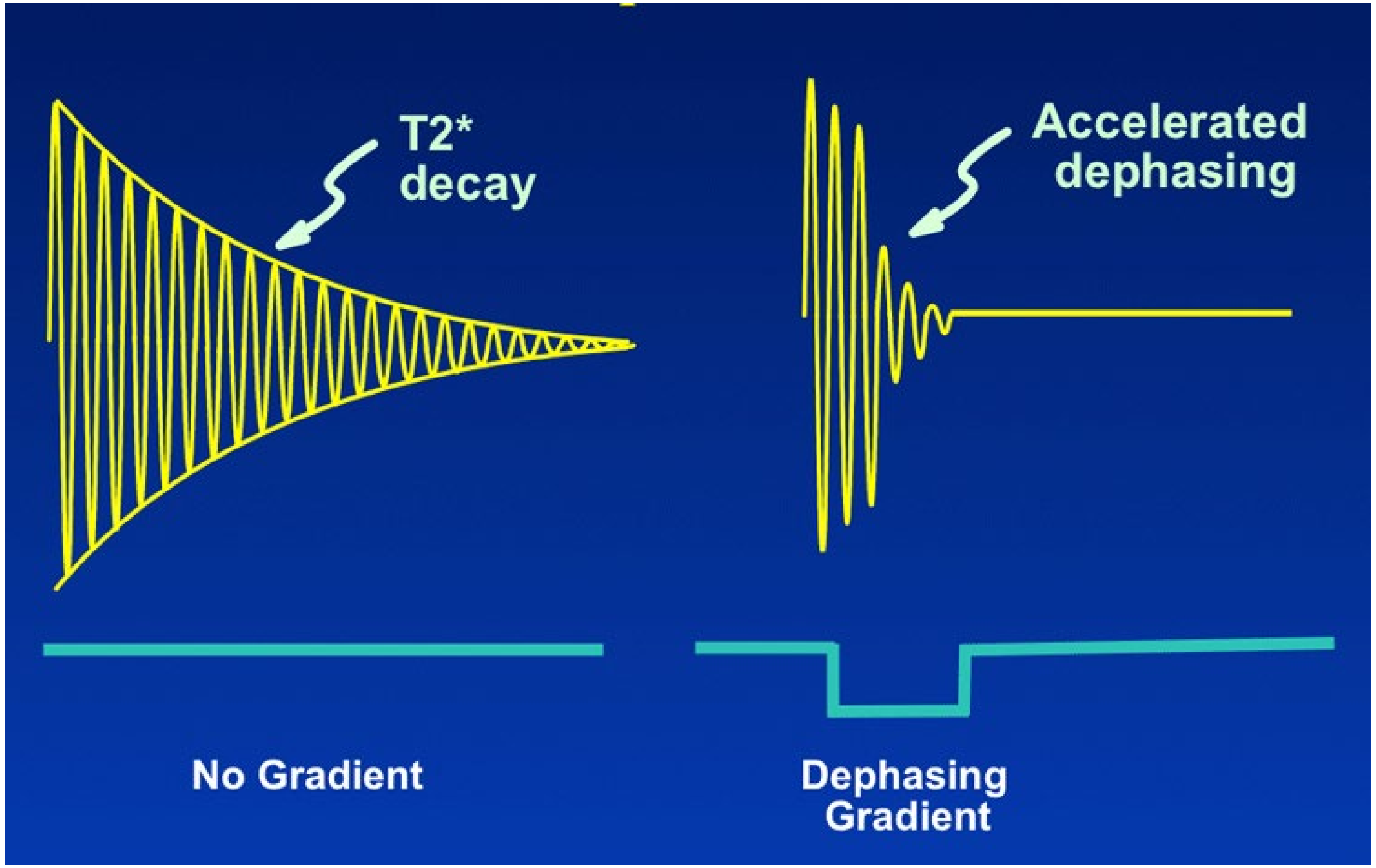
Fourier Transform

Signal Acquired



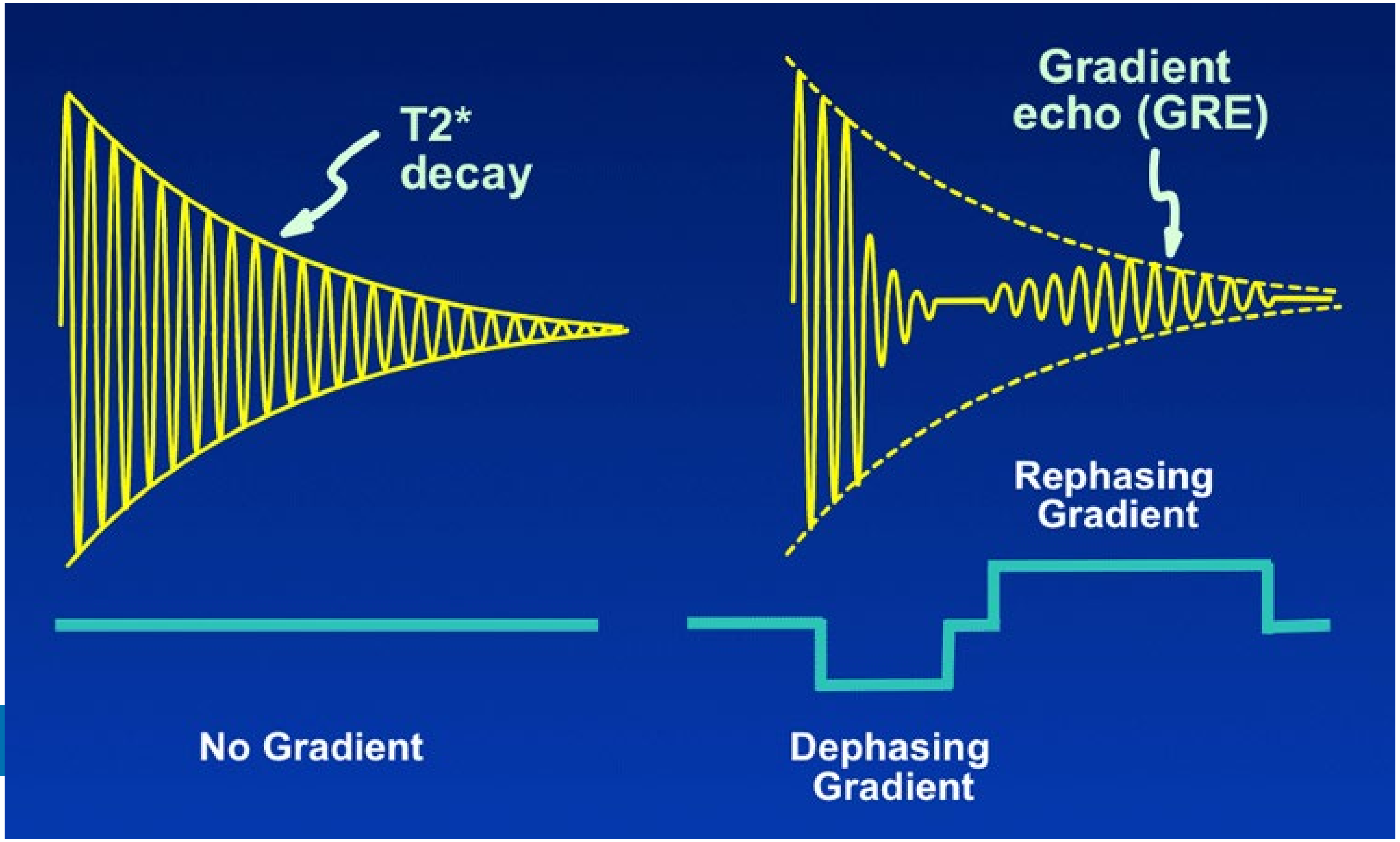
BASIC PULSE SEQUENCES (GRADIENT-ECHO)

- Apply a gradient – encode the FID – hardware limitations on gradient switch – first points are sampled during a time-varying gradients – artefacts
- Eliminate the first points – ↓ SNR
- Encode only positive values



BASIC PULSE SEQUENCES (GRADIENT-ECHO)

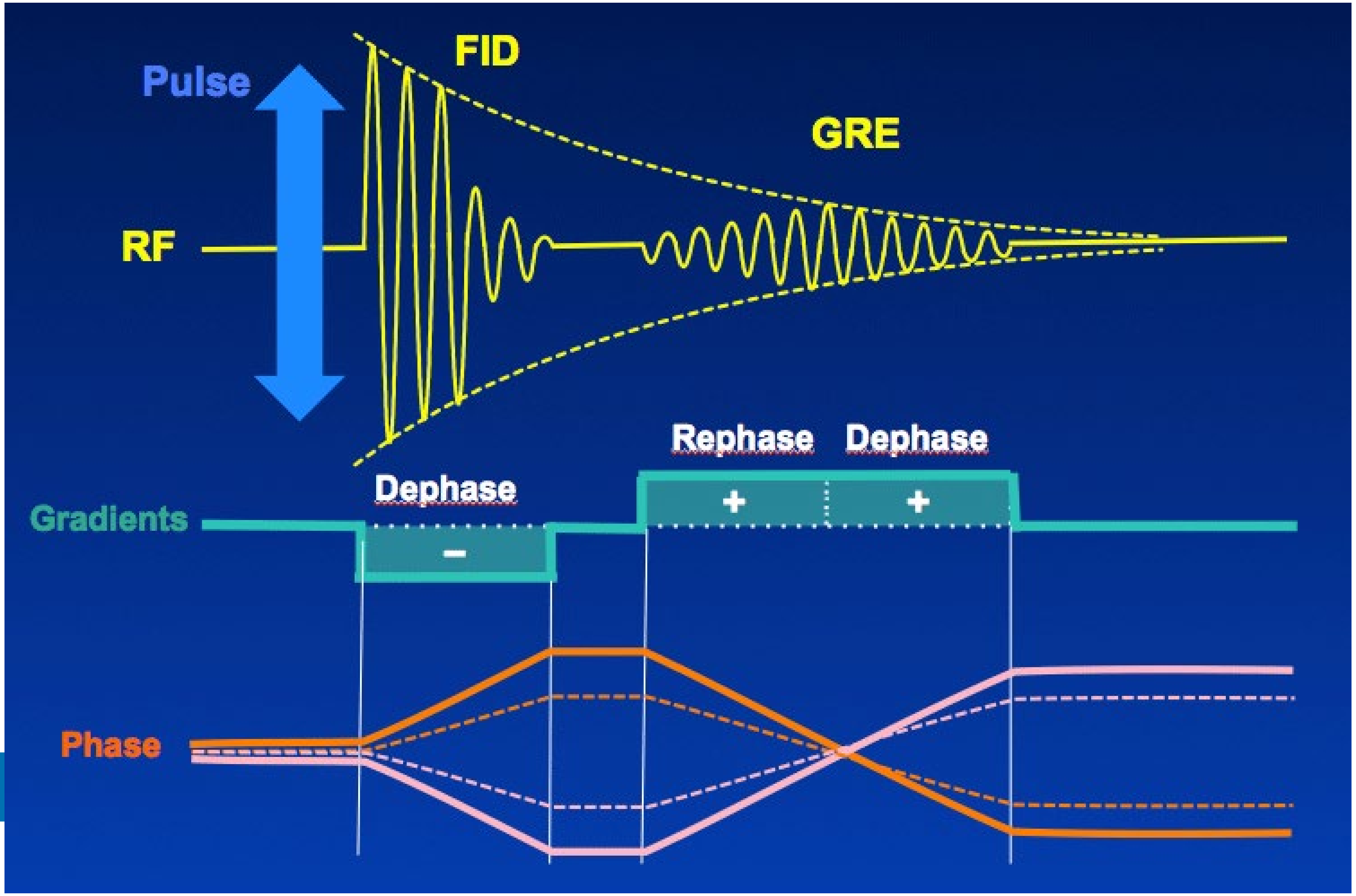
- A gradient-echo is a manipulation of the FID using a bipolar gradient
- First gradient lobe dephases the FID, while the second lobe rephases it at a later time, generating the echo.



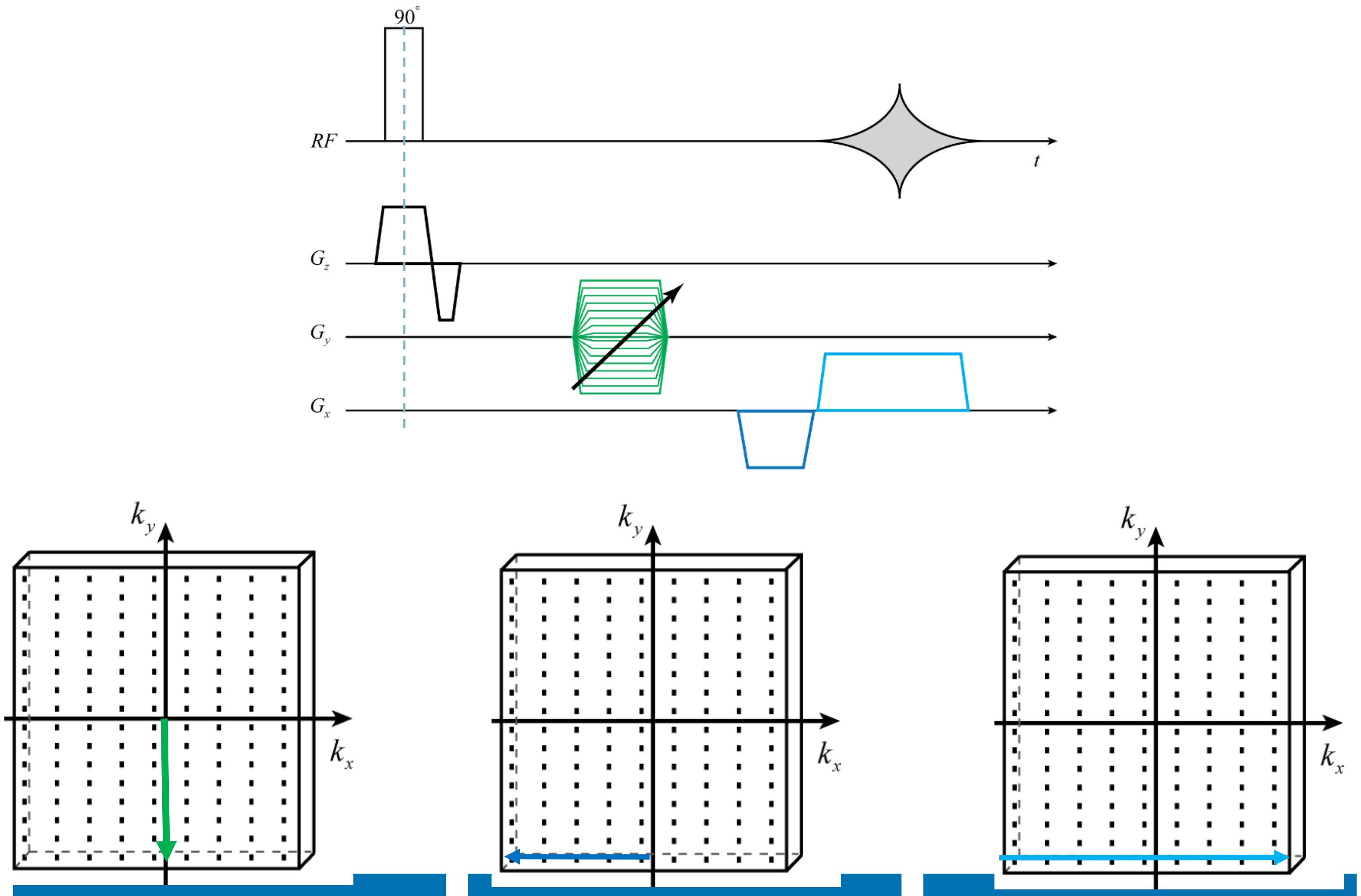
BASIC PULSE SEQUENCES (GRADIENT-ECHO)

- A gradient-echo is a manipulation of the FID using a bipolar gradient
- First gradient lobe dephases the FID, while the second lobe rephases it at a later time, generating the echo.

In MRI, this can be used to generate T_2^* and T_1 contrasts.

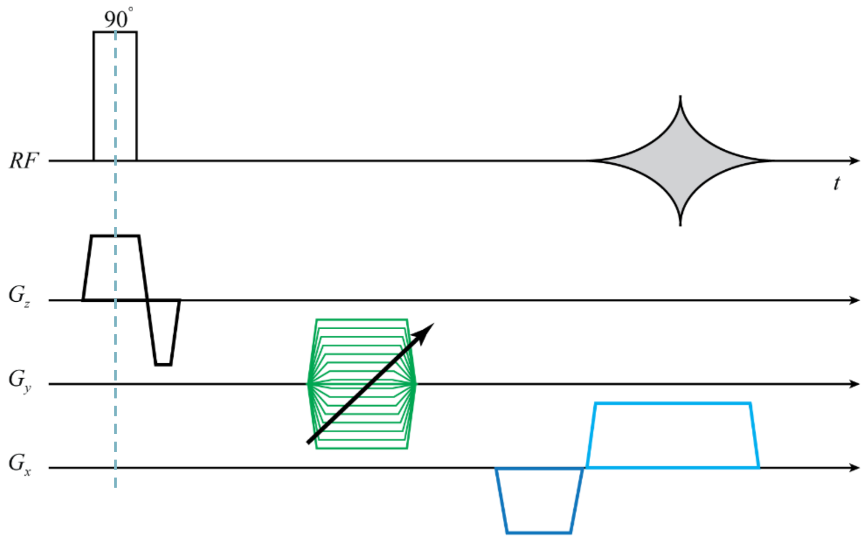


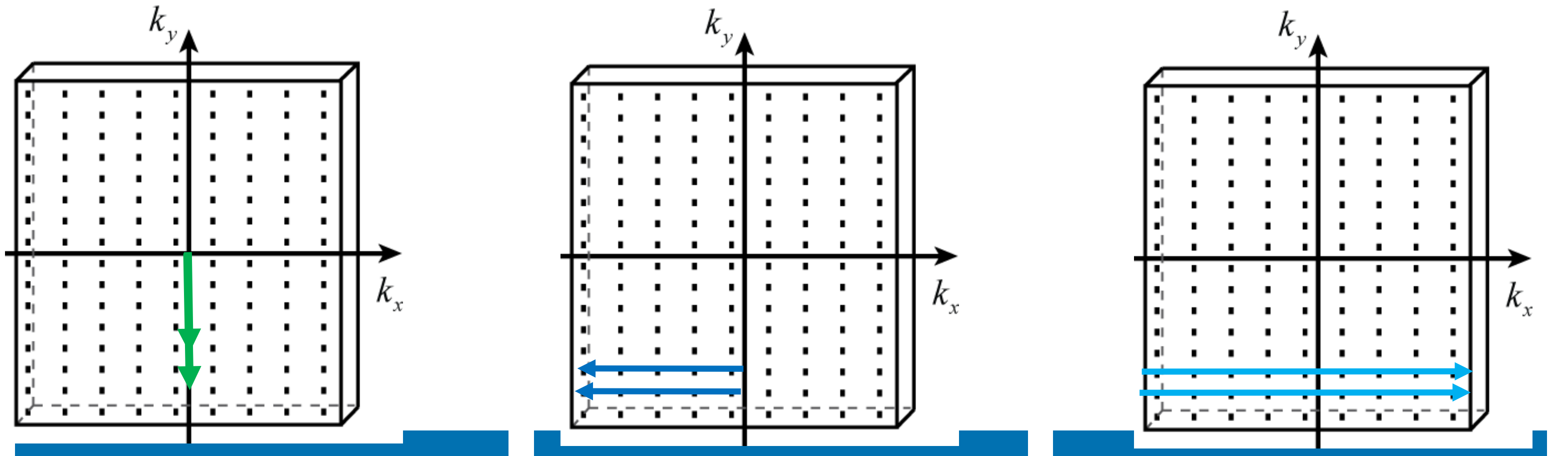
GRADIENT ECHO



One k-space line per TR

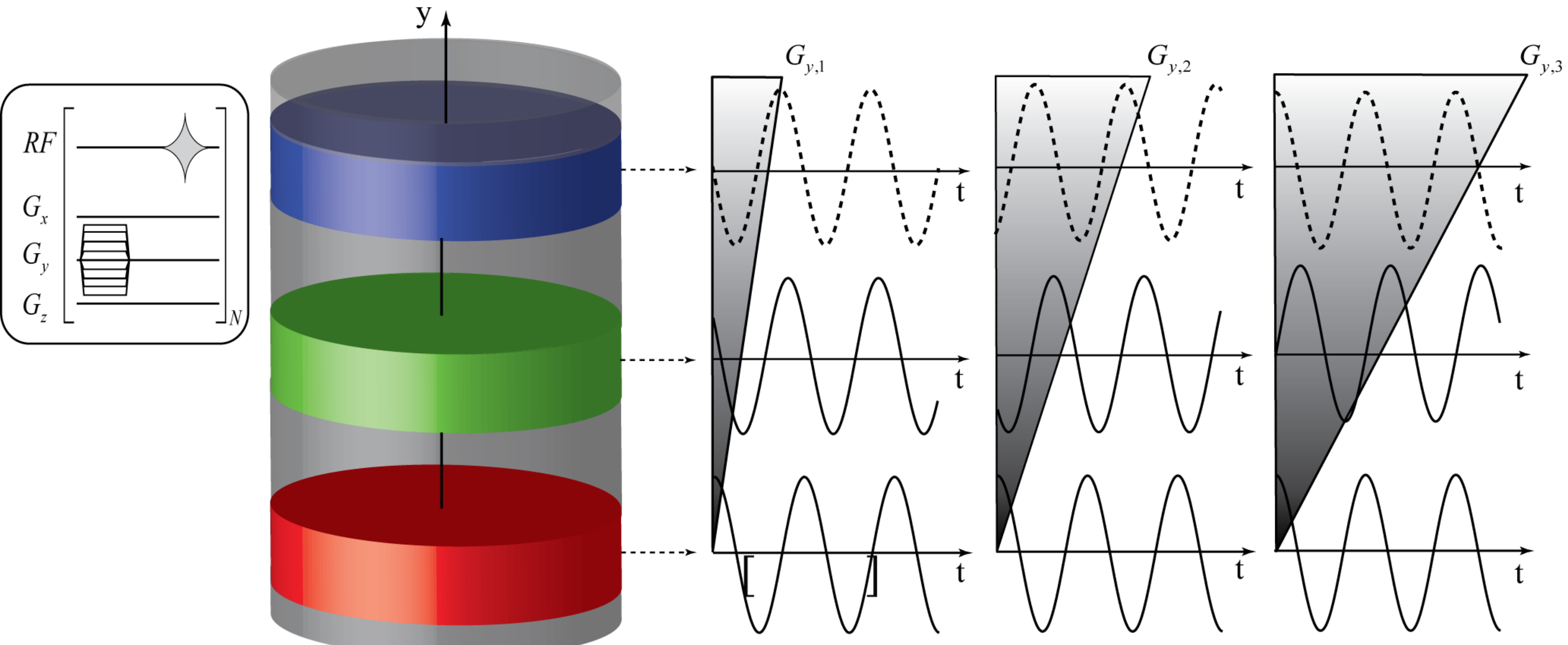
GRADIENT ECHO





One k-space line per TR

SPATIAL ENCODING – PHASE ENCODING



$$\phi\,(y) = \omega_y \cdot \tau_{pe} = \gamma \cdot y \cdot G_y \cdot \tau_{pe}$$

$$S(G_y, \tau_{pe}) \propto \iint_{slice} \rho(x,y) e^{-j\gamma G y y\, \tau_{pe}} dx\, dy$$

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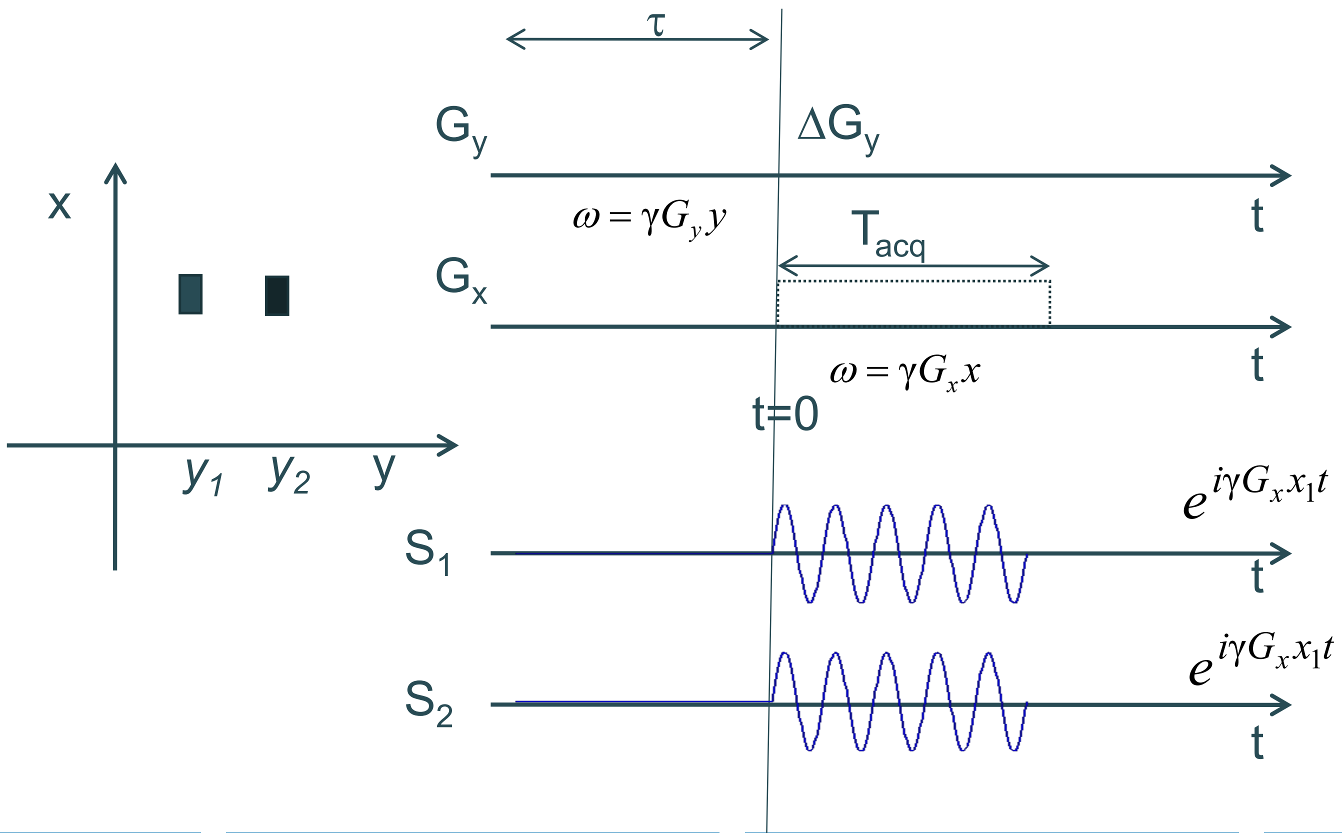
PHASE ENCODING

– In a single acquisition, frequency encoding indicates the position in one dimension. Encoding 2nd and 3rd dimension is usually accomplished via phase encoding (position is encoded in the phase of the NMR signal)



C I B M . C H

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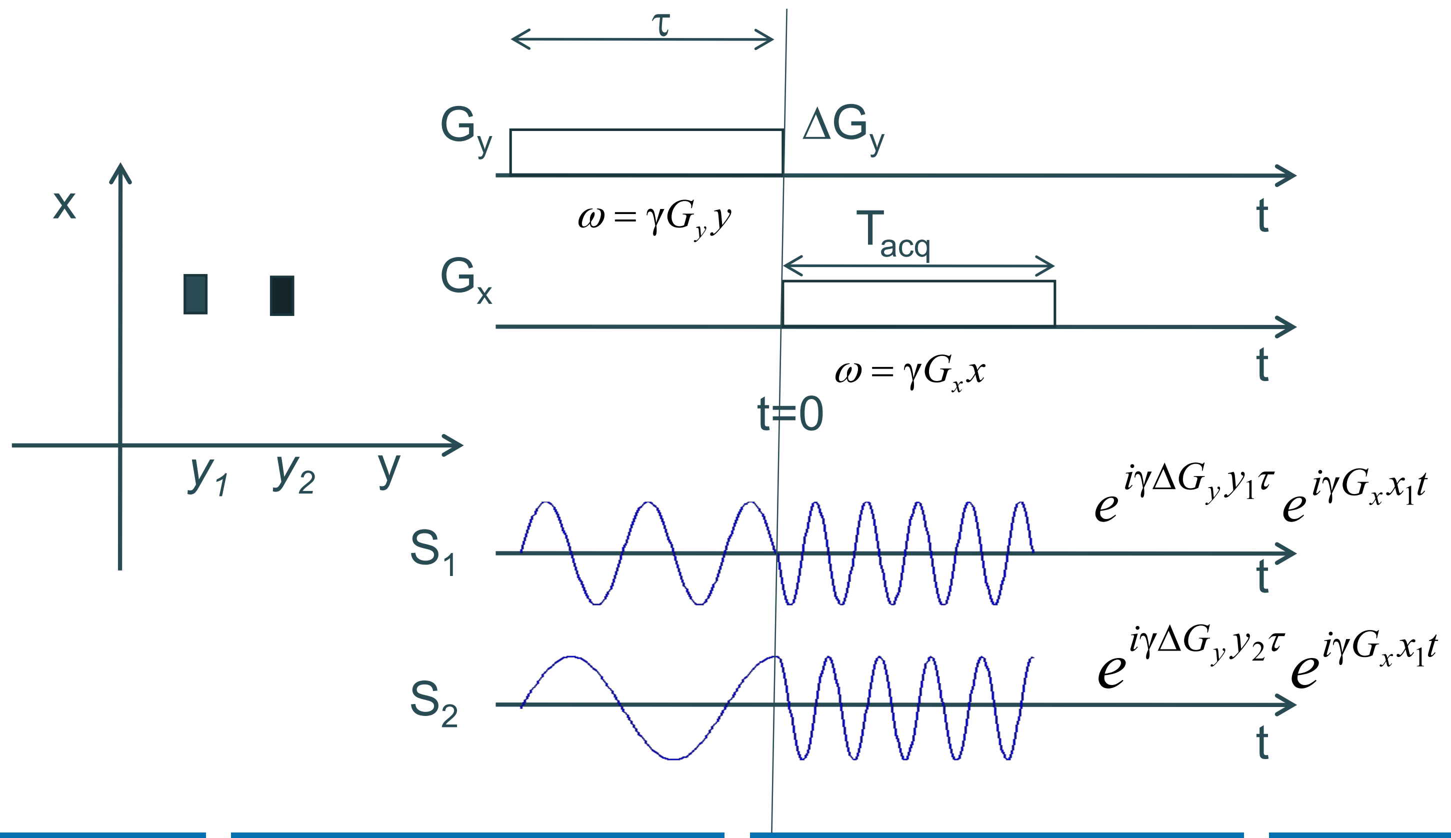

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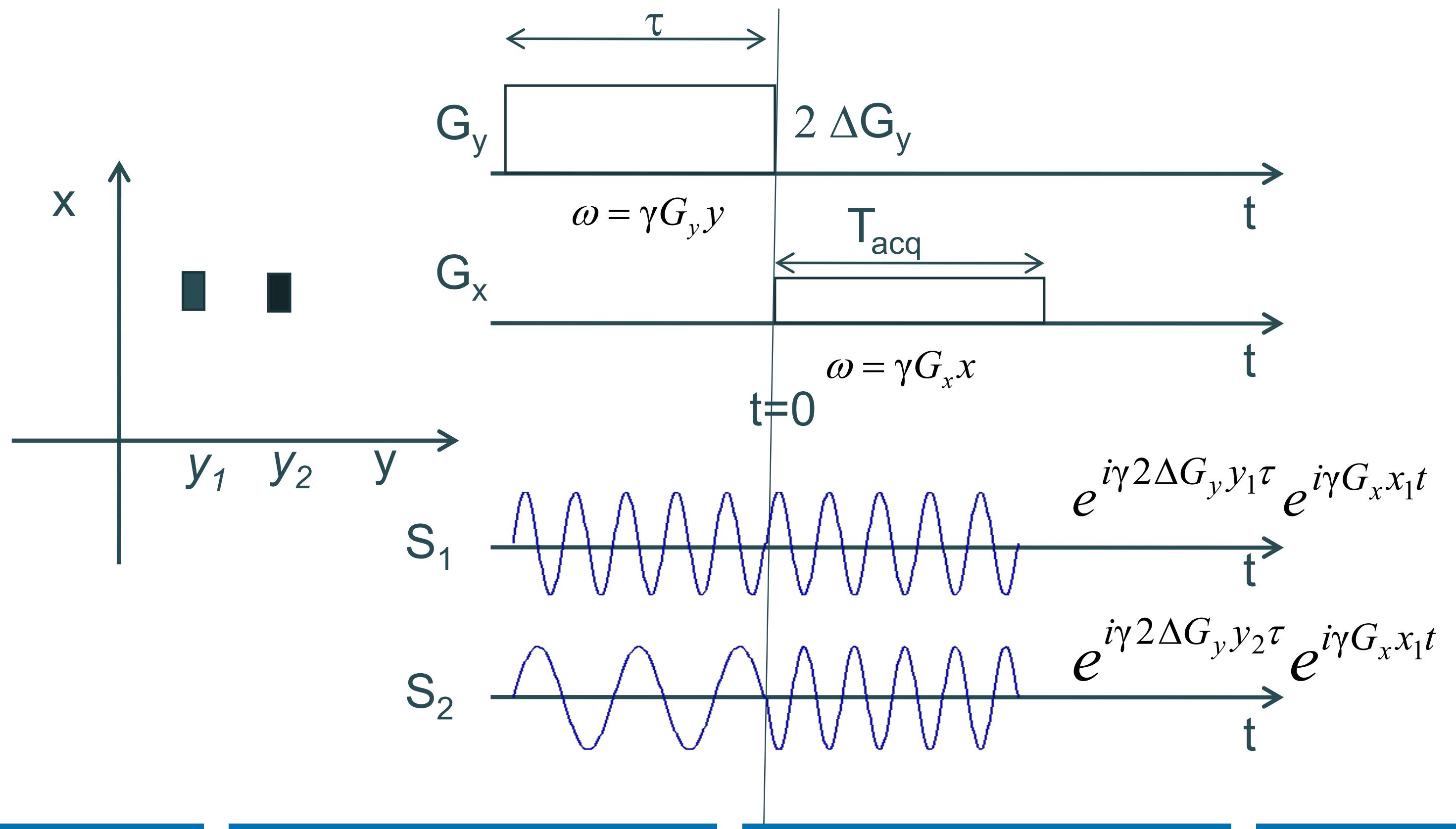


Phase of the signal depends on the y position

while

Frequency depends on the x position

We can not unambiguously identify y from a single measurement…

Multiple measurements are needed.

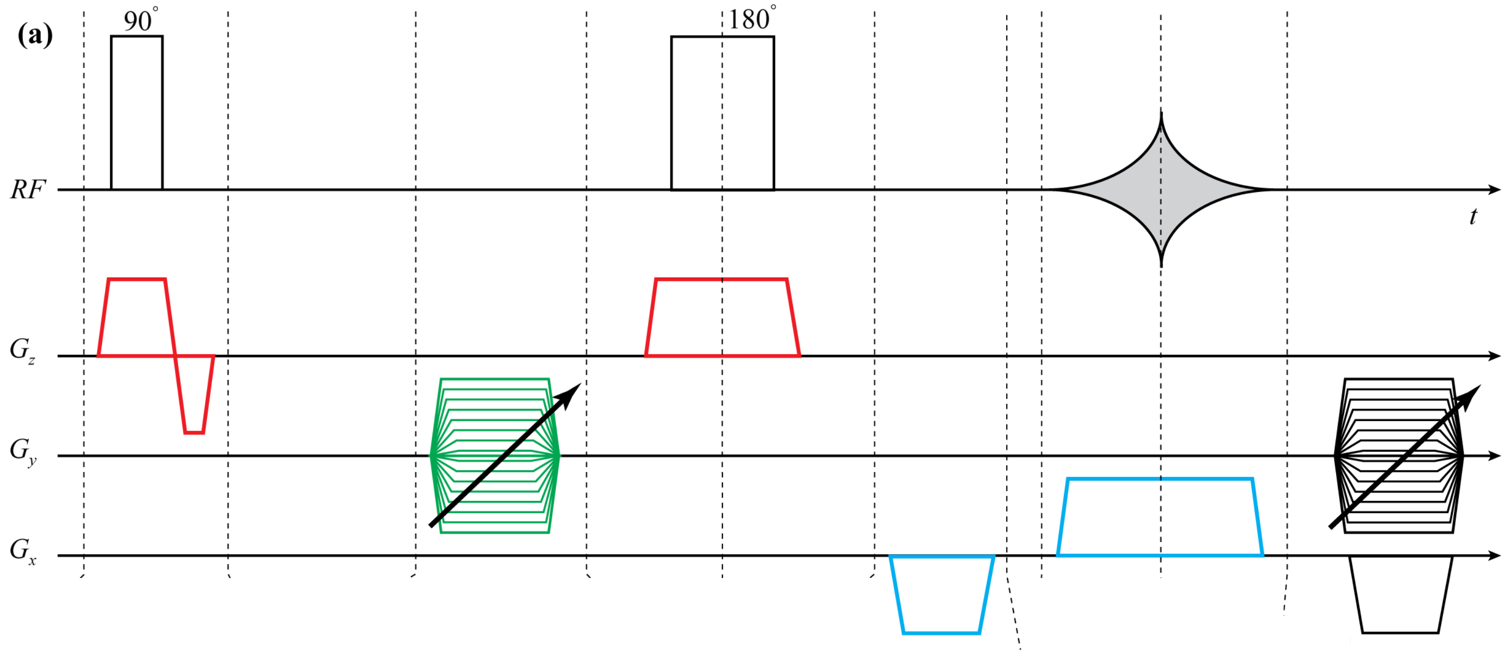
HOW TO ENCODE A 3D OBJECT?

HOW TO ENCODE A 3D OBJECT?

- Conventional 2D acquisitions
- Phase encoding based – (3D conventional acquisition)
- Slice selection based – (spectroscopy localization)

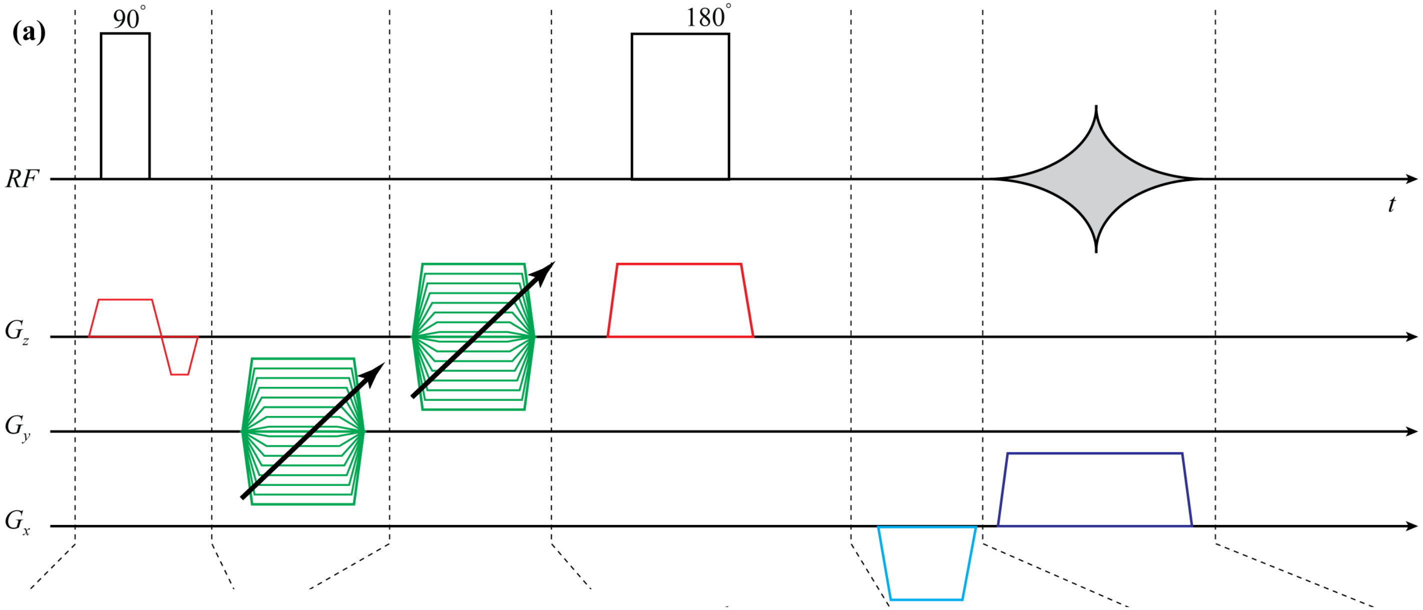
HOW TO ENCODE A 3D OBJECT?

- Conventional 2D acquisition



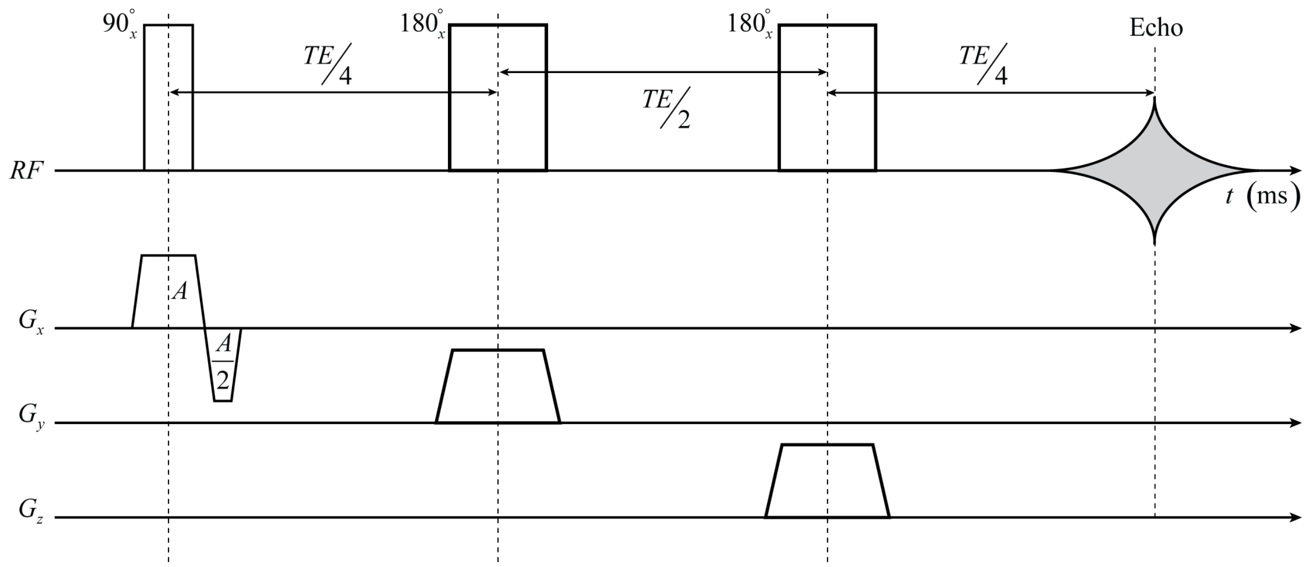
HOW TO ENCODE A 3D OBJECT?

- Phase encoding – Conventional 3D acquisition



HOW TO ENCODE A 3D OBJECT?

- Slice selection – Conventional spectroscopic localization



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SOME NOTIONS ABOUT K-SPACE

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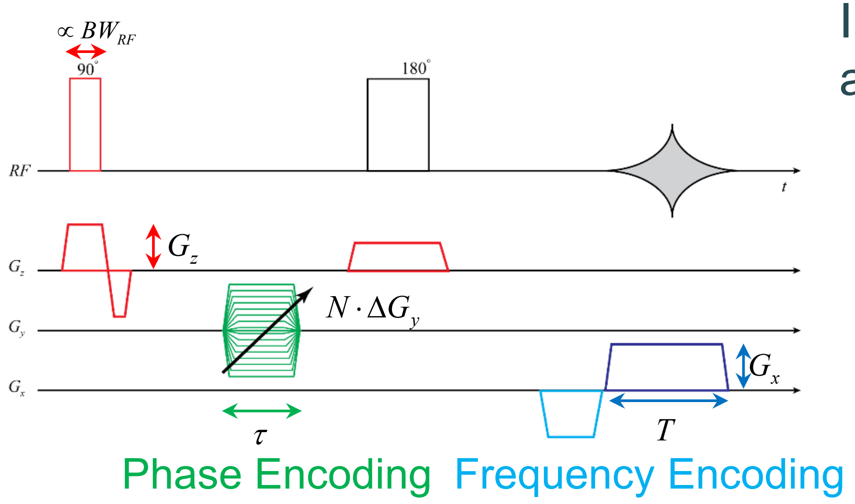

In the rotating frame, neglecting relaxation, the NMR signal can be written as:

$$S \propto \iint_{slice} \rho(x,x)dx\,dy$$

$$\omega = \gamma \cdot x \cdot G_x$$

$$S(Gx,t) \propto \iint_{slice} \rho(x,y)e^{-j\omega t}dx\,dy = \iint_{slice} \rho(x,y)e^{-j\gamma G_x xt}dx\,dy$$

$$\phi\,(y) = \omega_y \cdot \tau_{pe} = \gamma \cdot y \cdot G_y \cdot \tau_{pe}$$

$$S(G_y,\tau_{pe}) \propto \iint_{slice} \rho(x,x)e^{-j\gamma Gyy\,\tau_{pe}}dx\,dy$$

$$S(G_x,t,G_y,\tau_{pe}) \propto \iint_{slice} \rho(x,y)e^{-j\gamma Gxxt}\;e^{-j\gamma Gyy\,\tau_{pe}}dx\,dy$$

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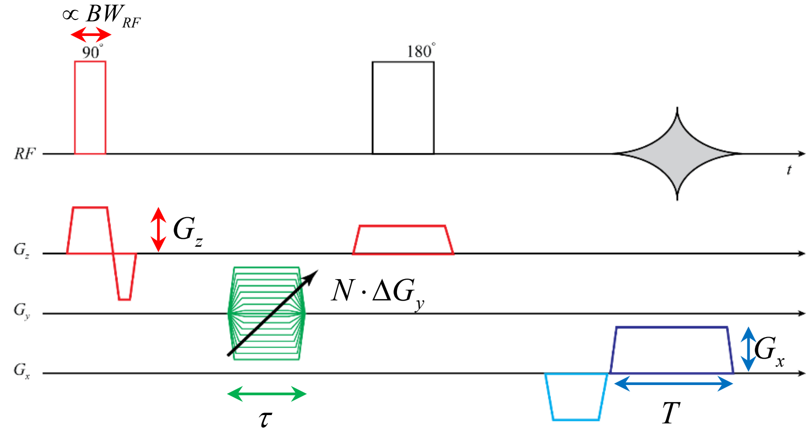
SOME NOTIONS ABOUT K-SPACE

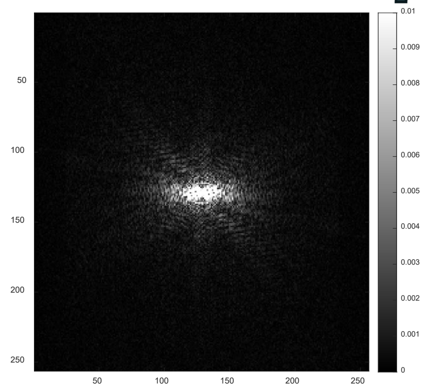


Phase Encoding Frequency Encoding

$$S(G_x, t, G_y, \tau_{pe}) \propto \iint_{slice} \rho(x,y) e^{-j\gamma G x x t}\, e^{-j\gamma G y y\, \tau_{pe}} dx\, dy$$

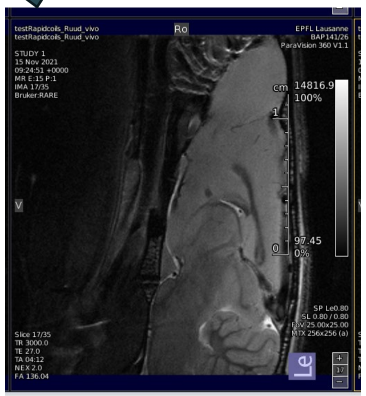
$$k_x = \frac{\gamma}{2\pi} \cdot G_x \cdot t \qquad k_y = \frac{\gamma}{2\pi} \cdot G_y \cdot \tau_{pe}$$

$$S(k_x, ky) \propto \iint_{slice} \rho(x,y)\, e^{-jk_x x}\, e^{-jk_y y} dx\, dy$$

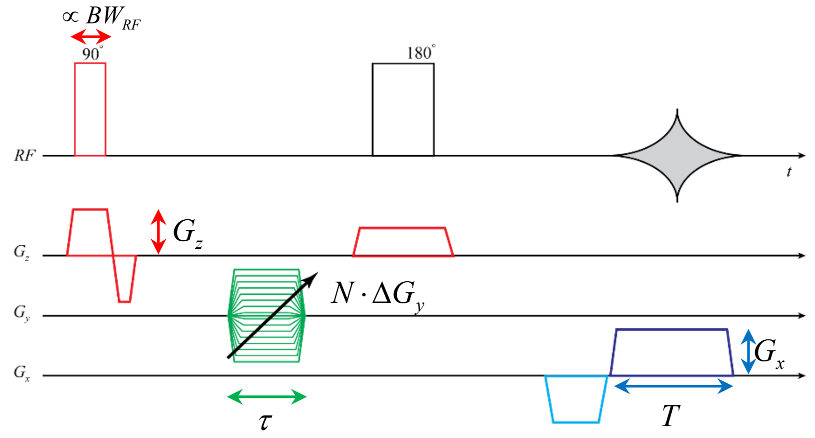


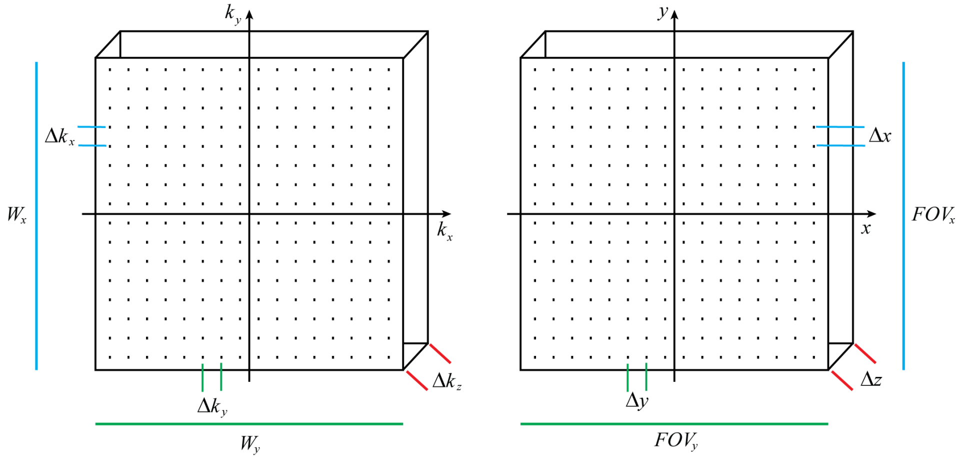
IFT

x,y



SOME NOTIONS ABOUT K-SPACE





Resolution

Slice Selection

$$FOV_z = \frac{BW_{RF}}{\gamma G_z}$$

Phase Encoding

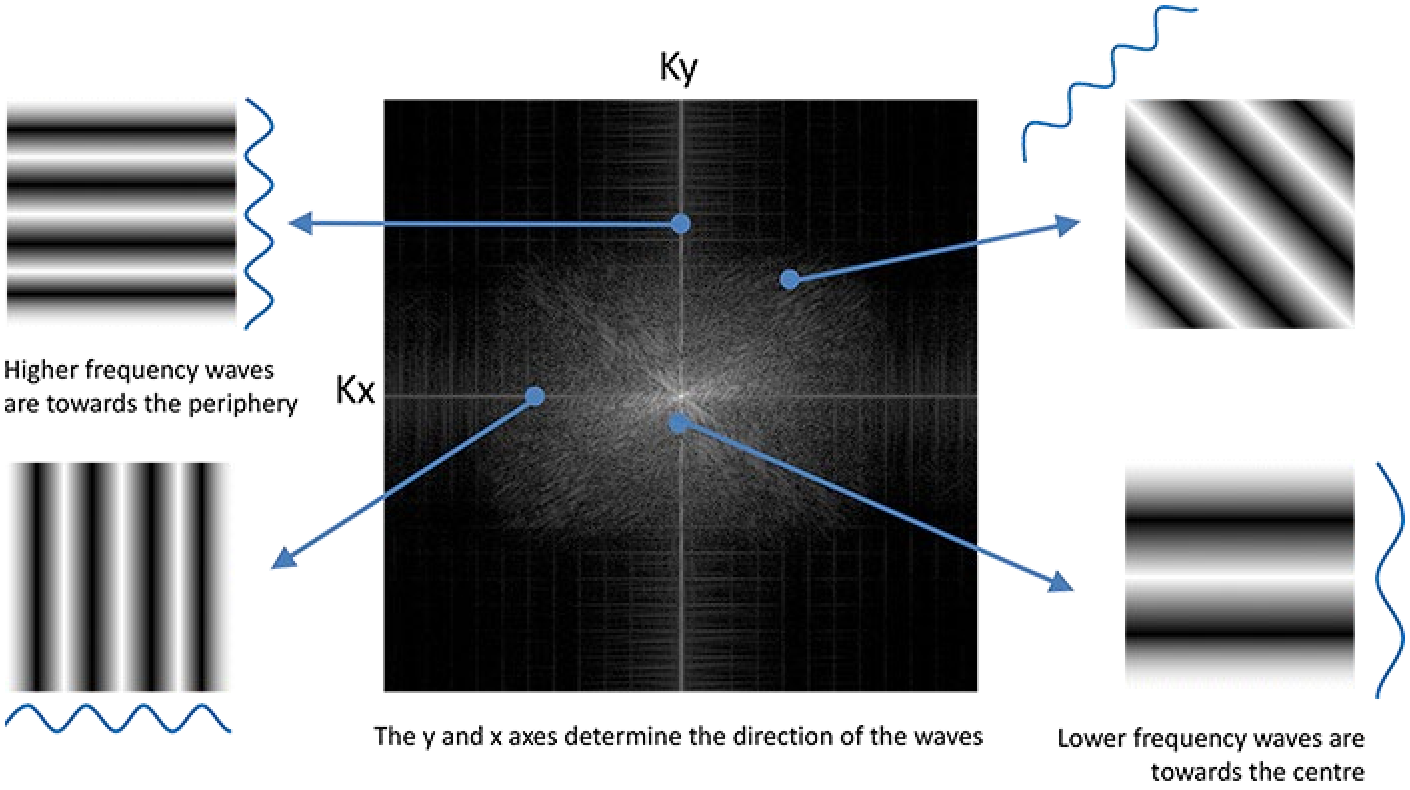
$$FOV_y = \frac{1}{\Delta k_y} = \frac{2\pi}{\gamma \Delta G_y \tau} \qquad \delta y = \frac{2\pi}{W_{k_y}} = \frac{2\pi}{\gamma \Delta G_y N \tau}$$

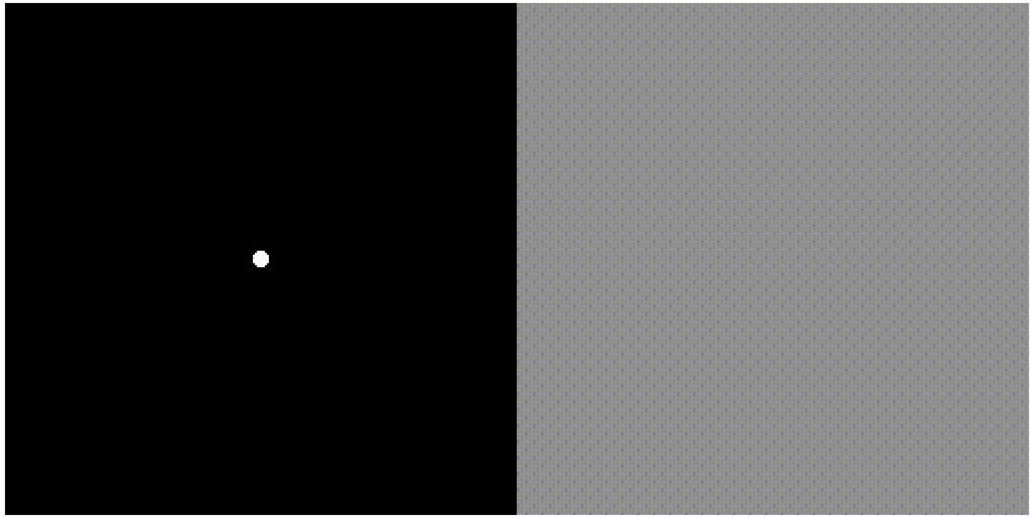
Frequency Encoding

$$FOV_x = \frac{1}{\Delta k_x} = \frac{2\pi}{\gamma G_x \Delta T} \qquad \delta x = \frac{2\pi}{W_{k_x}} = \frac{2\pi}{\gamma G_x T}$$

- Large FOV requires small steps in k
- ΔG_y τ creates a 2π cycle across the FOV
- High spatial resolution requires large Gradient x time product
- N/2 ΔG_y τ creates a π shift between two successive pixels

WHAT IS WHERE IN K-SPACE





Movie showing spatial frequencies corresponding to various points in *k*-space. (Courtesy Brian Hargreaves)

K-space - Radiology Cafe

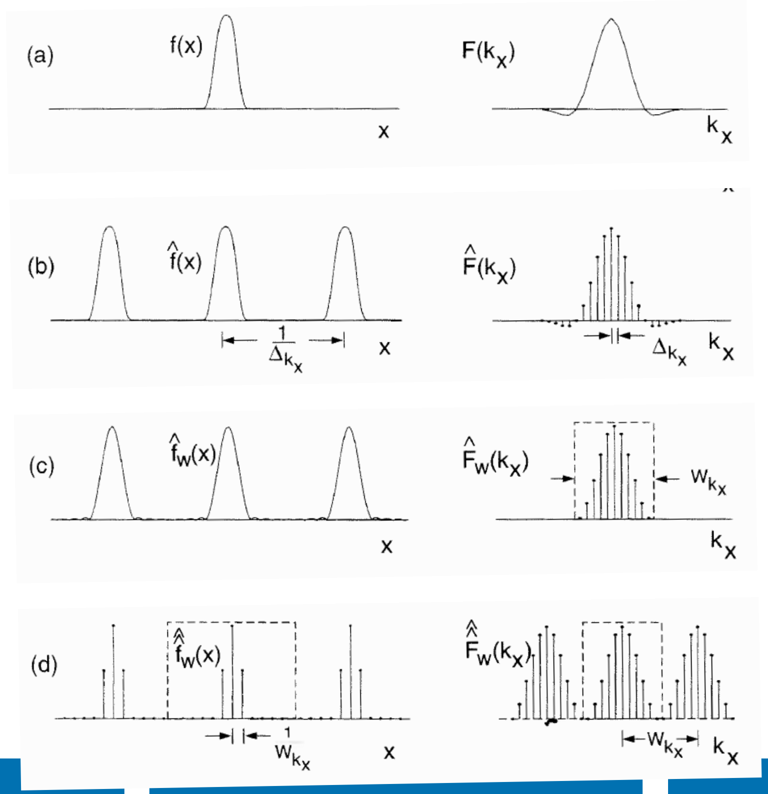
k-space: locations - Questions and Answers in MRI (mriquestions.com)

DISCRETE FOURIER TRANSFORM

FT ⟶ $$G(k)=\int_{-\infty}^{\infty} g(x)e^{-2\pi ikx}dx$$

DFT ⟶ $$G(\frac{p}{L})=\sum_{q=-n}^{n-1} g\left(\frac{qL}{2n}\right)e^{-2\pi pq/(2n)} \qquad p=-n,-n+1\ldots,n-1 \quad L=2n\Delta x$$

Reconstructed Image Signal sampled



ideally

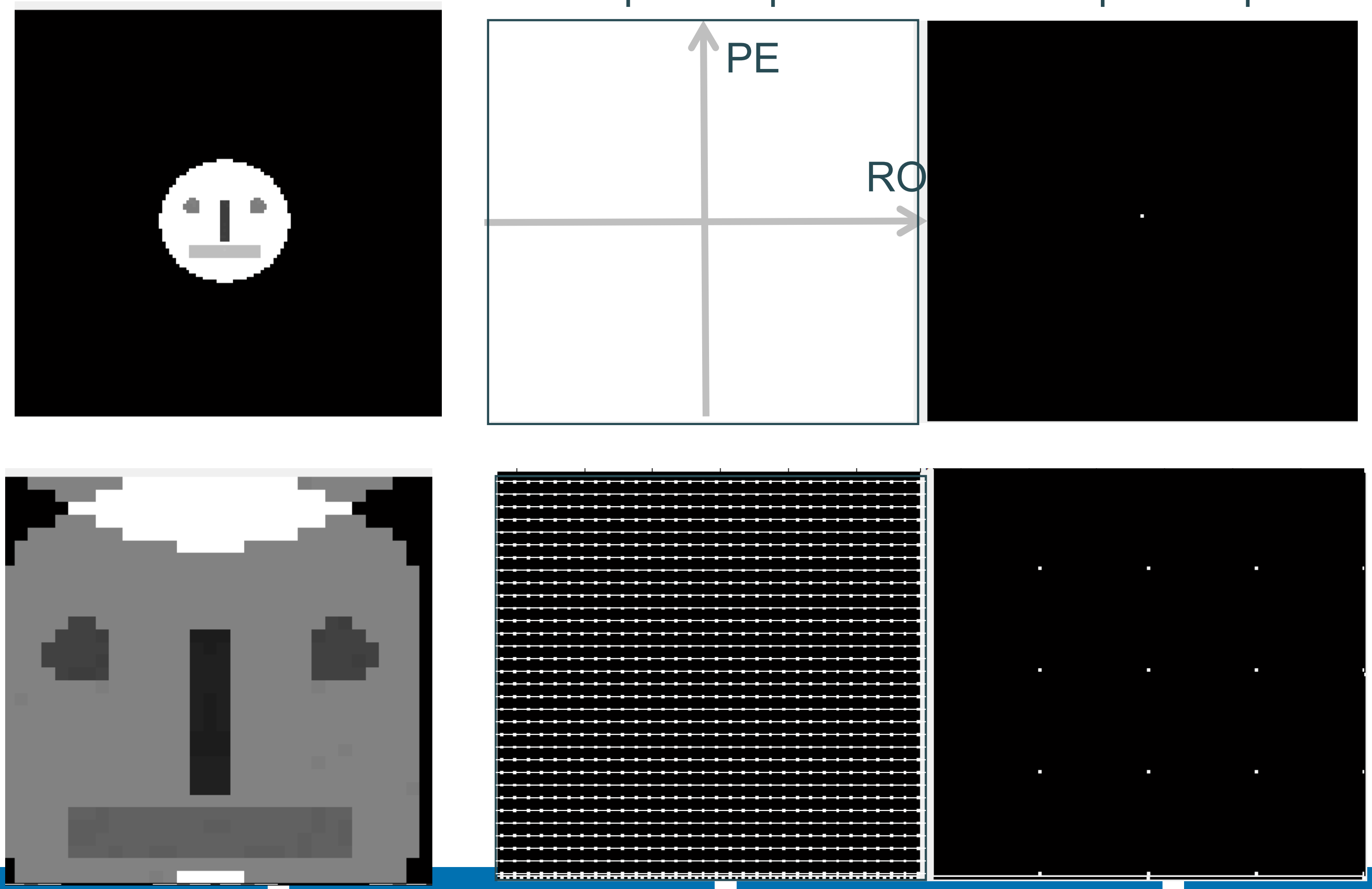
we can only sample every once in a while

we can only sample for so long

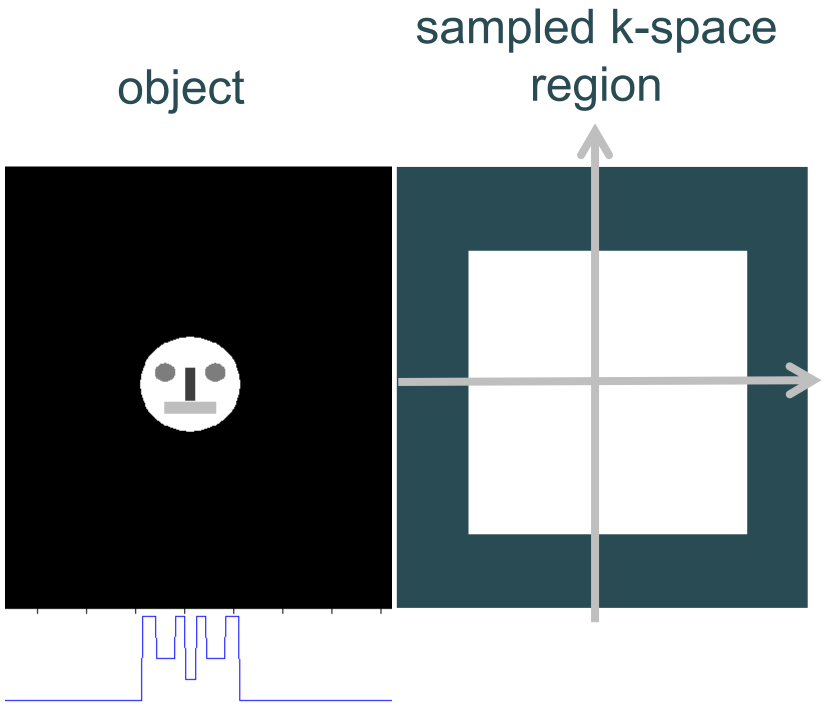
and we will only keep so much information

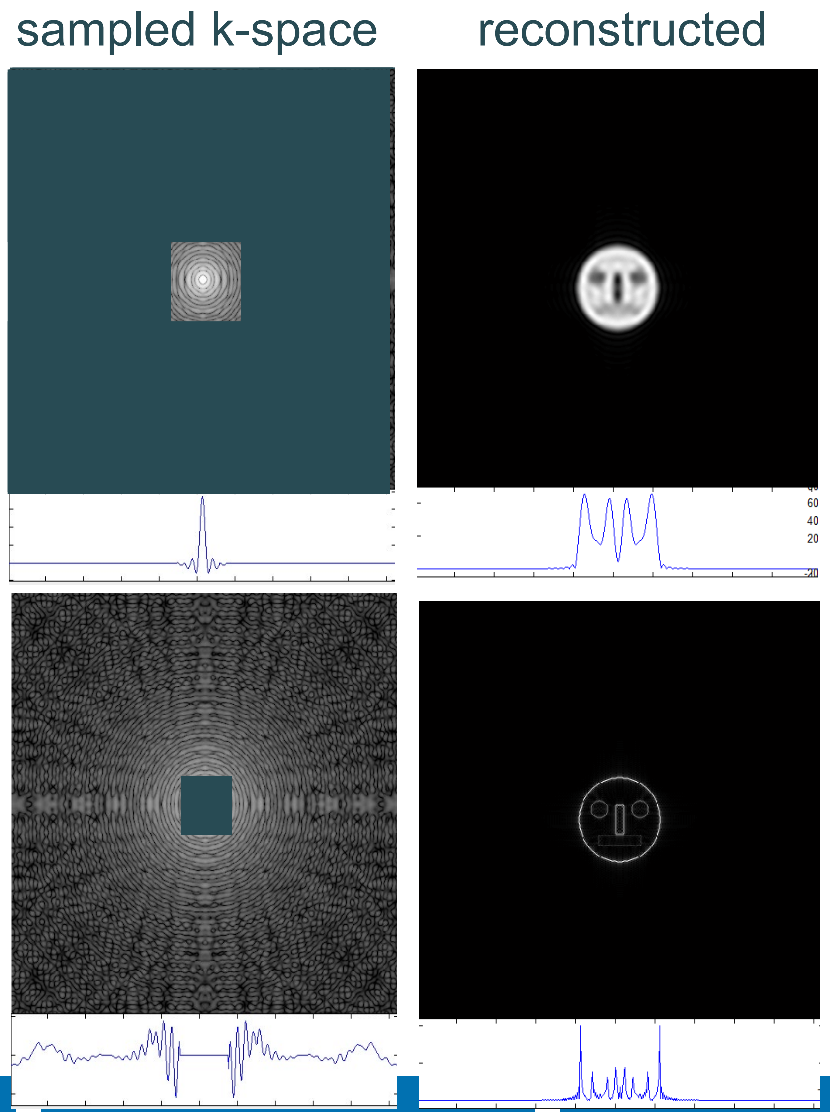
DISCRETE FOURIER TRANSFORM

reconstructed | sampled k-space | FT sampled k-space



WHAT IS WHERE IN K-SPACE





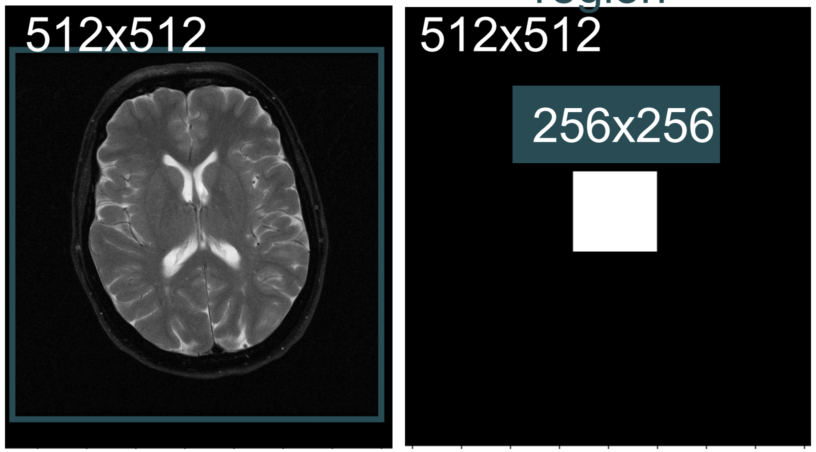
Low spatial frequencies (center of k-space) - **coarse structure**

High spatial frequencies (edge of k-space) - **detail**

WHAT IS WHERE IN K-SPACE

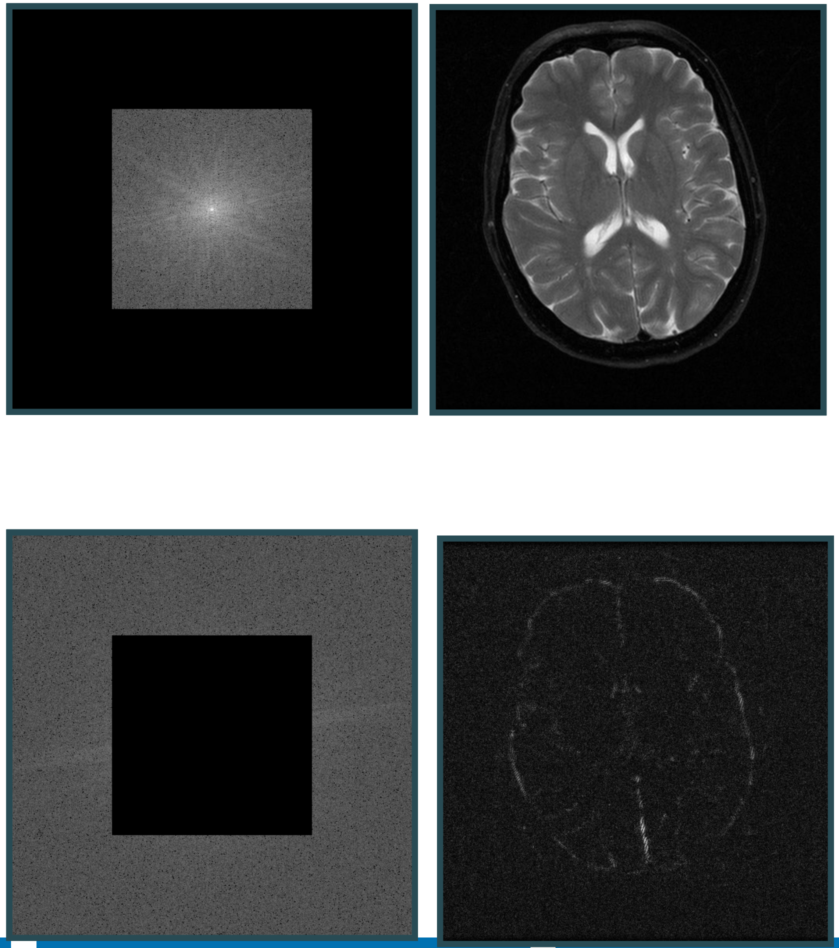
object

sampled k-space region



sampled k-space

reconstructed



Low spatial frequencies (center of k-space) - **coarse structure**

High spatial frequencies (edge of k-space) - **detail**

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SUMMARY & ACKNOWLEDGMENTS

MRI is incredibly rich & versatile

Slides:
Nicolas Kunz
Jose Marques
Cristina Cudalbu
Rolf Gruetter
Brayan Alves

Introductory NMR & MRI: Video 08: Magnetic Resonance Imaging (youtube.com)
11.1 - Echo formation in magnetic resonance (youtube.com)

Introduction to the Principles of MRI (Magnetic Resonance Imaging) (youtube.com)

Thank you for listening! Questions?

NEXT WEEKS

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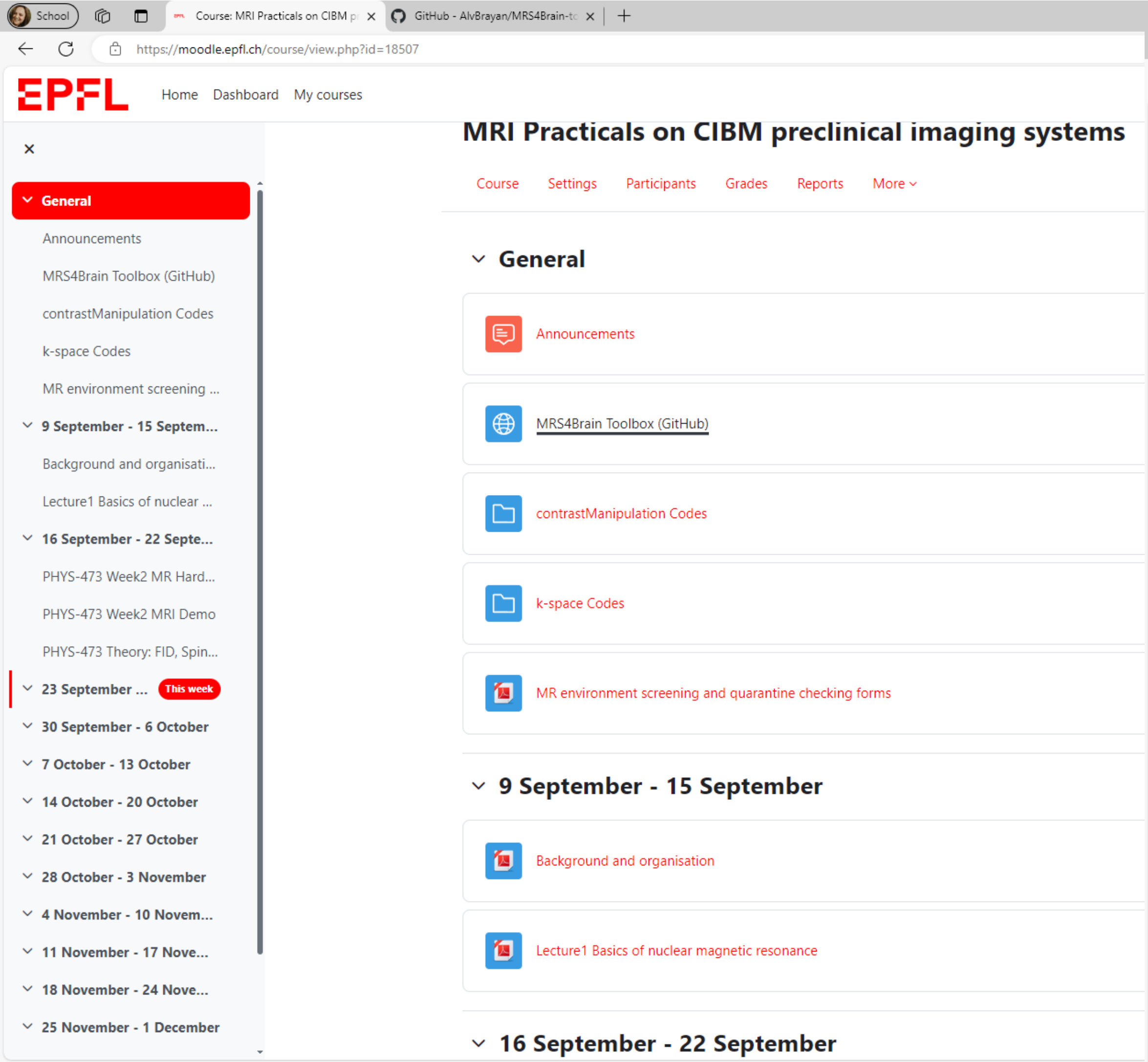

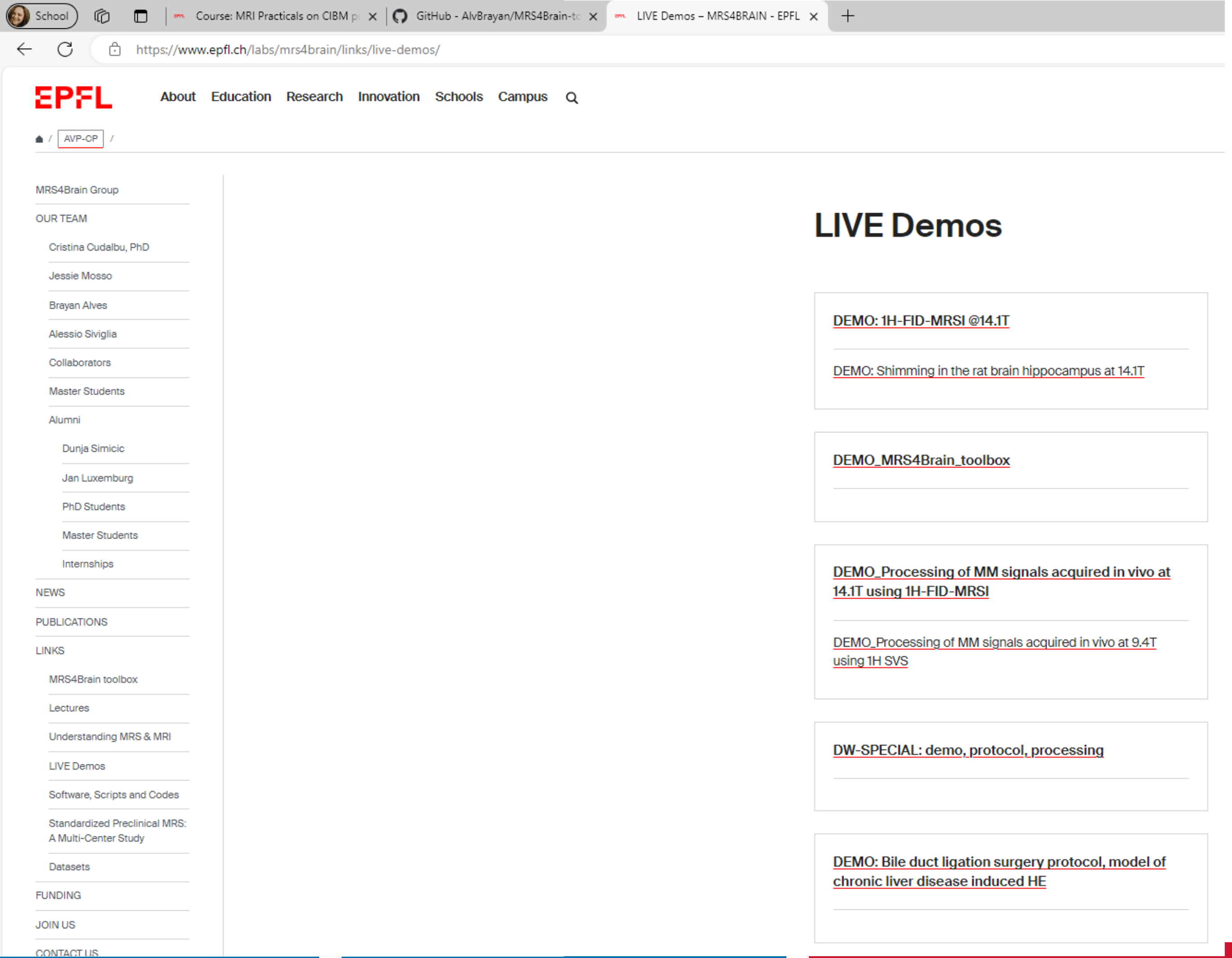

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Fondation Leenaards