



BASICS OF MRI: GRADIENTS AND KSPACE

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CIBM MRI EPFL AIT

PHYS 473

24th of September 2024



C I B M . C H

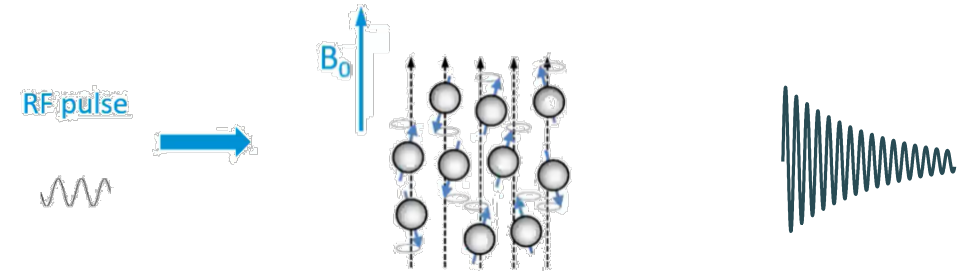
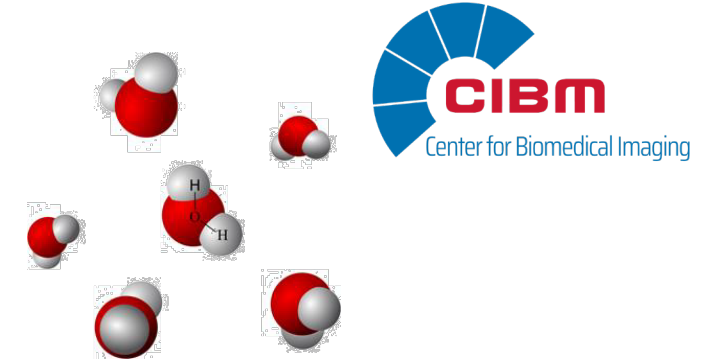
OUTLINE

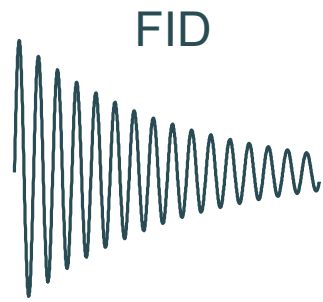
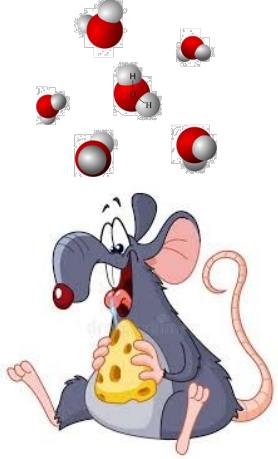
- Localization - Magnetic field gradients – gradient coils
- Spatial encoding
 - Slice selection
 - Frequency encoding
 - Phase encoding
- How to encode a 3D image
- Some notions about k-space
 - Discrete Fourier transform
 - What is where in k-space
 - How to fill the k-space

UNTIL NOW

Most MRI applications measure ^1H nuclei from water

- Static magnetic field (B_0)
- RF pulses
- FID – relaxation
 - M_{xy} decays, M_z grows (T_2 and T_1 relaxation)
- No spatial information - RF coils measure signal from entire body





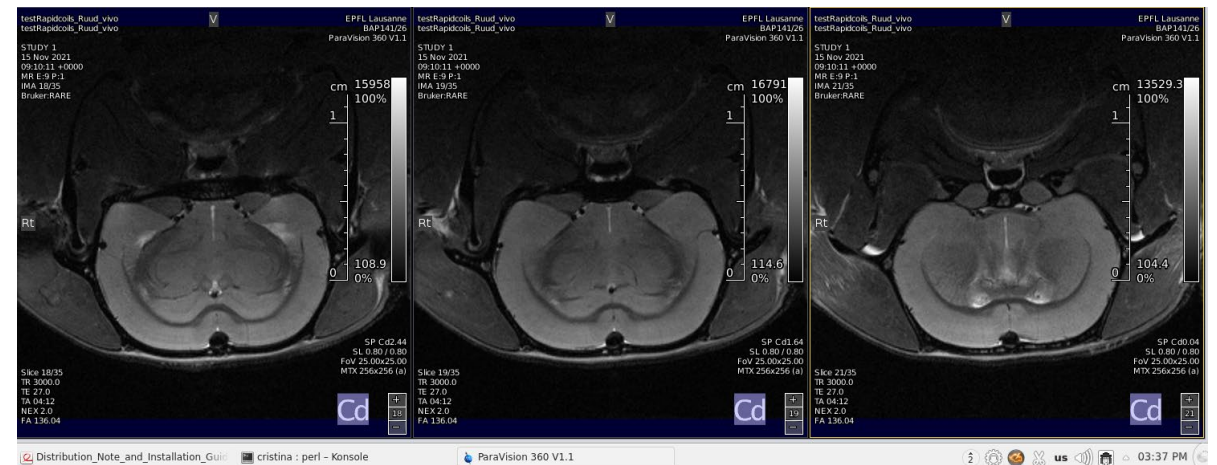
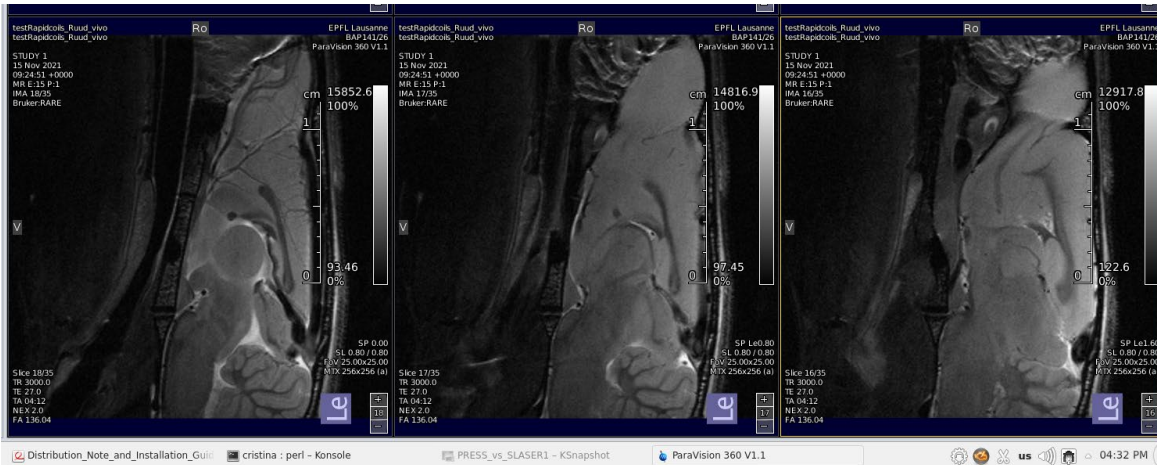
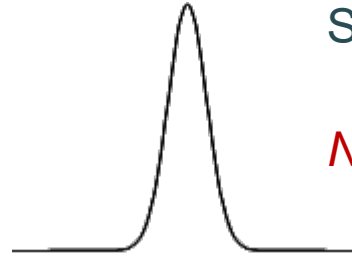
FT



?

Spectrum

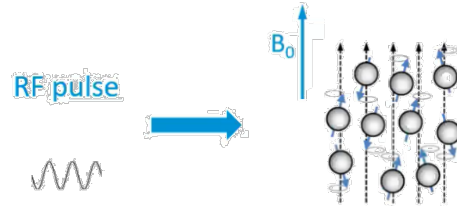
No spatial information



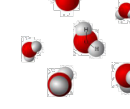
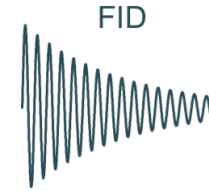
MRI – Magnetic Resonance Imaging

MAGNETIC RESONANCE IMAGING

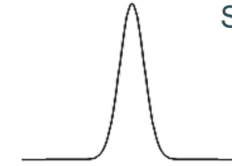
Equipment



No spatial information • RF coils measure signal from entire body



FT



Spectrum

Precession
Frequency

$$\omega_L = \gamma B_0$$

Static Magnetic Field

+ Magnetic field / Resonance frequency → position dependent

?

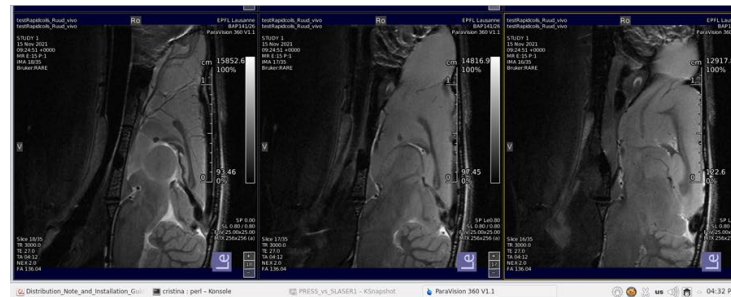
MRI Scanner Cutaway

Radio
Frequency (RF)
Coil

Gradient coils

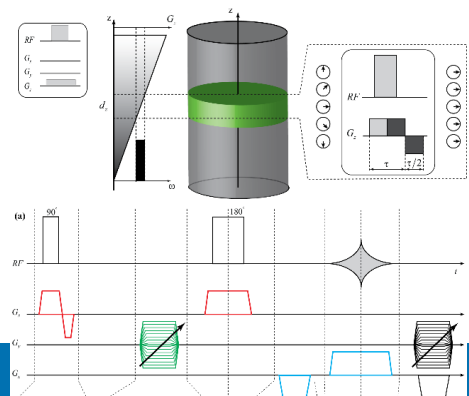
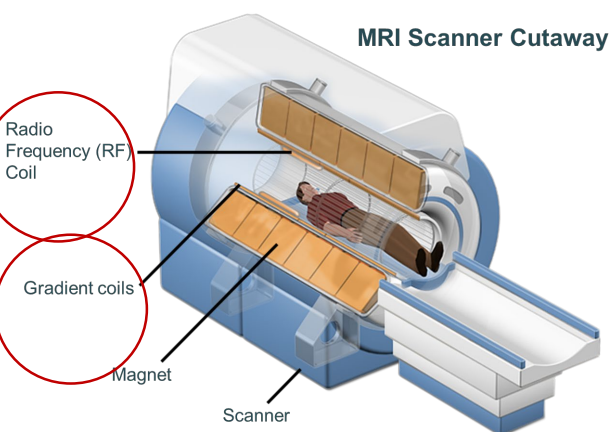
Magnet

Scanner

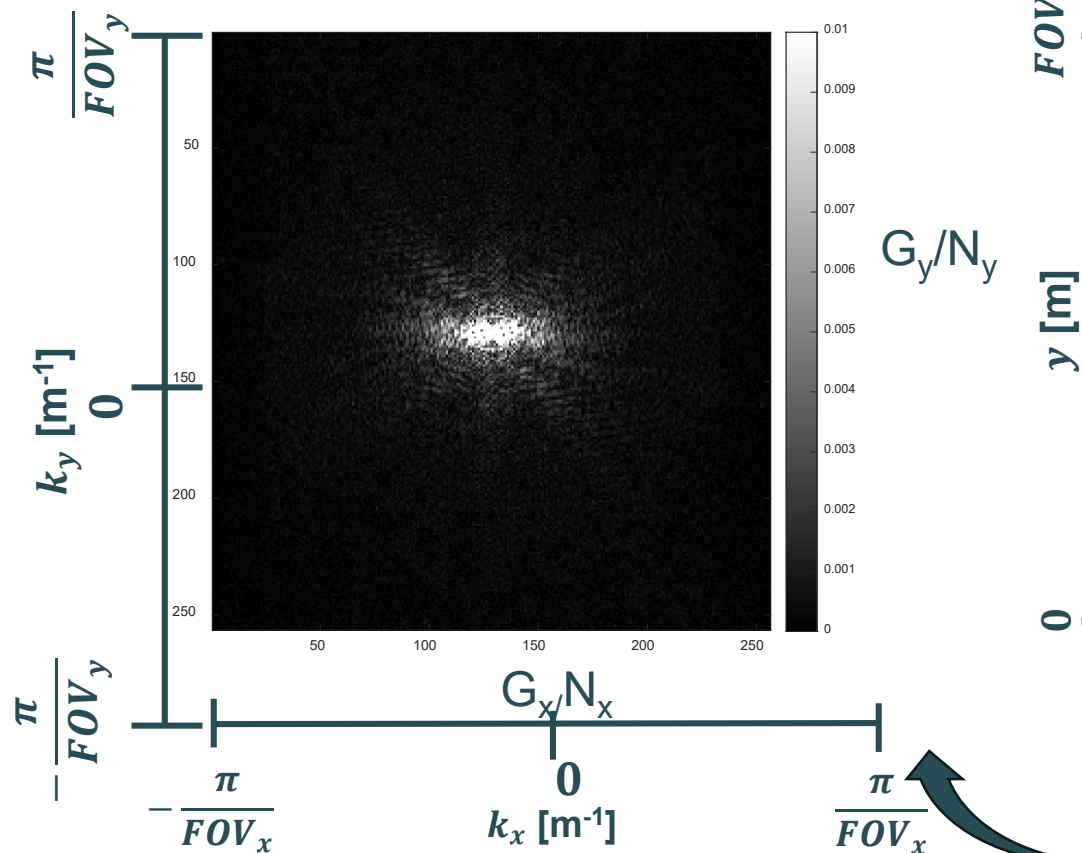


MRI – Magnetic Resonance Imaging

Object Space

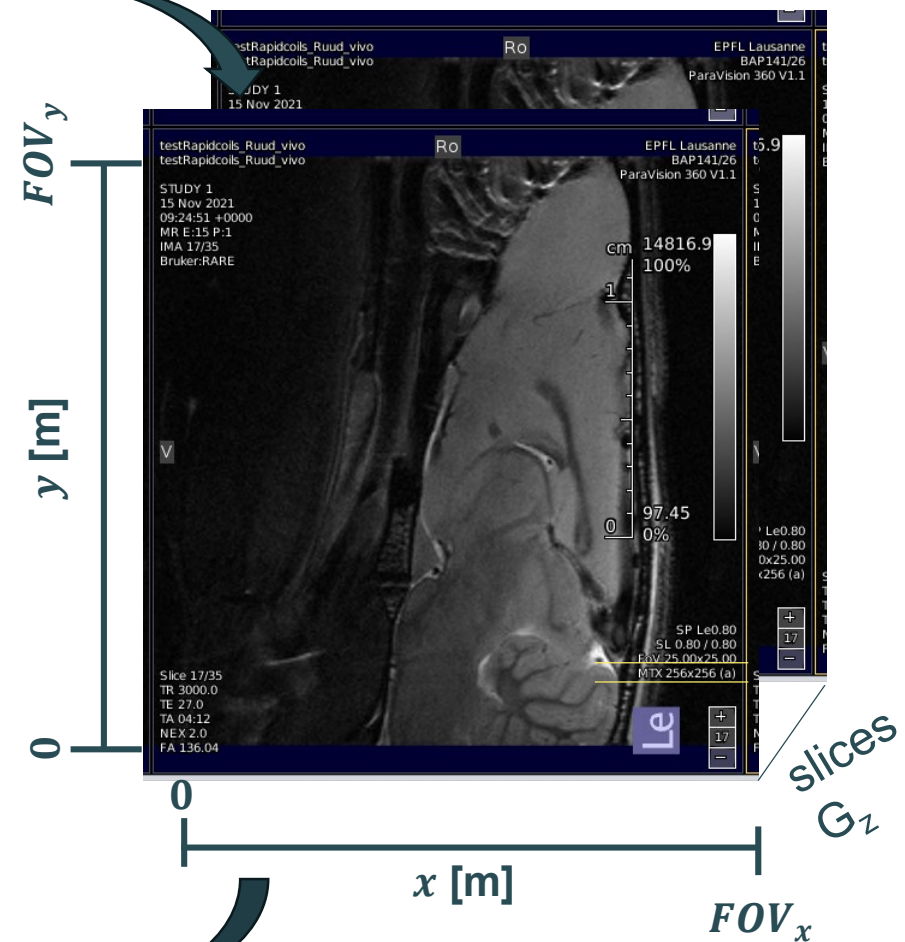


2D Kspace



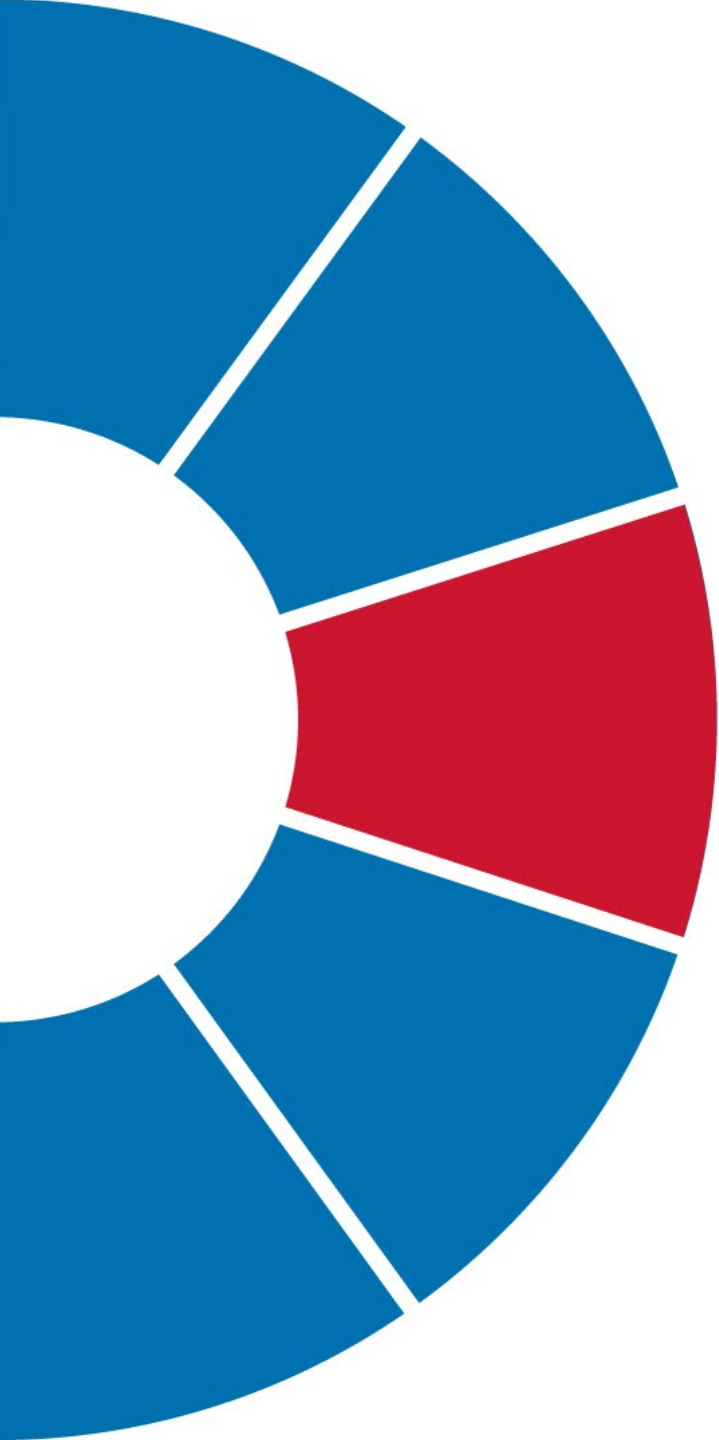
IFT_{x,y}

2D Image Space

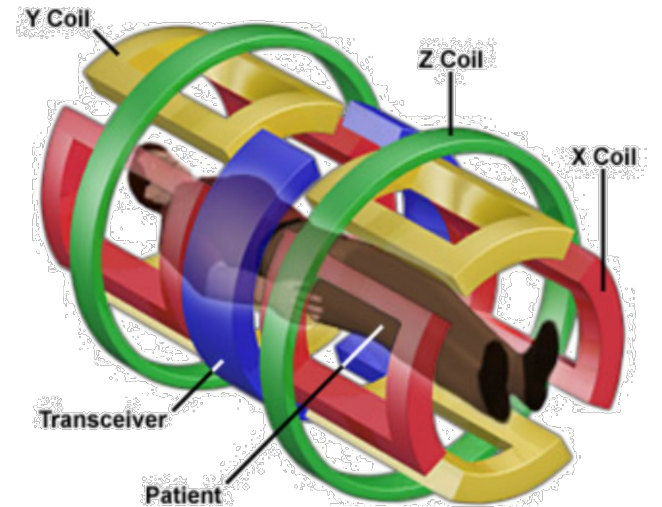


FT_{x,y}

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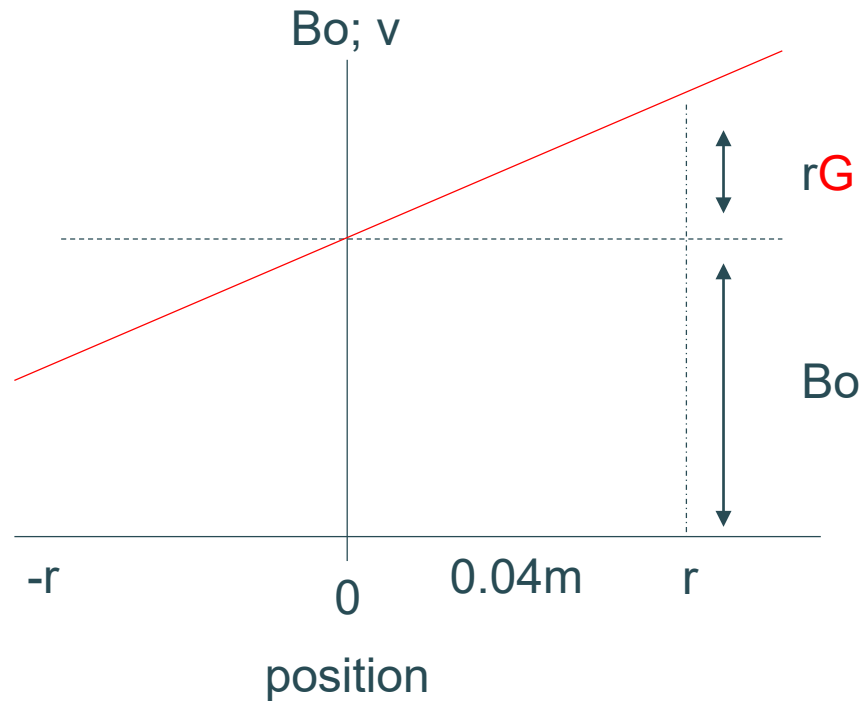


MAGNETIC FIELD GRADIENTS – GRADIENT COILS



HOW TO ENCODE SPATIAL POSITION ?

Magnetic field gradients - differentiate the signal across space



Magnetic field B along z varies spatially with x , y , and/or z :

$$\vec{G} \equiv \frac{dB_z}{d\vec{r}}$$

$$v = \gamma B_0$$

External magnetic field $B_0 = 3.0 \text{ Tesla (T)} = 30,000 \text{ Gauss (G)}$

Gyromagnetic ratio $\gamma(^1\text{H}) = 42.57 \text{ MHz/T} = 4257 \text{ Hz/G}$

Larmor frequency $v = 127.7 \text{ MHz}$

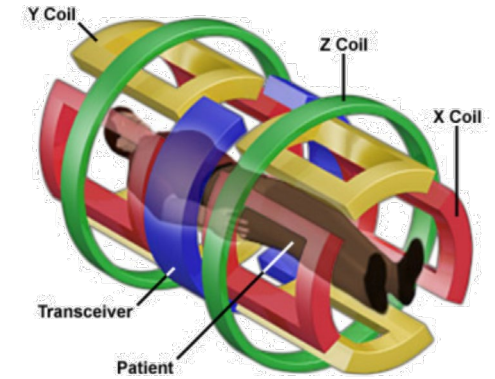
$$v = \gamma B_0 + \gamma r G$$

Position $r = 0.04 \text{ meters (m)} = 4 \text{ cm}$

Gradient $G = 0.05 \text{ T/m} = 5 \text{ G/cm} = 21.285 \text{ kHz/cm}$

Additional frequency offset = 85.14 kHz

$$\phi \text{ (radians)} = 2\pi v t$$



GRADIENT COILS

- Gradient coil generate a magnetic field along the main B_0 field varying linearly along one axis (x, y or z):

- $\vec{G}_x = x \cdot G_x \cdot \vec{e}_z$

- $\vec{G}_y = y \cdot G_y \cdot \vec{e}_z$

- $\vec{G}_z = z \cdot G_z \cdot \vec{e}_z$

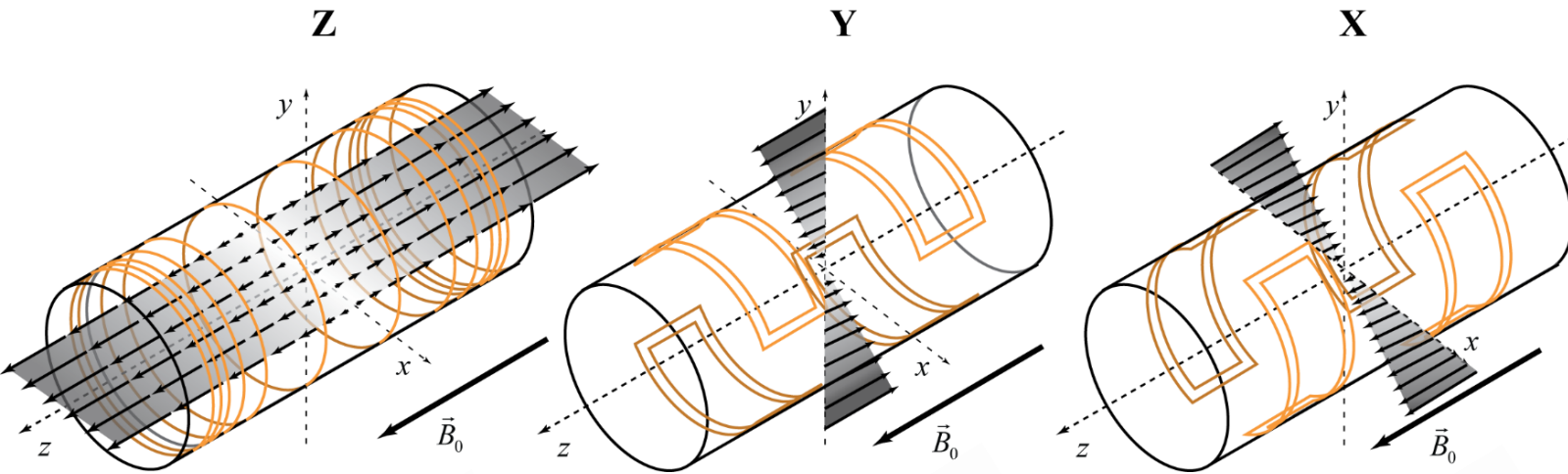
- By applying a (magnetic) gradient, we modify the resonance frequency locally:

- $\omega = \gamma \cdot B_0 + \gamma \cdot x \cdot G_x = \omega_0 + \underbrace{\gamma \cdot x \cdot G_x}_{\delta\omega}$

Or more generally if multiple gradients are applied at the same time

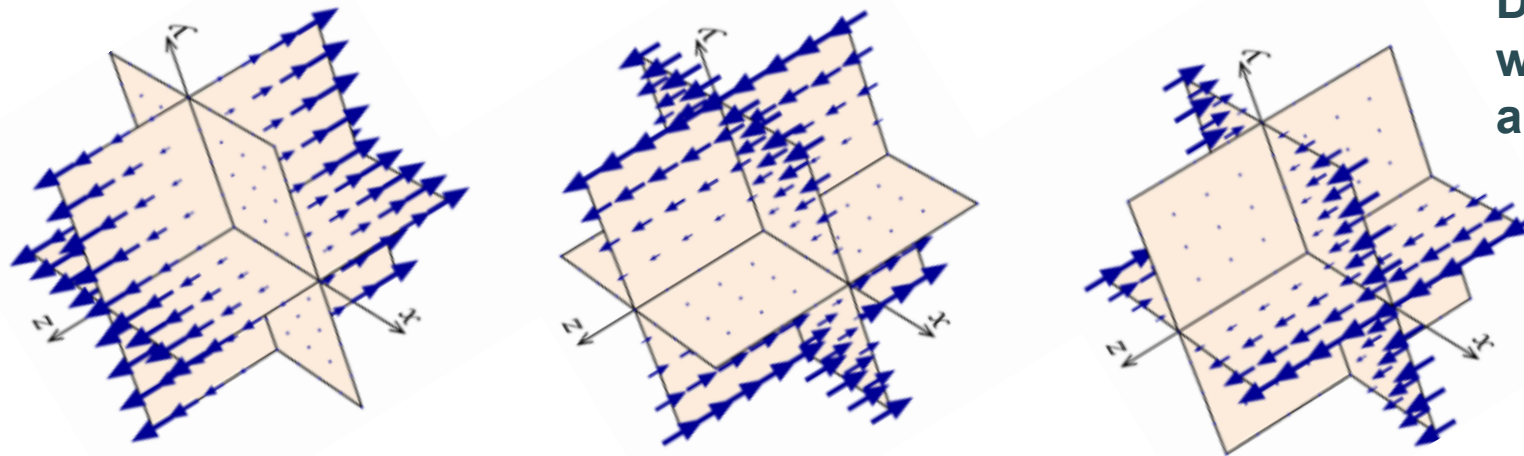
- $\omega = \omega_0 + \gamma \cdot (x \cdot G_x + y \cdot G_y + z \cdot G_z)$

GRADIENT COILS



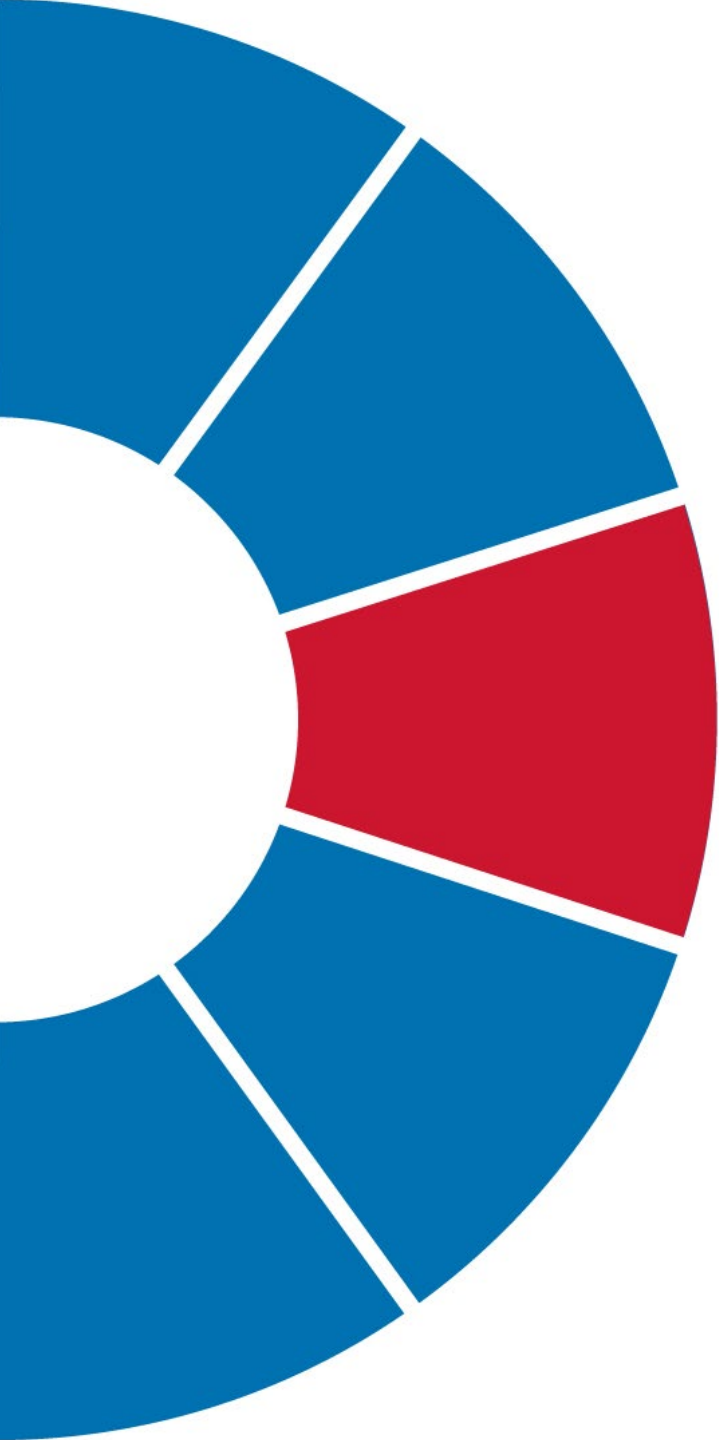
Gradient coils generate magnetic field parallel to B_0 – added to main B_0 which is on Z direction (arrows are parallel to B_0) = direction of the magnetic field generated by the gradient

Direction of gradient = direction in which the magnetic field varies = amplitude is varying along the 3 axes



GRADIENT COILS

- Amplitude G/cm or mT/m ($10.000\text{G}=1\text{T}$)
- Linearity (homogeneity)
- Rise time (0 to max amplitude, $120\text{ }\mu\text{s}$) – slew rate= $G_{\text{max}}/T_{\text{rise}}$
- Change current in B_0 – Lorentz force – acoustic noise in MRI



SPATIAL ENCODING

SPATIAL ENCODING

- ❑ MR relies on four different methods to spatially encode the NMR signal
- ❑ Gradient based methods:
 - ❑ Frequency Encoding (73' Lauterbur, 1973 Mansfield)
 - ❑ Slice selection (74' Mansfield)
 - ❑ Phase encoding (76' Ernst)
- ❑ Geometrical method:
 - ❑ RF-coil sensitivity profile (97' Sodickson, 99' Pruessman, 01' Larkman,...)

Lauterbur and Mansfield shared the 2003 Nobel Prize in Medicine

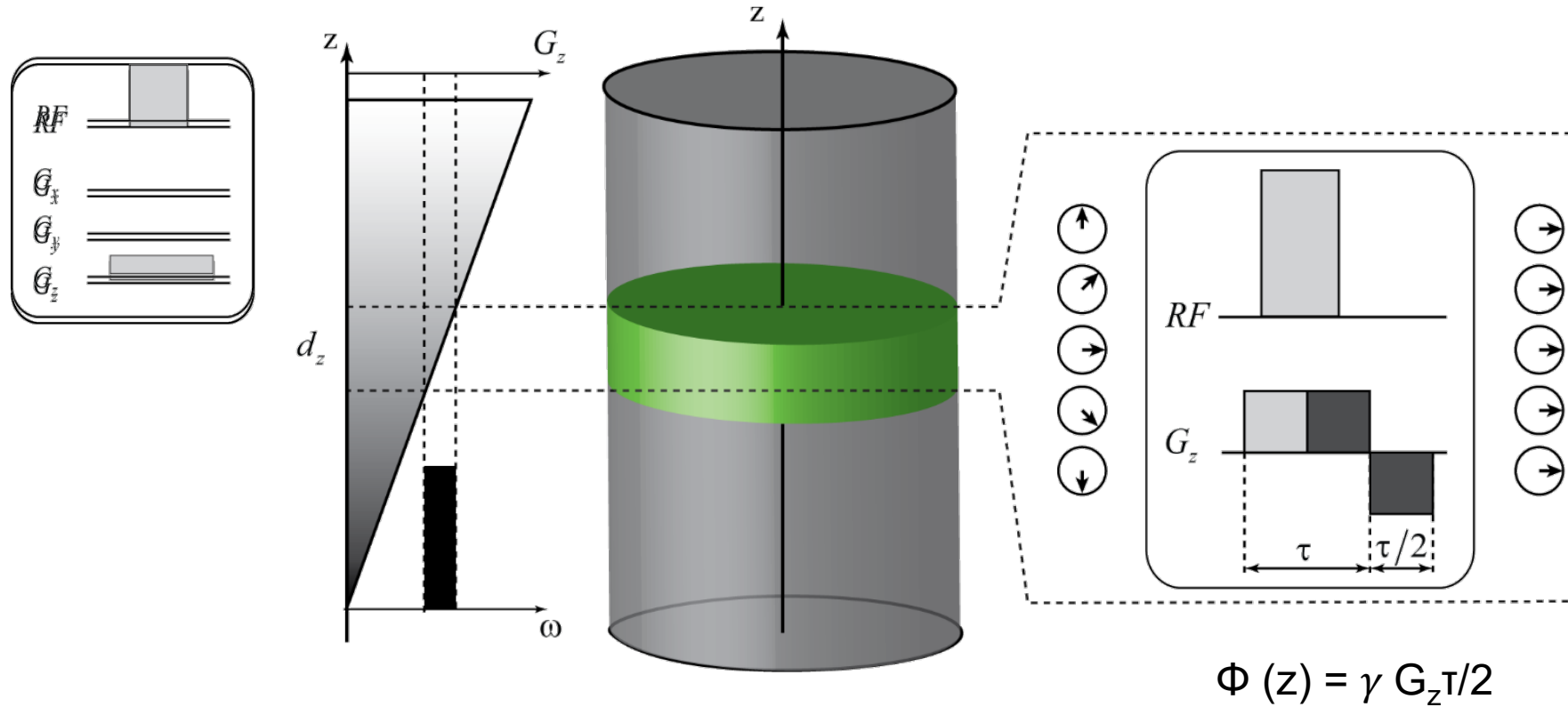


Peter Mansfield



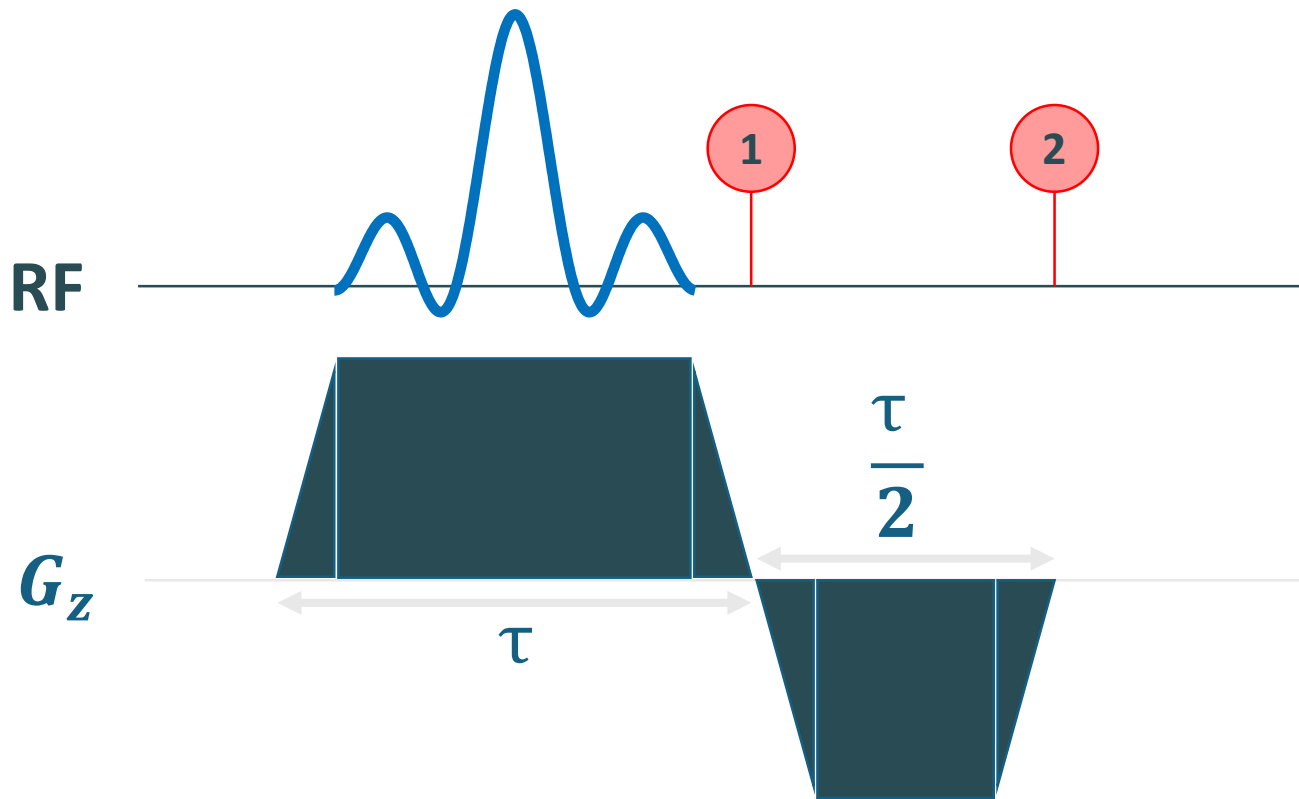
Paul
Lauterbur

SPATIAL ENCODING – SLICE SELECTION



$$S \propto \iint_{\text{slice}} \rho(x, y) dx dy$$

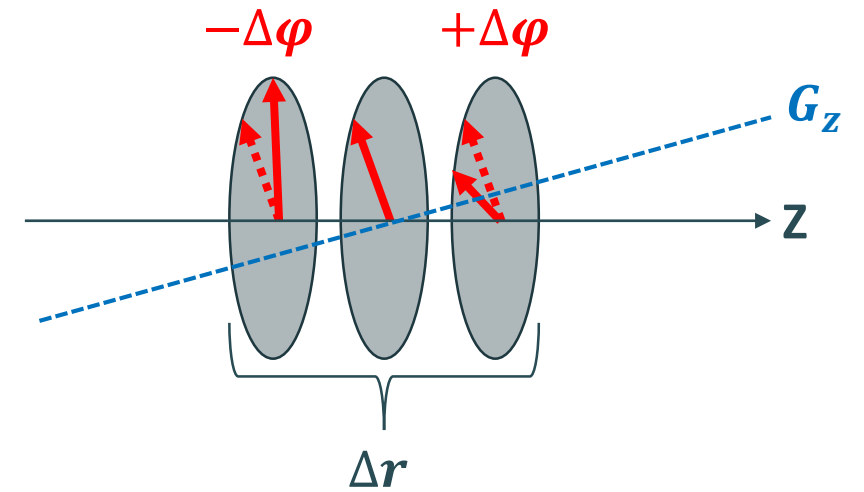
$$\delta z = \frac{BW_{RF}}{\gamma G_z}$$



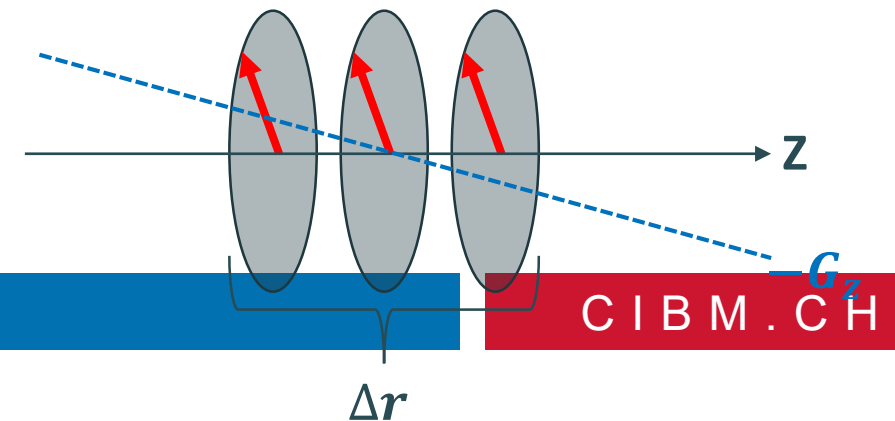
1

$$\Delta\varphi = \frac{\frac{\Delta r}{2} \gamma G_z \tau}{2}$$

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2



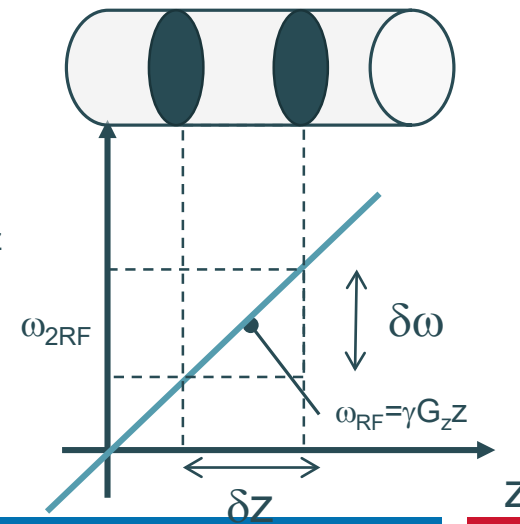
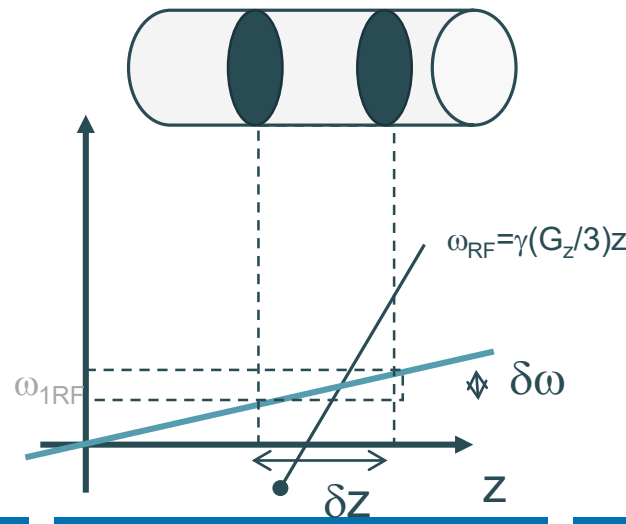
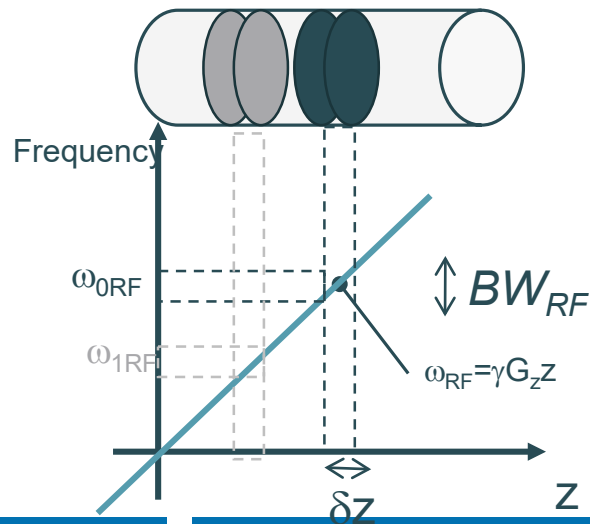
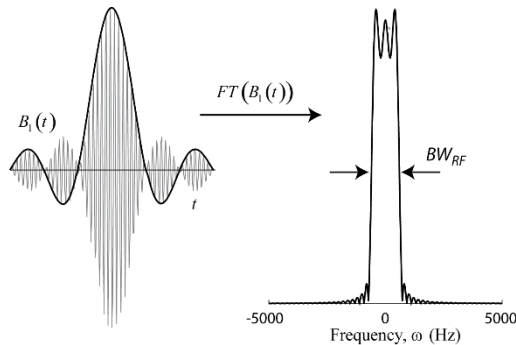
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SLICE SELECTION

- Gradient is applied during an RF-pulse, which is characterized by its excitation bandwidth (BW_{RF})

$$\delta z = \frac{BW_{RF}}{\gamma G_z}$$

- The slice thickness can be changed either by adjusting the pulse length or the gradient amplitude



EXAMPLE

$$\delta z = \frac{BW_{RF}}{\gamma G_z}$$

- 2mm slice
- 5mm out of the isocenter
- $BW_{RF} = 1000 \text{ Hz}$
- $G_z = ?$

- fixed G_z and a constant RF pulse,
- the slice position can only be adjusted by changing the transmitter frequency

$$\nu_{RF} = \nu_0 + \text{slice pos (cm)} \times G_z \text{ (kHz/cm)} = \nu_0 + 2.5 \text{ kHz}$$

$$G_z = \frac{1.0}{0.2} \text{ kHz cm}^{-1} = 5 \text{ kHz cm}^{-1} = 1.17 \text{ G cm}^{-1} = 11.7 \text{ mT m}^{-1}$$

$$\gamma = 4257.6 \text{ Hz G}^{-1}$$

While elimination of the gyromagnetic ratio makes the variables nucleus-specific, the use of uniform units (kHz and cm) greatly simplify calculations and avoids common mistakes.

- RF pulse frequency ν_{RF} (kHz),
- magnetic field strength $(\gamma/2\pi)B_0$ (kHz), and
- gradient strength $(\gamma/2\pi)G$ (kHz cm⁻¹)
- γ ($\text{rad s}^{-1} \text{ T}^{-1}$) = 267.522 ; $\gamma/2\pi$ (MHz T⁻¹) = 42.57

ORIENTATIONS

Dataset Browser x Examination - 19092024_ratMRSihighres_BA / 19092024_ratMRSihighres_BA x

STUDY 1
19 Sep 2024
09:55:25 +0200
MR E:6 P:1
IMA 8/20
Bruker:RARE

EPFL Lausanne
BAP141/26

Ro

STUDY 1
19 Sep 2024
09:44:42 +0200
MR E:5 P:1
IMA 10/20
Bruker:RARE

EPFL Lausanne
BAP141/26

V

STUDY 1
19 Sep 2024
09:44:42 +0200
MR E:5 P:1
IMA 11/20
Bruker:RARE

EPFL Lausanne
BAP141/26

V

cm 41024.1
100%
45.58
0%

Rt

SP D1.11
SL: 0.807/0.80
FoV 24.00x24.00
MTX 256x256 (a)

Slice 8/20
TR 3000.0
TE 27.0
TA 04:12
NEX 2.0
FA 180.0

V

SP Ro0.55
SL: 0.807/0.80
FoV 24.00x24.00
MTX 256x256 (a)

Slice 10/20
TR 3000.0
TE 27.0
TA 04:12
NEX 2.0
FA 180.0

Cd

SP Cd0.25
SL: 0.807/0.80
FoV 24.00x24.00
MTX 256x256 (a)

Slice 11/20
TR 3000.0
TE 27.0
TA 04:12
NEX 2.0
FA 180.0

Cd

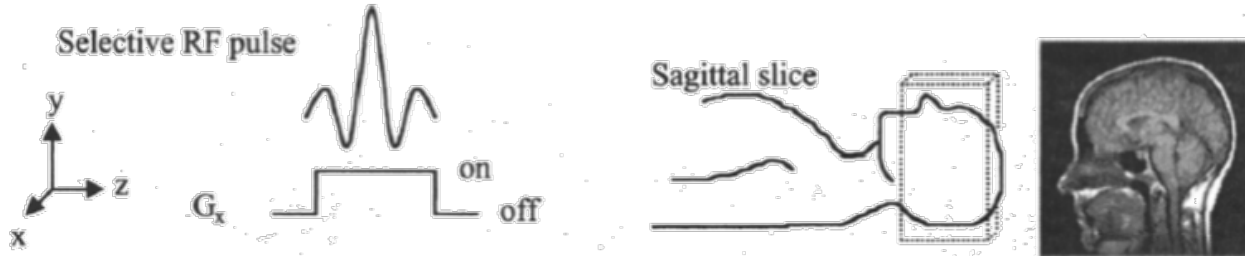
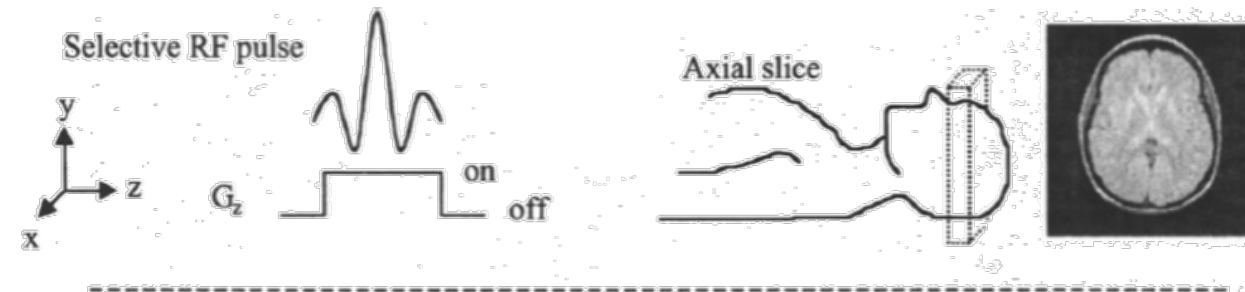
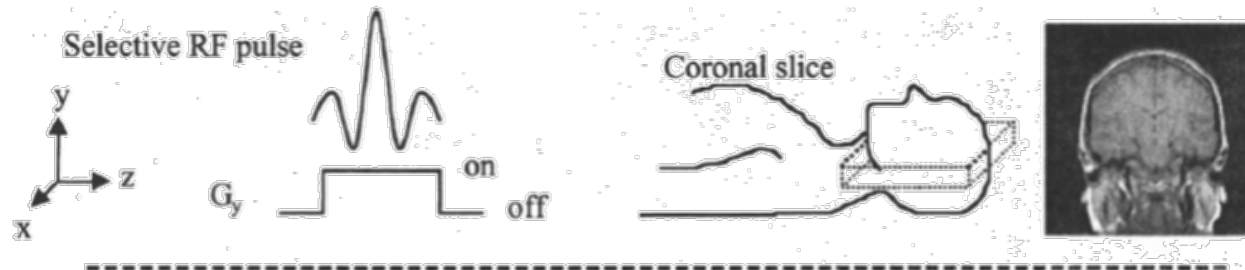
Open Apply Cancel

Ref	Instruction Name	Status	Duration
1	1_Localizer (E1)	SUCCEEDED	00:00:00
2	1_Localizer (E2)	SUCCEEDED	00:00:00
3	2_Localizer (E3)	SUCCEEDED	00:00:00
4	3_Localizer_multi_slice_10_s	SUCCEEDED	00:00:00
5	4_T2_TurboRARE_6_54K_08	SUCCEEDED	00:00:00
6	5_T2_TurboRARE_6_54K_Co	SUCCEEDED	00:00:00
7	5_T2_TurboRARE_6_54K_C	SUCCEEDED	00:00:00

Echo Time 27.00 ms
Repetition Time 4100.000 ms
Averages 10
Repetitions 1
Scan Time 0h13m40s0ms
Echo Spacing 9.000 ms
Rare Factor 6

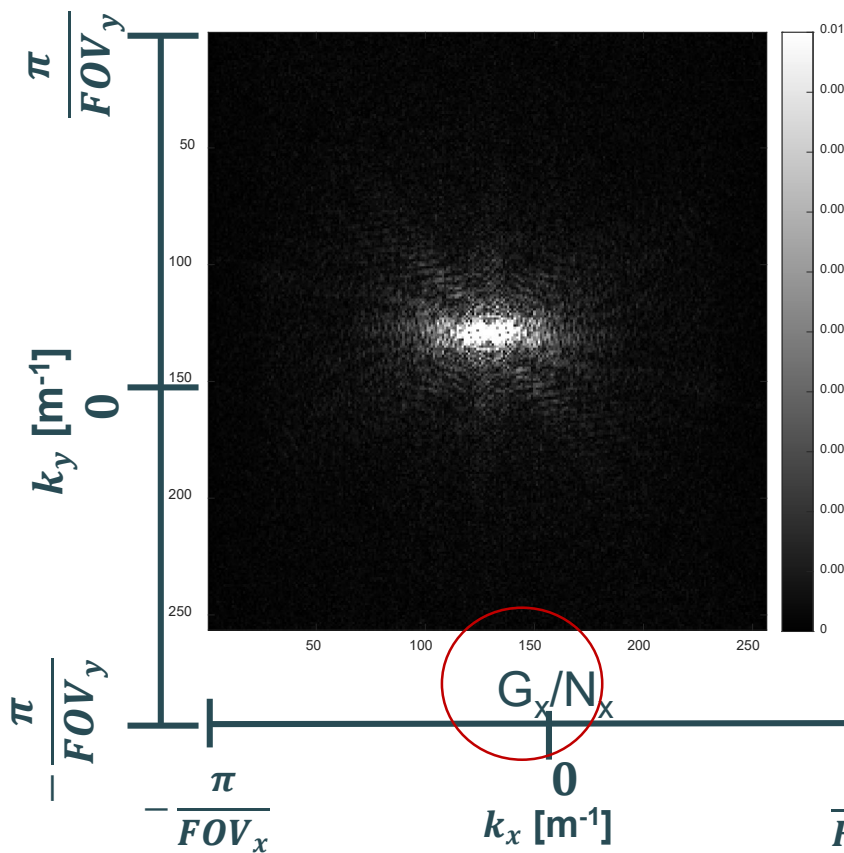
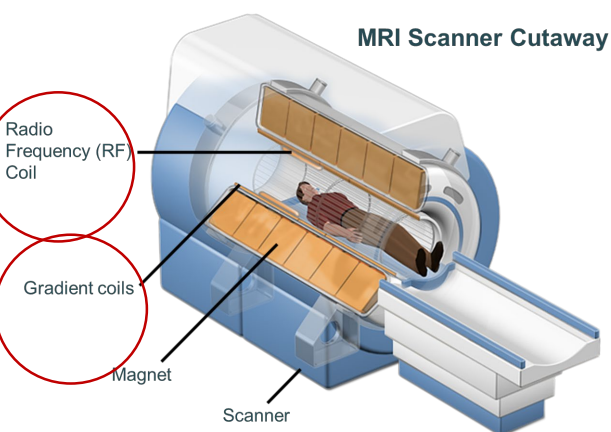
Slice Package 1 of 1
Slices 60
Slice Orientation ☐ Axial ☐ Sagittal ☒ Coronal
Read Orientation ☒ Le-Rt ☐ V-D ☐ Ro-Cd
Slice Thickness 0.200 mm
Image Size 128 128
Field of View 24.000 24.000 mm

ORIENTATIONS



2D Image Space

2D Kspace



IFT_{*x,y*}

 FOV_y y [m] G_y/N_y
$$\frac{\pi}{FOV_x}$$

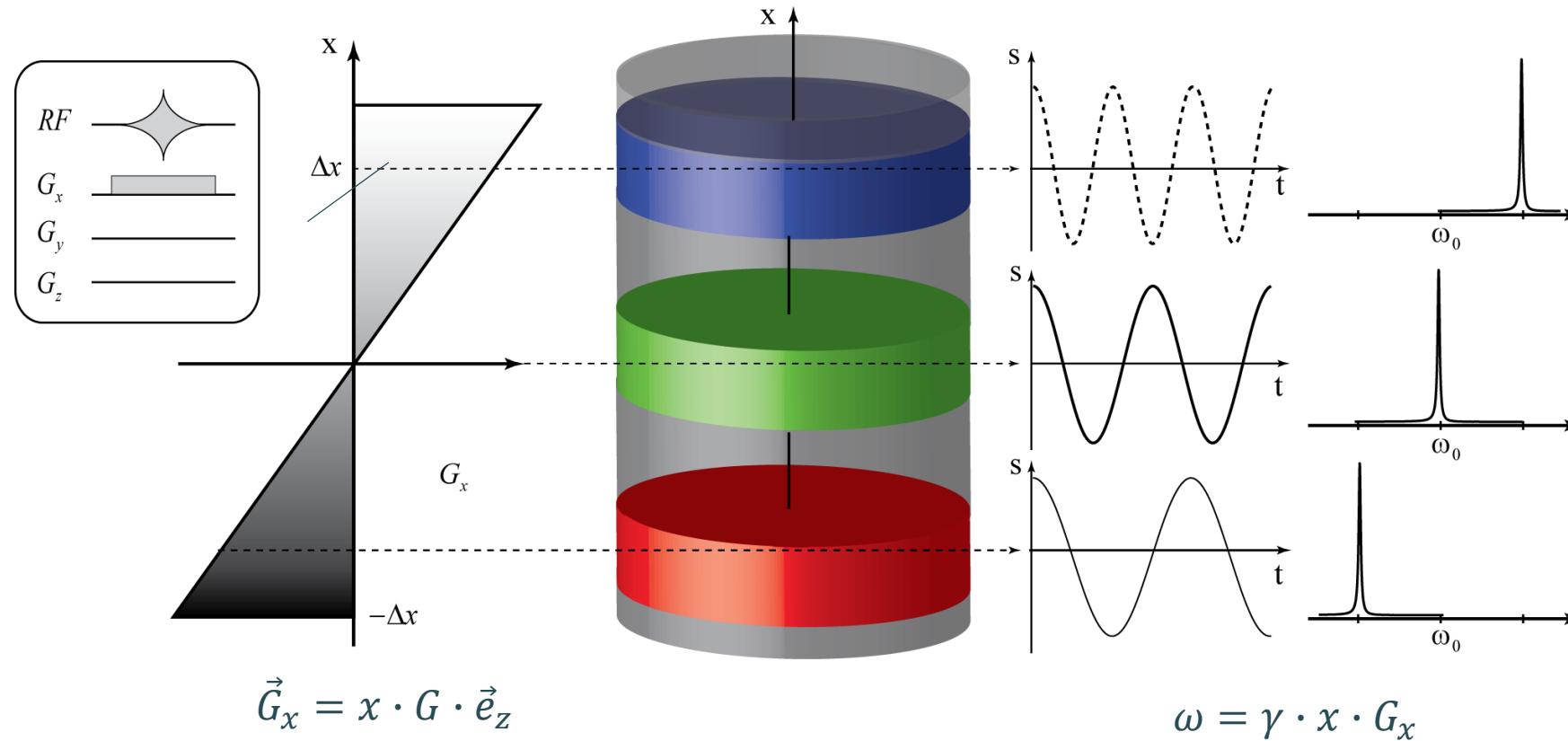
FT x,y

 FOV_x

slices
Gz

C I B M . C H

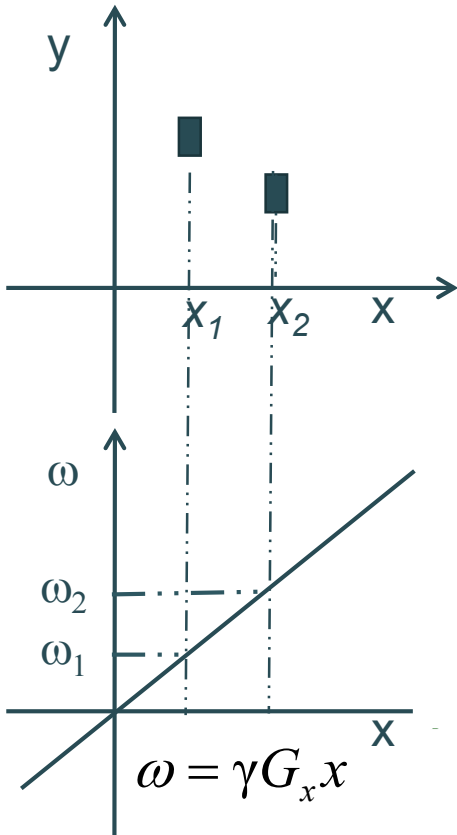
SPATIAL ENCODING – FREQUENCY ENCODING

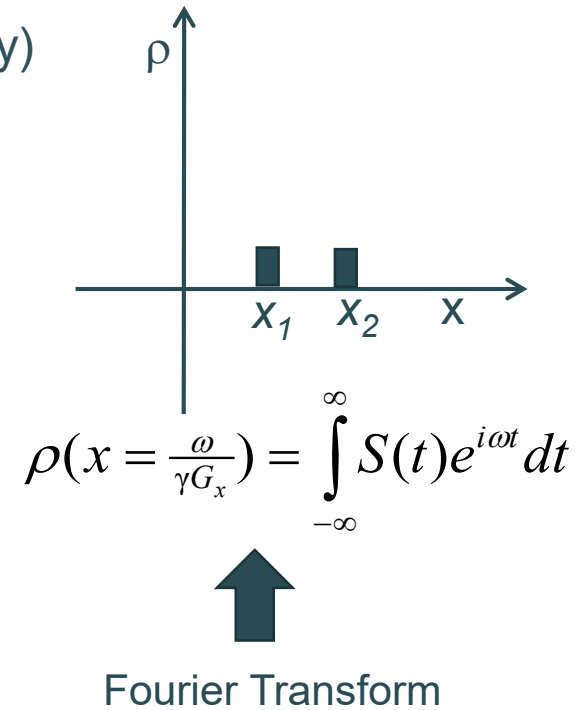


$$S(Gx, t) \propto \iint_{slice} \rho(x, y) e^{-j\omega t} dx dy = \iint_{slice} \rho(x, y) e^{-j\gamma G_x x t} dx dy$$

FREQUENCY ENCODING

- Consider the simple 2-dimensional object, $O(x,y)$

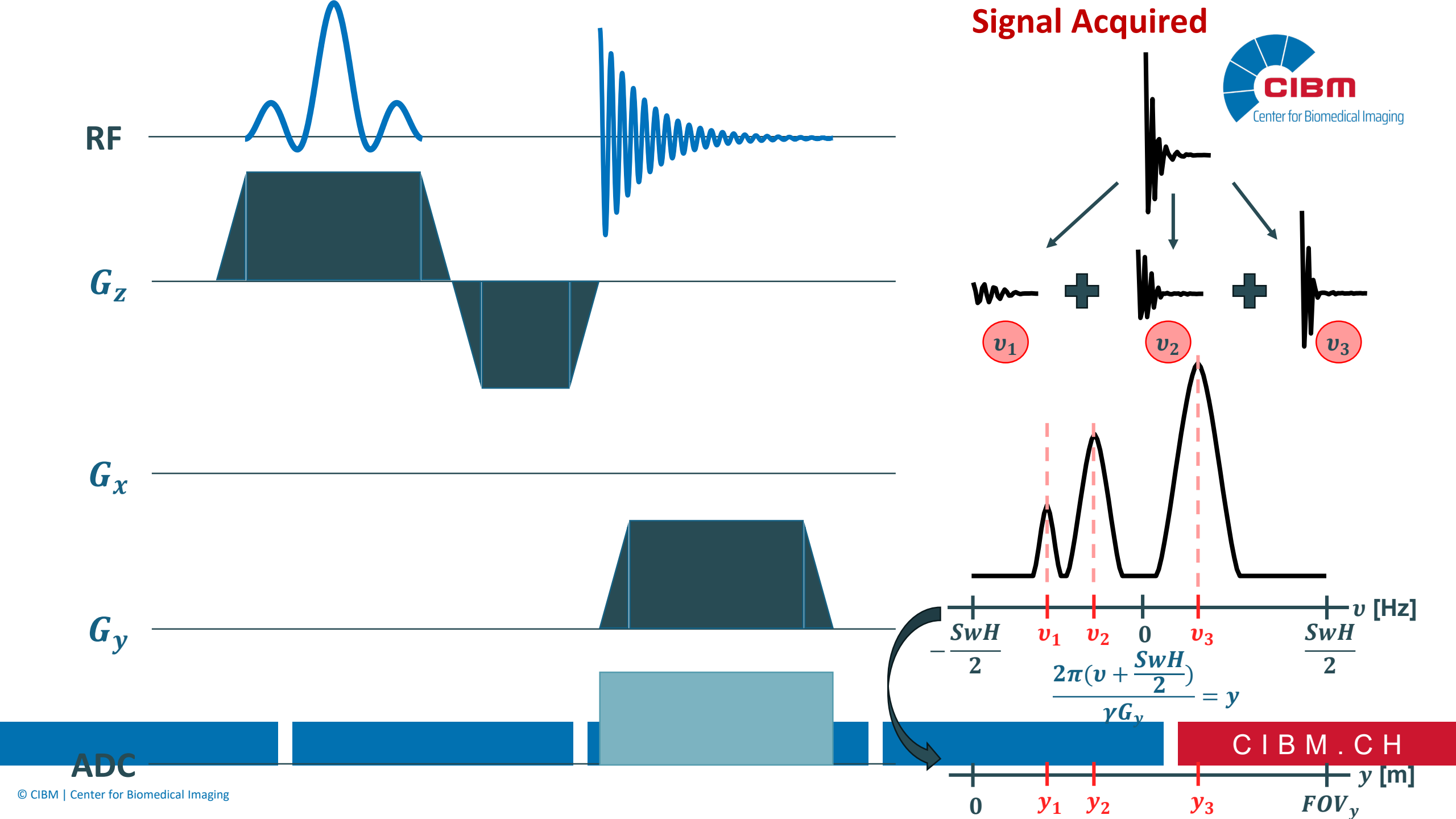




The diagram shows a 1D coordinate system with axes ρ and x . Two small black rectangles representing the density profile $\rho(x)$ are located at positions x_1 and x_2 along the x -axis. Below this, the Fourier transform equation is given:

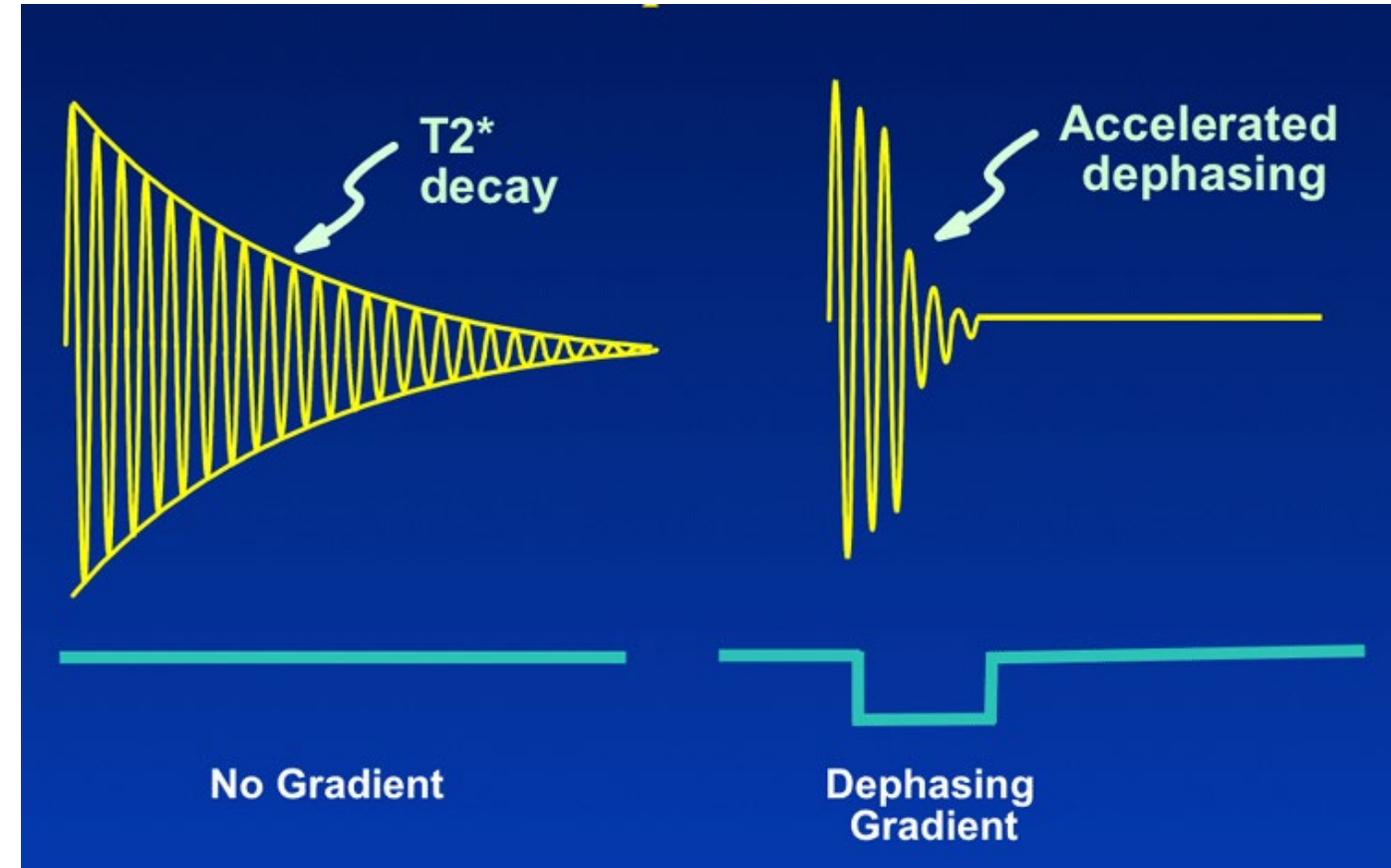
$$\rho(x = \frac{\omega}{\gamma G_x}) = \int_{-\infty}^{\infty} S(t) e^{i\omega t} dt$$

An upward-pointing arrow labeled "Fourier Transform" indicates the transformation from the spatial domain to the frequency domain.



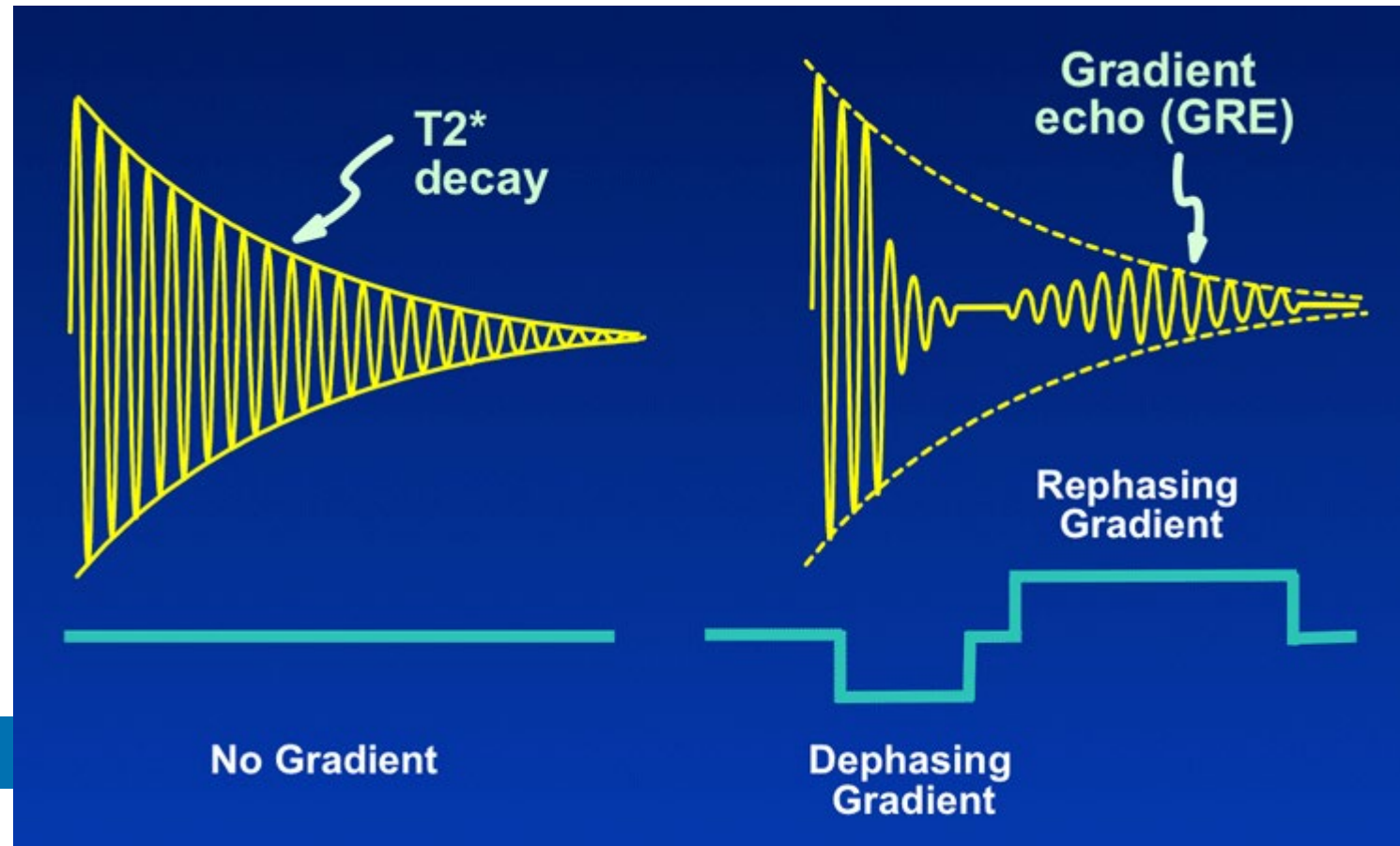
BASIC PULSE SEQUENCES (GRADIENT-ECHO)

- Apply a gradient – encode the FID – hardware limitations on gradient switch – first points are sampled during a time-varying gradients – artefacts
- Eliminate the first points – $\downarrow$ SNR
- Encode only positive values



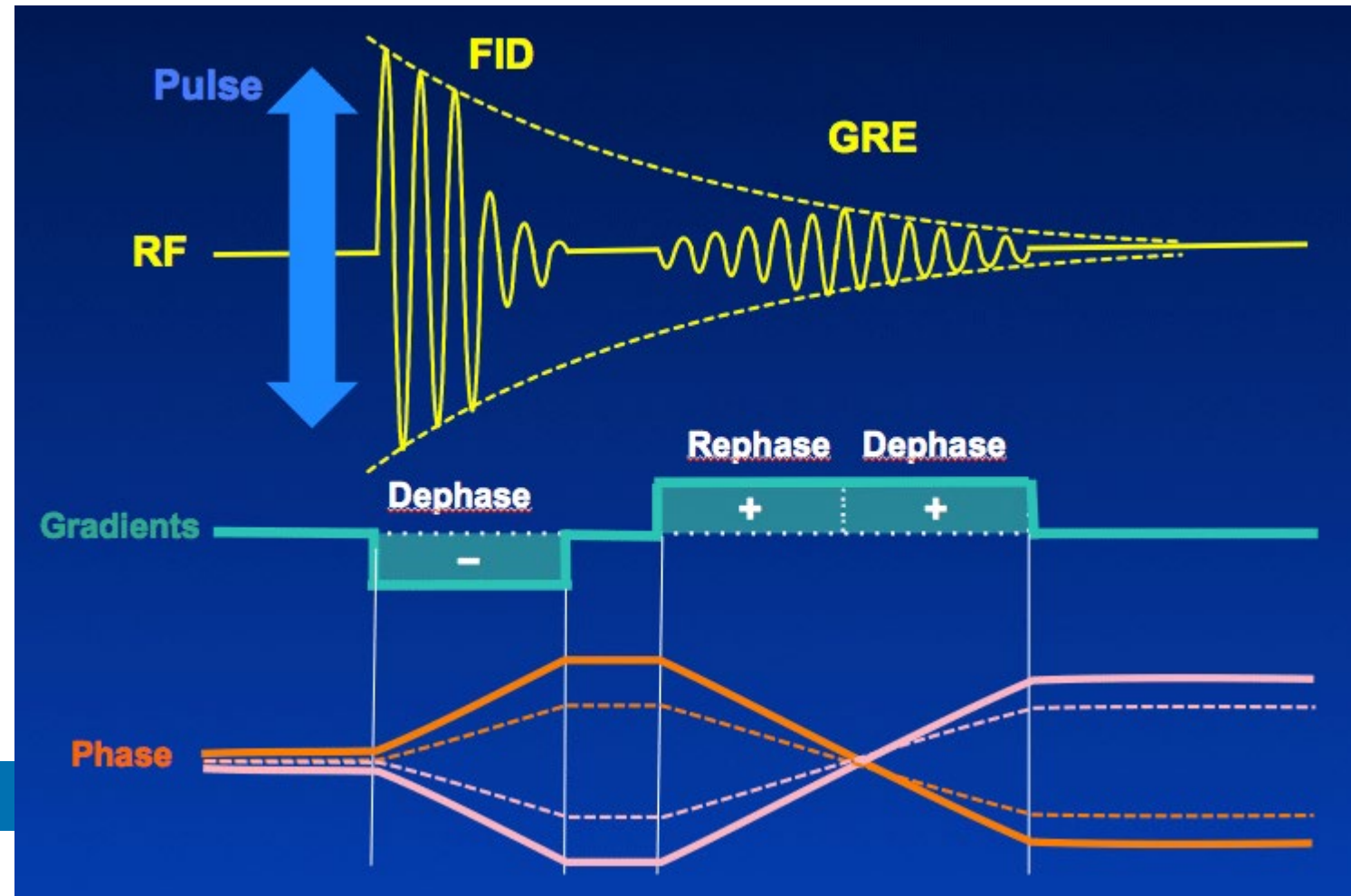
BASIC PULSE SEQUENCES (GRADIENT-ECHO)

- A gradient-echo is a manipulation of the FID using a bipolar gradient
- First gradient lobe dephases the FID, while the second lobe rephases it at a later time, generating the echo.



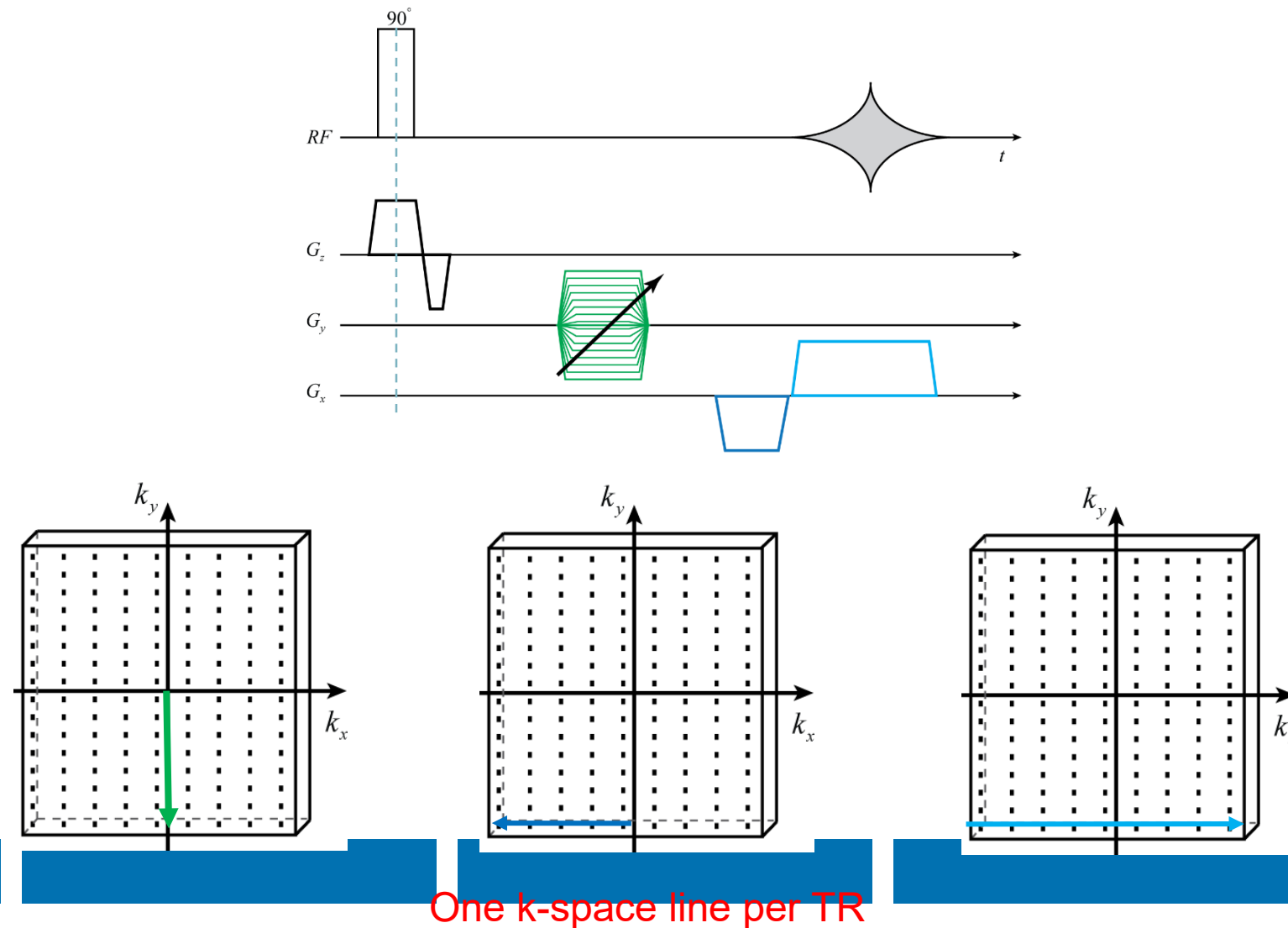
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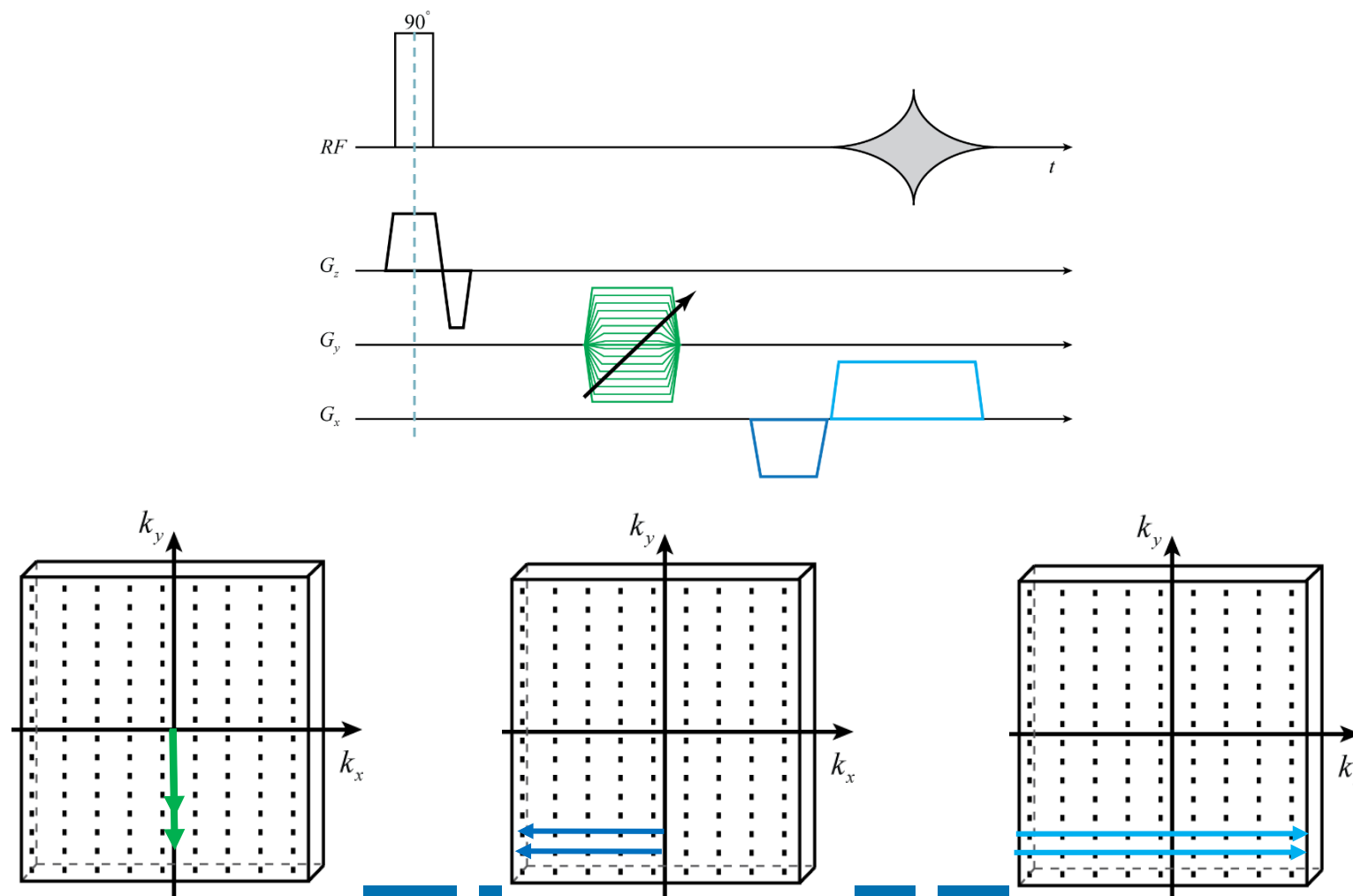


In MRI, this can be used to generate T_2^* and T_1 contrasts.

GRADIENT ECHO



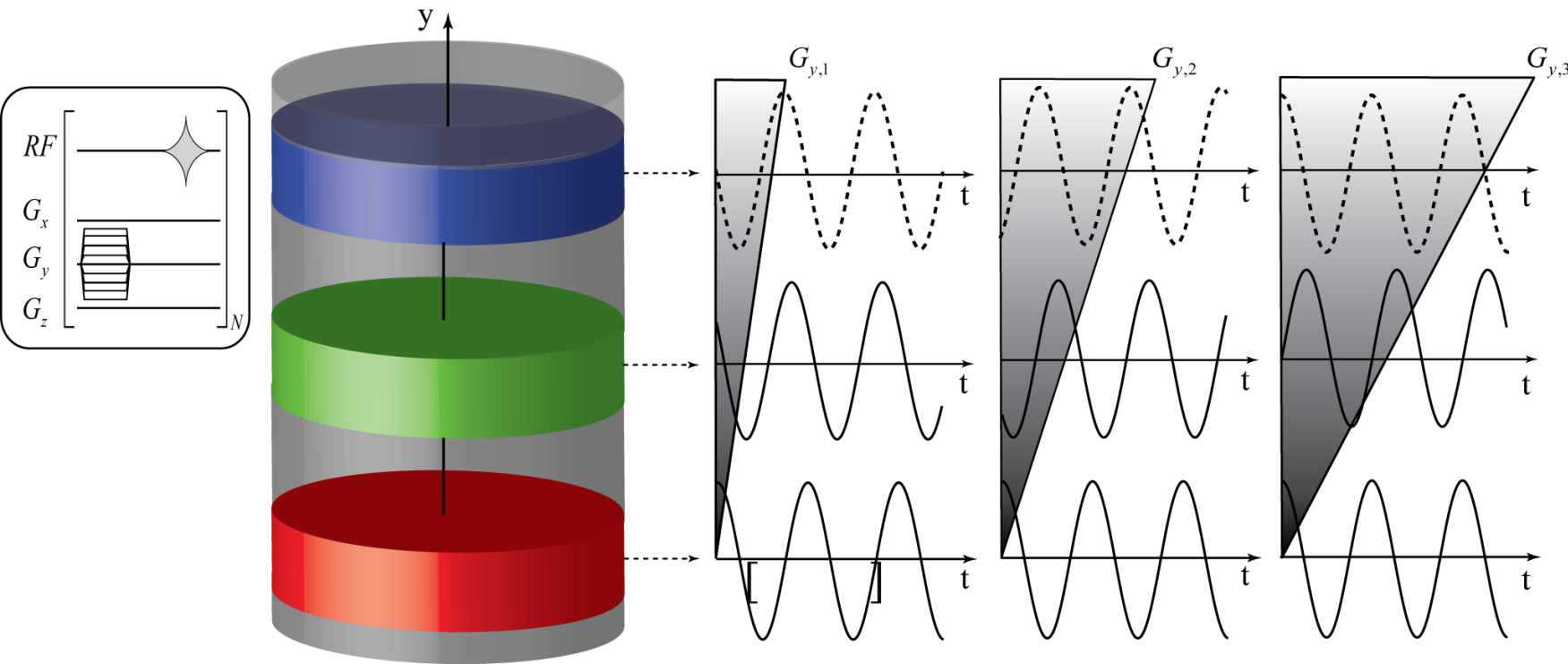
GRADIENT ECHO



One k-space line per TR

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SPATIAL ENCODING – PHASE ENCODING

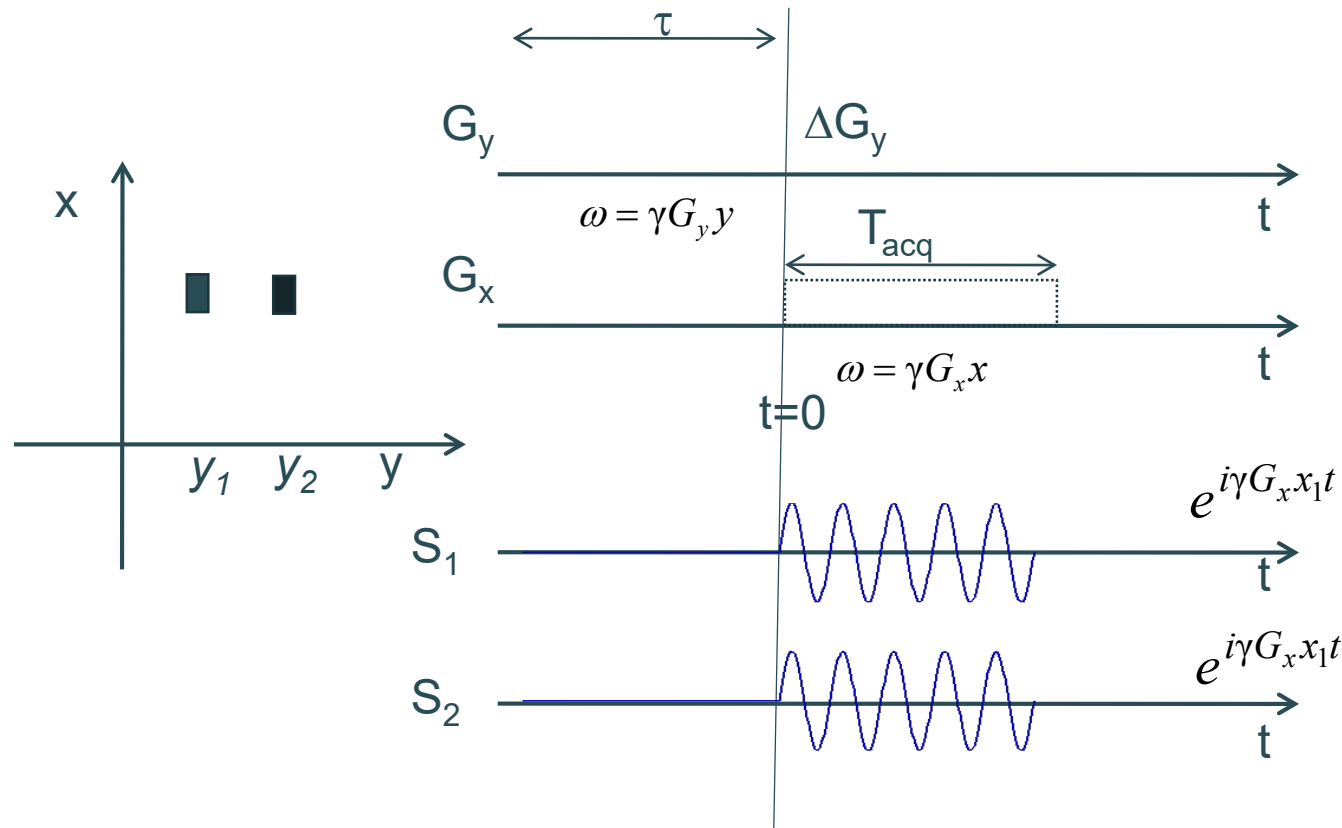


$$\Phi(y) = \omega_y \cdot \tau_{pe} = \gamma \cdot y \cdot G_y \cdot \tau_{pe}$$

$$S(G_y, \tau_{pe}) \propto \iint_{slice} \rho(x, y) e^{-j\gamma G_y y \tau_{pe}} dx dy$$

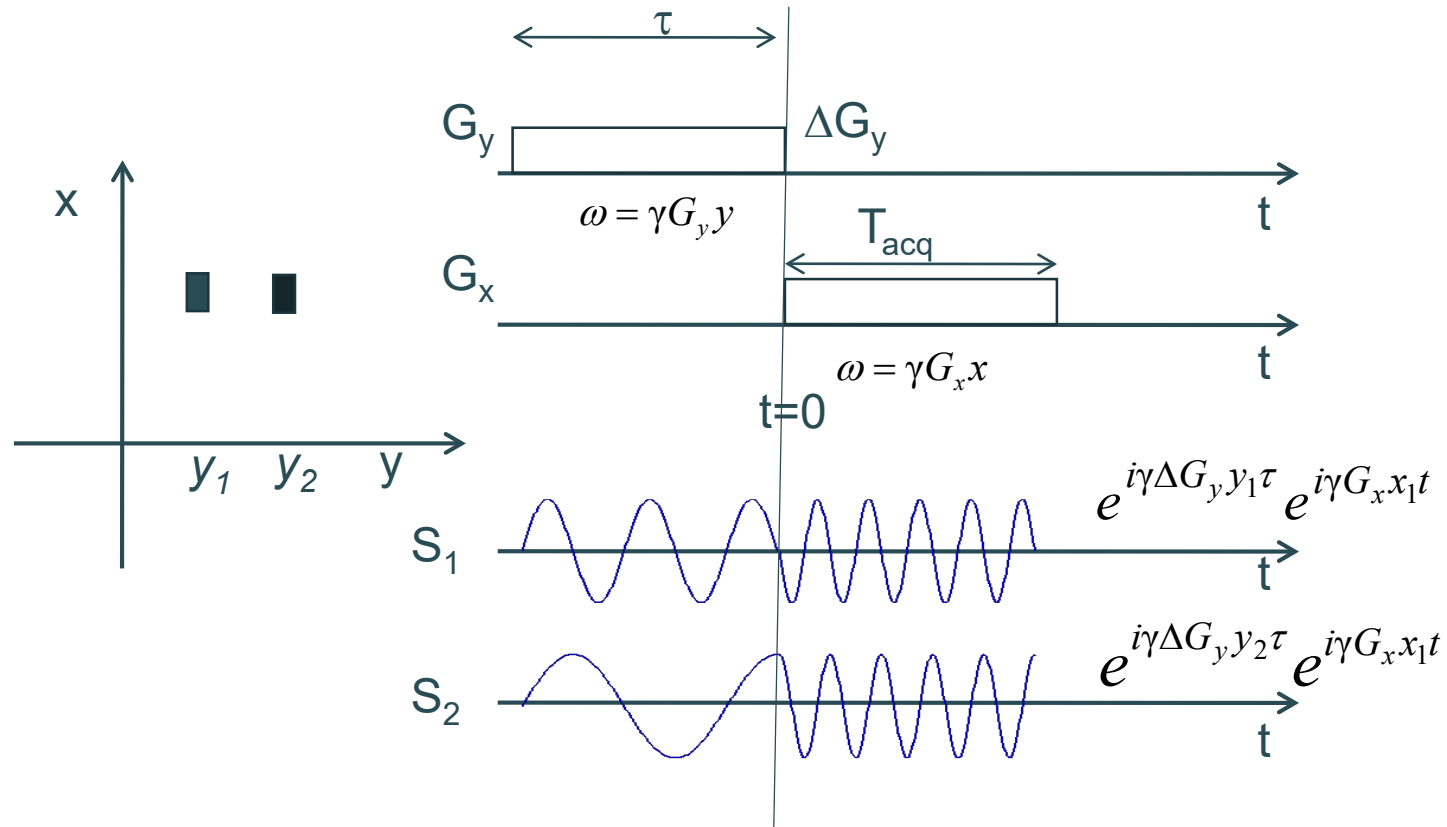
PHASE ENCODING

- In a single acquisition, frequency encoding indicates the position in one dimension. Encoding 2nd and 3rd dimension is usually accomplished via phase encoding (position is encoded in the phase of the NMR signal)



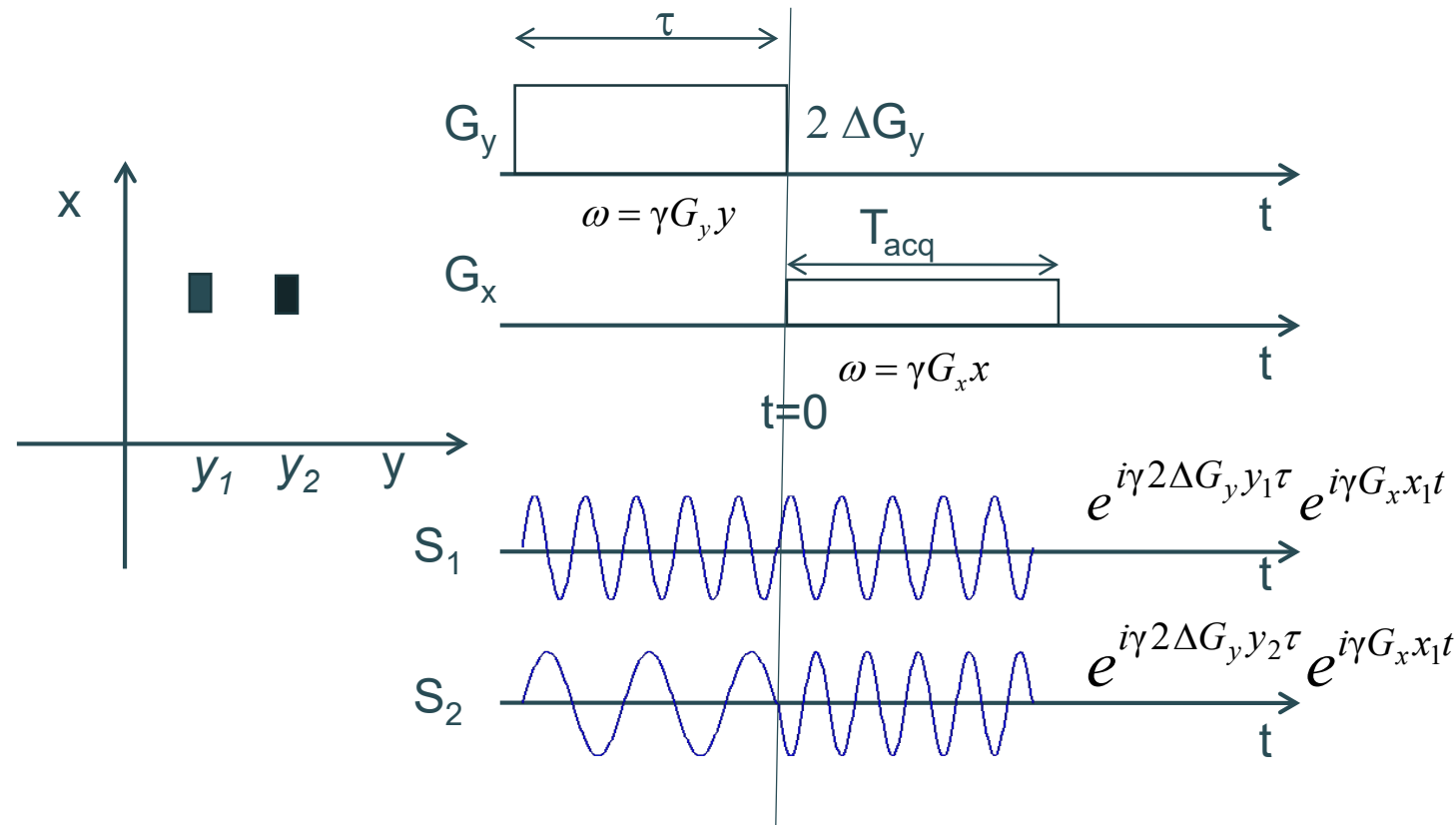
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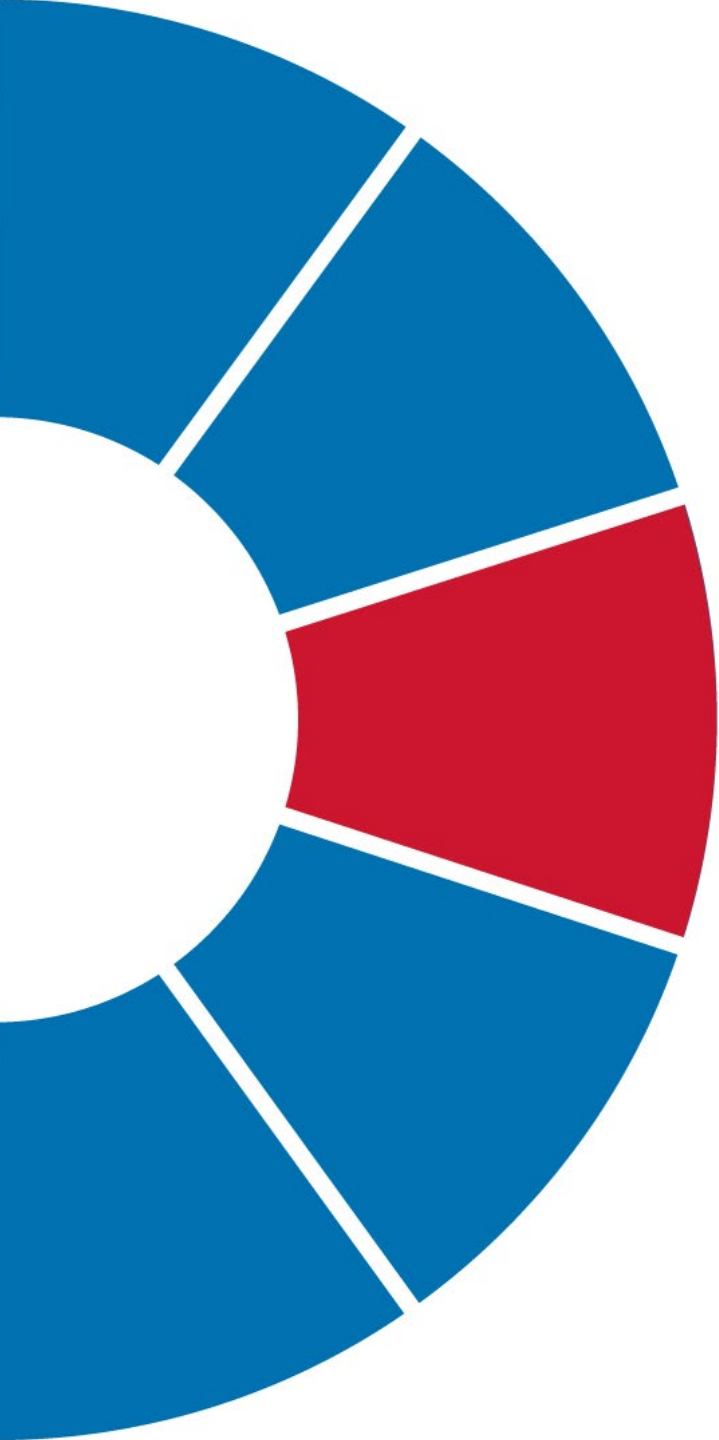
Phase of the signal depends on the y position

while

Frequency depends on the x position

We can not unambiguously identify y from a single measurement...

Multiple measurements are needed.



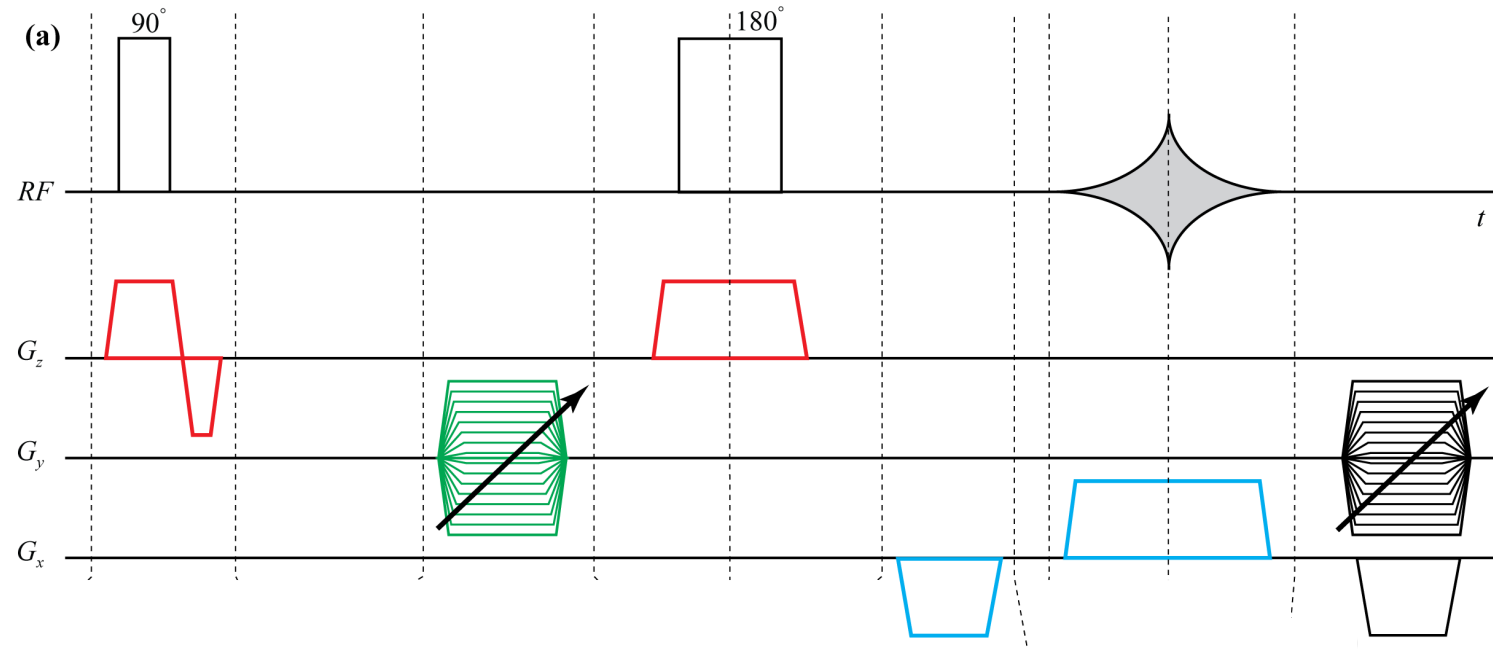
HOW TO ENCODE A 3D OBJECT?

HOW TO ENCODE A 3D OBJECT?

- Conventional 2D acquisitions
- Phase encoding based – (3D conventional acquisition)
- Slice selection based – (spectroscopy localization)

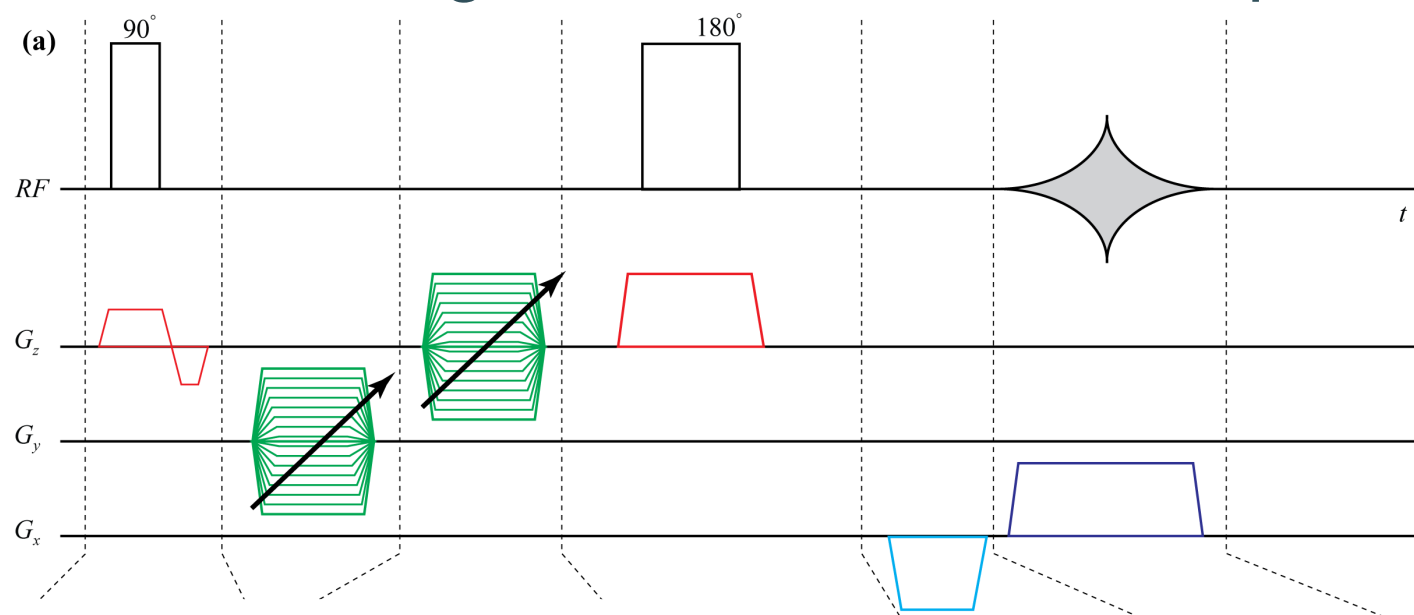
HOW TO ENCODE A 3D OBJECT?

- Conventional 2D acquisition



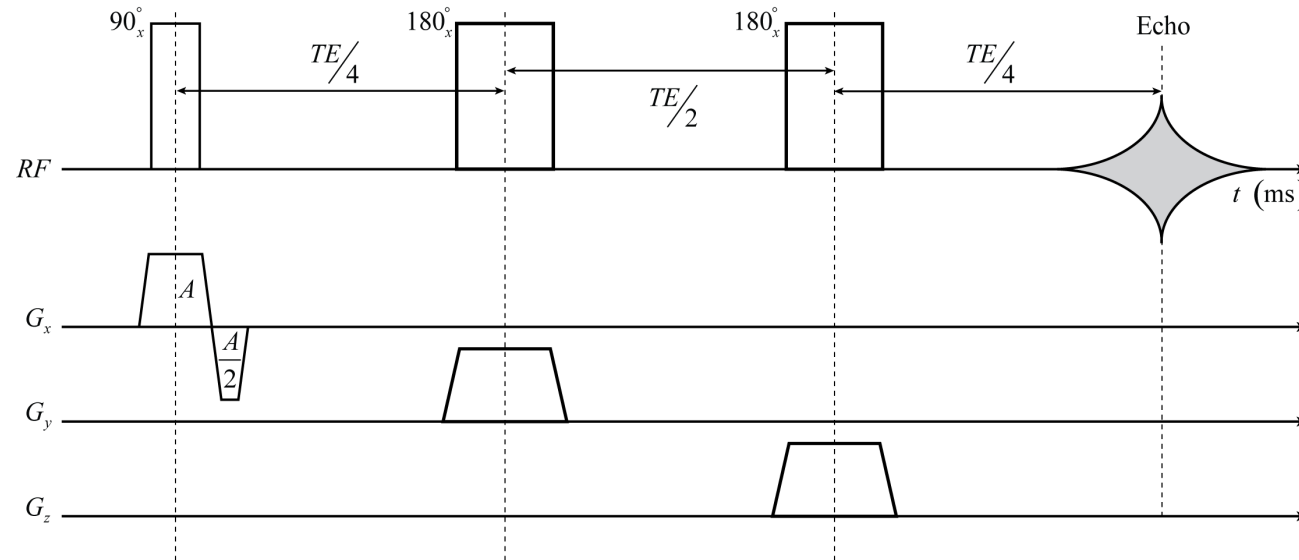
HOW TO ENCODE A 3D OBJECT?

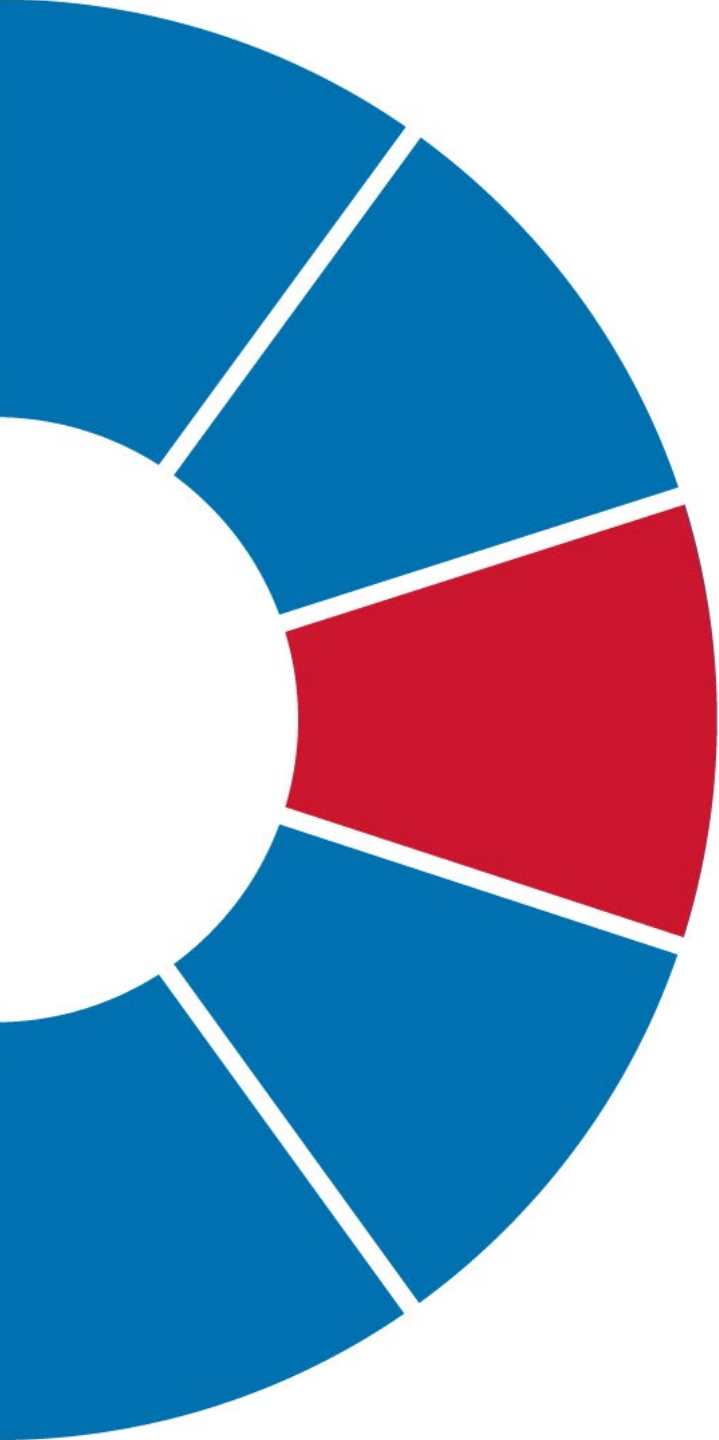
■ Phase encoding – Conventional 3D acquisition



HOW TO ENCODE A 3D OBJECT?

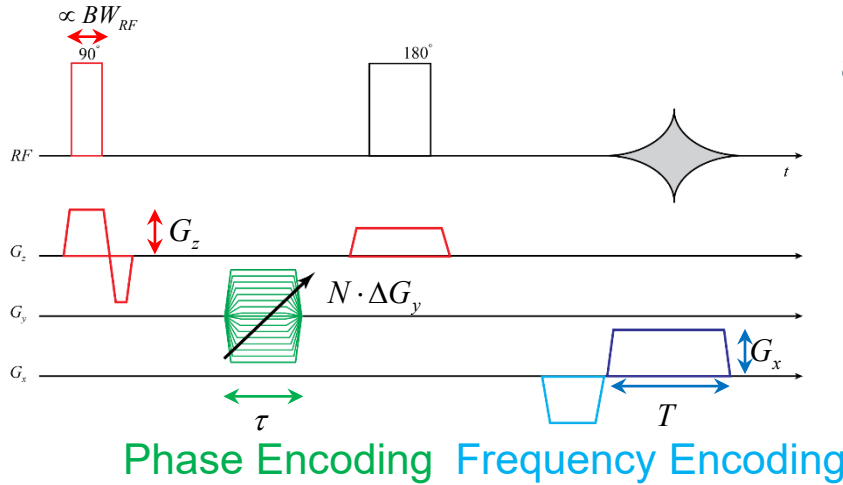
- Slice selection – Conventional spectroscopic localization





SOME NOTIONS ABOUT K-SPACE

SOME NOTIONS ABOUT K-SPACE



In the rotating frame, neglecting relaxation, the NMR signal can be written as:

$$S \propto \iint_{\text{slice}} \rho(x, y) dx dy$$

$$\omega = \gamma \cdot x \cdot G_x$$

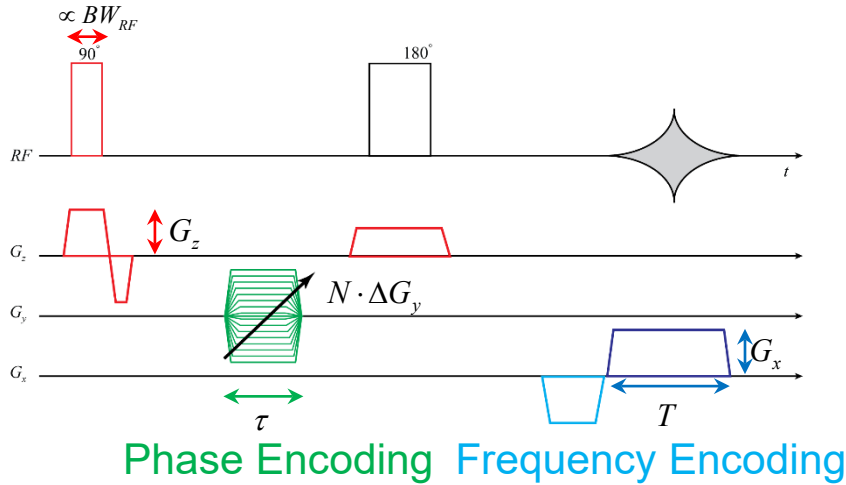
$$S(G_x, t) \propto \iint_{\text{slice}} \rho(x, y) e^{-j\omega t} dx dy = \iint_{\text{slice}} \rho(x, y) e^{-j\gamma G_x x t} dx dy$$

$$\phi(y) = \omega_y \cdot \tau_{pe} = \gamma \cdot y \cdot G_y \cdot \tau_{pe}$$

$$S(G_y, \tau_{pe}) \propto \iint_{\text{slice}} \rho(x, y) e^{-j\gamma G_y y \tau_{pe}} dx dy$$

$$S(G_x, t, G_y, \tau_{pe}) \propto \iint_{\text{slice}} \rho(x, y) e^{-j\gamma G_x x t} e^{-j\gamma G_y y \tau_{pe}} dx dy$$

SOME NOTIONS ABOUT K-SPACE

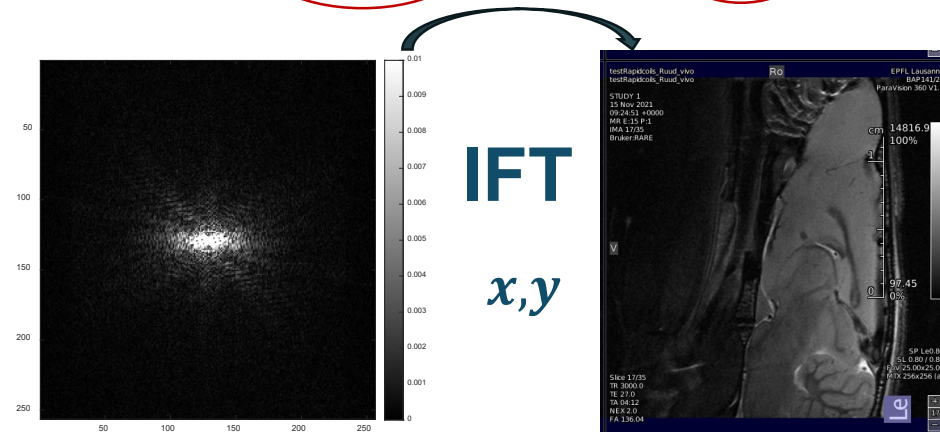


$$S(G_x, t, G_y, \tau_{pe}) \propto \iint_{\text{slice}} \rho(x, y) e^{-j\gamma G_x x t} e^{-j\gamma G_y y \tau_{pe}} dx dy$$

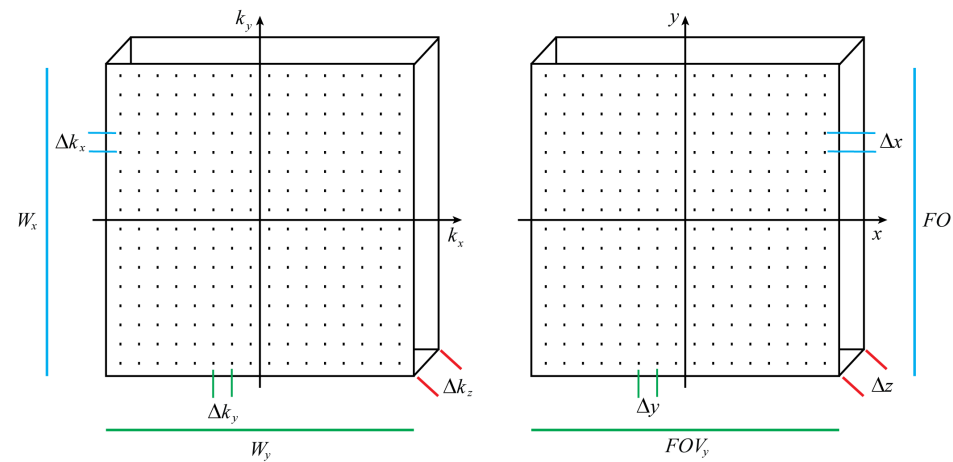
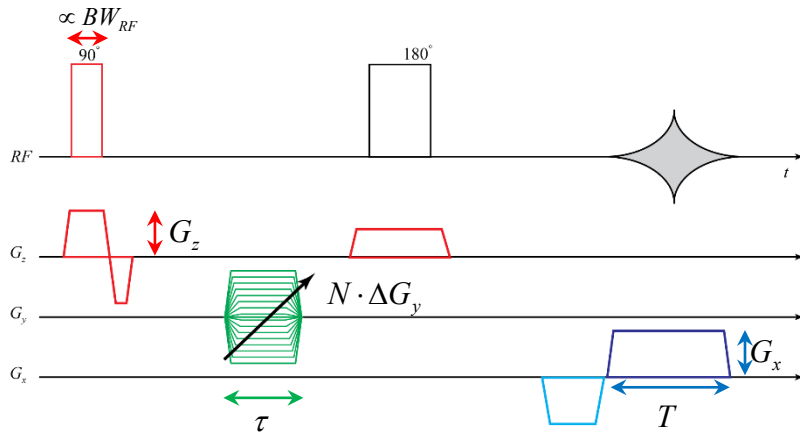
$$k_x = \frac{\gamma}{2\pi} \cdot G_x \cdot t$$

$$k_y = \frac{\gamma}{2\pi} \cdot G_y \cdot \tau_{pe}$$

$$S(k_x, k_y) \propto \iint_{\text{slice}} \rho(x, y) e^{-jk_x x} e^{-jk_y y} dx dy$$



SOME NOTIONS ABOUT K-SPACE



Slice Selection

Phase Encoding

Frequency Encoding

$$FOV_z = \frac{BW_{RF}}{\gamma G_z}$$

$$FOV_y = \frac{1}{\Delta k_y} = \frac{2\pi}{\gamma \Delta G_y \tau}$$

$$FOV_x = \frac{1}{\Delta k_x} = \frac{2\pi}{\gamma G_x \Delta T}$$

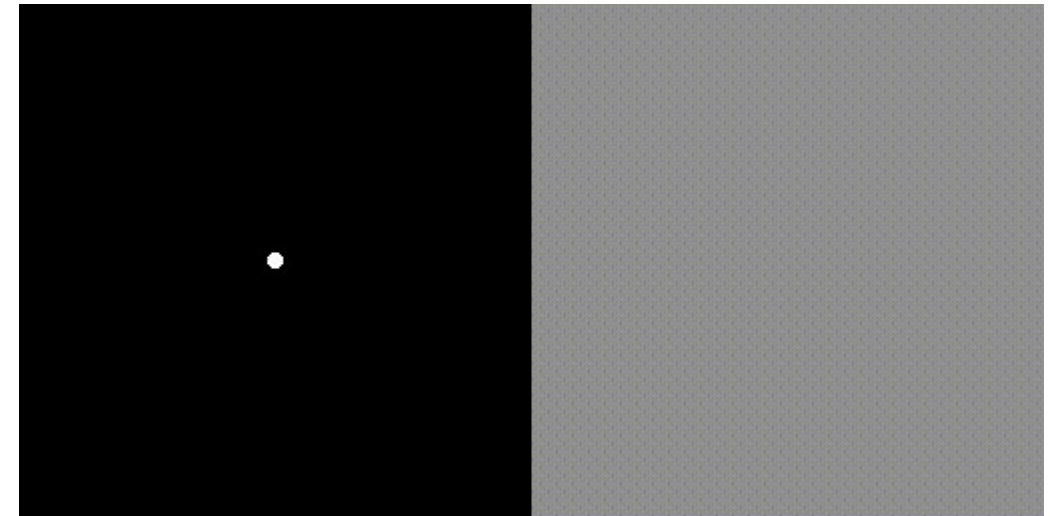
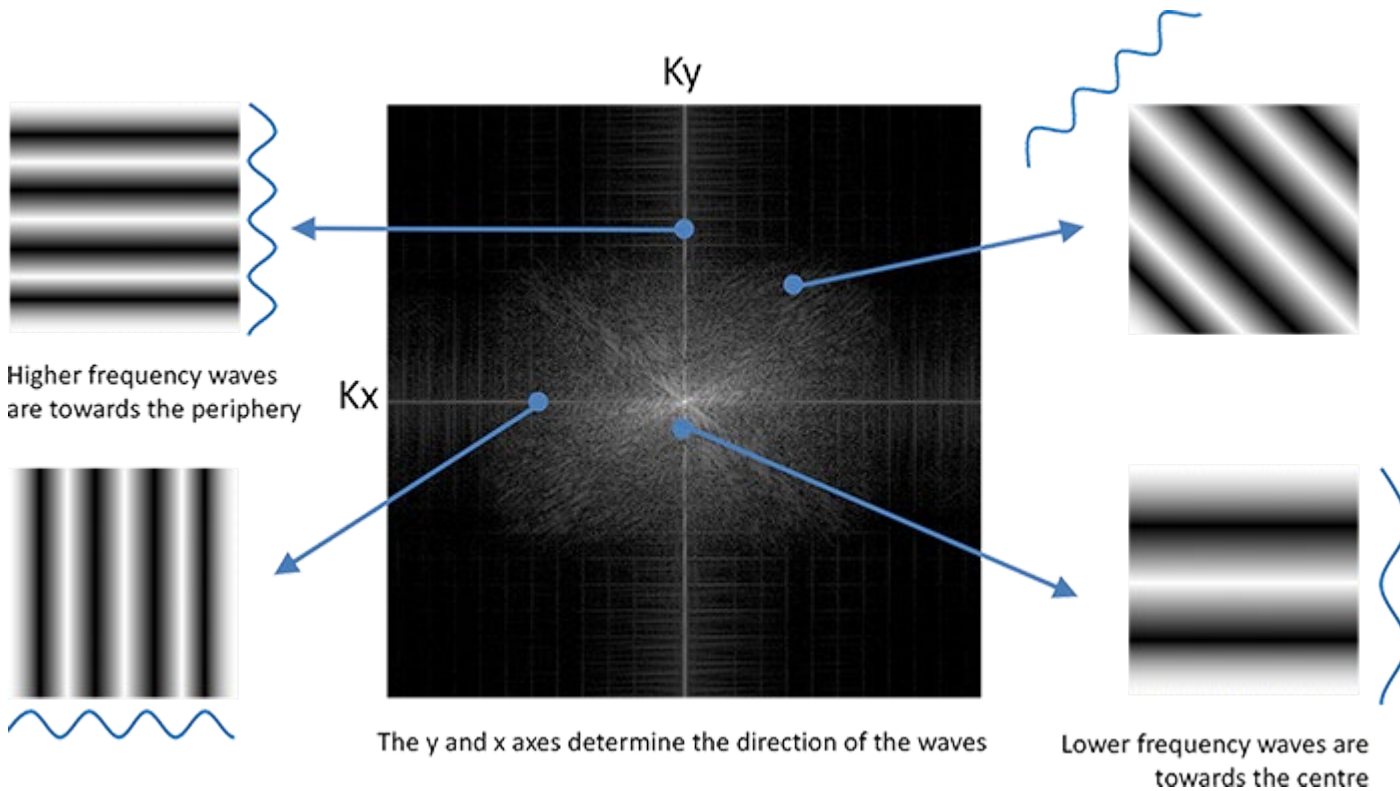
Resolution

$$\delta y = \frac{2\pi}{W_{k_y}} = \frac{2\pi}{\gamma \Delta G_y N \tau}$$

$$\delta x = \frac{2\pi}{W_{k_x}} = \frac{2\pi}{\gamma G_x T}$$

- Large FOV requires small steps in k
- $\Delta G_y \tau$ creates a 2π cycle across the FOV
- High spatial resolution requires large Gradient x time product
- $N/2 \Delta G_y \tau$ creates a π shift between two successive pixels

WHAT IS WHERE IN K-SPACE



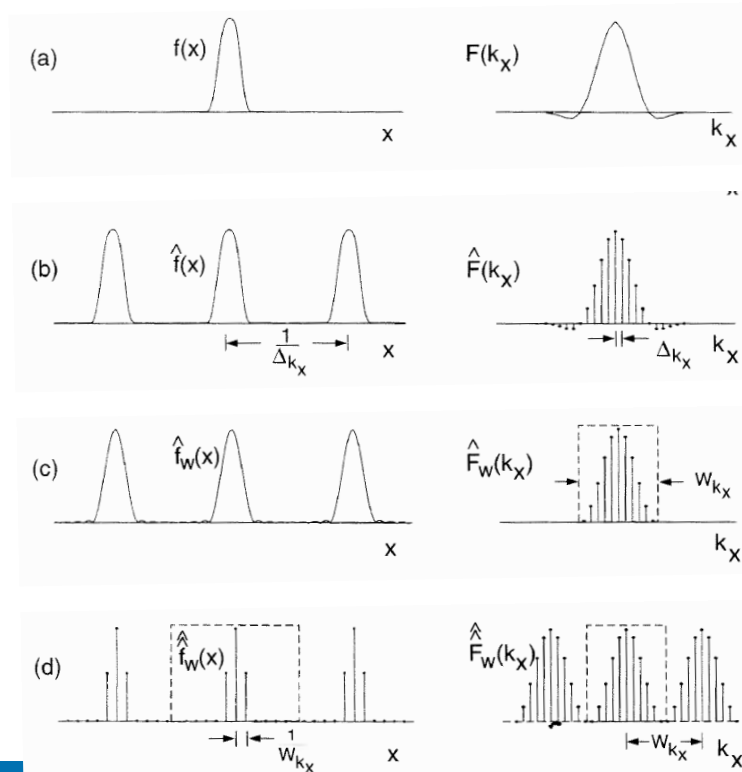
Movie showing spatial frequencies corresponding to various points in k -space. (Courtesy Brian Hargreaves)

DISCRETE FOURIER TRANSFORM

FT $\longrightarrow G(k) = \int_{-\infty}^{\infty} g(x) e^{-2\pi i k x} dx$

DFT $\longrightarrow G\left(\frac{p}{L}\right) = \sum_{q=-n}^{n-1} g\left(\frac{qL}{2n}\right) e^{-2\pi i p q / (2n)} \quad p = -n, -n+1, \dots, n-1 \quad L = 2n\Delta x$

Reconstructed Image Signal sampled



ideally

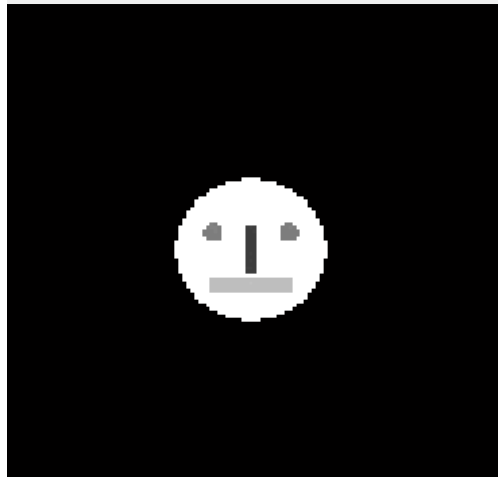
we can only sample every once in a while

we can only sample for so long

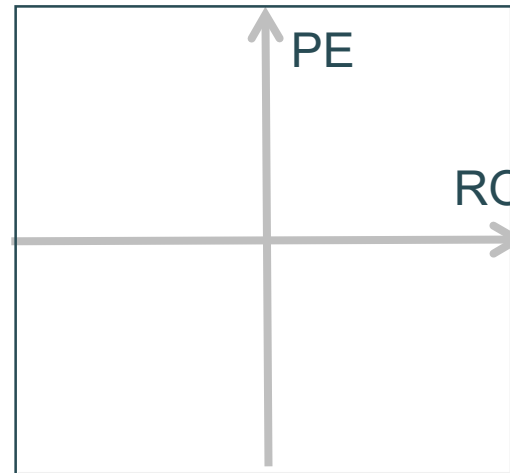
and we will only keep so much information

DISCRETE FOURIER TRANSFORM

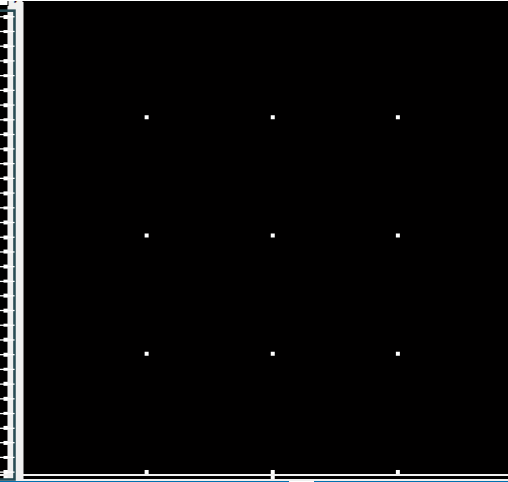
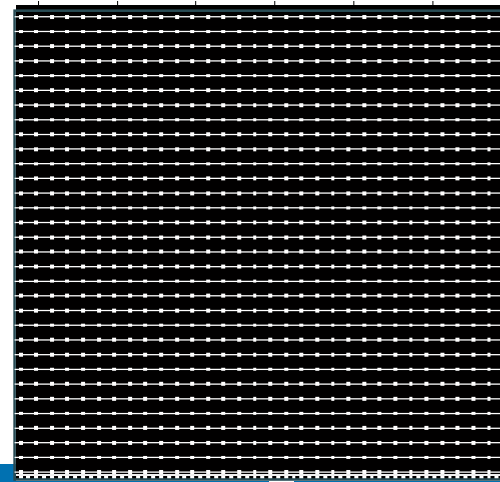
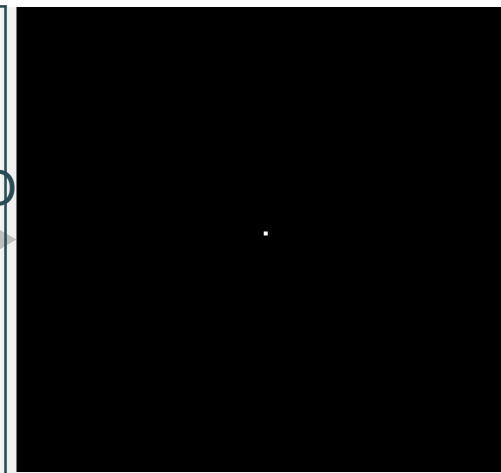
reconstructed



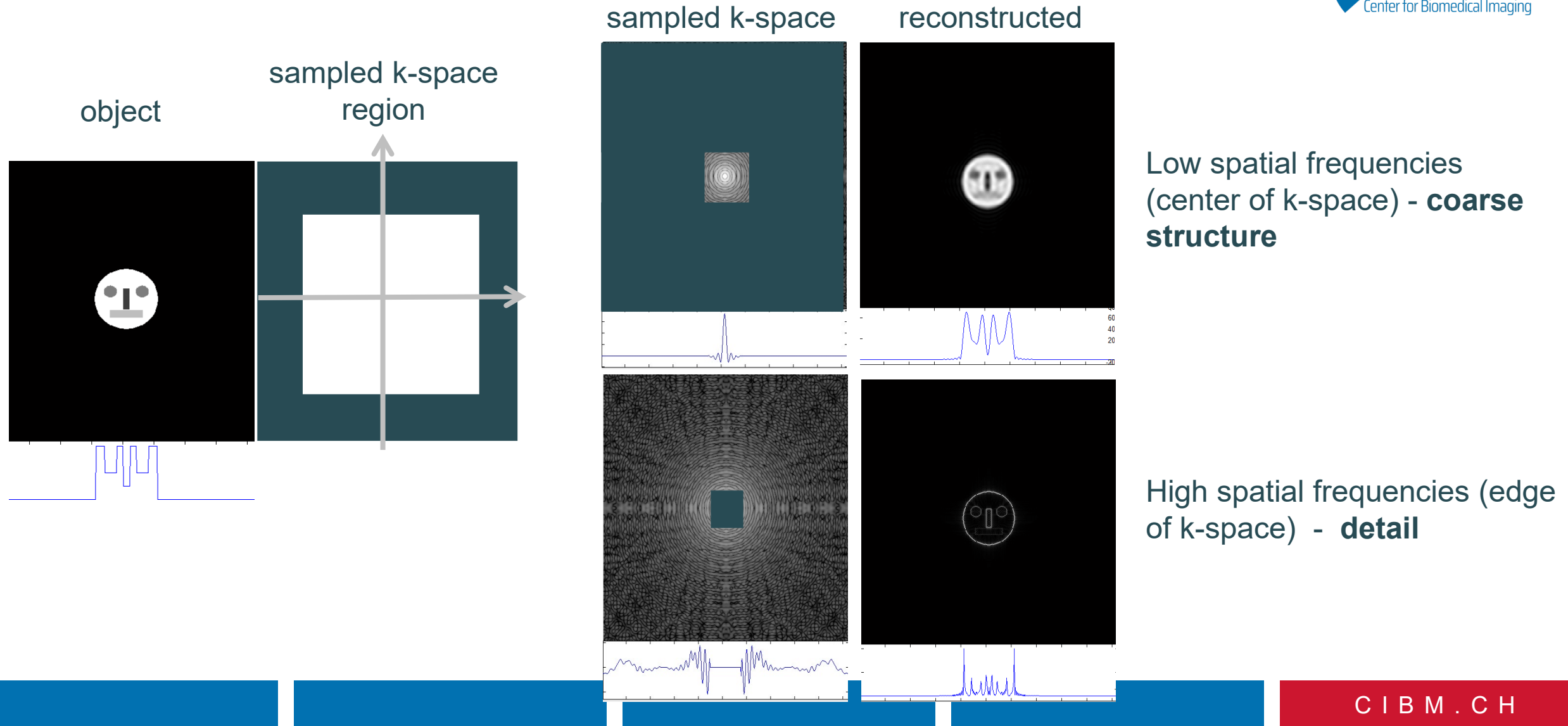
sampled k-space



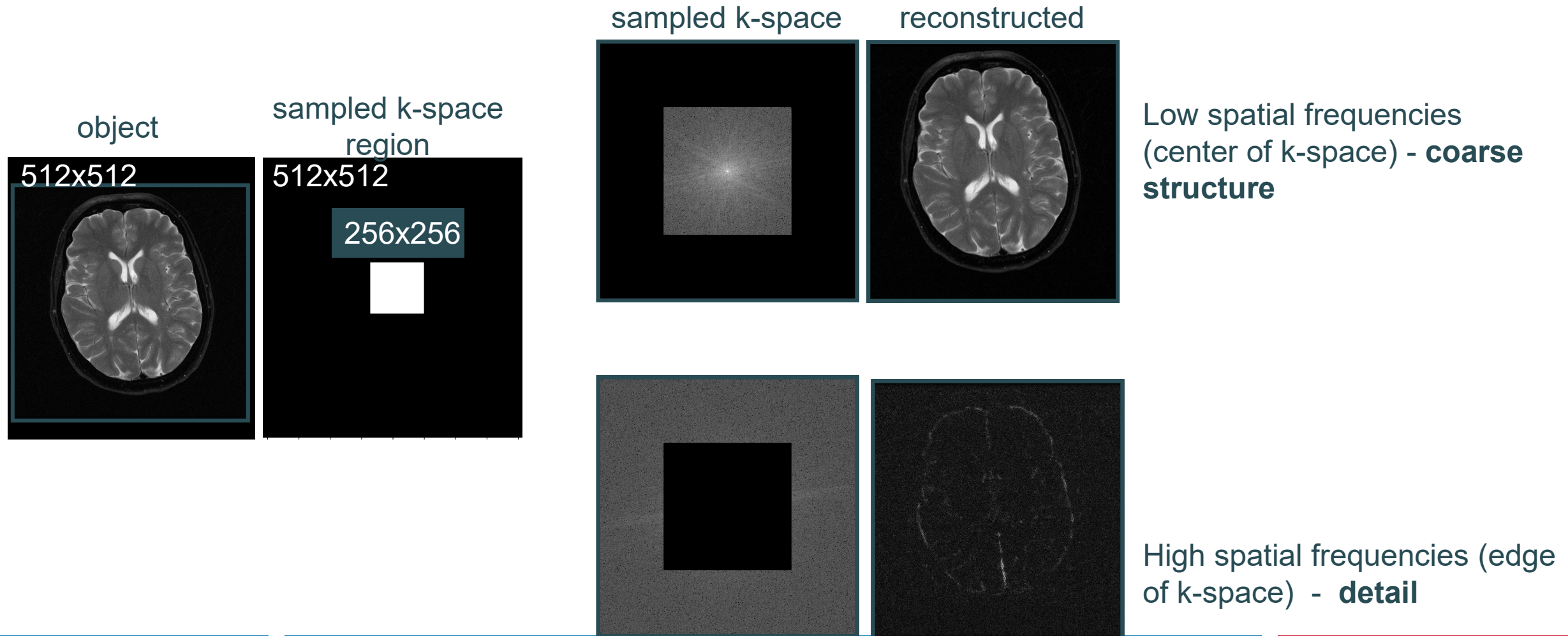
FT sampled k-space



WHAT IS WHERE IN K-SPACE



WHAT IS WHERE IN K-SPACE



SUMMARY & ACKNOWLEDGMENTS



MRI is incredibly rich & versatile

Slides:

Nicolas Kunz

Jose Marques

Cristina Cudalbu

Rolf Gruetter

Brayan Alves

[Introductory NMR & MRI: Video 08: Magnetic Resonance Imaging \(youtube.com\)](#)

[11.1 - Echo formation in magnetic resonance \(youtube.com\)](#)

[Introduction to the Principles of MRI \(Magnetic Resonance Imaging\) \(youtube.com\)](#)

Thank you for listening! Questions?

NEXT WEEKS



School

Course: MRI Practicals on CIBM p

GitHub - AlvBrayan/MRS4Brain-to

https://moodle.epfl.ch/course/view.php?id=18507

EPFL

Home Dashboard My courses

General

Announcements

MRS4Brain Toolbox (GitHub)

contrastManipulation Codes

k-space Codes

MR environment screening ...

9 September - 15 Septem...

Background and organisati...

Lecture1 Basics of nuclear ...

16 September - 22 Septe...

PHYS-473 Week2 MR Hard...

PHYS-473 Week2 MRI Demo

PHYS-473 Theory: FID, Spin...

23 September ... This week

30 September - 6 October

7 October - 13 October

14 October - 20 October

21 October - 27 October

28 October - 3 November

4 November - 10 Novem...

11 November - 17 Nove...

18 November - 24 Nove...

25 November - 1 December

MRI Practicals on CIBM preclinical imaging systems

Course Settings Participants Grades Reports More

General

Announcements

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MR environment screening and quarantine checking forms

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GitHub - AlvBrayan/MRS4Brain-to

LIVE Demos - MRS4BRAIN - EPFL

https://www.epfl.ch/labs/mrs4brain/links/live-demos/

EPFL

About Education Research Innovation Schools Campus

AVP-OP

MRS4Brain Group

OUR TEAM

Cristina Cudalbu, PhD

Jessie Mosso

Brayan Alves

Alessio Siviglia

Collaborators

Master Students

Alumni

Dunja Simicic

Jan Luxemburg

PhD Students

Master Students

Internships

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PUBLICATIONS

LINKS

MRS4Brain toolbox

Lectures

Understanding MRS & MRI

LIVE Demos

Software, Scripts and Codes

Standardized Preclinical MRS: A Multi-Center Study

Datasets

FUNDING

JOIN US

CONTACT US

LIVE Demos

DEMO: 1H-FID-MRSI @14.1T

DEMO: Shimming in the rat brain hippocampus at 14.1T

DEMO_MRS4Brain_toolbox

DEMO_Processing of MM signals acquired in vivo at 14.1T using 1H-FID-MRSI

DEMO_Processing of MM signals acquired in vivo at 9.4T using 1H SVS

DW-SPECIAL: demo, protocol, processing

DEMO: Bile duct ligation surgery protocol, model of chronic liver disease induced HE

CIBM.CH



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