Flavour Physics: Week: IV

Luismi Garcia

May 13, 2024

Problem 1. The NA62 experiment is a fix-target experiment focused on the study of charged Kaon decays. The proton from SPS at LHC collide with a fix-target, creating a bunch of hadrons. Kaons of approximately 75 GeV are selected and travel around 100 m before decaying. The golden mode of the experiment is the $K^+ \to \pi^+\nu\bar{\nu}$ decay, where only the entering K^+ and the exiting π^+ are detected. A powerful tool to distinguish such signal decay from potential backgrounds is the correlation between the missing mass squared (m^2_{miss}) and the momentum of the π^+ meson.

(a) A potential background is the $K^+ \to \pi^+\pi^0$ decay. Compute what is the missing mass for this background.

(b) Another potential background is the $K^+ \to \mu^+\bar{\nu}_\mu$ decay, where the μ^+ is misidentifed as a π^+. Compute the shape of such background in the squared missing mass vs π^+ p distribution, *i.e.* compute $m^2_{\text{miss}}(p_{\pi^+})$ (Hint: use Taylor series with the approximation $p >> m$: $\sqrt{1+x^2} \simeq 1 + x^2/2$)

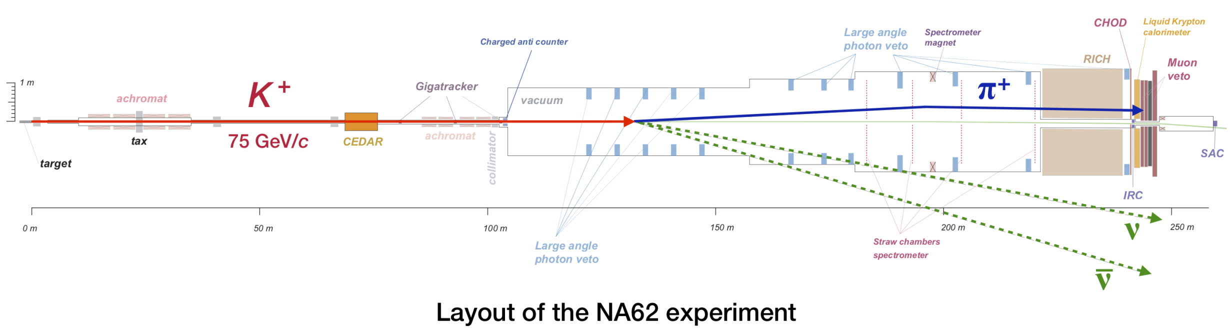

Figure 1: Sketch of the NA62 experiment with the overlaid $K^+ \to \pi^+\nu\bar{\nu}$ decay.

Solution

(a) The missing energy is computing from the four-momenta as

$$m^2_{\text{miss}} = (p_{\mathrm{K}^+} - p_{\pi^+})^2 = p^2_{\pi^0} = m^2_{\pi^0} = 0.018\,\mathrm{GeV}/c$$

(b) Due to the misindentification $\mu \to \pi$, the missing mass is shifted in the negative direction by:

$$\begin{aligned} m^2_{\text{miss}} &= (p_{\mathrm{K}^+} - p_{\mu^+})^2 - (p_{\mathrm{K}^+} - p_{\pi^+})^2 \\ &= m^2_{\mathrm{K}^+} + m^2_{\mu^+} + 2p_{\mathrm{K}^+}p_{\mu^+} - m^2_{\mathrm{K}^+} - m^2_{\pi^+} - 2p_{\mathrm{K}^+}p_{\pi^+} \\ &= m^2_{\mu^+} - m^2_{\pi^+} + 2E_{\mathrm{K}^+}E_{\mu^+} - 2\vec{p}_{\mathrm{K}^+}\vec{p}_{\mu^+} - 2E_{\mathrm{K}^+}E_{\pi^+} + 2\vec{p}_{\mathrm{K}^+}\vec{p}_{\pi^+} \end{aligned}$$

Now we use $E = \sqrt{(m^2 + \vec{p}^2)} = \|\vec{p}\|\sqrt{1 + \frac{m^2}{\vec{p}^2}} \simeq \|\vec{p}\|\,(1 + \frac{m^2}{\vec{p}^2})$, where we have used $\sqrt{1+x^2} \simeq 1 + x^2/2$. The momentum is correctly estimated as it is a measured quantity independent of the mass hypothesis, and, thus, $\vec{p}_{\pi^+} = \vec{p}_{\mu^+}$. Using all of this in our formula we get

$$\begin{aligned} E_{\mathrm{K}^+}E_{\mu^+} &= \|\vec{p}_{\mathrm{K}^+}\|\,\|\vec{p}_{\pi^+}\|\,(1 + \frac{m^2_{\mathrm{K}^+}}{2\vec{p}^2_{\mathrm{K}^+}})(1 + \frac{m^2_{\mu^+}}{2\vec{p}^2_{\pi^+}}) \\ E_{\mathrm{K}^+}E_{\pi^+} &= \|\vec{p}_{\mathrm{K}^+}\|\,\|\vec{p}_{\pi^+}\|\,(1 + \frac{m^2_{\mathrm{K}^+}}{2\vec{p}^2_{\mathrm{K}^+}})(1 + \frac{m^2_{\pi^+}}{2\vec{p}^2_{\pi^+}}) \end{aligned}$$

and introducing this in the previous formula:

$$\begin{aligned} m^2_{\text{miss}} &= m^2_{\mu^+} - m^2_{\pi^+} + 2E_{\mathrm{K}^+}E_{\mu^+} - 2\vec{p}_{\mathrm{K}^+}\vec{p}_{\mu^+} - 2E_{\mathrm{K}^+}E_{\pi^+} + 2\vec{p}_{\mathrm{K}^+}\vec{p}_{\pi^+} \\ &= m^2_{\mu^+} - m^2_{\pi^+} - 2\vec{p}_{\mathrm{K}^+}\vec{p}_{\pi^+}\left[\left(1 + \frac{m^2_{\mathrm{K}^+}}{2\vec{p}^2_{\mathrm{K}^+}}\right)\left(1 + \frac{m^2_{\pi^+}}{2\vec{p}^2_{\pi^+}}\right) - \left(1 + \frac{m^2_{\mathrm{K}^+}}{2\vec{p}^2_{\mathrm{K}^+}}\right)\left(1 + \frac{m^2_{\mu^+}}{2\vec{p}^2_{\pi^+}}\right)\right] \\ &= m^2_{\mu^+} - m^2_{\pi^+} - 2\,\|\vec{p}_{\mathrm{K}^+}\|\,\|\vec{p}_{\pi^+}\|\left[\frac{m^2_{\pi^+}}{2\vec{p}^2_{\pi^+}} - \frac{m^2_{\mu^+}}{2\vec{p}^2_{\pi^+}} + \mathcal{O}\left(\frac{m^2_i m^2_j}{\vec{p}^2_i\vec{p}^2_j}\right)\right] \\ &= m^2_{\mu^+} - m^2_{\pi^+} - \frac{\|\vec{p}_{\mathrm{K}^+}\|}{\|\vec{p}_{\pi^+}\|}(m^2_{\mu^+} - m^2_{\pi^+}) \\ &= (m^2_{\mu^+} - m^2_{\pi^+})(1 - \frac{\|\vec{p}_{\mathrm{K}^+}\|}{\|\vec{p}_{\pi^+}\|}) \end{aligned}$$

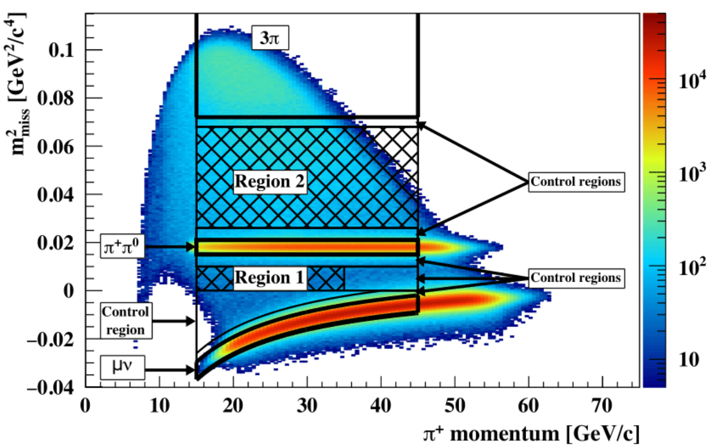

Figure 2: Missing energy as a function of the π^+ momentum for the signal $K^+ \to \pi^+\nu\bar{\nu}$ decay and potential backgrounds.

Problem 2. Write the tree level feynman diagram for the following decays, identify if they are color-suppressed or color-allowed. Write also the decay amplitude as a function of the CKM Wolfstein parameters:

(a) $B^0 \to D^+D^-$

(b) $B^0 \to \pi^+\pi^-$

(c) $B^0 \to \overline{D}^0\pi^0$

(d) $B^0 \to D^0\pi^0$

(e) $B^0 \to J/\psi K^0$

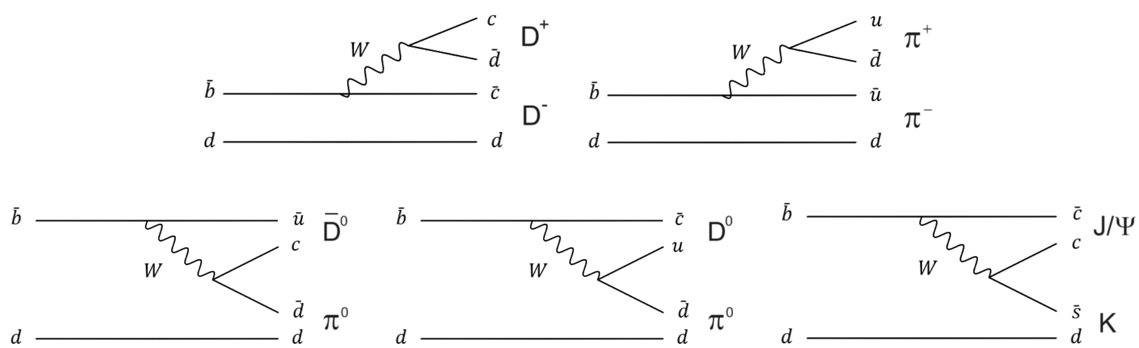

Figure 3: Feynman diagram for the (top) color allowed $B^0 \rightarrow D^+D^-$, $B^0 \rightarrow \pi^+\pi^-$ and (bottom) color suppressed $B^0 \rightarrow \overline{D}^0\pi^0$, $B^0 \rightarrow D^0\pi^0$ and $B^0 \rightarrow J/\psi K^0$ decays.

Solution

(a) $B^0 \rightarrow D^+D^-$: Color allowed, $\Gamma \propto A\lambda^3$

(b) $B^0 \rightarrow \pi^+\pi^-$: Color allowed, $\Gamma \propto A\lambda^3 e^{i\gamma}$

(c) $B^0 \rightarrow \overline{D}^0\pi^0$: Color suppressed, $\Gamma \propto A\lambda^2$

(d) $B^0 \rightarrow D^0\pi^0$: Color suppressed, $\Gamma \propto A\lambda^3 e^{i\gamma}$

(e) $B^0 \rightarrow J/\psi K^0$: Color suppressed, $\Gamma \propto A\lambda^2$