

# Flavour Physics: Week: VI

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**Problem 1.** A recent development in elementary particle physics is the discovery of charmed nonstrange mesons (called  $D^+$ ,  $D^0$ , and their charge conjugates) with masses around  $1870 \text{ MeV}/c^2$ .

- (a) Knowing the charge of charmed quark to be  $2/3$ , give the quark contents of the  $D^+$  and  $\bar{D}^0$  mesons
- (b) The  $D$  mesons decay weakly into ordinary mesons ( $\pi$ ,  $K$ , ...). Use  $\theta_c = 13.1^\circ$  to give estimates (with your reasoning) for the branching ratios of the following two-body decays:
  - $\frac{\mathcal{B}(D^0 \rightarrow K^- K^+)}{\mathcal{B}(D^0 \rightarrow K^- \pi^+)}$
  - $\frac{\mathcal{B}(D^0 \rightarrow \pi^- \pi^+)}{\mathcal{B}(D^0 \rightarrow K^- \pi^+)}$
  - $\frac{\mathcal{B}(D^0 \rightarrow K^+ \pi^-)}{\mathcal{B}(D^0 \rightarrow K^- \pi^+)}$
- (c) How would you show that the decay of  $D$  mesons is by means of weak interaction?
- (d) In a colliding beam at c.m. energy  $4.03 \text{ GeV}$ , a  $D^+$  meson (mass =  $1868.3 \text{ MeV}/c^2$ ) and a  $D^{*-}$  meson (mass =  $2008.6 \text{ MeV}/c^2$ ) are produced. The  $D^{*-}$  decays into a  $\bar{D}^0$  (mass =  $1863.3 \text{ MeV}/c^2$ ) and a  $\pi^-$ . What is the maximum momentum in the laboratory of the  $D^{*-}$  of the  $\pi^-$ ?

**Problem 2.** How well is satisfied unitarity from the measured CKM elements?

- (a) Check the unitarity condition and compute its uncertainty. Note: check the individual values from the PDG  
<https://pdg.lbl.gov/2024/web/viewer.html?file=../reviews/rpp2024-rev-ckm-matrix.pdf>  
(Hint: It suffice to prove that one of the rows is normalized)
- (b) Did the values change from 2021? (Hint: Look for the  $|V_{ud}|$  measurement and try to justify the change).

**Problem 3.** Consider the CKM Matrix in the Wolfenstein parametrization:

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \quad (1)$$

(a) Determine the Wolfenstein parameters  $(\lambda, A, \rho, \eta)$  from:

$$\begin{aligned} \Gamma(K^+ \rightarrow \mu^+ \nu_\mu)^{\text{exp}} &= 3.3793(79) \times 10^{-8} \text{ eV} \\ \Gamma(B^+ \rightarrow \tau^+ \nu_\tau)^{\text{exp}} &= 4.38(96) \times 10^{-8} \text{ eV} \\ \Delta M_{B_d}^{\text{exp}} &= 3.333(13) \times 10^{-10} \text{ MeV} \\ \Delta M_{B_s}^{\text{exp}} &= 1.1688(14) \times 10^{-8} \text{ MeV} \end{aligned}$$

using:

$$\begin{aligned} \Gamma(P \rightarrow l\nu) &= |V_{uq}|^2 \frac{f_P^2 m_P m_l^2}{16\pi v^4} \left(1 - \frac{m_l^2}{m_P^2}\right)^2 (1 + \delta_P) \\ \Delta M_{B_q} &= |V_{tb} V_{tq}|^2 \frac{m_{B_q} f_{B_q}^2 m_W^2}{12\pi^2 v^4} B_1^q S_1(m_b) \end{aligned}$$

(with  $P = \{K, B\}$  and  $q = \{d, s, b\}$  depending on the case), and the numerical parameters given in the table at the end of the exercise sheet.

Advise: Determine first the quantities  $(\{|V_{us}|, |V_{ub}|, |V_{tb}V_{td}|, |V_{tb}V_{ts}|\})$ , and then use the Wolfenstein parametrization to order  $\mathcal{O}(\lambda^4)$  to determine  $(\lambda, A, \rho, \eta)$ . You will need a computer.

- $v = 246.21965(6) \text{ GeV}$
- $m_{B_d} = 5.27963(15) \text{ GeV}$
- $G_F = 1.166 \cdot 10^{-5} \text{ GeV}^{-2}$
- $m_{B_s^0} = 5.36689(19) \text{ GeV}$
- $\alpha(m_b) = 1/132$
- $m_W = 80.379(12) \text{ GeV}$
- $m_\mu = 105.6583745(24) \text{ MeV}$
- $m_\tau = 1.77686(12) \text{ GeV}$
- $\delta_K = 0.0107(21)$
- $\delta_B \sim 0$
- $m_{K^\pm} = 493.677(16) \text{ MeV}$
- $m_{B^\pm} = 5.27932(14) \text{ GeV}$

- $f_{K^\pm} = 155.7(0.3) \text{ MeV}$
- $f_{B^\pm} = 190.0(1.3) \text{ MeV}$
- $f_{B_s^0} = 230.3(1.3) \text{ MeV}$
- $f_{B_d}^2 B_1^d = 0.0297(17) \text{ GeV}^2$
- $f_{B_s^0}^2 B_1^s = 0.0432(22) \text{ GeV}^2$
- $S_1(m_b) = 1.9848$
- $\lambda = 0.22537(46)$
- $A = 0.828(21)$
- $\rho = 0.194(24)$
- $\eta = 0.391(48)$
- $\tau_{B_s^0} = 1.527(11) : 10^{-12} \text{ s}$
- $1s = 1.5 \cdot 10^{24} \text{ GeV}^{-1}$
- $C_{10}^{SM}(m_b) = -4.309$

**Problem 4.** The  $B^0 \rightarrow \pi^-\pi^+$  decay involves change of flavour and, thus, grants access to element(s) of the CKM matrix.

- (a) Draw the Feynmann diagram for this decay, and point out what CKM matrix element ( $V_{ij}$ ) this decay is sensitive to.
- (b) The reconstruction of this decay involves the combination of two opposite charged tracks (particles). Look in the PDG potential backgrounds for this decay. Hint: look for decays of b-hadrons with similar mass to the  $B^0$  ( $B^0$ ,  $B_s^0$  and  $\Lambda_b^0$ ) to two tracks in the final state ( the  $\pi$  can be misidentificated as K or p).
- (c) Search for the branching ratio of these decays and compare them with that of the signal decay ( $B^0 \rightarrow \pi^-\pi^+$ ). Considering that  $B^0$  production is four times higher than  $B_s^0$  production at LHC ( $f_s/f_d(13\text{ TeV}) \sim 0.25$ ) and and three times higher than  $\Lambda_b^0$  ( $f_{\Lambda_b^0}/(f_d + f_u)(13\text{ TeV}) \sim 0.33$ ), what is the level of contamination that you can expect from these decays?
- (d) Considering the shift in mass due to the wrong reconstruction (incorrect mass hypothesis), and the different initial particle ( $B^0$  or  $\Lambda_b^0$ ), where would you expect the background reconstruction to be?