Problems

Question 13.1: Determining m_c and m_t from neutral meson mixing

Historically, the first estimates of m_c and m_t came from Δm_K and Δm_B, respectively.

1. Determine m_c using the theoretical expression for and the experimental value of Δm_K, and the values of the CKM elements. Put $m_u = 0$.

2. Explain why it is practically impossible to get an estimate for m_t using Δm_K.

3. To determine m_t using the theoretical expression and the experimental value of Δm_B, and the values of the CKM elements, we put $m_c = 0$ and use

$$\frac{\Delta m_B}{m_B} \approx \frac{B_B f_B^2}{m_B^2} \times \frac{g^4}{192\pi^2} \times \frac{m_B^2}{m_W^2} \times |V_{tb}V_{td}|^2 \times S(x_t, x_t). \quad \text{(Q13.22)}$$

For the values of the various parameters, we use

$$\begin{aligned} \Delta m_B &\approx 3.4 \times 10^{-13} \text{ GeV}, \\ \sqrt{B_B} f_B &\approx 0.22 \text{ GeV}, \\ m_B &\approx 5.28 \text{ GeV}, \\ g &\approx 0.65, \\ |V_{tb}V_{td}|^2 &\approx 6.4 \times 10^{-5}, \\ m_W &\approx 80 \text{ GeV}. \end{aligned} \quad \text{(Q13.23)}$$

In particular, we use

$$\frac{\Delta m_B}{m_B} \approx 6.3 \times 10^{-14}, \qquad \frac{f_B^2 B_B}{m_W^2} \approx 7.6 \times 10^{-6}. \quad \text{(Q13.24)}$$

Thus,

$$S(x_t, x_t) \approx \frac{(6.3 \times 10^{-14}) \times 192\pi^2}{(7.6 \times 10^{-6}) \times 0.65^4 \times 6.4 \times 10^{-5}} = 1.56. \quad \text{(Q13.25)}$$

We extract m_t by ploting $S(x_t, x_t)$ we see that $S(x_t, x_t) = 1.56$ correspond to $m_t^2/m_W^2 \approx 2.47$ which gives $m_t \approx 126$ GeV.

Compare your results to the direct measurements of m_c and m_t and comment on any difference

Question 13.2: Time scales in meson oscillations

In the following, we consider the four neutral meson systems, $P = K, D, B, B_s$. There are various time scales involved in neutral meson mixing: Δm, $\Delta\Gamma$, and Γ. Understanding the hierarchy (or lack of hierarchy) between them leads to insights and simplifications. We use the dimensionless quantities x and y, defined in Eq. (13.52), to understand the possible hierarchies between the relevant time scales.

$$x \equiv \frac{\Delta m}{\Gamma}, \qquad y \equiv \frac{\Delta\Gamma}{2\Gamma}, \tag{13.52}$$

Evaluate $\mathcal{P}(P^0 \to P^0)$ and $\mathcal{P}(P^0 \to \overline{P}^0)$ in the following limits:

1. $y = 0$ and $x \ll 1$ ("slow oscillation"). Keep the leading order terms in x in each case. Explain the result. Which of the four neutral meson systems fall into this category?

2. $y = 0$ and $x \gg 1$ ("fast oscillation"). Argue that in this case you can average over the oscillations. Give the time averaged oscillation to first order in $1/x$. Explain the result. Which of the four neutral meson systems fall into this category?

3. $y = 0$ and $x \sim 1$. In this case we have to use the full formula. Which of the four neutral meson systems fall into this category?

4. The generalization of Eq. (13.62), still keeping $\lambda_f = 0$, but taking into account finite x and y is

$$\begin{aligned}\mathcal{P}(P^0 \to P^0) &= \frac{1}{2}[\cosh(y\Gamma t) + \cos(x\Gamma t)],\\ \mathcal{P}(P^0 \to \overline{P}^0) &= \frac{1}{2}[2 - \cosh(y\Gamma t) - \cos(x\Gamma t)].\end{aligned} \tag{13.93}$$

   Assume $|y| \ll x \lesssim 1$. Argue that in this case the effect of the width difference is quadratic in y. Which of the four neutral meson systems fall into this category?

5. $y \sim x \ll 1$. Argue that in this case the width difference is as important as the mass difference. Which of the four neutral meson systems fall into this category?

6. $1 - |y| \ll 1$. Argue that in that case, independent of the value of x, at late times you can think about the state as the long-lived one. What can be considered "late times"? Which of the four neutral meson systems fall into this category?

There are few other limits that are not realized in the four neutral meson systems, and thus we do not elaborate on them.