Question 9.1: b and c lifetimes

In this question we study decay rates and the lifetimes of the b and c quarks. We only consider tree level decays. We denote by Q the b or c quark and by q the u, d and s quarks. We also approximate in the phase space factors $m_q = m_e = m_\mu = 0$.

The phase space factor for three-body decay with one massless particle is given by

$$\begin{aligned} f(x_1,x_2) = \sqrt{\lambda}_{12} \left[1 - 7(x_1+x_2) - 7(x_1^2+x_2^2) + x_1^3 + x_2^3 + x_1 x_2 (12 - 7(x_1+x_2))\right] \\ + 12\left[x_1^2 \ln \frac{(1+x_1-x_2+\sqrt{\lambda}_{12})^2}{4x_1} + x_2^2 \ln \frac{(1+x_2-x_1+\sqrt{\lambda}_{12})^2}{4x_2}\right] \\ - 12 x_1^2 x_2^2 \ln \frac{(1-x_2-x_1+\sqrt{\lambda}_{12})^2}{4x_2 x_1}, \end{aligned} \tag{9.43}$$

with

$$x_i \equiv \frac{m_i^2}{m_Q^2}, \qquad \lambda_{12} = 1 - 2(x_1+x_2) + (x_1-x_2)^2 \,. \tag{9.44}$$

For two massless particles, Eq. (9.43) reduces to the phase space factor in Eq. (9.31).

1. Consider semi-leptonic decays, $Q \to q\ell\bar{\nu}$. Draw the tree-level diagram and estimate the amplitude.

2. The decay width can be written as

$$\Gamma_Q = C m_Q^n |V_{ij}|^2 f(x_q, x_\ell). \tag{9.45}$$

   such that C is a constant, V_{ij} is the relevant CKM matrix element, $x_f \equiv m_f^2/m_Q^2$ and f is a dimensionless phase space factor (given for two massless and one massive final particles in Eq. (7.86)), with $f(0,0) = 1$. Find n.

$$\Gamma_\mu = \frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} f(m_e^2/m_\mu^2)(1+\delta_{\rm RC}), \qquad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x, \tag{7.86}$$

3. Draw the diagram of the $Q \to q_i \bar{q}_j q_k$ decay, and modify Eq. (9.45) to this case.

4. List the leading (in the CKM parameter λ) decay modes for the c and b quarks.

5. Estimate the BRs of the semi-leptonic modes

$$b \to ce\nu, \qquad c \to se\nu. \tag{9.46}$$

   For the rough estimate in this question, use the masses : $m_c = 1.27$ GeV, $m_b = 4.18$ GeV, $m_\tau = 1.79$ GeV. For b decays, negelecting the electron, muon, up, down and strange quark masses, the relevant phase space factors are

$$f(x_c, x_\tau) \approx 0.06, \quad f(x_c, x_c) \approx 0.18, \quad f(x_c, 0) \approx 0.51, \quad f(0,0) = 1. \tag{9.47}$$

For c decays, neglecting the electron, up and down quark masses, the relevant phase space factors are

$$f(x_s, 0) \approx 0.96, \quad f(x_s, x_\mu) \approx 0.91. \tag{9.48}$$

6. Using the value of the CKM elements from Eq. (9.5), estimate the ratio of the b and c lifetimes, τ_b/τ_c.

$$|V| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}. \tag{9.5}$$

7. Compare your estimates for the lifetimes and for the semi-leptonic BRs to the PDG values. To do so, assume that the b and c lifetimes and decay modes are given by those of the corresponding mesons, B^+ and D^+.