Flavour Physics: Week: VI

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Problem 1. The four generation SM

Consider an extension of the SM with an additional (4th) generation of quarks and leptons:

$$L_4(1,2)_{-1/2}\,, \quad E_4(1,2)_{-1/2}\,, \quad Q_4(3,2)_{+1/6}\,, \quad U_4(3,1)_{+2/3}\,, \quad D_4(3,1)_{-1/3}\,. \tag{1}$$

We denote the fourth generation mass eigenstates by t', b', τ', and ν'_τ in an obvious notation.

(a) Write down the Yukawa interactions. Use matrix notation, *e.g.*, use L_i where $i = 1, 2, 3, 4$ to denote the lepton doublets.

(b) What is the global symmetry of the model? What is the number of physical parameters related to the Yukawa terms? Separate them into masses, mixing angles and phases. (Hint: Read Sec. 7.5.3 Parameter counting in Yuval's book for further information).

(c) Is CP violated in the quark sector? Is it violated in the lepton sector?

Problem 2. (a) Derive the Wolfstein notation $\mathcal{O}(\lambda^3)$

$$V_{\text{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \tag{2}$$

using the CKM matrix as a function of the mixing angles and the CP phase

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{3}$$

and the equalities

$$s_{12} = \lambda\,, \qquad s_{23} = A\lambda^2\,, \qquad s_{13}e^{-i\delta} = A\lambda^3(\rho - i\eta), \tag{4}$$

Hint: Use Taylor expansion to express the elements c_{ij} as a function of the elements s_{ij}.

(b) Using the definition of the J_{CKM}

$$\mathcal{I}m(V_{ij}V_{kl}V_{il}^*V_{kj}^*) = J_{CKM}\sum_{m,n=1}^{3}\epsilon_{ikm}\epsilon_{jln}\,, \qquad (i,j,k,l = 1,2,3) \tag{5}$$

show that

$$J_{CKM} = \mathcal{I}m(V_{ud}V_{cs}V_{us}^*V_{cd}^*) \tag{6}$$

Problem 3. In the main text, we discuss the UT, that is the bd triangle,

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \tag{7}$$

. We mention that each of its sides is of $\mathcal{O}(\lambda^3)$, and each of its angles is of $\mathcal{O}(1)$. Here we discuss another unitarity triangle, the bs triangle.

(a) Write the equation for the bs UT.

(b) Draw the rescaled bs UT and label the angles as α_s, β_s and γ_s, in a similar way to our discussion of the rescaled bd triangle.

(c) Estimate, in terms of powers of λ, the size of each side of the bs UT.

Problem 4. Semileptonic decays and the CKM matrix.

A semileptonic decay is one where the final state has both hadrons and leptons. Here we consider semileptonic b decays.

(a) Draw the tree-level diagram for $\mathrm{b} \to \mathrm{ue}\overline{\nu}$ and estimate the size of the amplitude.

(b) Estimate the ratio $\frac{\Gamma(\mathrm{b} \to \mathrm{ue}\overline{\nu})}{\Gamma(\mathrm{b} \to \mathrm{ce}\overline{\nu})}$ as a function of CKM matrix elements. Neglect m_e and use $m_c/m_b \simeq 0.3$, and $m_u/m_b \simeq 0.002$. The phase space function for a three-body decay with one massive and two massless final particles is given by

$$f(x_f) = 1 - 8x_f + 8x_f^3 - x_f^4 - 12x_f^2 \log x_f, \quad x_f = \left(\frac{m_f}{m_b}\right)^2 \tag{8}$$

(c) The experimental value is $\frac{\Gamma(\mathrm{b} \to \mathrm{ue}\overline{\nu})}{\Gamma(\mathrm{b} \to \mathrm{ce}\overline{\nu})} \simeq 2 \times 10^{-2}$. Estimate the relevant ratio of CKM matrix elements. Comment on the agreement between your estimate and the best fit value for that ratio. (Hint: You will need to look for the value of relevant CKM elements in the PDG).