Exercises

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1 SHG in a GaAs optical waveguide

Nonlinear frequency conversion on *photonic integrated circuits* (PICs) opens new avenues compared to conventional bulk-optic based methods. In particular, the requirement of *birefrigence* to achieve phase matching is circumvented due to the concept of *effective refractive indices*. This means that materials exhibiting very high nonlinear tensor elements, but no birefringence, can be used for nonlinear processes. One such material is **Gallium Arsenide (GaAs)**. Assuming Kleinmann symmetry, the effective nonlinear d coefficient is $d_{14} \approx$ **120 pm/V** [2], almost two orders of magnitude higher than e.g. the nonlinear crystal LBO. In this exercise, you will simulate different aspects of second-harmonic generation (SHG) in a GaAs waveguide structure. A schematic of a *GaAs-on-insulator* waveguide is given in Fig. 1.

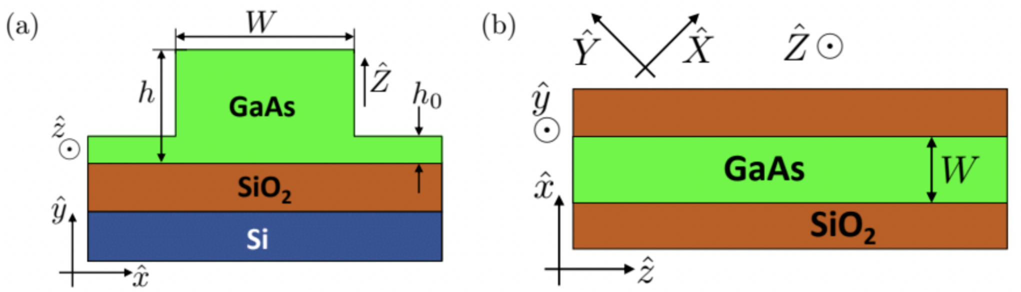

Figure 1: Schematic of a GaAs waveguide. a)

The fundamental requirement for achieving efficient frequency conversion is the notion of *phase matching*. This is quantified by requiring the wave vectors of the signal and pump to be exactly aligned. In the case of SHG, and for collinear systems such as an optical waveguide, this criterion simplifies to:

$$\Delta k = k_\mathrm{s} - 2k_\mathrm{p} \Leftrightarrow n_\mathrm{s} = n_\mathrm{p} \tag{1}$$

where $k_\nu = 2\pi n_\nu / \lambda_\nu$, with λ_ν being the wavelength and n_ν the refractive index of an optical mode ν, where $\nu = \{\mathrm{p}, \mathrm{s}\}$. The major difference between bulk and integrated nonlinear photonics is that in integrated structures, these indices can be matched by varying the *dimensionality* of the waveguiding structure. The particular index of a mode in a given waveguide configuration is known as the *effective refractive index*. By solving Maxwell's equations in the material cross-section, these effective refractive indices can be computed. They depend on the **wavelength** of light, and the **materials** and **dimensions** of the waveguide structure.

1.1 Simulating the waveguide

0. Setting up your numerical solver

(I) In the following exercises, you need to use a numerical *Finite Difference Method* (FDM) solver. This will solve Maxwell's equations in a grid defined by the user. You can choose to use either

Tidy3D or EMode Photonix. Both have free versions with the required functionality and Python interfacing.

(II) Install your preferred solver and test your connection.

1. Pump mode

(I) Using your numerical solver, simulate the waveguide structure in Fig. 1. Use the parameters $\lambda = 1960$ nm, $w = 1600$ nm, $h = 149$ nm and $h_0 = 15$ nm. The bottom SiO_2 layer is 3 µm thick. The materials can be found in solver's database.

(II) Inspect the simulated electric fields. How does each component look? These field distributions in waveguiding structures are called *optical modes*. Each mode is a solution to Maxwell's equation, indicating a stable, "supported" distribution of the electromagnetic field.

(III) Try to increase the number of modes simulated by your program. You will notice that the prefix of some modes change from **TE** to **TM**. This stands for *tranverse electric* and *transverse magnetic* respectively, and is the the analogue to free-space polarization states.

2. Signal mode

(I) Repeat exercise **1.** but with $\lambda = 980$, i.e. the second-harmonic wavelength. Compare the values of the calculated effective indices. Is phase matching achieved?

(II) Change the number of simulated modes such that you also solve for **TM** modes. Compare the simulated index of the *first* TM-mode (**TM-0**) with the *first* TE-mode of the pump (**TE-0**). Are they more similar than in the TE-0 - TE-0 case?

3. Sweeping the waveguide dimension

(I) During lithography, the height of the waveguide cannot be altered. However, the width can be accurately defined. In your simulation, sweep the waveguide width w from 1000 nm to 2000 nm. For each value, calculate the effective index of the **TE-0** mode of the the pump, λ_p = 1960 nm, and **TM-0** mode of the signal, λ_s = 980 nm. Plot the calculated effective indices as a function of waveguide width.

(II) Phase matching is achieved where the effective indices are equal. Is this condition fulfilled for any of the swept widths? If not, try increasing the resolution of your grid.

(III) Simulate the mode at the width where phase matching is achieved. Save the fields ($\mathfrak{e}_{x,y,z}$) in a file or variable for later use.

1.2 Calculating nonlinear conversion

In part 1 of the exercise, we demonstrated through simulation that phase-matching can be achieved between a TE and TM mode in a GaAs waveguide. This part of the exercise deals with calculating the efficiency of the SHG process.

4. Coupling

(I) To quantify how the pump and signal modes interact, a coupling coefficient is defined. This real and unit-less coupling coefficient is, in the special case of the zinc-blende point group (to which GaAs belongs), given by [1]:

$$\kappa = \frac{c\varepsilon_0}{2Q}\iint_{\mathbb{R}^2} (d\vec{v}_{\mathrm{s}}) \cdot \vec{\mathfrak{e}}_{\mathrm{s}}^{*}\, \mathrm{d}x\, \mathrm{d}y, \qquad \text{with: } d\vec{v}_{\mathrm{p}} = d_{14}\begin{pmatrix} 2\mathfrak{e}_{\mathrm{p},Y}\,\mathfrak{e}_{\mathrm{p},Z} \\ 2\mathfrak{e}_{\mathrm{p},X}\,\mathfrak{e}_{\mathrm{p},Z} \\ 2\mathfrak{e}_{\mathrm{p},X}\,\mathfrak{e}_{\mathrm{p},Y} \end{pmatrix} \tag{2}$$

The nonlinear tensor d is spatially varying and has the value of the nonlinear d_{14} coefficients where the material has an appreciable second-order susceptibility, and is negligible elsewhere. Notice the subscripts X, Y, Z, which is the nonlinear crystals coordinate system. The solutions given by your FDM solver will typically be in the global coordinate system (x, y, z). The transfer of coordinate system is:

$$\hat{X} = \frac{\hat{x}+\hat{z}}{\sqrt{2}} \quad ; \quad \hat{Y} = \frac{\hat{x}-\hat{z}}{\sqrt{2}} \quad ; \quad \hat{Z} = \hat{y}. \tag{3}$$

Calculate the coupling coefficient κ for your phase-matched waveguide. Let $Q = 1$ mW for normalization.

References

[1] Magnus L. Madsen, Emil Z. Ulsig, Sebastian Folsach, Pedro H. Godoy, Eric J. Stanton, and Nicolas Volet. Mid-infrared difference-frequency generation in algaas-on-insulator waveguides. *J. Opt. Soc. Am. B*, 40(7):1742–1748, Jul 2023.

[2] M Ohashi, T Kondo, R Ito, S Fukatsu, Y Shiraki, K Kumata, and SS Kano. Determination of quadratic nonlinear optical coefficient of $Al_x\, Ga_{1-x}$ As system by the method of reflected second harmonics. *J. Appl. Phys.*, 74(1):596–601, 1993.