Problem set 02

Nonlinear Optics for Quantum Technologies

February 27, 2025

2 First order degree of coherence

1. Consider:
$$|G^{(1)}(\tau)|^2 = | < E^-(0)E^+(\tau) > |^2 \leq | < E^-(0) > |^2 | < E^+(\tau) > |^2$$
$$= | < E^-(0) > |^2 | < E^+(0) > |^2 = | < E^- E^+(0) > |^2$$
where we used first Cauchy Schwartz inequality and then the fact that $|z|^2 = z^* z$ for any complex number.
Furthermore
$$|G^{(1)}(0)|^2 = | < E^-(0)E^+(0)) > |^2 = | < E^{+*}(0)E^+(0) > |^2$$
therefore
$$|g^{(1)}(\tau)|^2 \leq 1$$

2. See hand written solution

3. For homogenous broadening:
$$T_{coh} = \int_{-\infty}^{\infty} |g^{(1)}(\tau)|^2 d\tau$$
$$= \int_{-\infty}^{\infty} e^{-2\gamma|\tau|} d\tau = \frac{1}{\gamma}$$
For inhomogenous broadening:
$$T_{coh} = \int_{-\infty}^{\infty} e^{-\delta^2\tau^2} d\tau = \frac{\sqrt{\pi}}{\delta}$$

4. - Reminder:
   for $\omega > 0$ we have that $E(\omega) = \int_{-\infty}^{\infty} E^+(t) e^{i\omega t} dt$
   Let us consider now positive frequencies $\omega > 0$:
   $$PSD(\omega) = |E(\omega)|^2 = E^*(\omega) = \int_{-\infty}^{\infty} dt_1 E^{+*}(t_1) e^{-i\omega t_1} \int_{-\infty}^{\infty} dt_1 E^+(t_2) e^{-i\omega t_2}$$
   $$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dt_1 dt_2 E^{+*}(t_1)E^+(t_2) e^{i\omega(t_2-t_1)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dt_1 d\tau E^-(t_1)E^+(t_1+\tau) e^{i\omega\tau}$$
   $$= \int_{-\infty}^{\infty} G^{(1)}(\tau) e^{i\omega\tau}$$
   The same result is obtained similarly for negative frequencies.
   - For homogenous boradening:
   $$f(\omega) = \int_{-\infty}^{\infty} e^{-\gamma t} e^{i(\omega-\omega_0)t} dt = \frac{2\gamma}{\gamma^2 + (\omega-\omega_0)^2}$$
   For inhomogenous broadening:
   $$f(\omega) = \int_{-\infty}^{\infty} e^{-\delta^2 t^2} e^{i(\omega-\omega_0)t} dt = \frac{\sqrt{2\pi}}{\delta} e^{-\frac{\pi^2(\omega-\omega_0)^2}{\delta^2}}$$

- As shown in exercise 2 using an Mach-Zehnder interferometer one can measure the first order coherence of the light source, looking at the visibility of interference fringes versus delay. By Fourier transforming the obtained first order coherence one obtains the spectrum of the source.
- Since first order coherence and spectral distribution are related by fourier transform, by adding an narrowband pass filter to a light source we increase the first order coherence time, see for example `https://www.nature.com/articles/s41598-017-06215-x`.