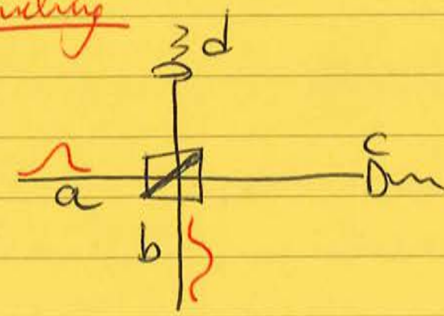
HOM Effect

Photon famously don't really interact.

But they do interfere. One of the coolest examples is Hong-Ou-Mandel (HOM) Interference.

Also might hear this called bunching

Can represent this as creation operators, $a^\dagger$ and $b^\dagger$, acting on vacuum:



$$|\psi\rangle_{in} = a^\dagger b^\dagger |0\rangle_{ab}$$

Let's assume our beam-splitter is 50/50 (but this works fine if it isn't). $a^\dagger$ and $b^\dagger$ transform as:

$$a^\dagger \rightarrow \frac{1}{\sqrt{2}} c^\dagger + \frac{1}{\sqrt{2}} d^\dagger$$

$$b^\dagger \rightarrow \frac{1}{\sqrt{2}} c^\dagger - \frac{1}{\sqrt{2}} d^\dagger$$

Relative phase shift here from BS operator

So after BS our state is:

$$|\psi\rangle_{out} = \left(\frac{1}{\sqrt{2}} c^\dagger + \frac{1}{\sqrt{2}} d^\dagger\right)\left(\frac{1}{\sqrt{2}} c^\dagger - \frac{1}{\sqrt{2}} d^\dagger\right)|0\rangle_{cd}$$

$$\Rightarrow |\psi\rangle_{out} = \frac{1}{2}\left[c^{\dagger 2}|0\rangle_{cd} - d^{\dagger 2}|0\rangle_{cd}\right]$$

$$-\frac{1}{2} c^\dagger d^\dagger |0\rangle + \frac{1}{2} c^\dagger d^\dagger |0\rangle$$

$\Rightarrow$ Relate to slides!

If the photons are identical in all attributes, last two terms cancel and we have:

$$|\Psi\rangle_{out} = \frac{1}{2}\left(|20\rangle_{cd} - |02\rangle_{cd}\right)$$

So both photons end up in either c or d.

Bunching

This underpins a lot of operations in photonic quantum info, including Bell State Measurements.

Can also be used as a diagnostic tool to see if your photons are indistinguishable.

What does this look like more in the 'real world'?

In reality there might be some time delay, so we can look at the detector clicks we get as this changes.

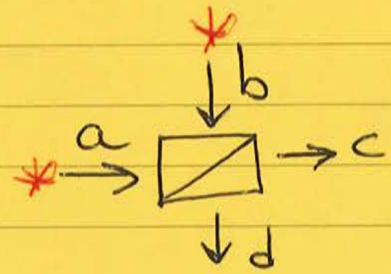
Bell States and Measurements

If we know we should receive a Bell State, how can we measure this?

Start with:

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle - |VH\rangle\right)$$

Propagate through our 50/50 BS



$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_a|V\rangle_b - |V\rangle_a|H\rangle_b\right)$$

$$a^\dagger \rightarrow \frac{1}{\sqrt{2}}\left(c^\dagger + d^\dagger\right)$$

$$b^\dagger \rightarrow \frac{1}{\sqrt{2}}\left(+c^\dagger \mp d^\dagger\right)$$

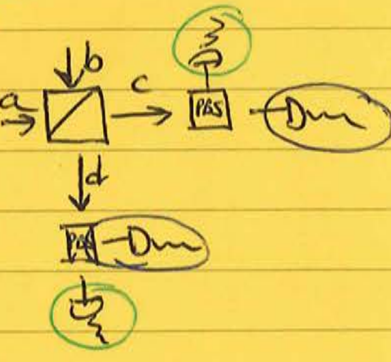
$$|\psi^-\rangle \rightarrow \frac{1}{2\sqrt{2}}\Big[\cancel{-|HV\rangle_c} + |H\rangle_c|V\rangle_d - |V\rangle_c|H\rangle_d + \cancel{|HV\rangle_d} + \cancel{|HV\rangle_c} - |V\rangle_c|H\rangle_d + |H\rangle_c|V\rangle_d \cancel{-|HV\rangle_d}\Big]$$

$$\Rightarrow |\psi^-\rangle \Rightarrow \frac{1}{\sqrt{2}}\left(|H\rangle_c|V\rangle_d - |V\rangle_c|H\rangle_d\right)$$

Mapped polarisation to path

In contrast:

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_a|V\rangle_b + |V\rangle_a|H\rangle_b\right)$$



$$|\psi^+\rangle \Rightarrow -\frac{1}{\sqrt{2}}\left(|HV\rangle_c - |HV\rangle_d\right)$$

look for "coincidences" (coincident clicks).

What about $|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|HH\rangle \pm |VV\rangle\right)$?

$$|\phi^{\pm}\rangle \rightarrow -\frac{1}{2\sqrt{2}}\left(\left[|H\rangle_c|H\rangle_c \pm |V\rangle_c|V\rangle_c\right] - \left[|H\rangle_d|H\rangle_d \pm |V\rangle_d|V\rangle_d\right]\right)$$

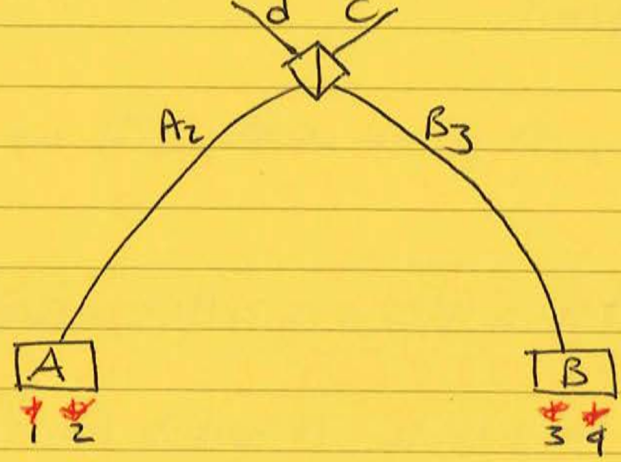
No coincident clicks.

Only difference is phases in front of $|VV\rangle$ terms.

We can't detect this with click pattern (without nonlinear optics/ancillas).

And so a Bell State Measurement with only linear optics (and no ancillas) can only be 50% efficient.

Entanglement Swapping



$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_{A1}|V\rangle_{A2} - |V\rangle_{A1}|H\rangle_{A2}\right)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_{B3}|V\rangle_{B4} - |V\rangle_{B3}|H\rangle_{B4}\right)$$

Photon in A2 transforms as: $A_2^{+} \to \frac{1}{\sqrt{2}}(c^{+} + d^{+})$
Photon in B3 transforms as: $B_3^{+} \to \frac{1}{\sqrt{2}}(-c^{+} + d^{+})$

Combined state at the beginning is $|\psi\rangle = |\psi_1\rangle|\psi_2\rangle$:

$$|\psi\rangle = \frac{1}{2}\Big[|H\rangle_{A1}|V\rangle_{A2}|H\rangle_{B3}|V\rangle_{B4} -$$

$$|H\rangle_{A1}|V\rangle_{A2}|V\rangle_{B3}|H\rangle_{B4} - |V\rangle_{A1}|H\rangle_{A2}|H\rangle_{B3}|V\rangle_{B4} +$$

$$|V\rangle_{A1}|H\rangle_{A2}|V\rangle_{B3}|H\rangle_{B4}\Big]$$

Let's look at what happens if we perform a BSM on A2 and B3.

First, A2 and B3 will go through our 50/50 BS, and we see how these transform.

Sub in the transforms, do some maths, and we find our state is now:

$$|\psi\rangle = \frac{1}{4}\Big[|H\rangle_{A1}\left(-|VH\rangle_c + |V\rangle_c|H\rangle_d - |V\rangle_d|H\rangle_c + |VH\rangle_d\right)|V\rangle_{B4}$$

$$- |H\rangle_{A1}\left(-|VV\rangle_c + |V\rangle_c|V\rangle_d - |V\rangle_d|V\rangle_c + |VV\rangle_d\right)|H\rangle_{B4}$$

$$- |V\rangle_{A1}\left(-|HH\rangle_c + |H\rangle_c|H\rangle_d - |H\rangle_d|H\rangle_c + |HH\rangle_d\right)|V\rangle_{B4} +$$

$$|V\rangle_{A1}\left(-|HV\rangle_c + |H\rangle_c|V\rangle_d - |H\rangle_d|V\rangle_c + |HV\rangle_d\right)|H\rangle_{B4}\Big].$$

Let's look for coincidence clicks between H and V photons in ~~the same~~ different paths (~~both~~ one in c ~~or~~ and ~~both~~ one in d). Use 4 detectors.

These four terms survive, so our state after the measurement is:

$$|\psi\rangle = \frac{1}{2}\Big[|H\rangle_{A1}|V\rangle_{B4} - |V\rangle_{A1}|H\rangle_{B4} \Big] = |\psi^-\rangle$$

Which is entangled!

TIME-BIN ENTANGLEMENT (1).

Encode information in timing degree of freedom.

In going through crystal, photon pairs are created in either early or late time-bin.

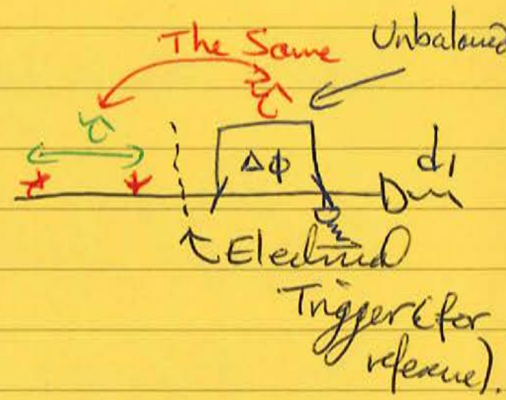
But we don't know which.

State is then:

$$|\psi\rangle = |t\rangle_s |t\rangle_i + |t+\tau\rangle_s |t+\tau\rangle_i$$

Time-bin entangled state ($|\Phi^+\rangle$).

How do we measure this?



Interferometers (Franson).

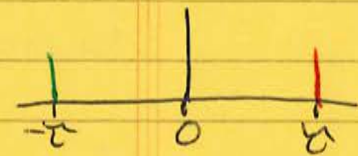
Consider 1 photon,

$$|\psi\rangle \propto |E\rangle + e^{i\phi}|L\rangle$$

- Draw this as poles on Bloch Sphere (Z basis).
- Superpositions are on Equator.

3 different 'timing' options.

• Early photon goes short => $-\tau$
↳ equivalent to measuring $|E\rangle\langle E|$



• Late photon goes long => τ
↳ Equivalent to measuring $|L\rangle\langle L|$

• Early goes long, late goes short => 0

Click on our detector d1 is equivalent to projection onto $|X^+\rangle = \frac{1}{\sqrt{2}}(|E\rangle + |L\rangle)$

So if we input state $|X\rangle = \frac{1}{\sqrt{2}}(|E\rangle + |L\rangle)$, will get click on this detector (for $t=0$) with 100% prob.

Detector d2 is equivalent to projection on $|X^-\rangle$

By changing $\Delta\phi$, can change which port the photon will exit.

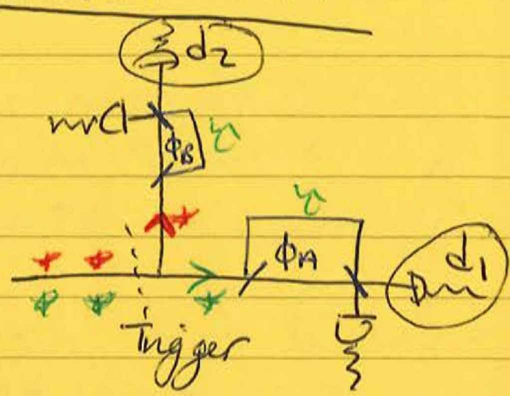
So changes basis you're measuring in (X^+ to Y^+ to X^- to Y^-) => Go around Equator.

Peak at $t=0$, measured on just 1 detector, goes up and down.

TIME-BIN ENTANGLEMENT (2).

Now to our 2-photon state.
$|\psi\rangle \propto \frac{1}{\sqrt{2}}(|EE\rangle + e^{i\phi}|LL\rangle)$.

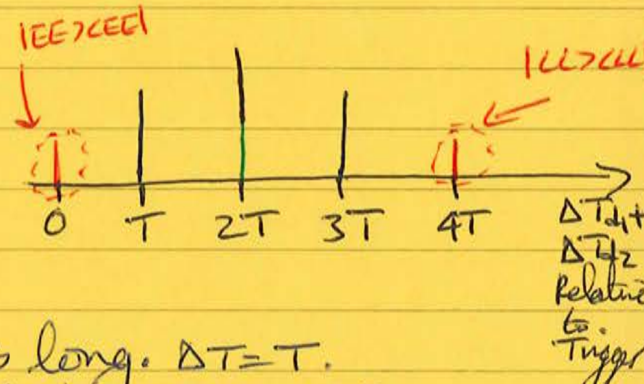
Now need the arm difference on both interferometers to match the bin spacing.



We now want to look at coincidences between outputs (e.g. d_1 and d_2) and when this occurred relative to our trigger signal (Triple Coincidence).

5 possible timing outcomes.

• Created in $|E\rangle$, both go short, $\Delta T = 0$
↳ Measurement in $|EE\rangle\langle EE|$
Independent of phase.



• 11, one goes short one goes long. $\Delta T = T$.
↳ Measurement of one photon in $|E\rangle$ and one photon in $|\phi\rangle$, superposition state. Example: $|EX^+\rangle\langle EX^+|$. d_1 clicked early, d_2 at $\Delta T = T$, and $\phi_B = 0$. Changes to ϕ_B changes $X^+ \to X^-$ so peak height will change.

• 11, both go long. $\Delta T = 2T$.
• Also have created late and both go short.

Now both are superposition measurements $|\phi^A\phi^B\rangle\langle\phi^A\phi^B|$.
For $\phi^A = 0 = \phi^B$, Measurement in $|x^+x^+\rangle\langle x^+x^+|$.
By changing ϕ^A and ϕ^B change the state.

- Created late, one short one long, $\Delta T = 3T$.
  Get $|L\phi\rangle\langle L\phi|$.

- " , Both go long, measurement in $|LL\rangle\langle LL|$.

ENERGY-TIME ENTANGLEMENT

Start with a crystal, shine a laser on it, emits a pair of photons.

What does this look like in time?

All these different time modes within laser coherence.

Photons can be created in any of these, so the state is distributed over all of these modes.

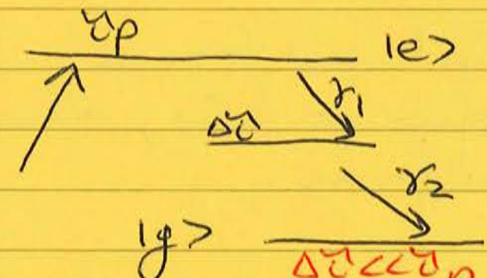
What does this look like?

$|\psi\rangle \propto \sum_t |t,t\rangle_{s,i}$

Moment of emission is unpredictable within τ_p, but creation times are highly correlated.

3-level Scheme

We can also view this as a 3-level system.



Imagine system with long τ_p, don't know when photons are emitted.

But with very short $\Delta\tau$, we know that as soon as γ_1 is emitted, γ_2 will be emitted very quickly afterwards.

So for this state $|\psi\rangle \propto \sum_t |t, t\rangle_{s,i}$:

We don't know exactly when they were created, but they are created at the same time (to within their bandwidths). They are energy-time entangled.

Measurement: Franson Interferometer.

We can once again measure this with Franson Interferometer, but the situation is now slightly different.

First, how long should our interferometer be?
With time bin, it was clear interferometer had to be same as time bin separation.

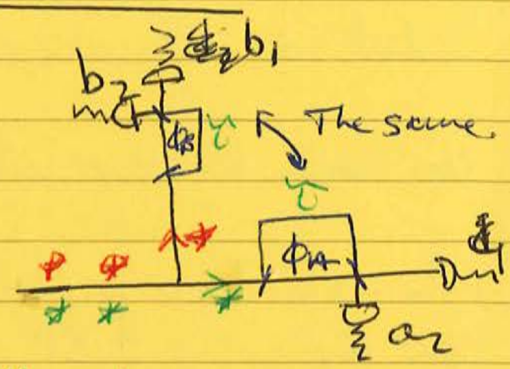
With CW, as it's continuous, we have to essentially choose to pick out ~~a~~ a particular time mode, with separation equal to interferometer length.

Just need to make sure this choice of τ is > photon coherence time, otherwise we'll have single photon interference.

Let's now look at what happens when our state propagates through the interferometer.

ENERGY-TIME MEASUREMENT

We have made τ longer than the photon coherence so there is no first-order interference.

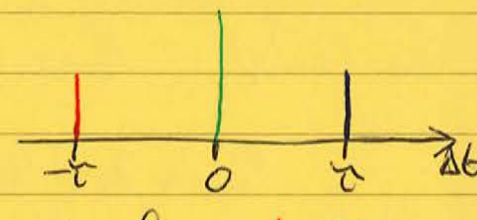


However a photon pair can still produce second-order interference effects when we look at coincidence detections between two outputs, a_1 and b_1.

3 Possible Timing Outcomes (as no trigger).

As there is no trigger, the only timing outcomes are from the photons with respect to each other.

(i) One goes short (A) and the other goes long (B) $E_a L_b$



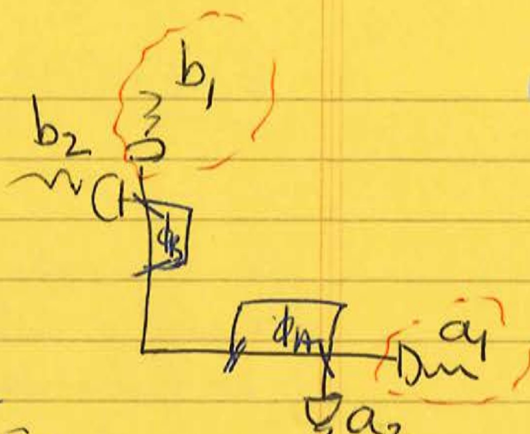
(ii) One goes short (B) and the other goes long. $L_a E_b$.

(iii) Both go short or both go long, $E_a E_b$ and $L_a L_b$.

For (i) and (ii) we can tell exactly what happened from the detector clicks (know who went long/short).

For (iii), we can't tell whether they went short or long.

What does this mean?



Go back to our interferometers:

When we have a coincidence click with time difference $\pm\Delta\tau$, we can ignore this.

If we keep those terms with $\Delta\tau = 0$, we have the state:

$$|\psi\rangle \propto |S\rangle_a |S\rangle_b + e^{i(\phi_A+\phi_B)} |L\rangle_a |L\rangle_b$$

From our point of view, both of these states give the same coincident click pattern, and so they can interfere
=> with the clicks proportional to $\phi_A+\phi_B$.

More rigorously, a photon in each of the paths transforms at the final beam splitter as:

$$S_A \to \frac{1}{\sqrt{2}}\left(a_1^\dagger|0\rangle + a_2^\dagger|0\rangle\right), \quad L_A \to \frac{1}{\sqrt{2}}\left(+a_1^\dagger|0\rangle - a_2^\dagger|0\rangle\right)$$

And the same for B.

So our state $|\psi\rangle$ becomes:

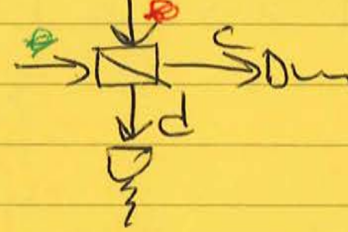
$$|\psi\rangle \propto \frac{1}{4}\Big[a_1^\dagger b_1^\dagger\left(1+e^{i(\phi_A+\phi_B)}\right)|00\rangle + a_2^\dagger b_2^\dagger\left(1+e^{i(\phi_A+\phi_B)}\right)|00\rangle$$
$$+ a_2^\dagger b_1^\dagger\left(1-e^{i(\phi_A+\phi_B)}\right)|00\rangle + a_1^\dagger b_2^\dagger\left(1-e^{i(\phi_A+\phi_B)}\right)|00\rangle\Big]$$

And so if we look at coincidences between a_1 and b_1, we will see the number of coincidences oscillate with $\cos(\phi_A+\phi_B)$.

BELL STATE MEASUREMENTS (TIME).

Start with $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|t\rangle|t+\tau\rangle - |t+\tau\rangle|t\rangle\right)$

Now the BSM is done by <u>time-difference</u> in clicks.



After 50/50 BS we have;

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|t\rangle_c|t+\tau\rangle_d - |t+\tau\rangle_c|t\rangle_d\right)$$

So if we measure either
Click in c at time t, <u>and</u> in d at time $t+\tau$
<u>OR</u>
Click in c at time $t+\tau$, <u>and</u> in d at time t,
we know we have $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|t\rangle|t+\tau\rangle - |t+\tau\rangle|t\rangle\right)$

For $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|t\rangle|t+\tau\rangle + |t+\tau\rangle|t\rangle\right)$

This becomes;

$$|\psi\rangle = -\frac{1}{\sqrt{2}}\left(|t\rangle_c|t+\tau\rangle_c - |t\rangle_d|t+\tau\rangle_d\right)$$

So <u>either</u> click in c at t, <u>and</u> in c at $t+\tau$
<u>OR</u>
Click in d at t, <u>and</u> in d at $t+\tau$.

Why is this difficult to measure?

For $|\phi^\pm\rangle = \frac{1}{\sqrt{2}}\left(|t\rangle|t\rangle \pm |t+\tau\rangle|t+\tau\rangle\right)$:

$$|\phi^\pm\rangle \Rightarrow -\frac{1}{2\sqrt{2}}\Big[\left(|tt\rangle_{cc} - |tt\rangle_{dd}\right) \mp \left(|t+\tau\rangle_c|t+\tau\rangle_c - |t+\tau\rangle_d|t+\tau\rangle_d\right)\Big]$$

Both photons exit in the <u>same path</u> at the <u>same time</u>.

Again, phase can't be detected with click pattern.

So we again have 50% efficiency.

Entanglement-Swapping with Energy-Time

How do we do this?

General state from source is:

$$|\psi\rangle_A \propto \sum_t |t,t\rangle_A$$

Combined state is:

All 4 photons at the same time.

$$|\psi\rangle = |\psi\rangle_A\,|\psi\rangle_B$$

$$\Rightarrow |\psi\rangle \propto \overbrace{\sum_t |t,t\rangle_A |t,t\rangle_B}^{\text{All 4 photons at the same time.}} + \underbrace{\sum_t \sum_{\tau>0} \Big(\underbrace{|t,t\rangle_A |t+\tau,t+\tau\rangle_B}_{\text{source B later}} + \underbrace{|t+\tau,t+\tau\rangle_A |t,t\rangle_B}_{\text{Source A later}}\Big)}_{\text{Photons arriving at different times.}}$$

Photons arriving at different times.

How do we do a BSM?

By choosing this, post-select relevant events.

Time Measurement

One detector clicks at t, one at $t+\tau$, corresponds to $|\psi^-\rangle$ measurement at BS.

Remaining signal photons, A1 and B4, projected into:

$$|\psi\rangle \propto |t\rangle_{A1}|t+\tau\rangle_{B4} - |t+\tau\rangle_{A1}|t\rangle_{B4}.$$

This is now a time-bin entangled state.

This state can be analyzed once again by interferometers.

Here, interferometers erase the time information (don't know if it's early state going long, or late going through early).

Erasing time information means the previously temporally distinct events can now interfere.