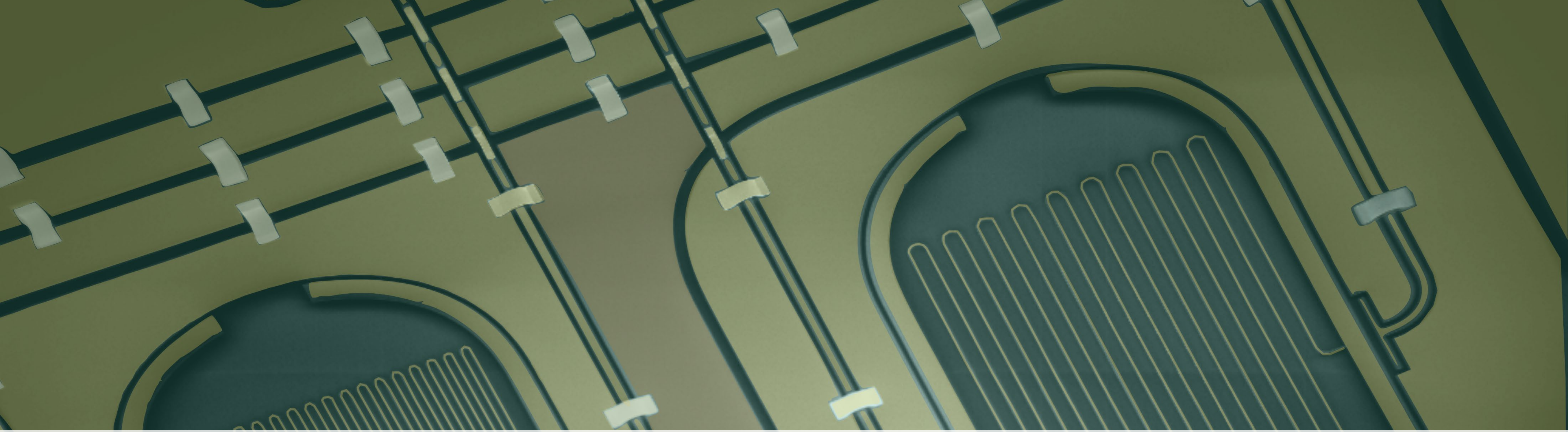
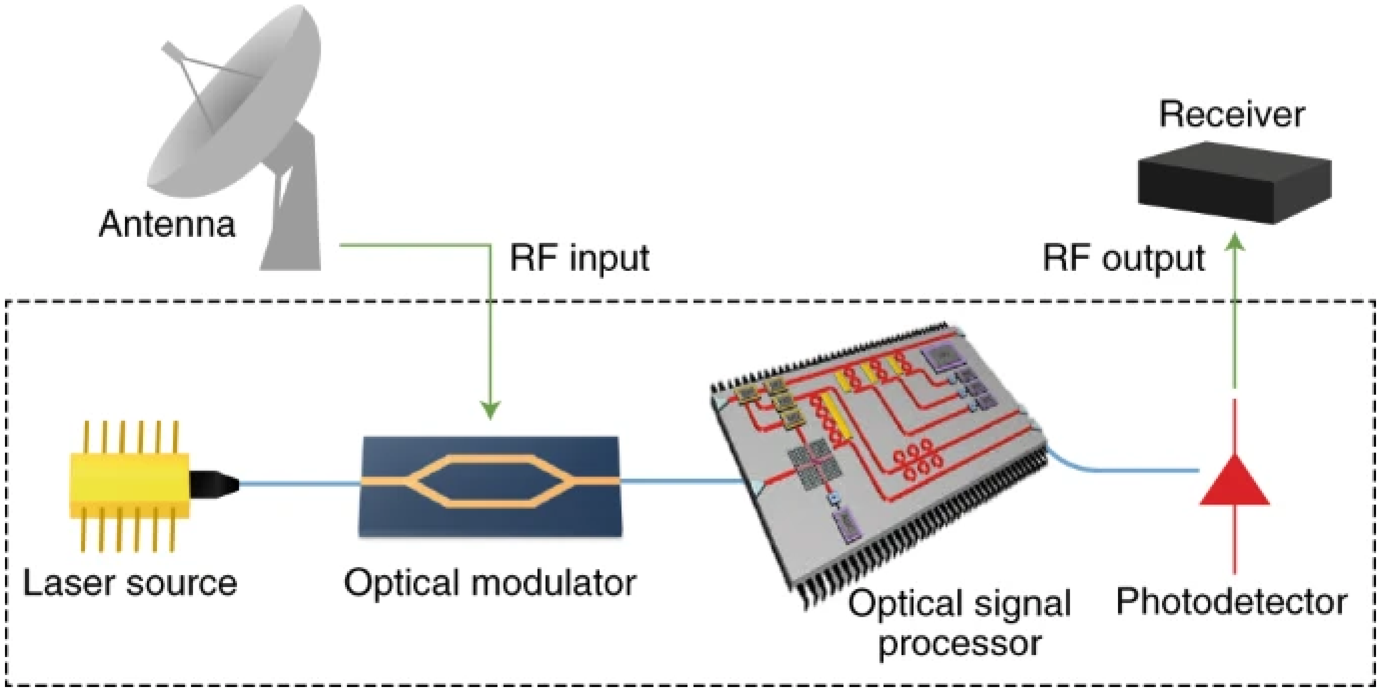


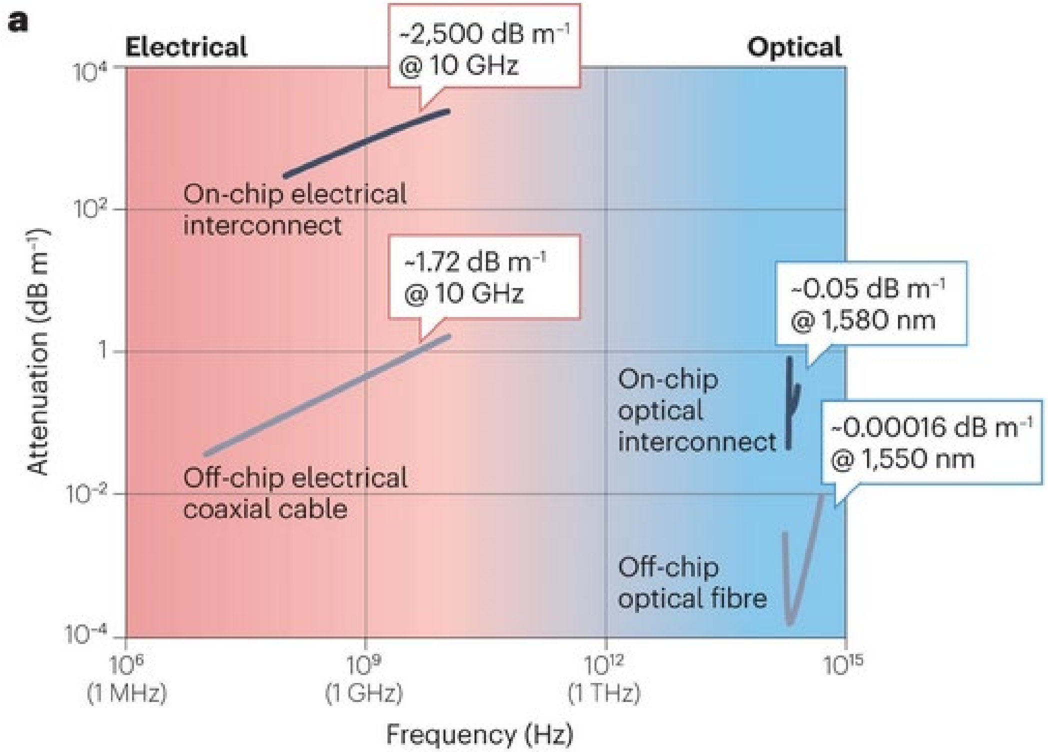
Nonlinear optics for quantum technologies: Microwave-optical frequency conversion

EPFL

Terence Blésin, Wil Kao, Christophe Galland

RF-optical link: Why?





- Low propagation loss → transmission over long distance
- Low thermal noise → low error rate in communication/computing/sensing
- Modulation/demodulation of signals → processing
- Large bandwidth → faster signals
- Multiplexing → SWaP

[1] D. Marpaung, J. Yao, and J. Capmany, Integrated microwave photonics, Nature Photonics **13**, 2 (2019).

[2] P. L. McMahon, The physics of optical computing, Nat Rev Phys 1 (2023).

RF-optical link: How?

EPFL

Typical fiber-coupled phase modulator

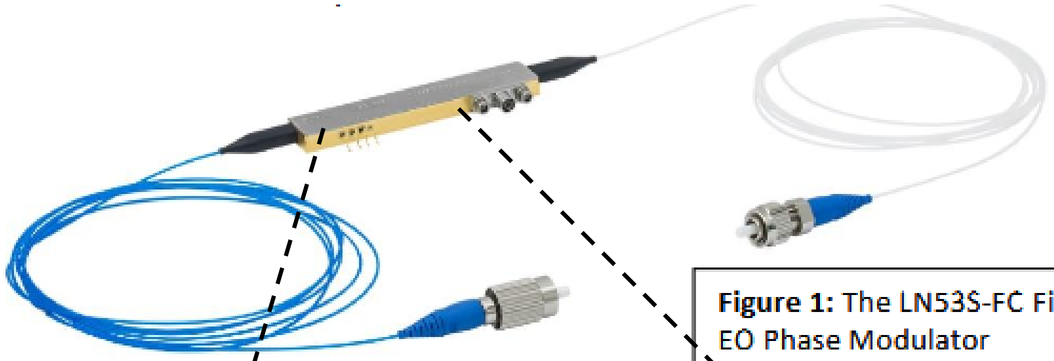
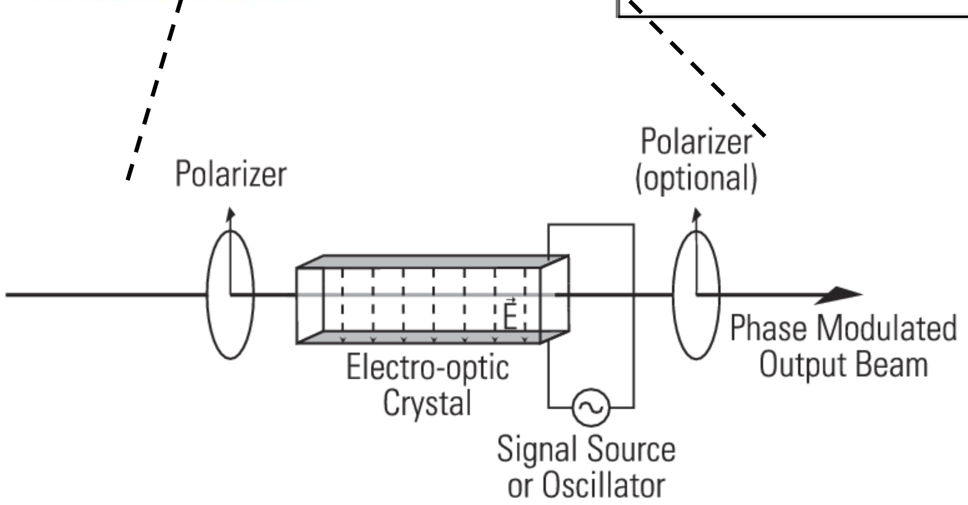


Figure 1: The LN53S-FC Fiber-Coupled EO Phase Modulator



Phase modulation, sideband generation and Bessel functions

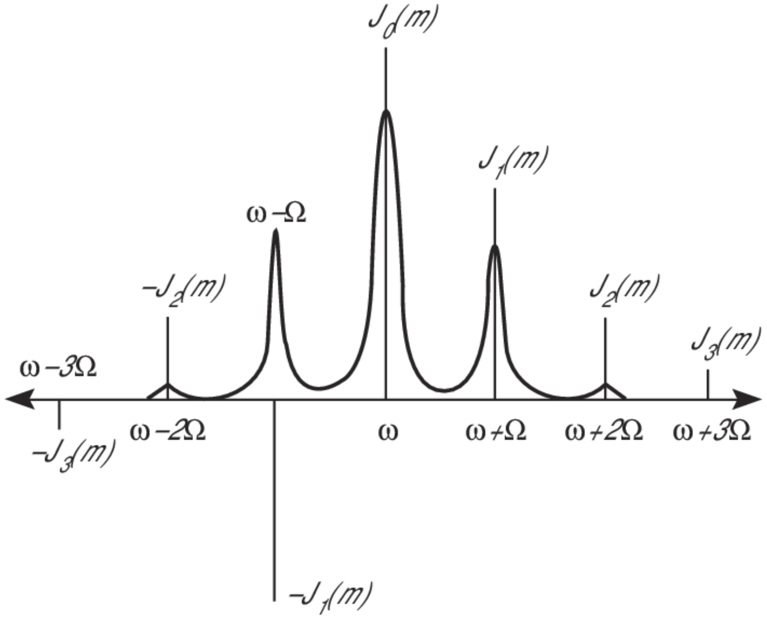
$$E_m(t) = E_o \cos\left(2\pi f_o t + \phi_o \sin(2\pi f_m t)\right),$$

$$E_m(t) = Re\{E_o exp(i2\pi f_o t) exp(i\phi_o sin 2\pi f_m t)\}$$

$$E_m(t) = E_o \sum_{n=-\infty}^{\infty} J_n(\phi_o) cos[(2\pi f_o + n2\pi f_m)t]$$

Modulation depth

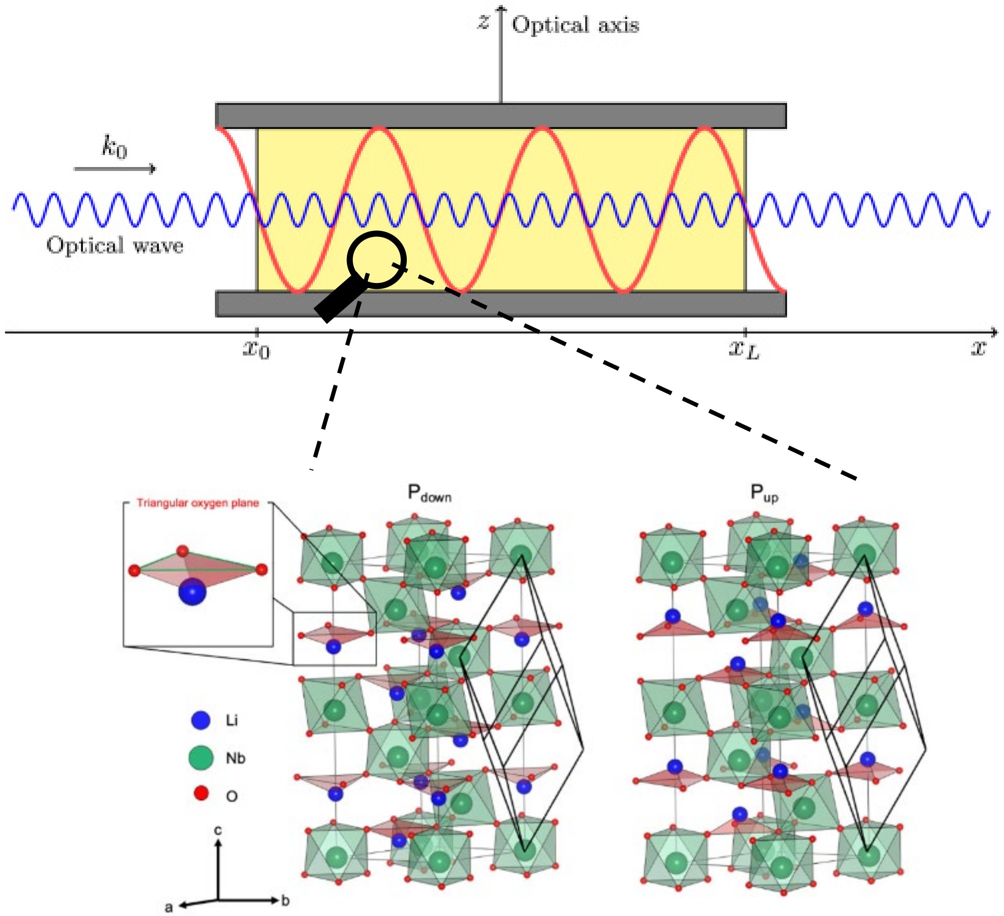
$$\frac{\phi_o}{\pi} = \frac{V_{pp}}{2V_\pi}$$



https://www.thorlabs.com/images/TabImages/EO_Phase_Modulator_RF_Source_Option.pdf

https://www.newport.com/f/standard-phase-modulators

RF-optical link: How?



Typical electro-optic material used in phase modulators: lithium niobate $LiNbO_3$

Broken inversion symmetry → three-wave mixing process enabled

$$P_i = \varepsilon_0 \chi_{ij} E_j + 2 d_{ijk} E_j E_k + 4 \chi_{ijkl} E_j E_k E_l + \cdots ,$$

Link between the second order susceptibility coefficient and the Pockels coefficients:

$$d_{ijk} = -\frac{\varepsilon_{ii}\varepsilon_{jj}}{4\varepsilon_0} r_{ijk},$$

Modification of the optical indicatrix through Pockels effect
→ electric field induced optical phase retardation

$$\eta_{ij}(\varepsilon) = \eta_{ij}(0) + \sum_k r_{ijk} E_k,$$

[1] A. Yariv, *Quantum Electronics*, 2nd Edition (Wiley, New York, NY, 1975).

[2] D. B. Horoshko, M. M. Eskandary, and S. Y. Kilin, Quantum model for traveling-wave electro-optical phase modulator, J. Opt. Soc. Am. B, JOSAB **35**, 2744 (2018).

[3] I. Kim and A. A. Demkov, Linear electro-optic effect in trigonal $\mathrm{LiNb}{\mathrm{O}}_{3}$: A first-principles study, Phys. Rev. Mater. **8**, 025202 (2024).

RF-optical link: How?

Optical impermeability change:

$$\Delta\left|\frac{1}{n_i^2}\right| = \sum_{j=1}^{3} r_{ij} E_j$$

Optical refractive index change:

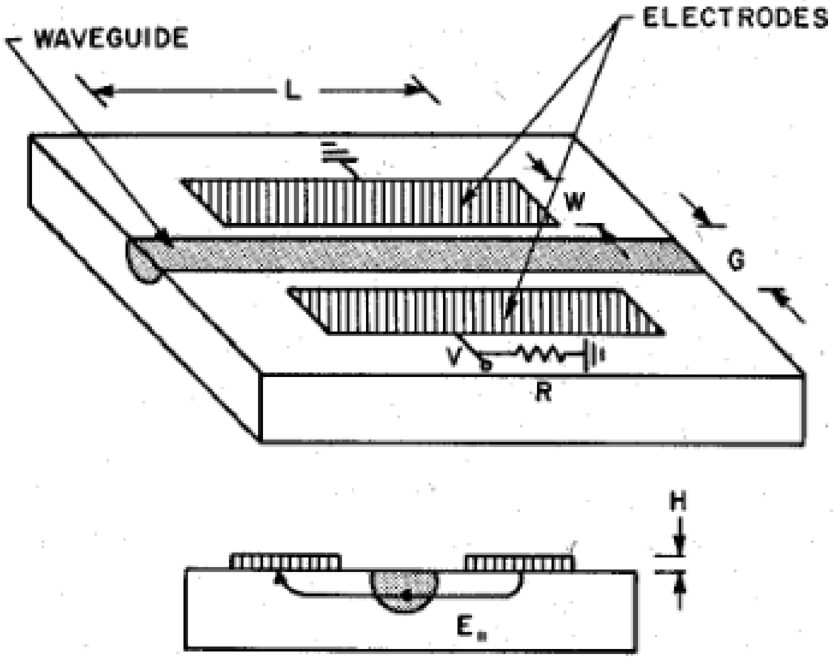
$$(\Delta n)_i = -\frac{n^2}{2} \sum_{j=1}^{3} r_{ij} E_j$$

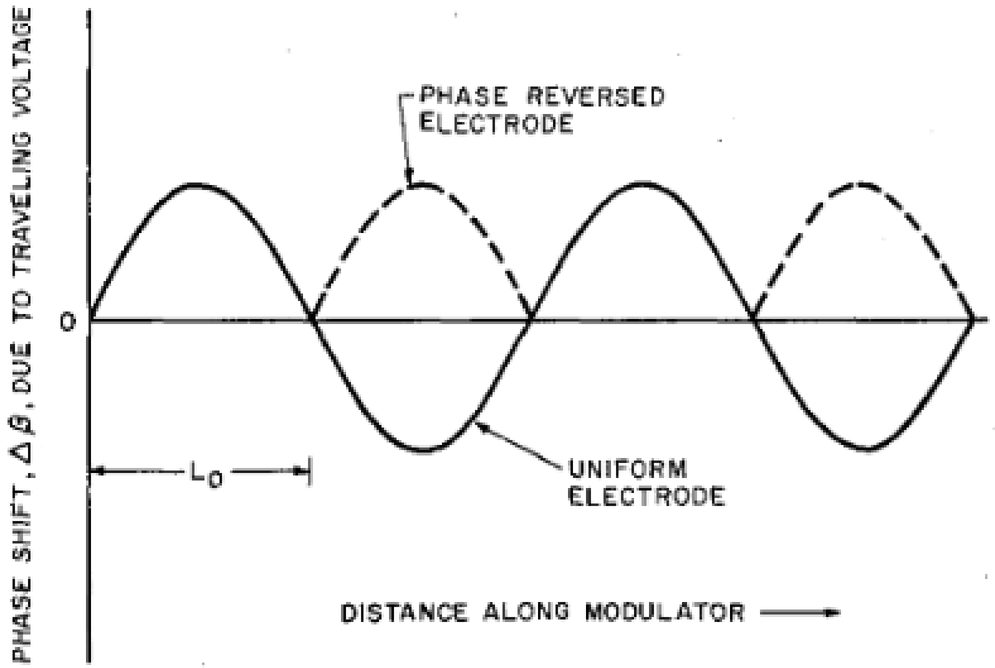
Average index change:

$$\overline{\Delta n}(V) = \frac{-n^3 r}{2} \frac{V}{G} \Gamma$$

Influence of the mode volume

$$\Gamma = \frac{G}{V} \iint E |E'|^2 \, dA$$



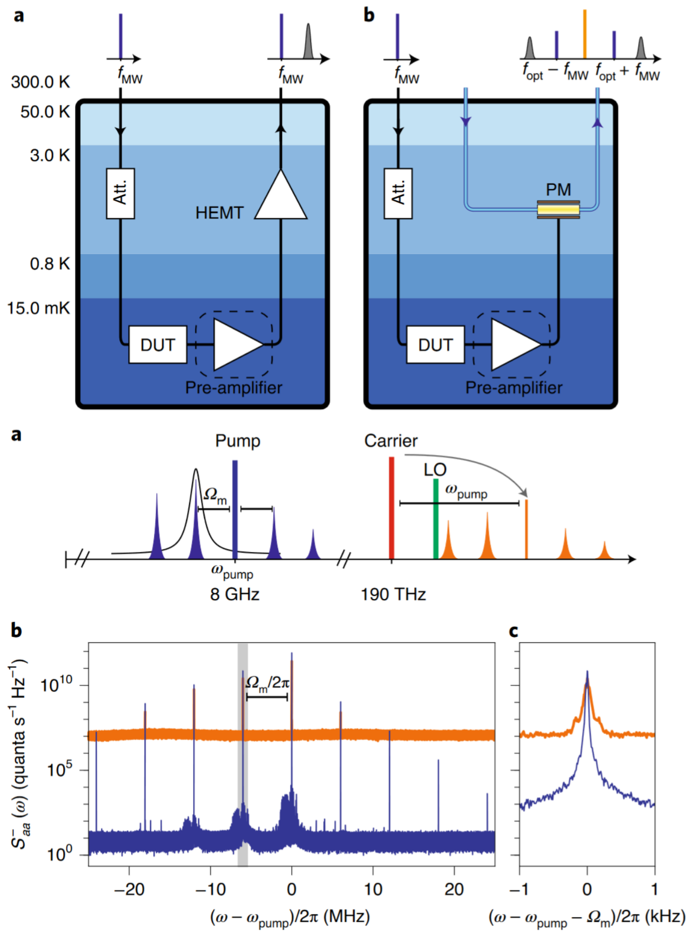


[1] R. C. Alferness, Waveguide Electrooptic Modulators, IEEE Transactions on Microwave Theory and Techniques **30**, 1121 (1982).

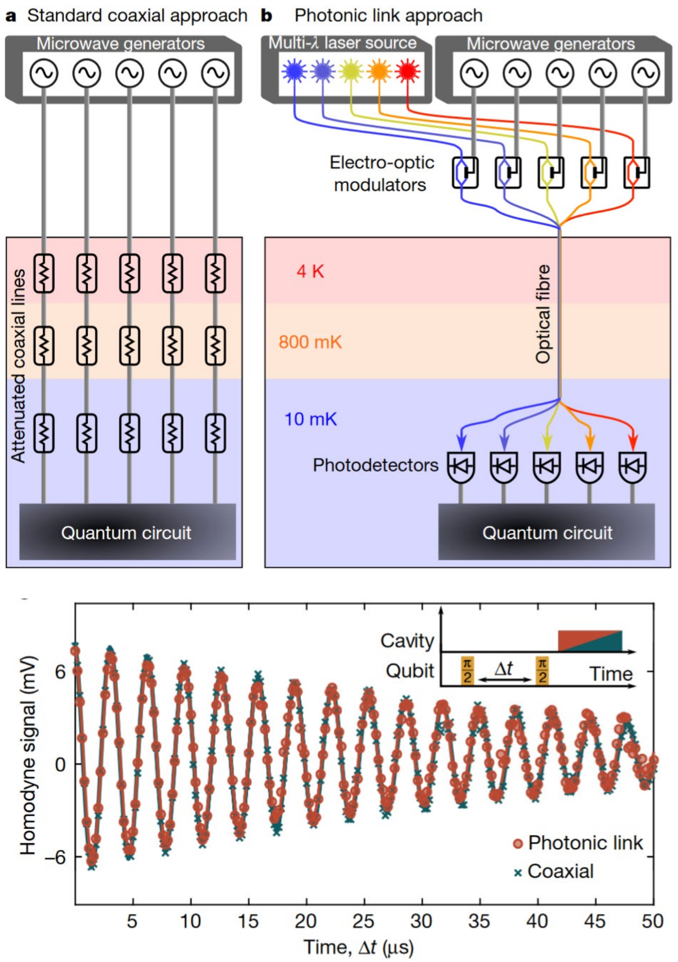
Optically driven microwave qubit operations

EPFL

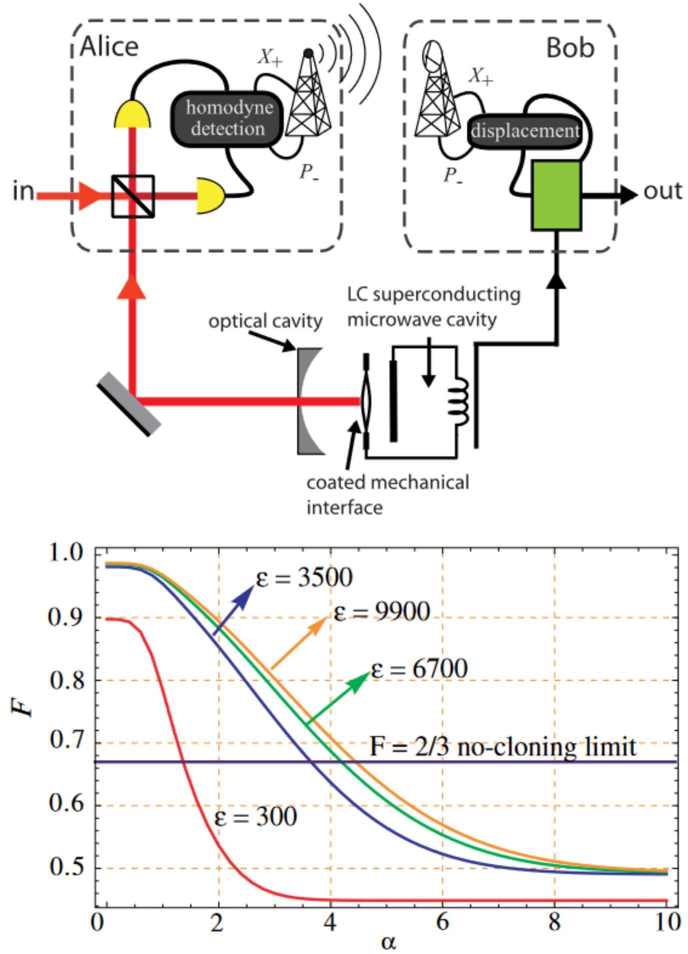
Optical readout



Optical control



Remote state transfer

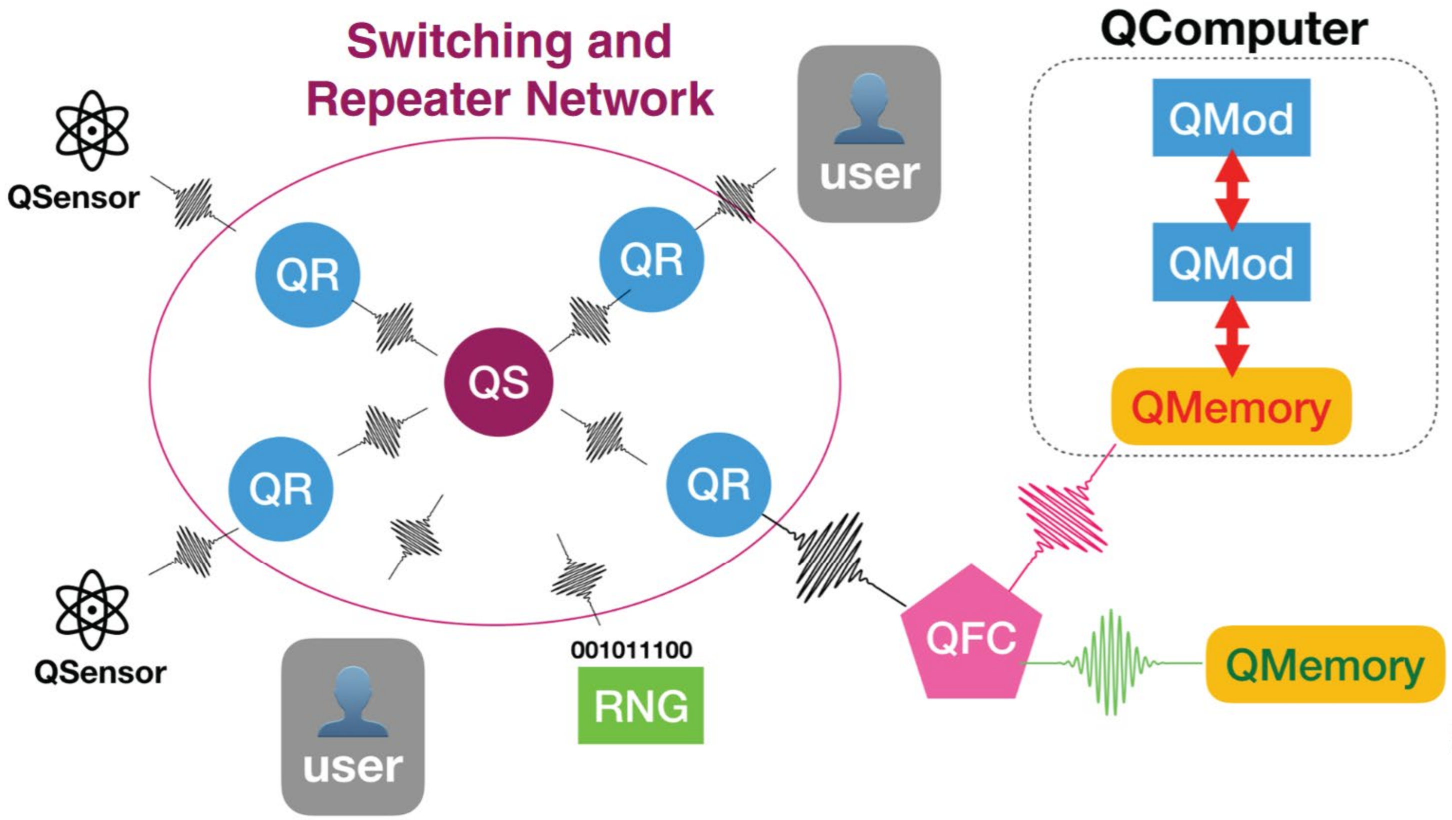


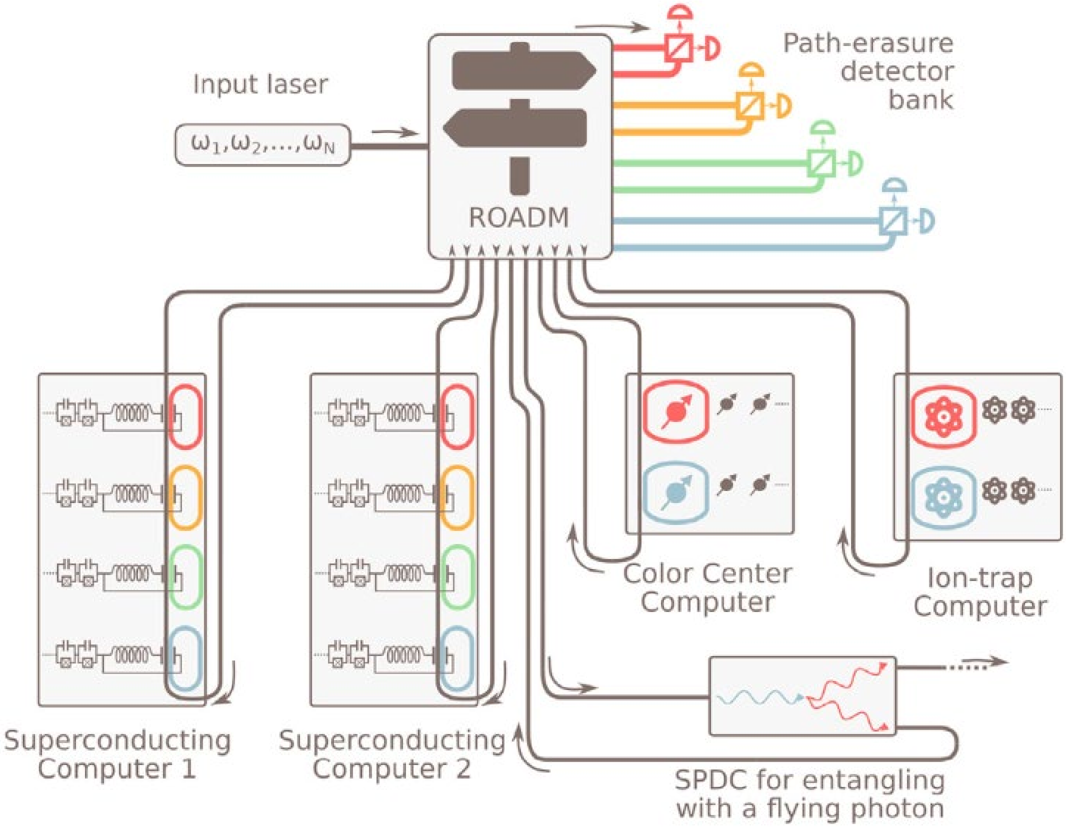
[1] A. Youssefi, I. Shomroni, Y. J. Joshi, N. R. Bernier, A. Lukashchuk, P. Uhrich, L. Qiu, and T. J. Kippenberg, *A Cryogenic Electro-Optic Interconnect for Superconducting Devices*, Nat Electron **4**, 326 (2021).

[2] F. Lecocq, F. Quinlan, K. Cicak, J. Aumentado, S. A. Diddams, and J. D. Teufel, *Control and Readout of a Superconducting Qubit Using a Photonic Link*, Nature **591**, 575 (2021).

[3] Sh. Barzanjeh, M. Abdi, G. J. Milburn, P. Tombesi, and D. Vitali, *Reversible Optical-to-Microwave Quantum Interface*, Phys. Rev. Lett. **109**, 130503 (2012).

RF-optical link: Quantum networks





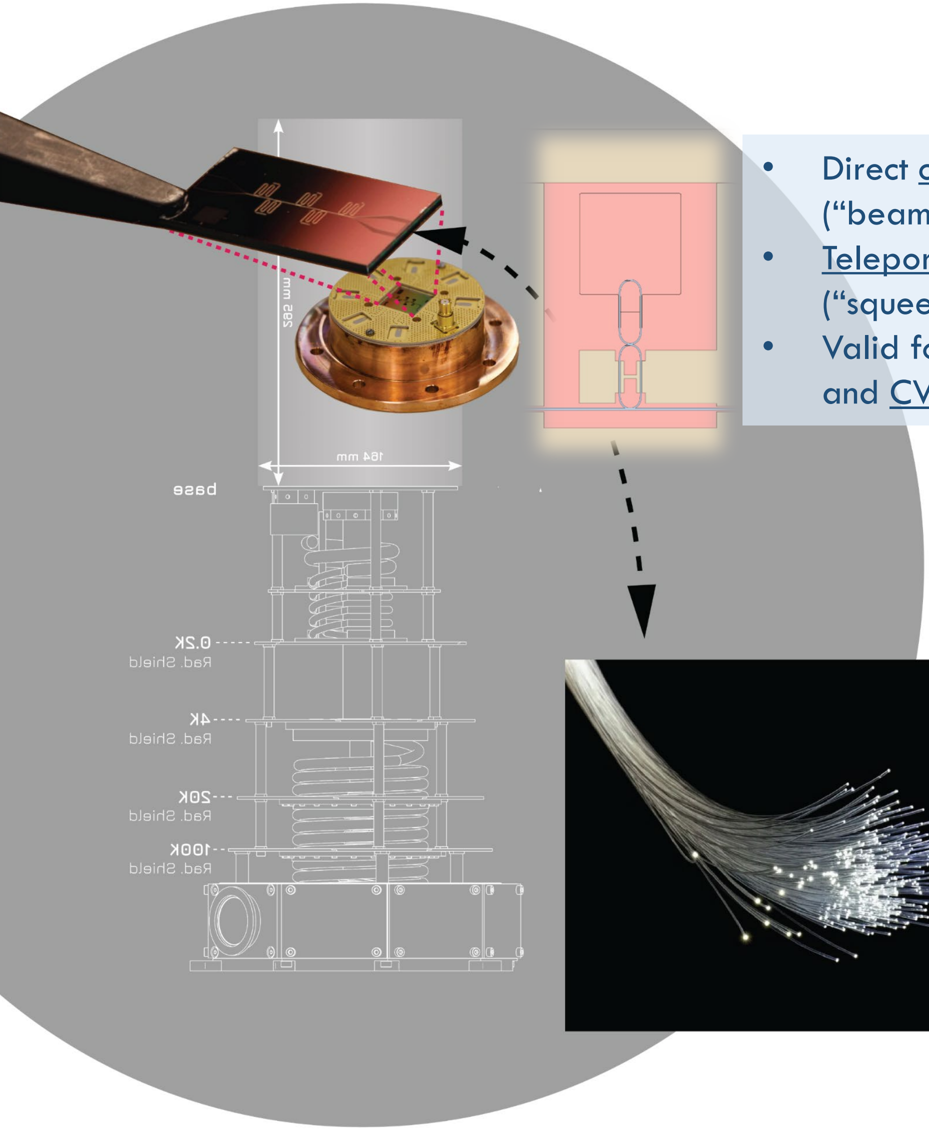
[1] H. J. Kimble, The quantum internet, Nature **453**, 7198 (2008).

[2] D. Awschalom et al., *Development of Quantum Interconnects (QuICs) for Next-Generation Information Technologies*, PRX Quantum **2**, 017002 (2021).

[3] S. Krastanov, H. Raniwala, J. Holzgrafe, K. Jacobs, M. Lončar, M. J. Reagor, and D. R. Englund, Optically Heralded Entanglement of Superconducting Systems in Quantum Networks, Phys. Rev. Lett. **127**, 040503 (2021).

[4] X. Xu et al., Optomechanical Microwave-to-Optical Photon Transducer Chips: Empowering the Quantum Internet Revolution, Micromachines **15**, 4 (2024).

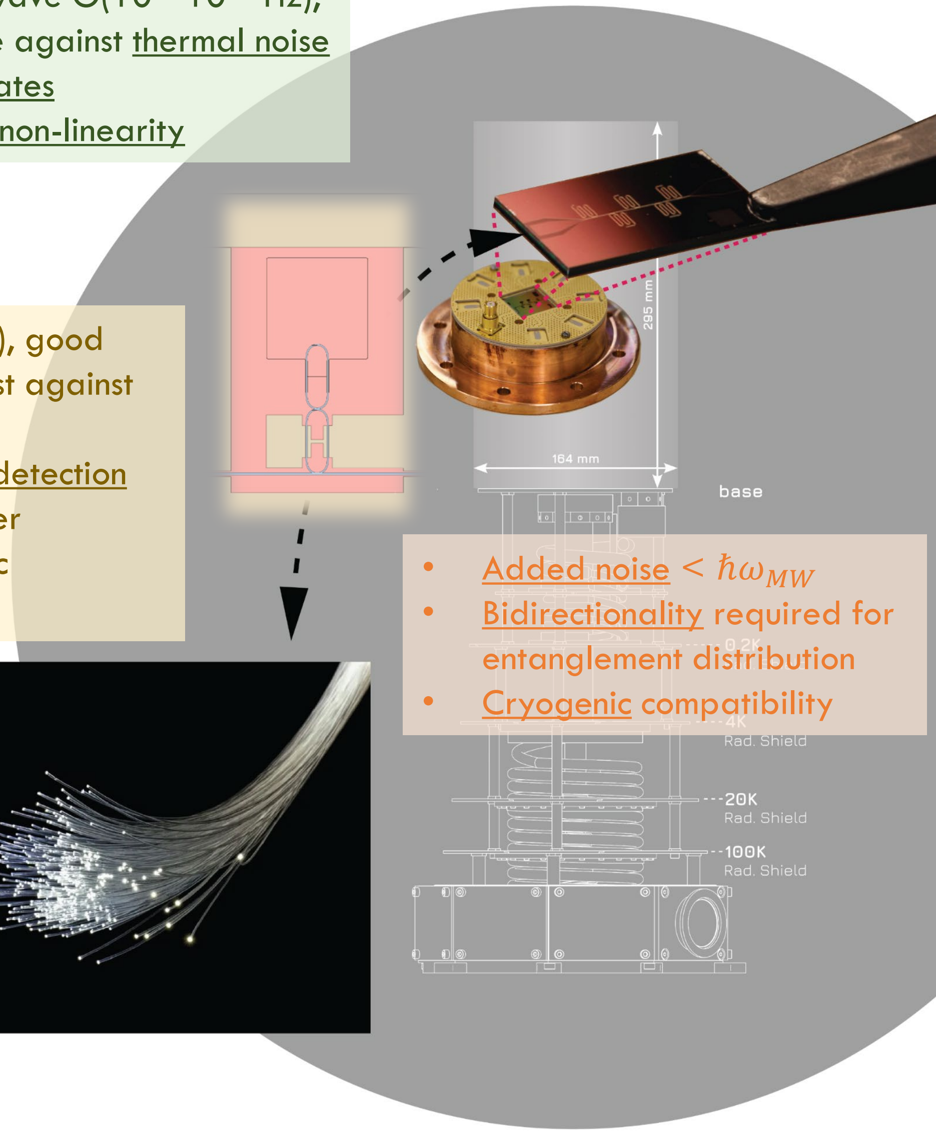
Coherent interconnect for quantum networking



- Direct conversion ("beam splitter")
- Teleportation ("squeezer")
- Valid for both discrete and CV encoding

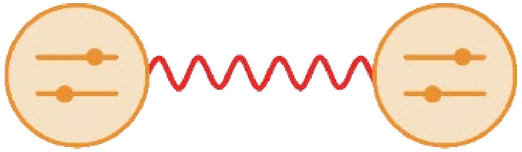
- Microwave O(10^9–10^{10} Hz), fragile against thermal noise
- Fast gates
- Large non-linearity

- Optics O(10^{14} Hz), good flying qubit, robust against thermal noise
- Quantum-limited detection and photon counter
- Mature fiber-optic infrastructure

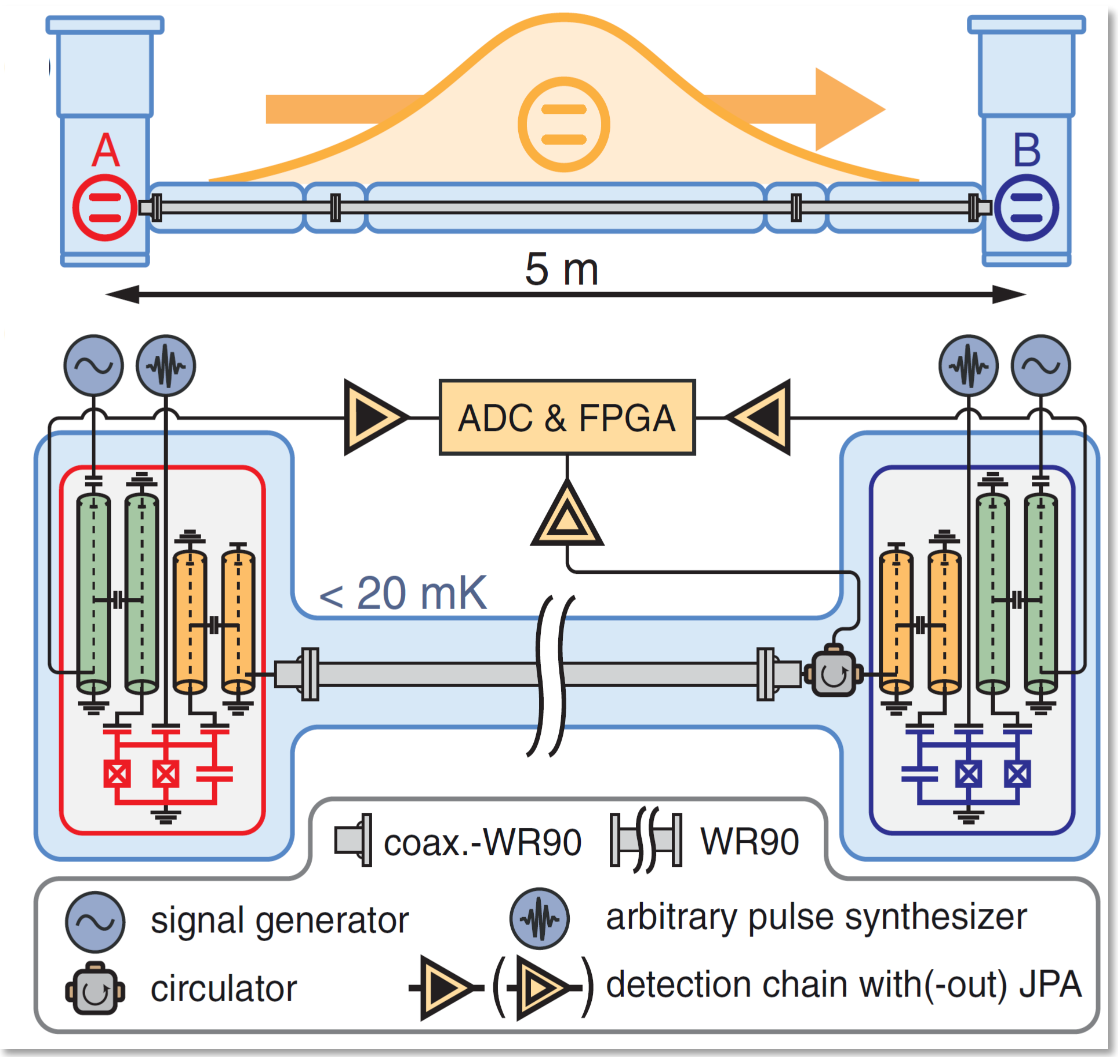


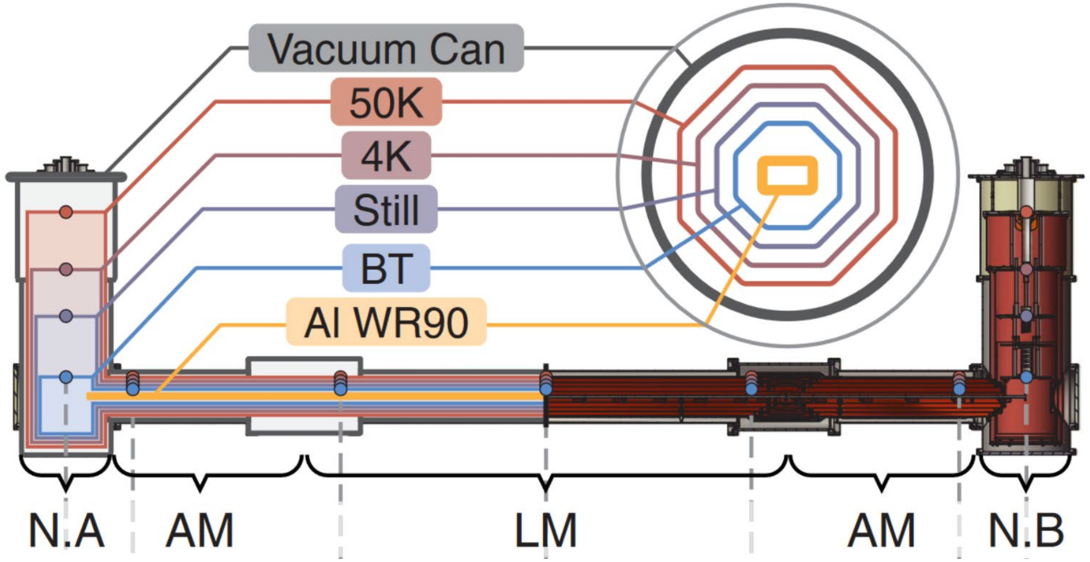
- Added noise $< \hbar\omega_{MW}$
- Bidirectionality required for entanglement distribution
- Cryogenic compatibility

Quantum repeater/memory node(s)



Quantum network without transducers?

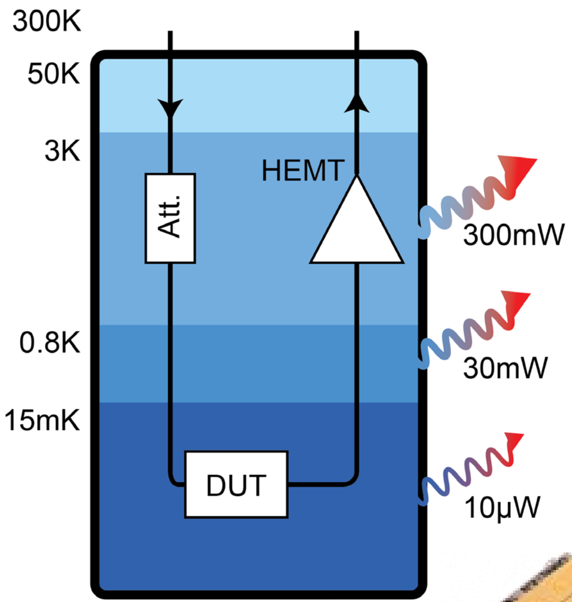




[1] P. Magnard *et al.*, "Microwave Quantum Link between Superconducting Circuits Housed in Spatially Separated Cryogenic Systems," *Phys. Rev. Lett.*, vol. 125, no. 26, p. 260502 (2020)

Cryogenic (classical) interconnects for microwave qubits

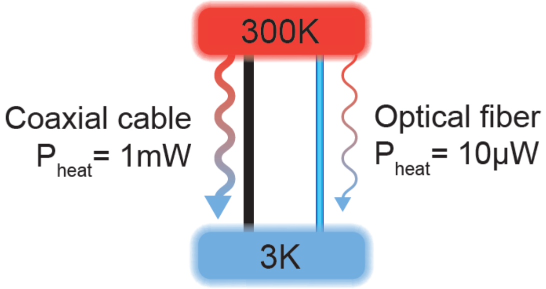
Courtesy of IBM

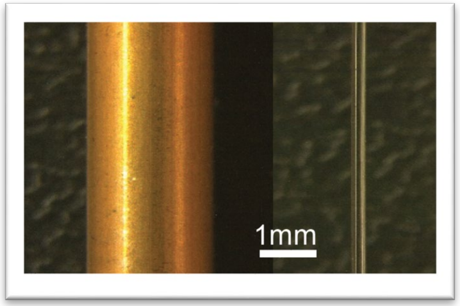


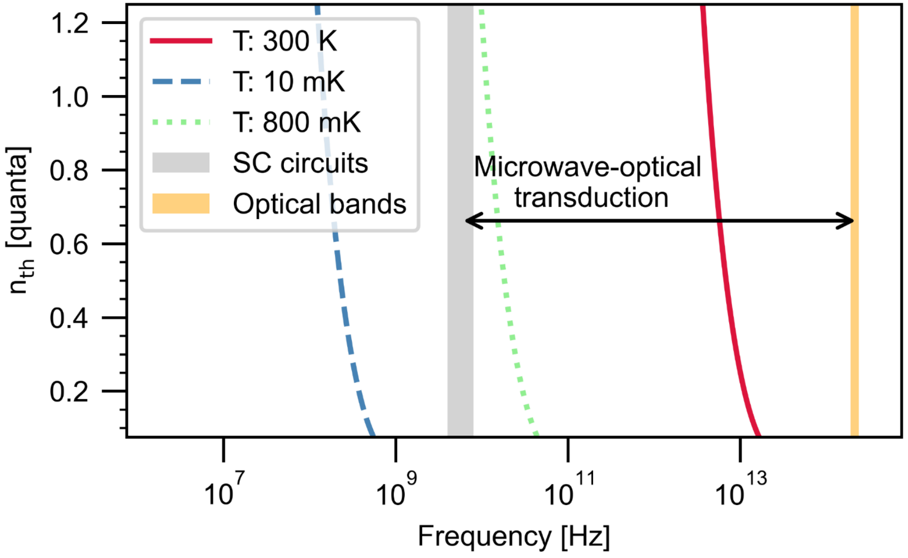
- High electron mobility transistors
- Typical added noise ~ 10 quanta/s.Hz
- Bandwidth ~ 4-8 GHz
- Working at 3K
- Typically, 10mW heating
- Each coaxial cable adds 1mW heat load

Issues with Coaxial cables

- High thermal conductance
- High loss
- Space limitation







[1] A. Youssefi, I. Shomroni, Y. J. Joshi, N. R. Bernier, A. Lukashchuk, P. Uhrich, L. Qiu, and T. J. Kippenberg, *A Cryogenic Electro-Optic Interconnect for Superconducting Devices*, Nat Electron **4**, 326 (2021).

Added noise in a transducer

Reminder on squeezing and the uncertainty limit:

Uncertainty relation: $R = R_1 + iR_2$

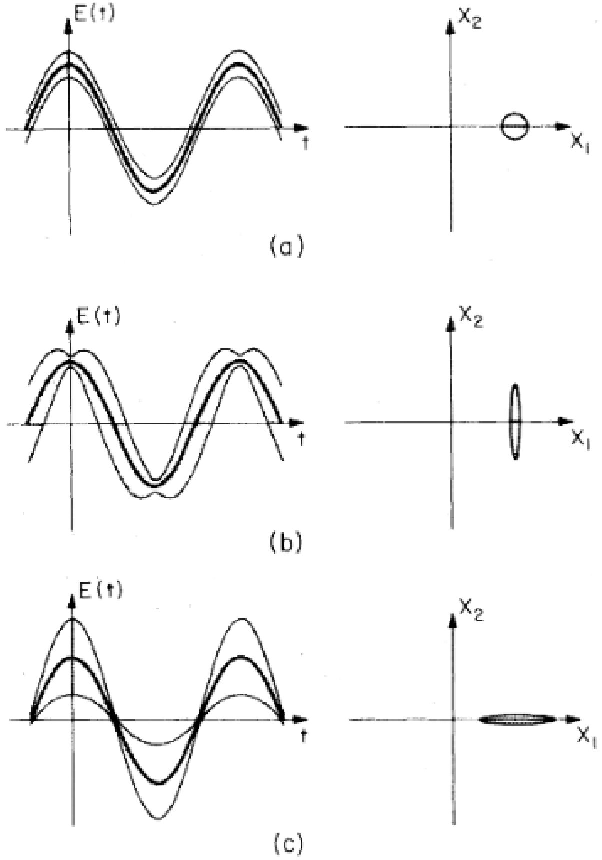
$$|\Delta R|^2 \geq \tfrac{1}{2}|\langle [R, R^\dagger]\rangle|$$

Conventional squeezing example:

$a = X_1 + iX_2 \qquad [X_1, X_2] = i/2$

$$\Delta X_1 \Delta X_2 \geq \tfrac{1}{4}$$

$$|\Delta a|^2 = (\Delta X_1)^2 + (\Delta X_2)^2 \geq \tfrac{1}{2}$$



Microwave → Amplifier/Transducer → Light

Amplifier/transducer input/output:

$$a_I = X_1 + iX_2$$
$$b_O = Y_1 + iY_2$$
$$b_O = Ma_I + La_I^\dagger + \mathscr{F}$$

Requirements to satisfy bosonic commutation relations:

$$1 = |M|^2 - |L|^2 + [\mathscr{F}, \mathscr{F}^\dagger]$$
$$[\,|\Delta\mathscr{F}|_{op}^2 \geq \tfrac{1}{2}|1 - |M|^2 + |L|^2|$$

Added noise during the amplification/conversion process:

$$|\Delta b_O|^2 = G|\Delta a_I|^2 + |\Delta\mathscr{F}|_{op}^2$$
$$A \equiv |\Delta\mathscr{F}|_{op}^2/G \qquad A \geq \tfrac{1}{2}|1 \mp G^{-1}|$$
$$|\Delta b_O|^2 = G(|\Delta a_I|^2 + A) \geq \tfrac{1}{2}G + \tfrac{1}{2}|G \mp 1|$$

Uncertainty limit for amplifiers/transducers:
- phase insensitive $A \geq \tfrac{1}{2}|1 - G^{-1}|$
- phase sensitive $A_1 A_2 \geq \tfrac{1}{16}|1 - (G_1 G_2)^{-1/2}|^2$

[1] C. M. Caves, Quantum limits on noise in linear amplifiers, Phys. Rev. D **26**, 1817 (1982).

Input-output formalism in quantum optics

Bosonic system coupled to a bath

$$H = H_{sys} + H_B + H_{int} ,$$
$$H_B = \hbar \int_{-\infty}^{\infty} d\omega\, \omega b^\dagger(\omega) b(\omega) ,$$
$$H_{int} = i\hbar \int_{-\infty}^{\infty} d\omega\, \kappa(\omega)[b^\dagger(\omega)c - c^\dagger b(\omega)] ,$$

Bosonic commutation relation for the bath operator

$$[b(\omega), b^\dagger(\omega)] = \delta(\omega - \omega')$$

Langevin equations of motion:

$$\dot{b}(\omega) = -i\omega b(\omega) + \kappa(\omega)c ,$$
$$\dot{a} = -\frac{i}{\hbar}[a, H_{sys}] + \int d\omega\, \kappa(\omega)\{b^\dagger(\omega)[a,c] - [a,c^\dagger]b(\omega)\}$$

Formal solution of the dynamics of the bath

$$b(\omega) = e^{-i\omega(t-t_0)} b_0(\omega) + \kappa(\omega) \int_{t_0}^{t} e^{-i\omega(t-t')} c(t')dt' .$$
$$\dot{a} = -\frac{i}{\hbar}[a, H_{sys}] + \int d\omega\, \kappa(\omega)\{e^{i\omega(t-t_0)} b^\dagger(\omega)[a,c] - [a,c^\dagger]e^{-i\omega(t-t_0)} b_0(\omega)\}$$
$$+ \int d\omega [\kappa(\omega)]^2 \int_{t_0}^{t} dt' \{e^{i\omega(t-t')} c^\dagger(t')[a,c] - [a,c^\dagger]e^{-i\omega(t-t')} c(t')\} .$$

Definition of the input/output bath field:

$$b_{in}(t) = \frac{1}{\sqrt{2\pi}} \int d\omega\, e^{-i\omega(t-t_0)} b_0(\omega)$$
$$[b_{in}(t), b_{in}^\dagger(t')] = \delta(t - t')$$

Definition of the output bath field:

$$b_{out}(t) = \frac{1}{\sqrt{2\pi}} \int d\omega\, e^{-i\omega(t-t')} b_1(\omega)$$

First Markov approximation:

$$\kappa(\omega) = \sqrt{\gamma/2\pi} .$$
$$\int d\omega\, b(\omega) = b_{in}(t) + \frac{\sqrt{\gamma}}{2} c(t)$$
$$\dot{a} = -\frac{1}{\hbar}[a, H_{sys}] - \left[[a,c^\dagger]\left[\frac{\gamma}{2}c + \sqrt{\gamma} b_{in}(t)\right] - \left[\frac{\gamma}{2}c^\dagger + \sqrt{\gamma} b_{in}^\dagger(t)\right][a,c]\right]$$

Standard form of the Heisenberg-Langevin equation of motion for a bosonic mode coupled to a single bath including losses

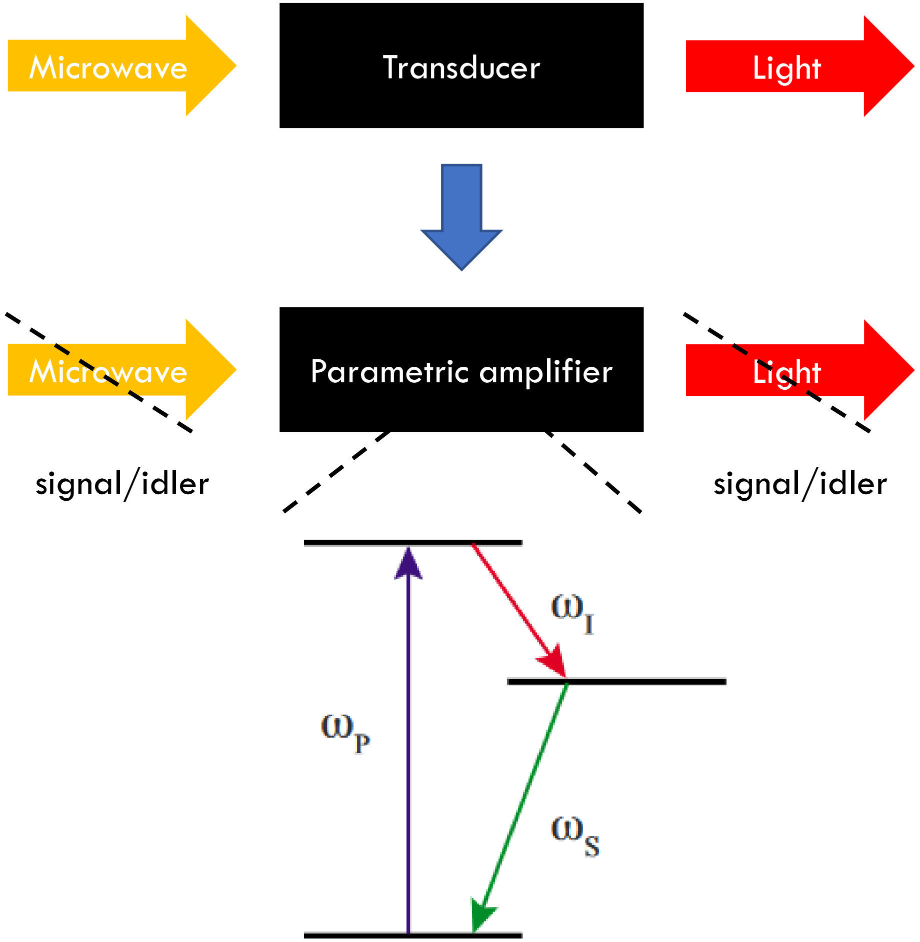
$$\dot{a} = -i\omega_0 a - \frac{\gamma}{2}a - \sqrt{\gamma} b_{in}(t)$$

Input-output relation for a damped quantum system:

$$b_{out}(t) - b_{in}(t) = \sqrt{\gamma} c(t)$$

[1] C. W. Gardiner and M. J. Collett, Input and output in damped quantum systems: Quantum stochastic differential equations and the master equation, Phys. Rev. A **31**, 3761 (1985).

Example: parametric amplifier



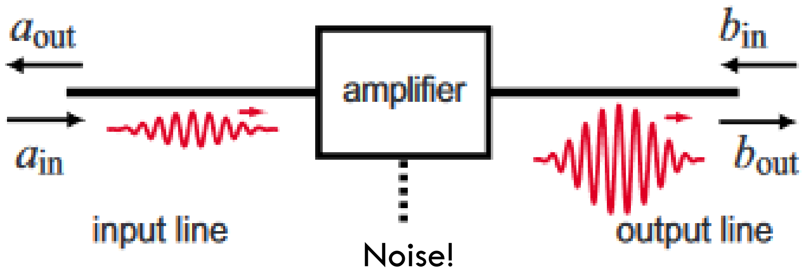
$$\hat{H}_{\text{sys}} = \hbar(\omega_P \hat{a}_P^\dagger \hat{a}_P + \omega_I \hat{a}_I^\dagger \hat{a}_I + \omega_S \hat{a}_S^\dagger \hat{a}_S) + i\hbar\eta(\hat{a}_S^\dagger \hat{a}_I^\dagger \hat{a}_P - \hat{a}_S \hat{a}_I \hat{a}_P^\dagger). \quad (5.13)$$

Amplifier output with added noise: $\hat{b} = \sqrt{G}\hat{a} + \hat{\mathcal{F}},$

Bound on the added noise: $(\Delta b)^2/G \geqslant (\Delta a)^2 + \frac{1}{2}.$

$$(\Delta b)^2 = G(\Delta a)^2 + \tfrac{1}{2}\langle\{\hat{\mathcal{F}},\hat{\mathcal{F}}^\dagger\}\rangle \geqslant G(\Delta a)^2 + \tfrac{1}{2}|\langle[\hat{\mathcal{F}},\hat{\mathcal{F}}^\dagger]\rangle|$$
$$\geqslant G(\Delta a)^2 + |G-1|/2. \quad (5.9)$$

Noise operator: $\hat{\mathcal{F}} = \sqrt{G-1}\hat{d}^\dagger$ $\quad [\hat{\mathcal{F}},\hat{\mathcal{F}}^\dagger] = 1 - G$



[1] A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, and R. J. Schoelkopf, Introduction to quantum noise, measurement, and amplification, Rev. Mod. Phys. **82**, 1155 (2010).

Example: parametric amplifier

Three-wave mixing Hamiltonian:

$$\hat{H}_{\mathrm{sys}} = \hbar(\omega_P \hat{a}_P^\dagger \hat{a}_P + \omega_I \hat{a}_I^\dagger \hat{a}_I + \omega_S \hat{a}_S^\dagger \hat{a}_S) + i\hbar\eta(\hat{a}_S^\dagger \hat{a}_I^\dagger \hat{a}_P - \hat{a}_S \hat{a}_I \hat{a}_P^\dagger). \quad (5.13)$$

Strong classical pump limit: $\hat{a}_P = \psi_P e^{-i\omega_P t} = \psi_P e^{-i(\omega_\mathrm{I}+\omega_S)t}$,

Multi-photon coupling rate: $\lambda \equiv \eta\psi_P$.

$$\hat{H}_{\mathrm{sys}} = \hbar(\omega_\mathrm{I} \hat{a}_\mathrm{I}^\dagger \hat{a}_\mathrm{I} + \omega_S \hat{a}_S^\dagger \hat{a}_S) + i\hbar\lambda(\hat{a}_S^\dagger \hat{a}_\mathrm{I}^\dagger e^{-i(\omega_\mathrm{I}+\omega_S)t} - \hat{a}_S \hat{a}_\mathrm{I} e^{+i(\omega_\mathrm{I}+\omega_S)t}),$$

Interaction in the rotating frames for the signal and idler:

$$\hat{V}_{\mathrm{sys}} = i\hbar\lambda(\hat{a}_S^\dagger \hat{a}_\mathrm{I}^\dagger - \hat{a}_S \hat{a}_\mathrm{I}).$$

Equations of motion without loss (cf exercise session):

$$\dot{\hat{a}}_S = +\lambda \hat{a}_\mathrm{I}^\dagger, \quad \dot{\hat{a}}_\mathrm{I}^\dagger = +\lambda \hat{a}_S,$$

Input-output relation: $\hat{b}_{S,\mathrm{out}} = \hat{b}_{S,\mathrm{in}} + \sqrt{\kappa_S}\hat{a}_S$.

Langevin equations with loss and external coupling:

$$\dot{\hat{a}}_S = -(\kappa_S/2)\hat{a}_S + \lambda \hat{a}_\mathrm{I}^\dagger - \sqrt{\kappa_S}\hat{b}_{S,\mathrm{in}},$$

$$\dot{\hat{a}}_\mathrm{I}^\dagger = -(\kappa_\mathrm{I}/2)\hat{a}_\mathrm{I}^\dagger + \lambda \hat{a}_S - \sqrt{\kappa_\mathrm{I}}\hat{b}_{\mathrm{I,in}}^\dagger.$$

Steady state coupled mode system:

$$\hat{a}_S = (2\lambda/\kappa_S)\hat{a}_\mathrm{I}^\dagger - (2/\sqrt{\kappa_S})\hat{b}_{S,\mathrm{in}},$$

$$\hat{a}_\mathrm{I}^\dagger = (2\lambda/\kappa_\mathrm{I})\hat{a}_S - (2/\sqrt{\kappa_\mathrm{I}})\hat{b}_{\mathrm{I,in}}^\dagger.$$

Cooperativity (square root):

$$Q \equiv 2\lambda/\sqrt{\kappa_\mathrm{I}\kappa_S}$$

$$\hat{b}_{S,\mathrm{out}} = \frac{Q^2+1}{Q^2-1}\hat{b}_{S,\mathrm{in}} + \frac{2Q}{Q^2-1}\hat{b}_{\mathrm{I,in}}^\dagger$$

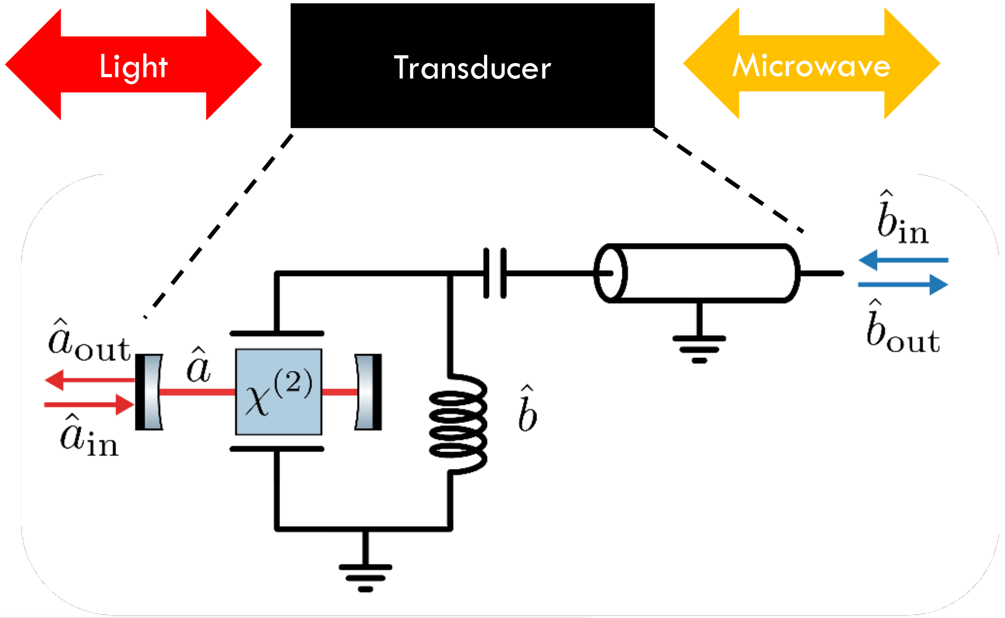
Gain/efficiency:

$$-\sqrt{G_0} = (Q^2+1)/(Q^2-1)$$

$$\hat{b}_{S,\mathrm{out}} = -\sqrt{G_0}\hat{b}_{S,\mathrm{in}} - \sqrt{G_0-1}\hat{b}_{\mathrm{I,in}}^\dagger.$$

[1] A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, and R. J. Schoelkopf, Introduction to quantum noise, measurement, and amplification, Rev. Mod. Phys. **82**, 1155 (2010).

Cavity electro-optics for microwave-optical transduction



Pockels effect: $\Delta\eta_{ij} = \sum_k r_{ijk} E_k$

Optical impermeability: $\eta = \dfrac{1}{n^2}$

Optical indicatrix: $\displaystyle\sum_{i,j=1}^{3} \eta_{ij} = 1$

Electro-optic interaction:

$$\hat{H}_{\mathrm{EO}} = -\hbar \underbrace{\frac{1}{\tau_{\mathrm{round-trip}}}\hat{\phi}}_{\Delta\omega_{\mathrm{opt}}} \hat{a}^\dagger \hat{a}$$

Phase shift:

$$\hat{\phi} = \frac{\omega_{\mathrm{opt}} n^3 r l}{cd} \hat{V}$$

Voltage operator:

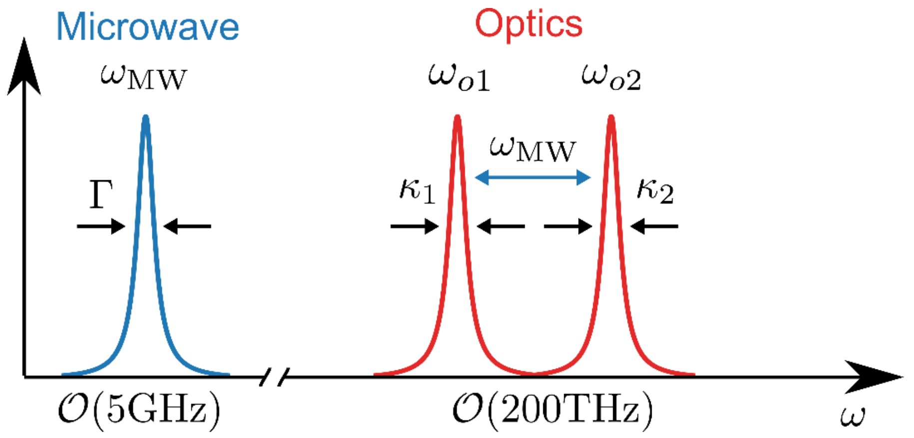
$$\hat{V} = \underbrace{\sqrt{\frac{\hbar\omega_{\mathrm{MW}}}{2C}}}_{V_{\mathrm{ZPF}}} (\hat{b} + \hat{b}^\dagger)$$

Cavity perturbation: $\dfrac{\partial\omega_{\mathrm{opt}}}{\partial V} = \dfrac{-\omega}{2\epsilon_0} \dfrac{\iiint \langle D|\frac{\partial\eta}{\partial V}|D\rangle d\mathcal{V}}{\iiint \langle E|\epsilon|E\rangle d\mathcal{V}}$

$$g_0^{\mathrm{EO}} = V_{\mathrm{ZPF}} \frac{\partial\omega_{\mathrm{opt}}}{\partial V} = \sqrt{\frac{\hbar\omega_m}{2C}} \frac{\omega_o}{2\epsilon_0} \frac{\partial}{\partial V} \frac{\iiint \sum_{ijk} D_i^o r_{ijk} E_k^m D_j^o d\mathcal{V}}{\iiint \sum_{ij} E_i^o \epsilon_{ij} E_j^o d\mathcal{V}}$$

"How large is the frequency shift induced on the optical cavity by a single microwave photon?"

$$\hat{H}_{\mathrm{EO}} = -\hbar g_0^{\mathrm{EO}} \hat{a}^\dagger \hat{a} \left(\hat{b} + \hat{b}^\dagger\right)$$



[1] M. Tsang, *Cavity Quantum Electro-Optics*, Phys. Rev. A **81**, 063837 (2010).

[2] H. A. Haus and W. Huang, *Coupled-Mode Theory*, Proceedings of the IEEE **79**, 1505 (1991).

Electro-optic interaction

Electro-optic interaction

$$\hat{H}_{\mathrm{EO}} = -\hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger)$$

Linearization

$$\hat{a} \approx \bar{a} + \delta\hat{a}$$

- Average amplitude $\bar{a} = \langle \hat{a} \rangle$
- Fluctuations $\langle \delta\hat{a} \rangle = 0$

$$\hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger) \approx \hbar g_0 \left[\cancel{|\bar{a}|^2 + \delta\hat{a}^\dagger \delta\hat{a}} + \bar{a}\delta\hat{a}^\dagger + \bar{a}^*\delta\hat{a}\right](\hat{b} + \hat{b}^\dagger)$$

- $\hbar g_0 |\bar{a}|^2 (\hat{b} + \hat{b}^\dagger)$ *Voltage shift*
- $\hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger) \rightarrow \hbar g_0 \left[\bar{a}\delta\hat{a}^\dagger + \bar{a}^*\delta\hat{a}\right](\hat{b} + \hat{b}^\dagger)$ *Linearized EO Hamiltonian*
- $\hbar g_0 \delta\hat{a}^\dagger \delta\hat{a}(\hat{b} + \hat{b}^\dagger)$ *Neglected* (at least factor $|\bar{a}|$)

Voltage shift

Frequency shift per volt

$$G = -\frac{\partial \omega_{\mathrm{cav}}}{\partial V} = \frac{g_0}{V_{\mathrm{ZPF}}}$$

Optical resonance shift

$$\omega_c^{\mathrm{new}} = \omega_c^{\mathrm{old}} + G\delta\bar{V}$$

Radiation pressure-induced polarization

$$\bar{Q}_{\mathrm{RP}} = \hbar G |\bar{a}|^2$$

Average voltage shift

$$\delta\bar{V} = \frac{\bar{Q}_{\mathrm{RP}}}{(L\omega_m^2)^{-1}}$$

Optomechanical interaction

Optomechanical interaction

$$\hat{H}_{\mathrm{OM}} = -\hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger)$$

Linearization

$$\hat{a} \approx \bar{a} + \delta\hat{a}$$

- Average amplitude $\bar{a} = \langle \hat{a} \rangle$
- Fluctuations $\langle \delta\hat{a} \rangle = 0$

$$\hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger) \approx \hbar g_0 \left[\cancel{|\bar{a}|^2 + \delta\hat{a}^\dagger \delta\hat{a}} + \bar{a}\delta\hat{a}^\dagger + \bar{a}^*\delta\hat{a}\right](\hat{b} + \hat{b}^\dagger)$$

- $\hbar g_0 |\bar{a}|^2 (\hat{b} + \hat{b}^\dagger)$ *Shift of the displacement origin*
- $\hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger) \to \hbar g_0 \left[\bar{a}\delta\hat{a}^\dagger + \bar{a}^*\delta\hat{a}\right](\hat{b} + \hat{b}^\dagger)$ *Linearized OM Hamiltonian*
- $\hbar g_0 \delta\hat{a}^\dagger \delta\hat{a}(\hat{b} + \hat{b}^\dagger)$ *Neglected* (at least factor $|\bar{a}|$)

Displacement shift

Frequency shift per displacement

$$G = -\frac{\partial \omega_{\mathrm{cav}}}{\partial x} = \frac{g_0}{x_{\mathrm{ZPF}}}$$

Optical resonance shift

$$\omega_c^{\mathrm{new}} = \omega_c^{\mathrm{old}} + G\delta\bar{x}$$

Average radiation pressure force

$$\bar{F}_{\mathrm{RP}} = \hbar G |\bar{a}|^2$$

Average mechanical shift

$$\delta\bar{x} = \frac{\bar{F}_{\mathrm{RP}}}{m_{\mathrm{eff}}\omega_m^2}$$

[1] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, *Cavity Optomechanics*, Rev. Mod. Phys. **86**, 1391 (2014).

Radiation pressure interaction

EPFL

Optomechanical interaction $\hat{H}_{\mathrm{OM}} = -\hbar g_0 \hat{a}^\dagger \hat{a} \left(\hat{b} + \hat{b}^\dagger\right) \Rightarrow -\hbar g \left(\delta\hat{a} + \delta\hat{a}^\dagger\right)\left(\hat{b} + \hat{b}^\dagger\right)$

Linearized optical field amplitude $\hat{a} \approx \sqrt{\bar{n}_c} + \delta\hat{a}$

Effective coupling strength $g = \sqrt{\bar{n}_c} g_0$

Direct conversion ("beam splitter")

$$\Delta = -\omega_m \longrightarrow \hat{H}_{\mathrm{OM}}^{(\mathrm{BS})} = -\hbar g \left(\delta\hat{a}^\dagger\hat{b} + \delta\hat{a}\hat{b}^\dagger\right)$$

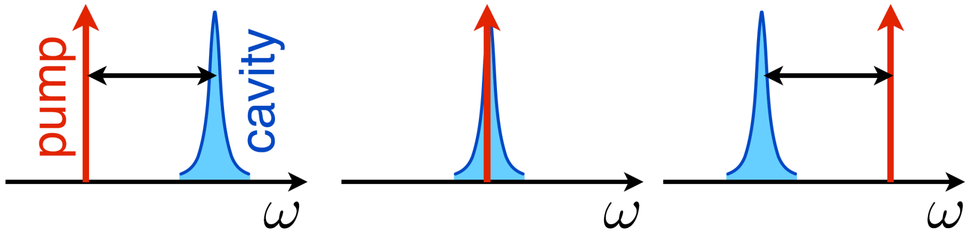
QND measurement

$$\Delta = 0 \longrightarrow \hat{H}_{\mathrm{OM}}^{(\mathrm{QND})} = -\hbar g \frac{2}{x_{\mathrm{ZPF}}} \hat{X}_{\mathrm{opt}} \hat{x}_m$$

Entanglement ("two-mode squeezing")

$$\Delta = \omega_m \longrightarrow \hat{H}_{\mathrm{OM}}^{(\mathrm{TMS})} = -\hbar g \left(\delta\hat{a}\hat{b} + \delta\hat{a}^\dagger\hat{b}^\dagger\right)$$

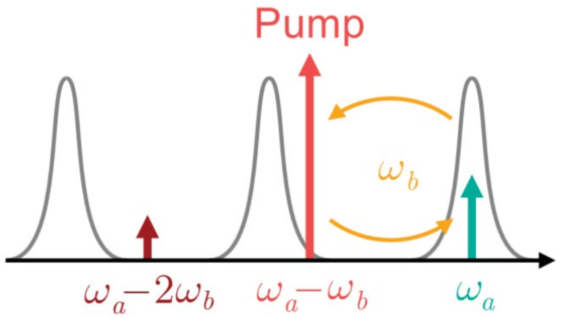
Laser-cavity detuning $\Delta = \omega_L - \omega_0^{\mathrm{opt}}$

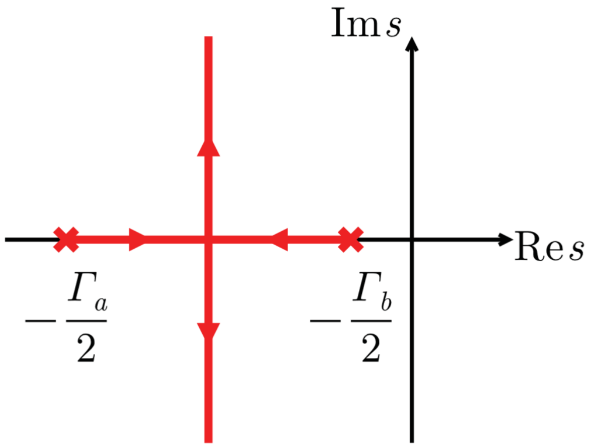


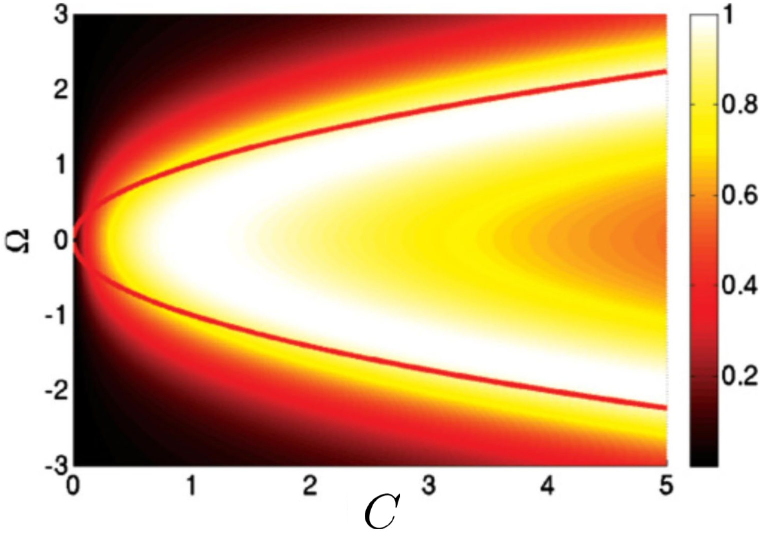
$\Delta = -\Omega_{\mathrm{m}}$	$\Delta = 0$	$\Delta = +\Omega_{\mathrm{m}}$
beam-splitter (cooling)	QND	squeezer (entanglement)
$\delta\hat{a}^\dagger\hat{b} + \hat{b}^\dagger\delta\hat{a}$	$\hat{x}_a\hat{x}_b$	$\delta\hat{a}^\dagger\hat{b}^\dagger + \hat{b}\delta\hat{a}$

[1] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, *Cavity Optomechanics*, Rev. Mod. Phys. **86**, 1391 (2014).

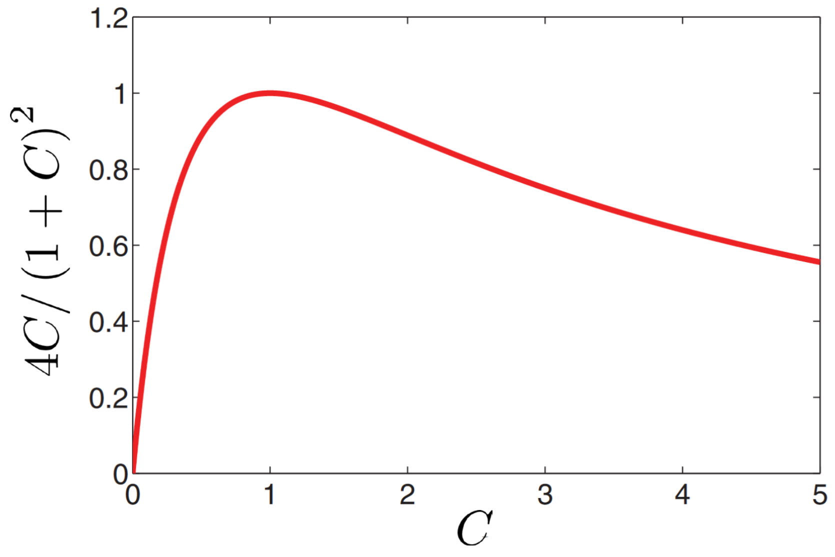
Direct conversion/"beam-splitter" (anti-Stokes process)

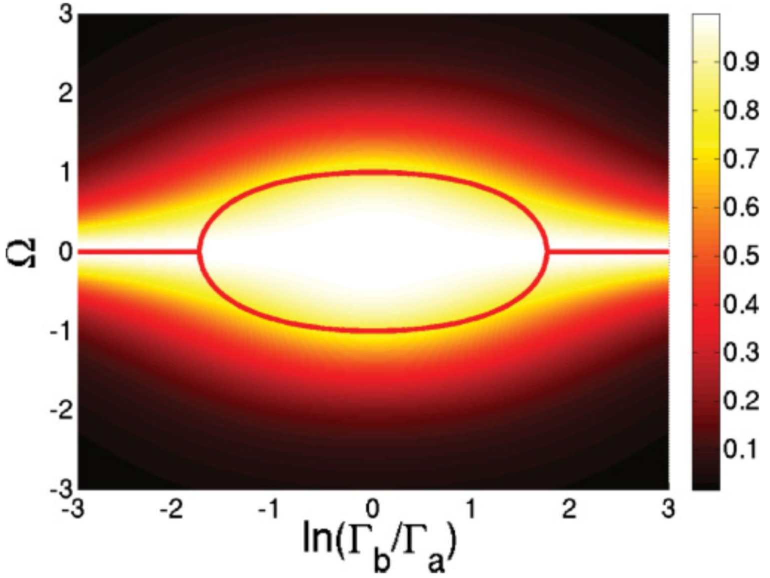






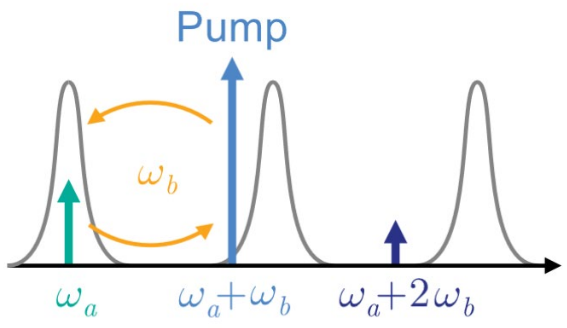
Normal mode splitting

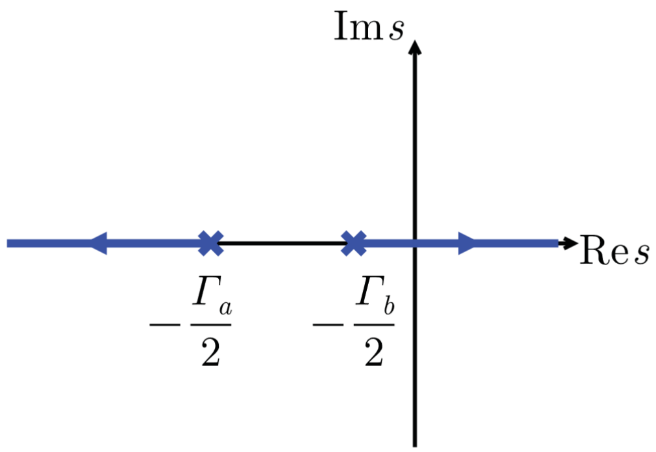


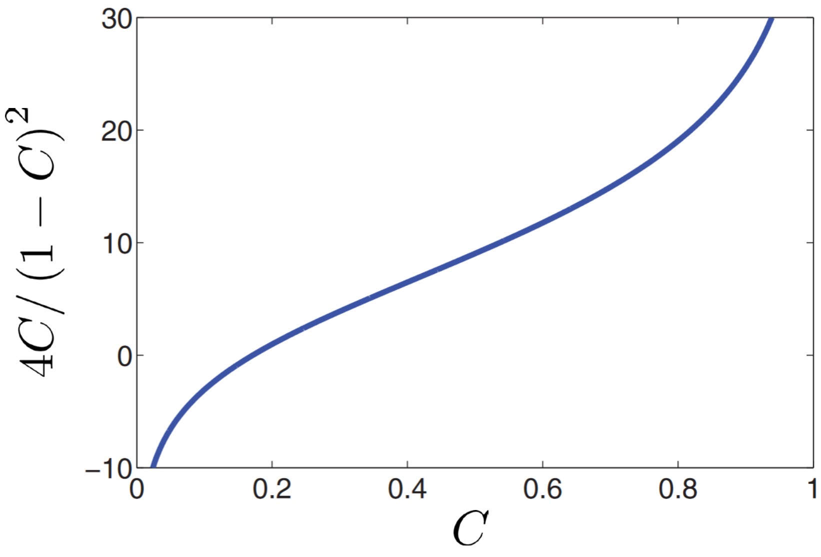


[1] M. Tsang, *Cavity Quantum Electro-Optics. II. Input-Output Relations between Traveling Optical and Microwave Fields*, Phys. Rev. A **84**, 043845 (2011).

Two-mode squeezing (Stokes process)

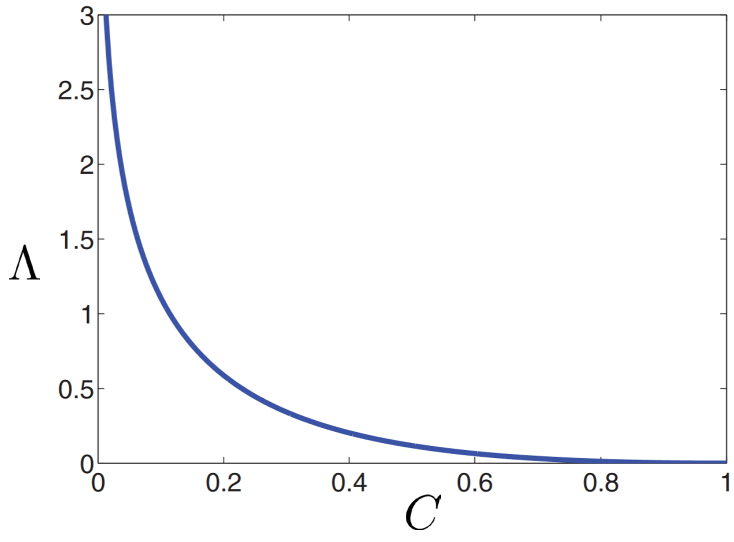






Non-classicality parameter

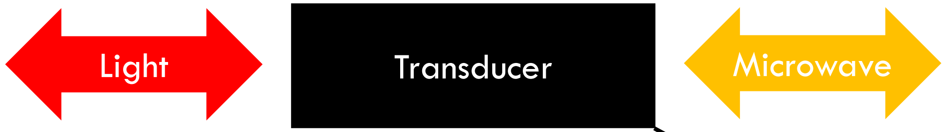
$$\Lambda = \log\left(4C/(1+C)^2\right)$$



[1] M. Tsang, *Cavity Quantum Electro-Optics. II. Input-Output Relations between Traveling Optical and Microwave Fields*, Phys. Rev. A **84**, 043845 (2011).

Transducer internal dynamics

EPFL



$$\hat{H}_{\text{EO}} \approx -\hbar g \left(\delta\hat{a} + \delta\hat{a}^\dagger\right)\left(\hat{b} + \hat{b}^\dagger\right)$$

Equations of motion:

$$\frac{d}{dt}\hat{a}(t) = \left(-i\omega_{\text{opt}} - \frac{\kappa_{\text{opt}}}{2}\right)\hat{a}(t) + ig\left(\hat{b}(t) + \hat{b}^\dagger(t)\right) + \sqrt{\kappa_{\text{opt,ex}}}\hat{a}_{\text{in}}(t)$$

$$\frac{d}{dt}\hat{b}(t) = \left(-i\omega_{\text{m}} - \frac{\kappa_{\text{m}}}{2}\right)\hat{b}(t) + ig^*\hat{a}(t) + ig\hat{a}^\dagger(t) + \sqrt{\kappa_{\text{m,ex}}}\hat{b}_{\text{in}}(t)$$

<u>Fourier transform:</u>

$$\hat{a}[\omega] = \mathcal{F}\left(\hat{a}(t)\right)[\omega] = \int_{-\infty}^{\infty} \hat{a}(t)e^{i\omega t}dt$$

$$\mathcal{F}\left(\frac{d}{dt}\hat{a}(t)\right)[\omega] = -i\omega\hat{a}[\omega]$$

$$\mathcal{F}\left(\hat{a}^\dagger(t)\right)[\omega] = \left(\hat{a}[-\omega]\right)^\dagger \equiv \hat{a}^\dagger[\omega]$$

Frequency domain:

$$\hat{a}[\omega] = \chi_{\text{opt}}[\omega]\left(ig\hat{b}[\omega] + ig\hat{b}^\dagger[\omega] + \sqrt{\kappa_{\text{opt,ex}}}\hat{a}_{\text{in}}[\omega]\right)$$

$$\hat{b}[\omega] = \chi_{\text{m}}[\omega]\left(ig^*\hat{a}[\omega] + ig\hat{a}^\dagger[\omega] + \sqrt{\kappa_{\text{m,ex}}}\hat{b}_{\text{in}}\hat{b}_{\text{in}}[\omega]\right)$$

Susceptibilities:

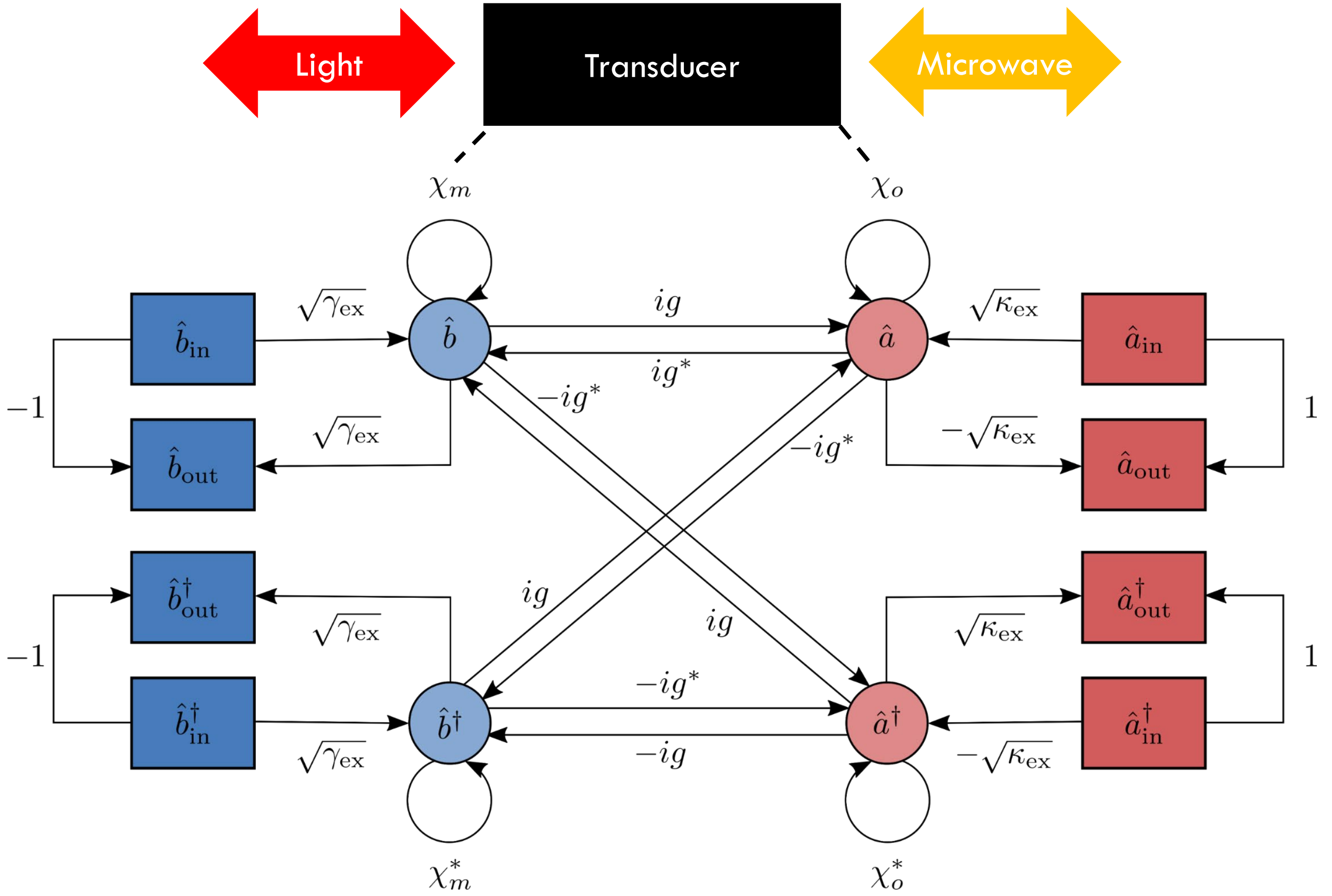
$$\chi_o[\omega] = \frac{1}{-i(\omega-\omega_o)+\frac{\kappa}{2}}$$

$$\chi_m[\omega] = \frac{1}{-i(\omega-\omega_m)+\frac{\gamma}{2}}$$

For an alternative approach using rotating frames, see the Supplementary material of

[1] T. Blésin, W. Kao, A. Siddharth, R. N. Wang, A. Attanasio, H. Tian, S. A. Bhave, and T. J. Kippenberg, Bidirectional microwave-optical transduction based on integration of high-overtone bulk acoustic resonators and photonic circuits, Nat Commun **15**, 6096 (2024).

Signal flow graph – Link to internal dynamics



Internal dynamics:

$$\hat{a}[\omega] = \chi_{\mathrm{opt}}[\omega]\left(ig\hat{b}[\omega] + ig\hat{b}^\dagger[\omega] + \sqrt{\kappa_{\mathrm{ex}}}\hat{a}_{\mathrm{in}}[\omega]\right)$$

$$\hat{b}[\omega] = \chi_{\mathrm{m}}[\omega]\left(ig^*\hat{a}[\omega] + ig\hat{a}^\dagger[\omega] + \sqrt{\gamma_{\mathrm{ex}}}\hat{b}_{\mathrm{in}}\hat{b}_{\mathrm{in}}[\omega]\right)$$

Input-output relations:

$$\hat{a}_{\mathrm{out}} = \hat{a}_{\mathrm{in}} - \sqrt{\kappa_{\mathrm{ex}}}\hat{a} \qquad \text{(in transmission)}$$

$$\hat{b}_{\mathrm{out}} = -\hat{b}_{\mathrm{in}} + \sqrt{\gamma_{\mathrm{ex}}}\hat{b} \qquad \text{(in reflection)}$$

$$\hat{a}^\dagger_{\mathrm{out}}\hat{a}_{\mathrm{out}} = \underbrace{|S_{\hat{a}_{\mathrm{out}}\hat{a}_{\mathrm{in}}}|^2 + |S_{\hat{a}_{\mathrm{out}}\hat{a}^\dagger_{\mathrm{in}}}|^2 + |S_{\hat{a}_{\mathrm{out}}\hat{b}_{\mathrm{in}}}|^2 + |S_{\hat{a}_{\mathrm{out}}\hat{b}^\dagger_{\mathrm{in}}}|^2}_{\text{Fundamental added noise}} + \underbrace{\left(|S_{\hat{a}_{\mathrm{out}}\hat{a}_{\mathrm{in}}}|^2 + |S_{\hat{a}_{\mathrm{out}}\hat{a}^\dagger_{\mathrm{in}}}|^2\right)\hat{a}^\dagger_{\mathrm{in}}\hat{a}_{\mathrm{in}}}_{\text{Transmission}} + \underbrace{\left(|S_{\hat{a}_{\mathrm{out}}\hat{b}_{\mathrm{in}}}|^2 + |S_{\hat{a}_{\mathrm{out}}\hat{b}^\dagger_{\mathrm{in}}}|^2\right)\hat{b}^\dagger_{\mathrm{in}}\hat{b}_{\mathrm{in}}}_{\text{Conversion}} + \underbrace{\ldots}_{\text{High order terms}}$$

[1] T. Blésin, H. Tian, S. A. Bhave, and T. J. Kippenberg, Quantum coherent microwave-optical transduction using high-overtone bulk acoustic resonances, Phys. Rev. A **104**, 052601 (2021).

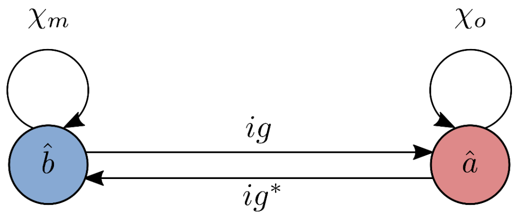
Signal flow graph – Feedback loops

$$\hat{a}[\omega] = \chi_{\mathrm{opt}}[\omega]\left(ig\hat{b}[\omega] + ig\hat{b}^\dagger[\omega] + \sqrt{\kappa_{\mathrm{opt,ex}}}\hat{a}_{\mathrm{in}}[\omega]\right)$$

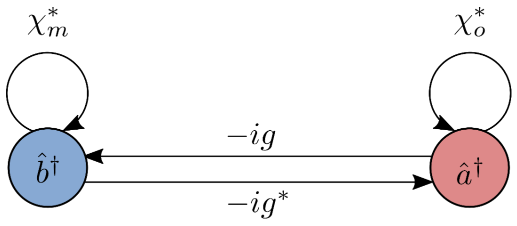
$$\hat{b}[\omega] = \chi_{\mathrm{m}}[\omega]\left(ig^*\hat{a}[\omega] + ig\hat{a}^\dagger[\omega] + \sqrt{\kappa_{\mathrm{m,ex}}}\hat{b}_{\mathrm{in}}\hat{b}_{\mathrm{in}}[\omega]\right)$$

2 nodes

Anti-Stokes process

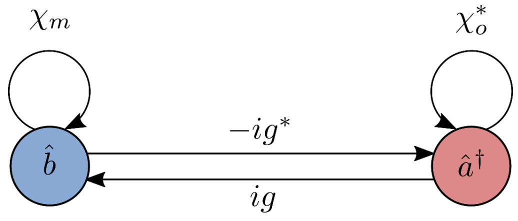


$-|g|^2\chi_o\chi_m$

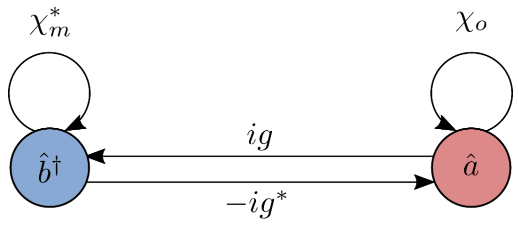


$-|g|^2\chi_o^*\chi_m^*$

Stokes process



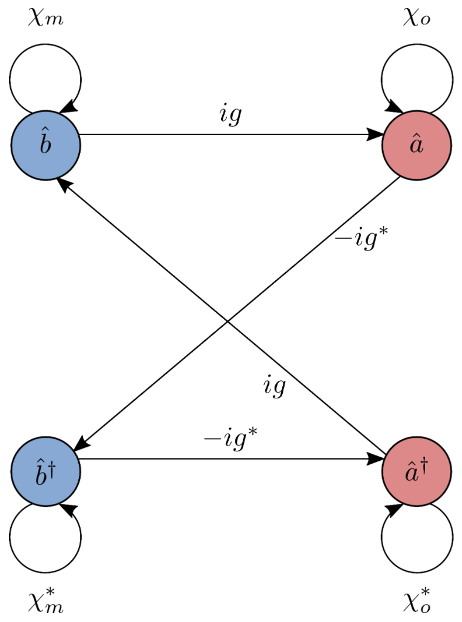
$|g|^2\chi_o^*\chi_m$



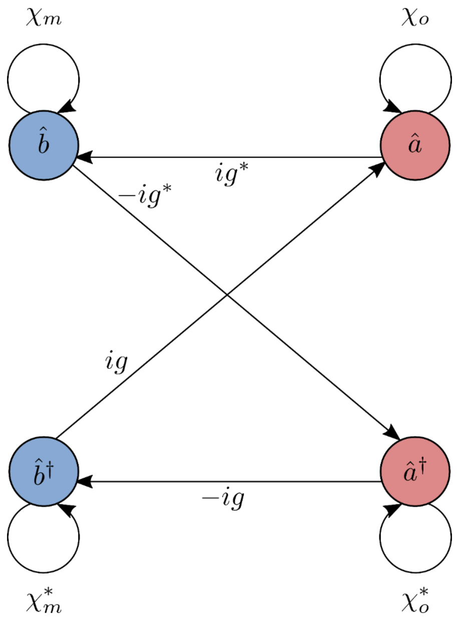
$|g|^2\chi_o\chi_m^*$

4 nodes

Cross-talk between Stokes and anti-Stokes processes



$|g|^4|\chi_m|^2|\chi_o|^2$

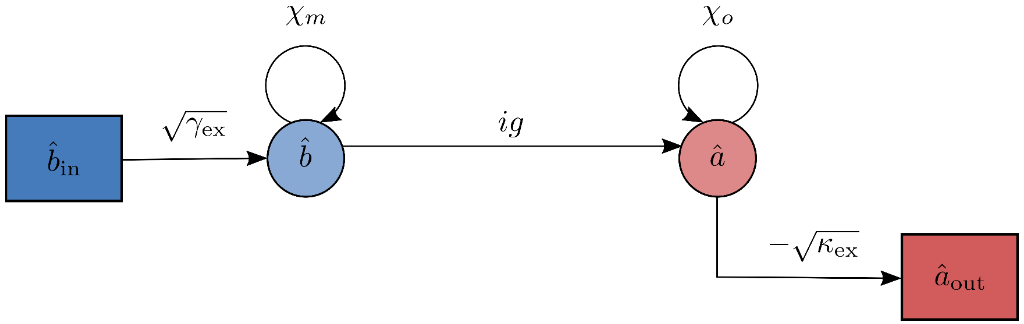


$|g|^4|\chi_m|^2|\chi_o|^2$

Signal flow graph – Up-conversion forward paths

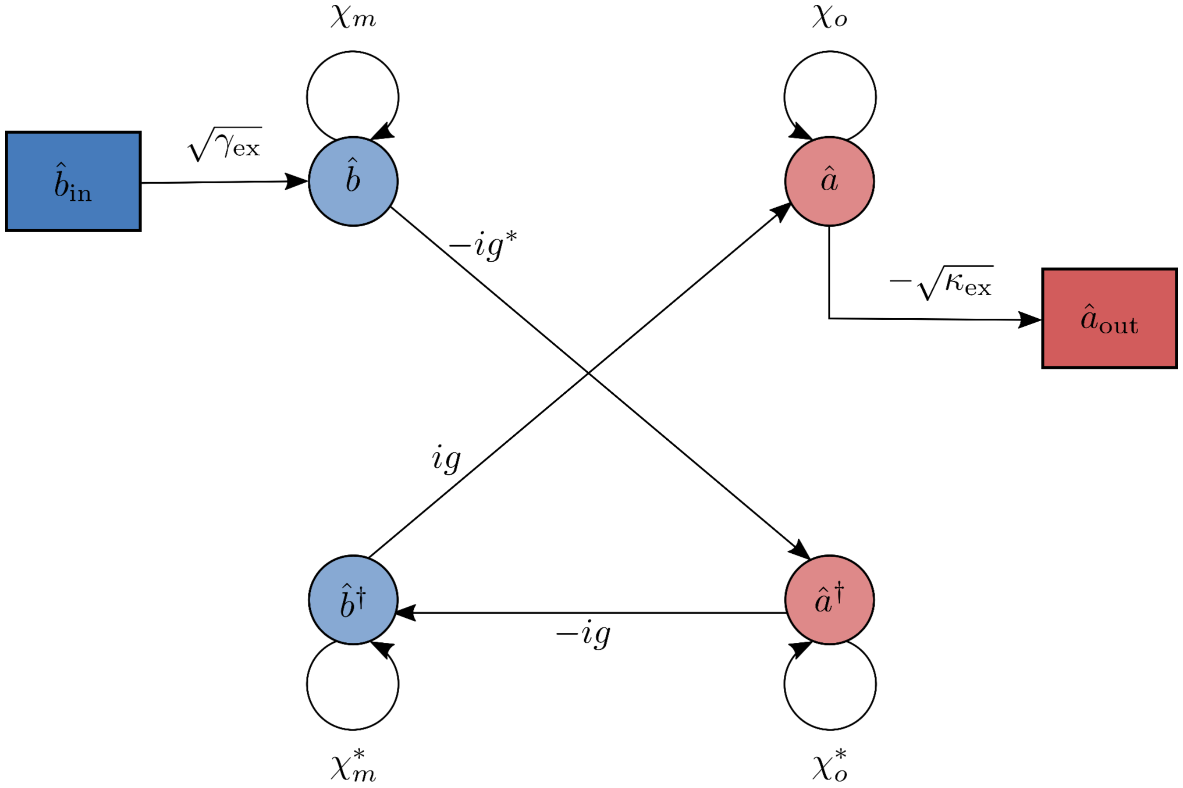
EPFL

Direct conversion (anti-Stokes)



$$-i\sqrt{\kappa_{\mathrm{ex}}}\sqrt{\gamma_{\mathrm{ex}}}g\chi_o\chi_m$$

Two-mode squeezing (Stokes)



$$i\sqrt{\kappa_{\mathrm{ex}}}\sqrt{\gamma_{\mathrm{ex}}}g|g|^2|\chi_o|^2|\chi_m|^2$$

Microwave-to-optical scattering coefficient

$$S_{\hat{a}_{\mathrm{out}}\hat{b}_{\mathrm{in}}} = \frac{-i\sqrt{\kappa_{\mathrm{ex}}}\sqrt{\gamma_{\mathrm{ex}}}g\chi_o\chi_m\left(1-|g|^2\chi_o^*\chi_m^*\right)}{1+|g|^2\chi_o\chi_m+|g|^2\chi_o^*\chi_m^*-|g|^2\chi_o^*\chi_m-|g|^2\chi_o\chi_m^*-2|g|^4|\chi_m|^2|\chi_o|^2}$$

[1] T. Blésin, H. Tian, S. A. Bhave, and T. J. Kippenberg, Quantum coherent microwave-optical transduction using high-overtone bulk acoustic resonances, Phys. Rev. A **104**, 052601 (2021).

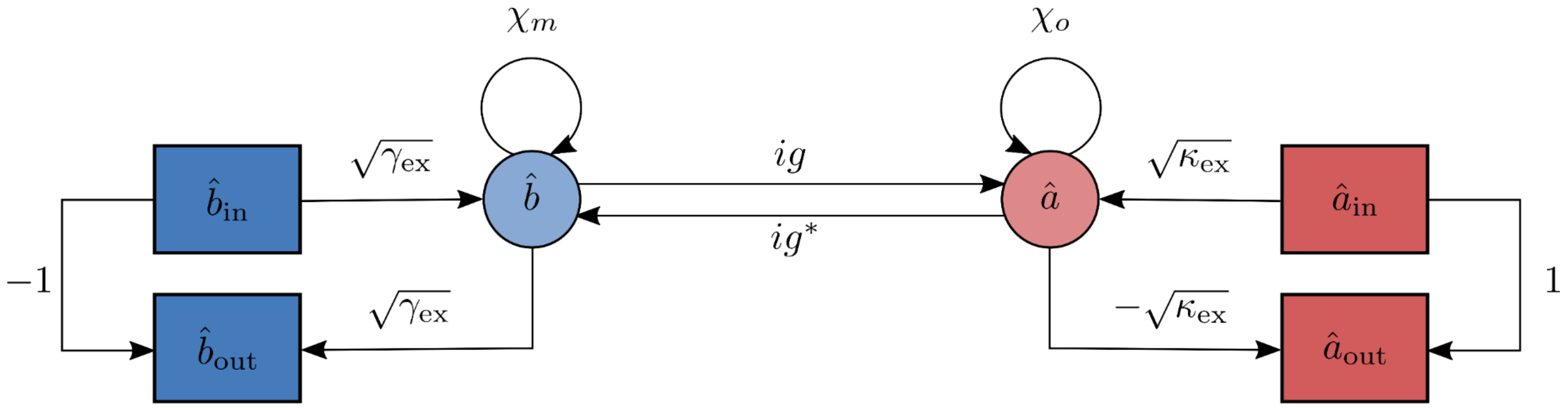
Simple model: Doubly resonant transducer (anti-Stokes process)

EPFL

Figures of merit of transducers:
- Conversion efficiency
- Added noise
- Conversion bandwidth
- Heat load/power consumption

Converted signal $\delta\hat{a}_{\mathrm{out}}[\omega] \approx \sqrt{\eta}\left(\hat{b}_{\mathrm{in}}[\omega] + \hat{f}_{\mathrm{added}}[\omega]\right)$

Added noise $\sqrt{\eta}\hat{f}_{\mathrm{added}}[\omega] = G_{\hat{a}_{\mathrm{out}}\hat{a}_{\mathrm{in}}}[\omega]\delta\hat{a}_{\mathrm{in}}[\omega] + G_{\hat{a}_{\mathrm{out}}\hat{f}_{\mathrm{opt}}}[\omega]\hat{f}_{\mathrm{opt}}[\omega] + G_{\hat{a}_{\mathrm{out}}\hat{f}_{\mathrm{MW}}}[\omega]\hat{f}_{\mathrm{MW}}[\omega]$



$$|S_{\hat{a}_{\mathrm{out}}\hat{b}_{\mathrm{in}}}|^2 \approx \frac{\kappa_{\mathrm{ex}}\gamma_{\mathrm{ex}}|g|^2|\chi_o|^2|\chi_m|2}{|1+|g|^2\chi_o\chi_m|^2}$$

Quantum amplifiers uncertainty relation

$$N_{\mathrm{added}} \geq \frac{1}{2}\left|1-\eta^{-1}\right|$$

C. Caves *Physical Review D* 26 1817-1839 (1982)

On-chip conversion efficiency:

$$\eta_{\mathrm{on-chip}} \approx \eta_{\mathrm{opt}}\eta_{\mathrm{m}}\frac{4C}{(1+C)^2}$$

$\kappa_{\mathrm{ex}}\chi_o \downarrow$ $\gamma_{\mathrm{ex}}\chi_m \downarrow$

$|g|^2\chi_o\chi_m|^2 \downarrow$

Conversion bandwidth:

$$\Gamma = \gamma + \frac{4g^2}{\kappa}$$

Extraction efficiencies:

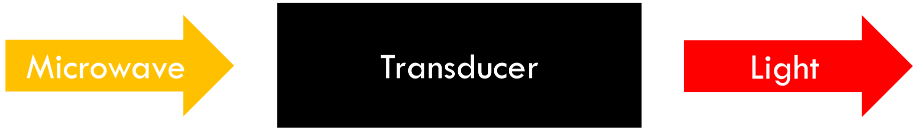
$$\eta_{\mathrm{opt}} = \frac{\kappa_{\mathrm{ex}}}{\kappa} \quad \eta_{\mathrm{m}} = \frac{\gamma_{\mathrm{ex}}}{\gamma}$$

Cooperativity:

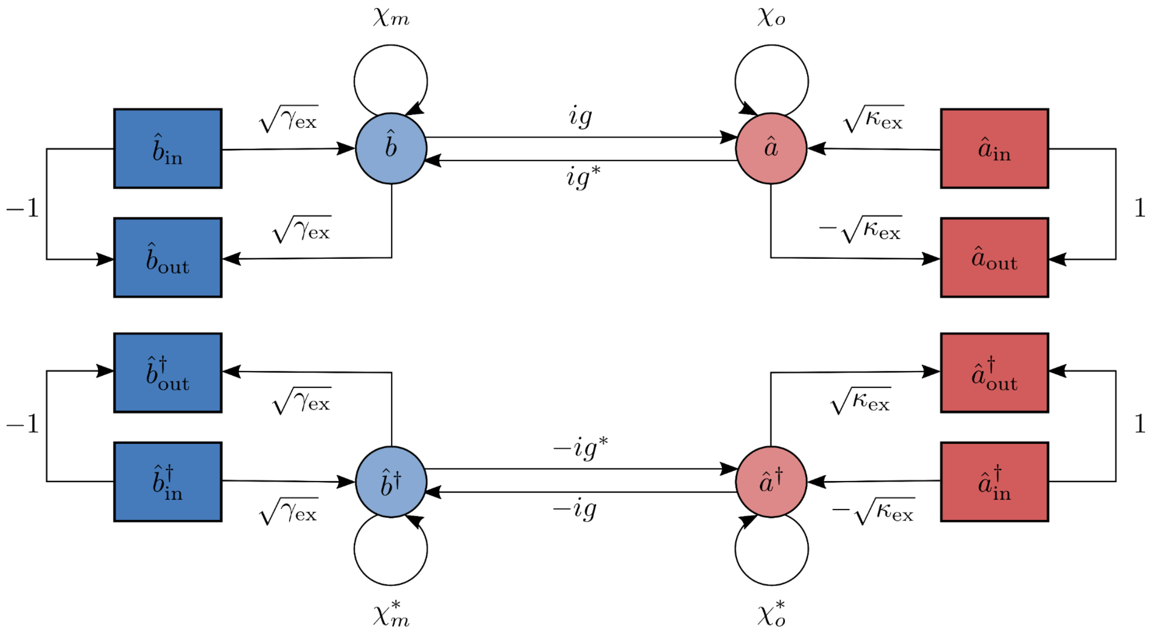
$$C_0 = \frac{4g_0^2}{\kappa\gamma} \quad C = \frac{4g^2}{\kappa\gamma} = \frac{4g_0^2 n_c}{\kappa\gamma}$$

Up-conversion efficiency

$$\hat{a}^\dagger_{\mathrm{out}}\hat{a}_{\mathrm{out}} = |S_{\hat{a}_{\mathrm{out}}\hat{a}_{\mathrm{in}}}|^2 + |S_{\hat{a}_{\mathrm{out}}\hat{a}^\dagger_{\mathrm{in}}}|^2 + |S_{\hat{a}_{\mathrm{out}}\hat{b}_{\mathrm{in}}}|^2 + |S_{\hat{a}_{\mathrm{out}}\hat{b}^\dagger_{\mathrm{in}}}|^2 + \left(|S_{\hat{a}_{\mathrm{out}}\hat{a}_{\mathrm{in}}}|^2 + |S_{\hat{a}_{\mathrm{out}}\hat{a}^\dagger_{\mathrm{in}}}|^2\right)\hat{a}^\dagger_{\mathrm{in}}\hat{a}_{\mathrm{in}} + \left(|S_{\hat{a}_{\mathrm{out}}\hat{b}_{\mathrm{in}}}|^2 + |S_{\hat{a}_{\mathrm{out}}\hat{b}^\dagger_{\mathrm{in}}}|^2\right)\hat{b}^\dagger_{\mathrm{in}}\hat{b}_{\mathrm{in}} + \dots$$

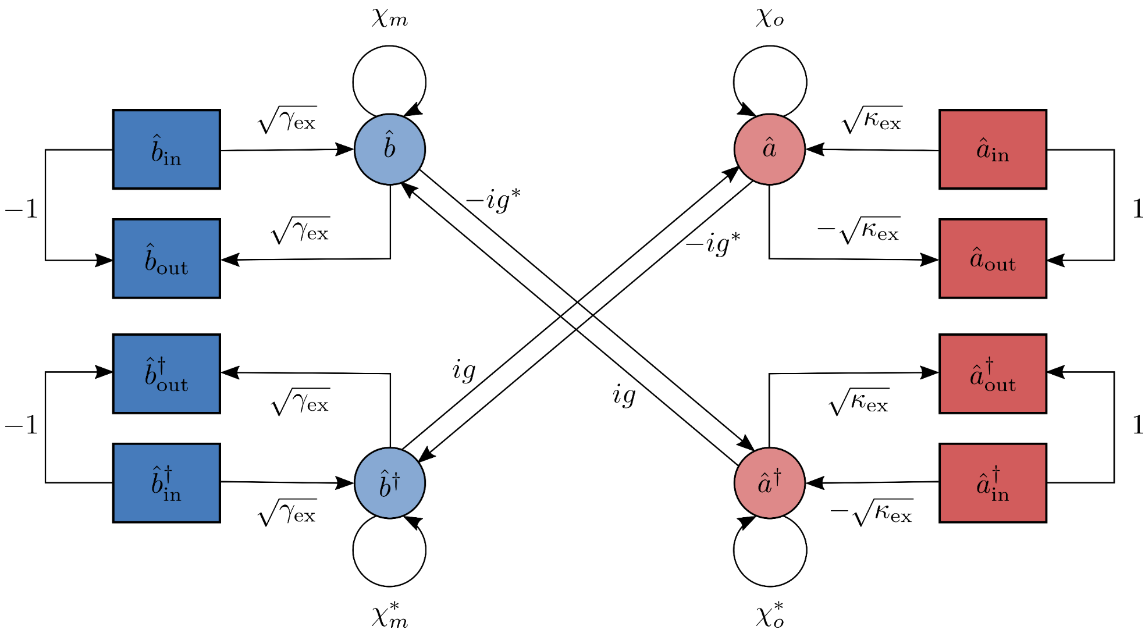


Direct conversion (anti-Stokes)



$$S_{\hat{a}_{\mathrm{out}}\hat{b}_{\mathrm{in}}} \approx \frac{-i\sqrt{\kappa_{\mathrm{ex}}}\sqrt{\gamma_{\mathrm{ex}}}\,g\chi_o\chi_m}{1+|g|^2\chi_o\chi_m}$$

Two-mode squeezing (Stokes)



$$S_{\hat{a}^\dagger_{\mathrm{out}}\hat{b}_{\mathrm{in}}} \approx \frac{i\sqrt{\kappa_{\mathrm{ex}}}\sqrt{\gamma_{\mathrm{ex}}}\,g^*\chi_o^*\chi_m}{1-|g|^2\chi_o^*\chi_m}$$

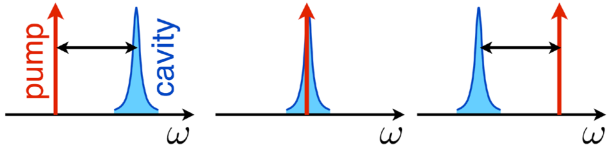
Summary: Radiation pressure interaction for transduction

Optomechanical interaction $\hat{H}_{\mathrm{OM}} = -\hbar g_0 \hat{a}^\dagger \hat{a} \left(\hat{b} + \hat{b}^\dagger\right)$

Linearized optical field amplitude $\hat{a} \approx \sqrt{\bar{n}_c} + \delta\hat{a}$

Effective coupling strength $g = \sqrt{\bar{n}_c} g_0$

Laser-cavity detuning $\Delta = \omega_L - \omega_{\mathrm{opt}}$

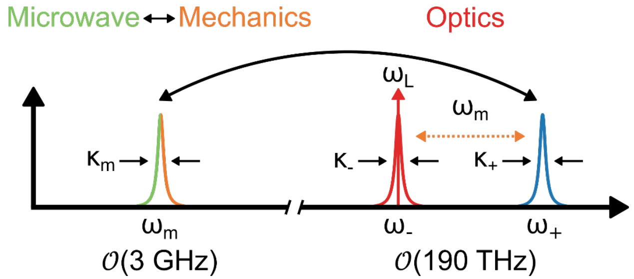


$\Delta = -\Omega_{\mathrm{m}}$	$\Delta = 0$	$\Delta = +\Omega_{\mathrm{m}}$
beam-splitter (cooling)	QND	squeezer (entanglement)
$\delta\hat{a}^\dagger\hat{b} + \hat{b}^\dagger\delta\hat{a}$	$\hat{x}_a\hat{x}_b$	$\delta\hat{a}^\dagger\hat{b}^\dagger + \hat{b}\delta\hat{a}$

[1] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, *Cavity Optomechanics*, Rev. Mod. Phys. **86**, 1391 (2014).

Anti-Stokes process: direct conversion ("beam splitter")

$$\Delta = -\omega_m \longrightarrow \hat{H}_{\mathrm{OM}}^{(\mathrm{BS})} = -\hbar g \left(\delta\hat{a}^\dagger\hat{b} + \delta\hat{a}\hat{b}^\dagger\right)$$

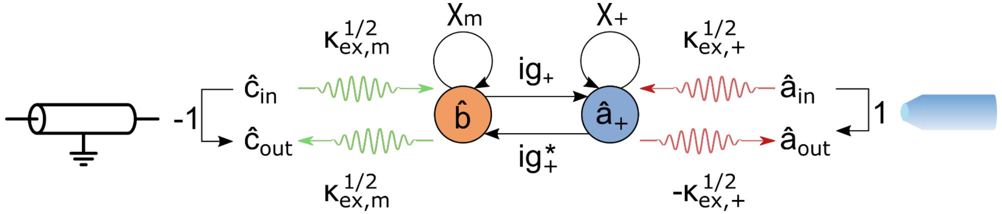


<u>Conversion efficiency:</u>

$$\eta_+[\omega] \approx \frac{\kappa_{\mathrm{ex},+}}{\kappa_+}\frac{\kappa_{\mathrm{ex,m}}}{\kappa_m}\frac{4C_+}{(1+C)^2}$$

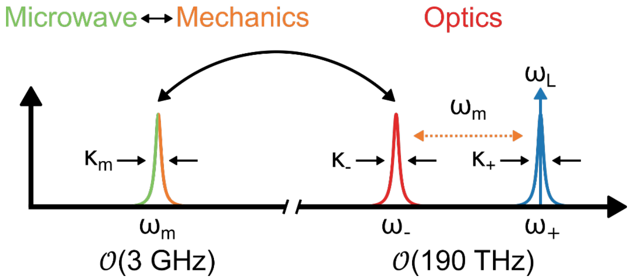
<u>Added noise:</u>

$$n_{\mathrm{add}} \geq 0$$



Stokes process: Entanglement ("two-mode squeezing")

$$\Delta = \omega_m \longrightarrow \hat{H}_{\mathrm{OM}}^{(\mathrm{TMS})} = -\hbar g \left(\delta\hat{a}\hat{b} + \delta\hat{a}^\dagger\hat{b}^\dagger\right)$$

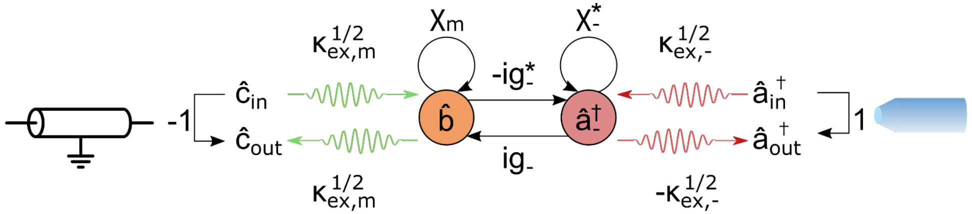


<u>Conversion gain:</u>

$$\eta_-[\omega] \approx \frac{\kappa_{\mathrm{ex},-}}{\kappa_-}\frac{\kappa_{\mathrm{ex,m}}}{\kappa_m}\frac{4C_-}{(1-C_-)^2}$$

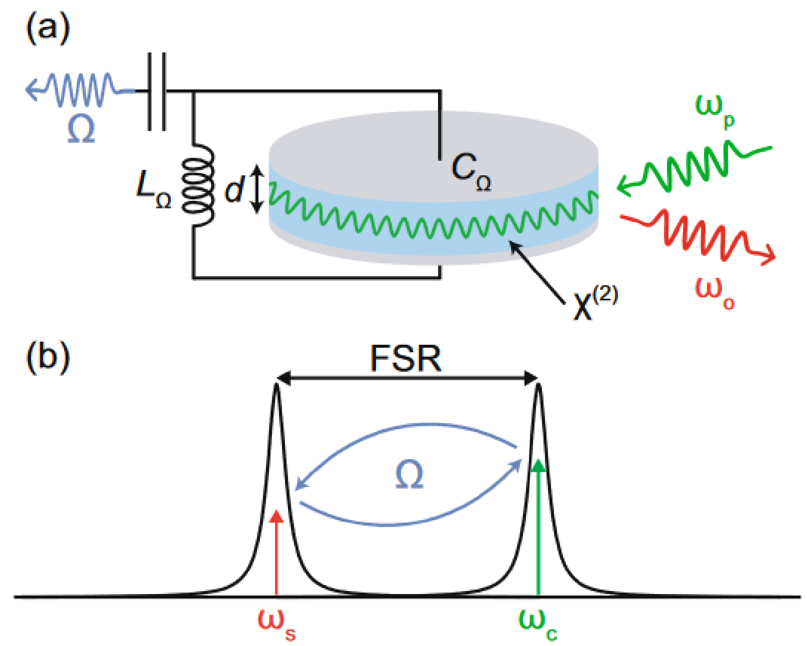
<u>Added noise:</u>

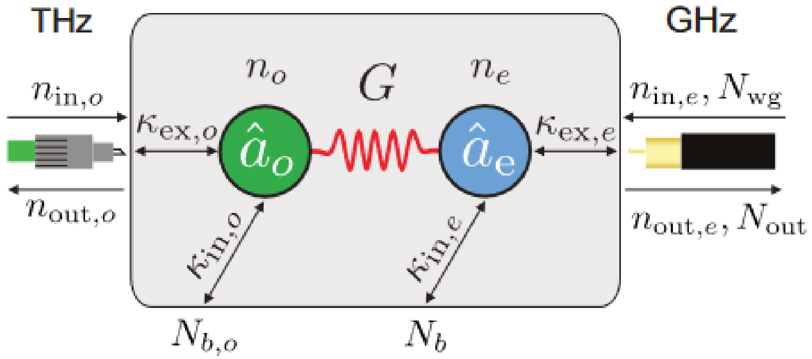
$$n_{\mathrm{add}} \geq 1$$

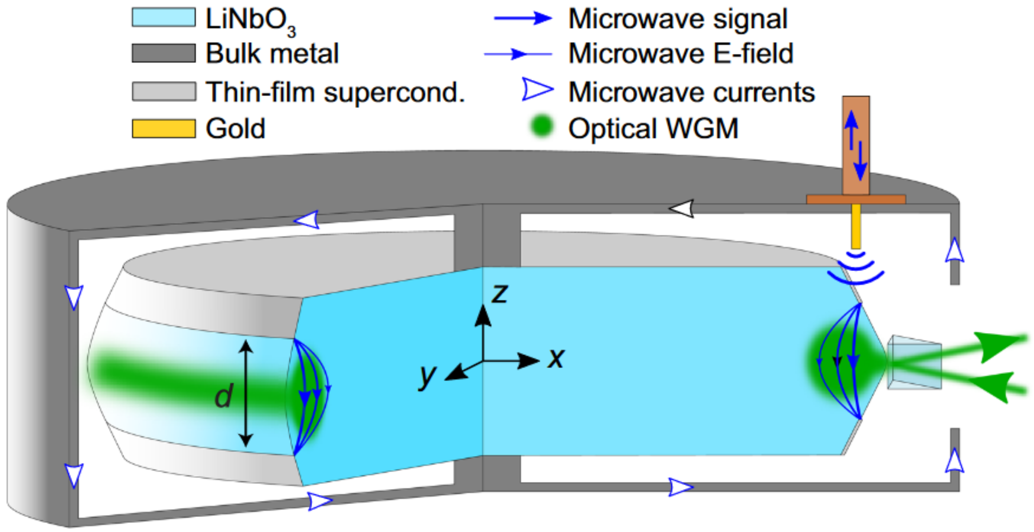


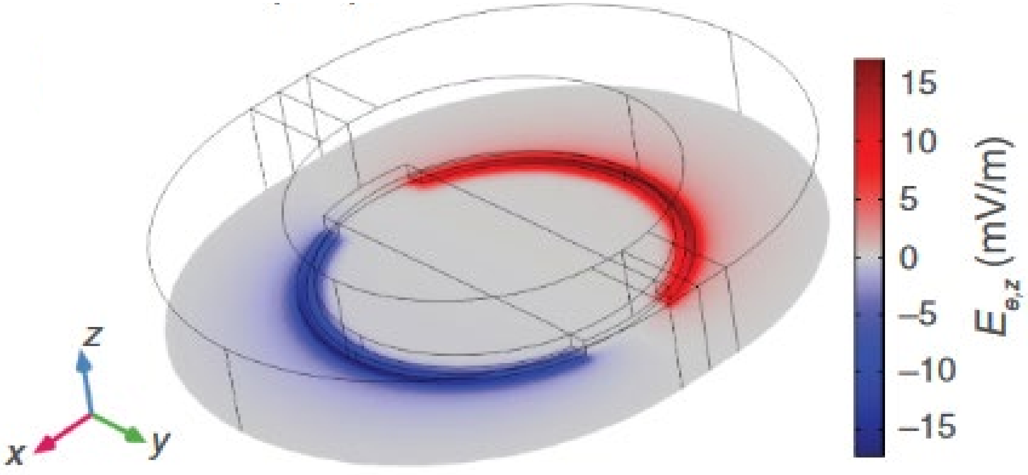
[1] T. Blésin, W. Kao, A. Siddharth, R. N. Wang, A. Attanasio, H. Tian, S. A. Bhave, and T. J. Kippenberg, Bidirectional microwave-optical transduction based on integration of high-overtone bulk acoustic resonators and photonic circuits, Nat Commun **15**, 6096 (2024).

Lithium niobate bulk transducer (IST Austria)









[1] A. Rueda, W. Hease, S. Barzanjeh, and J. M. Fink, Electro-optic entanglement source for microwave to telecom quantum state transfer, Npj Quantum Inf **5**, 1 (2019).

[2] W. Hease, A. Rueda, R. Sahu, M. Wulf, G. Arnold, H. G. L. Schwefel, and J. M. Fink, Bidirectional Electro-Optic Wavelength Conversion in the Quantum Ground State, PRX Quantum **1**, 020315 (2020).

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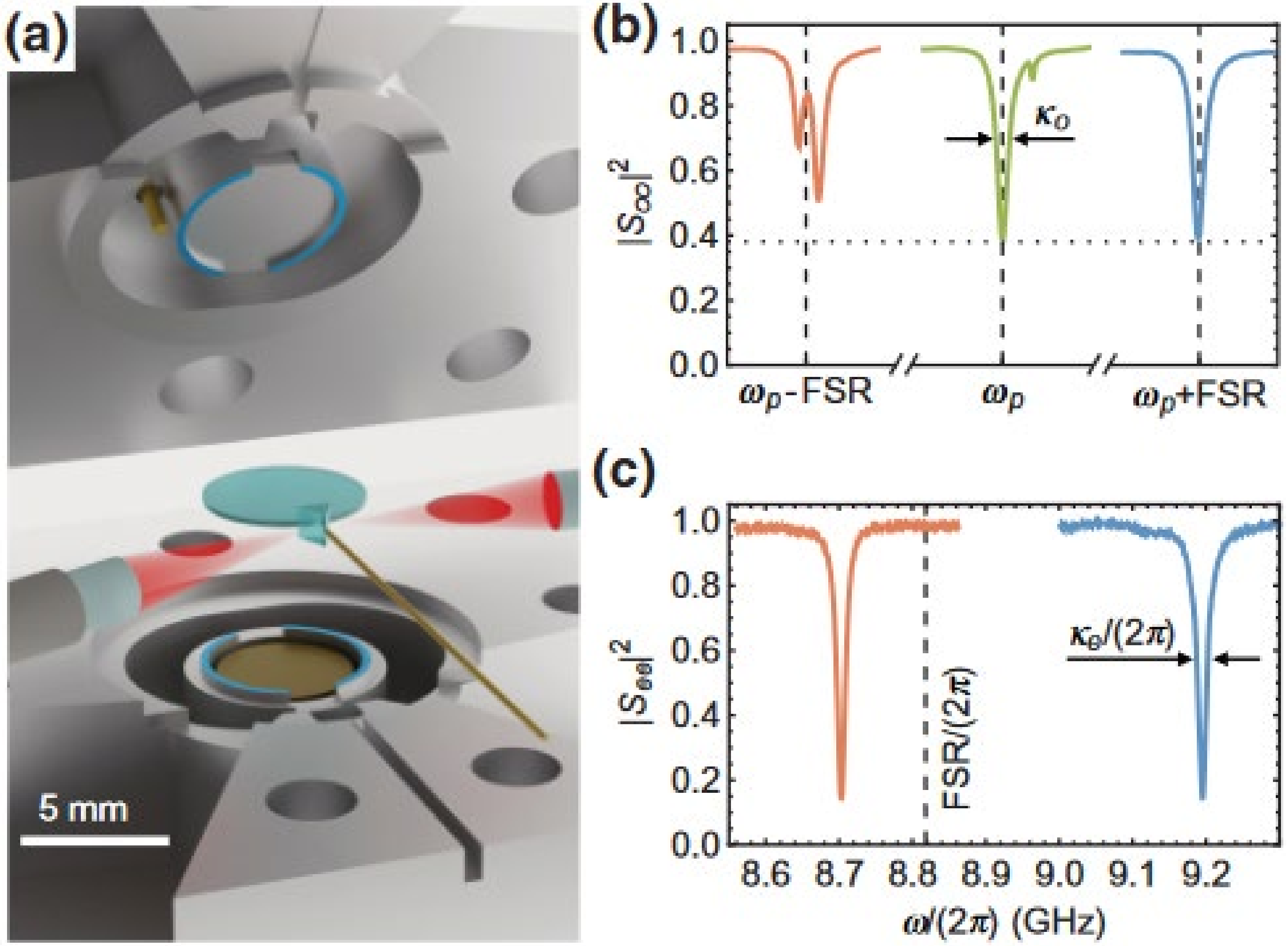

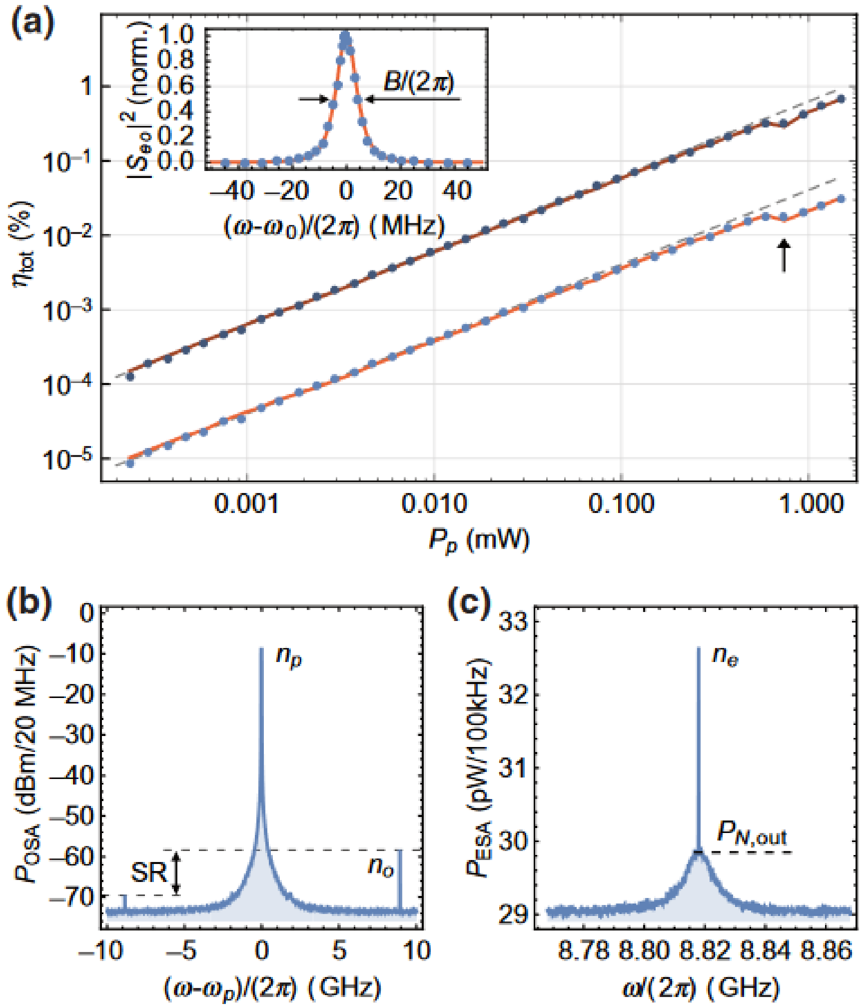

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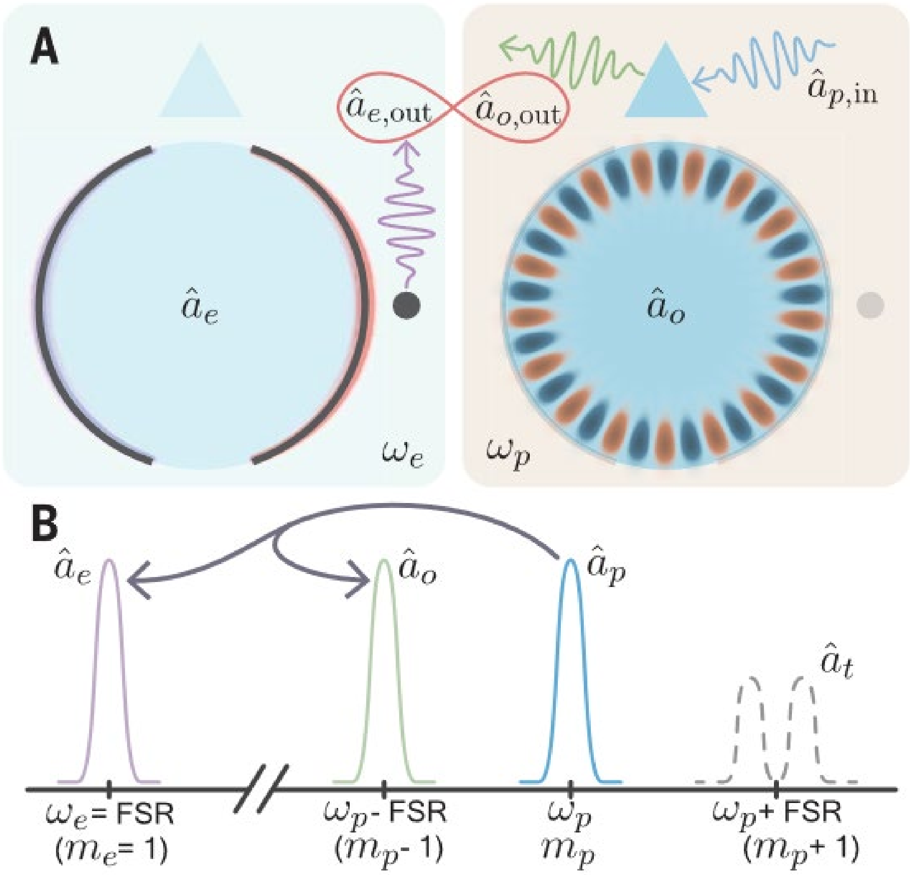

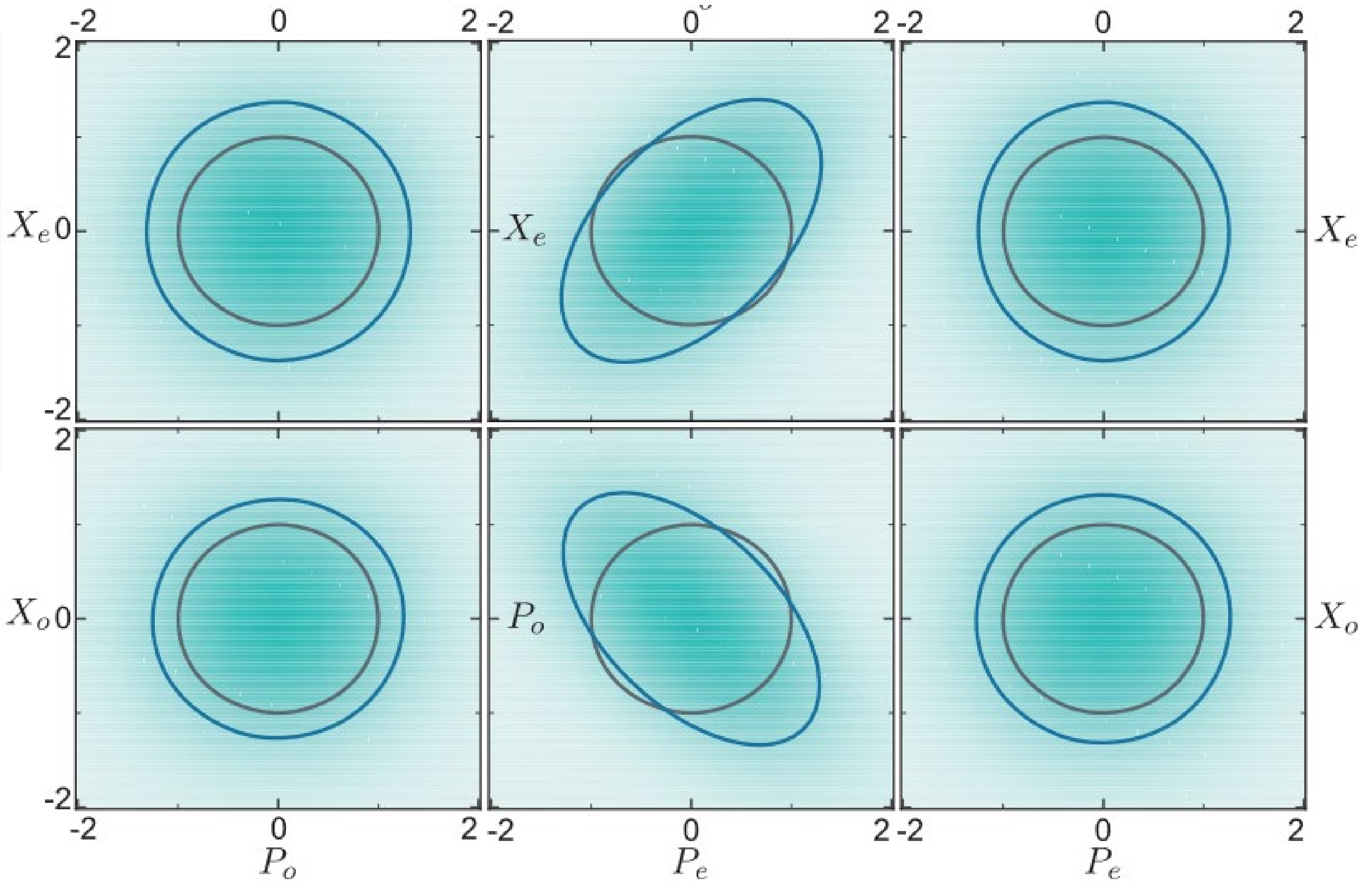

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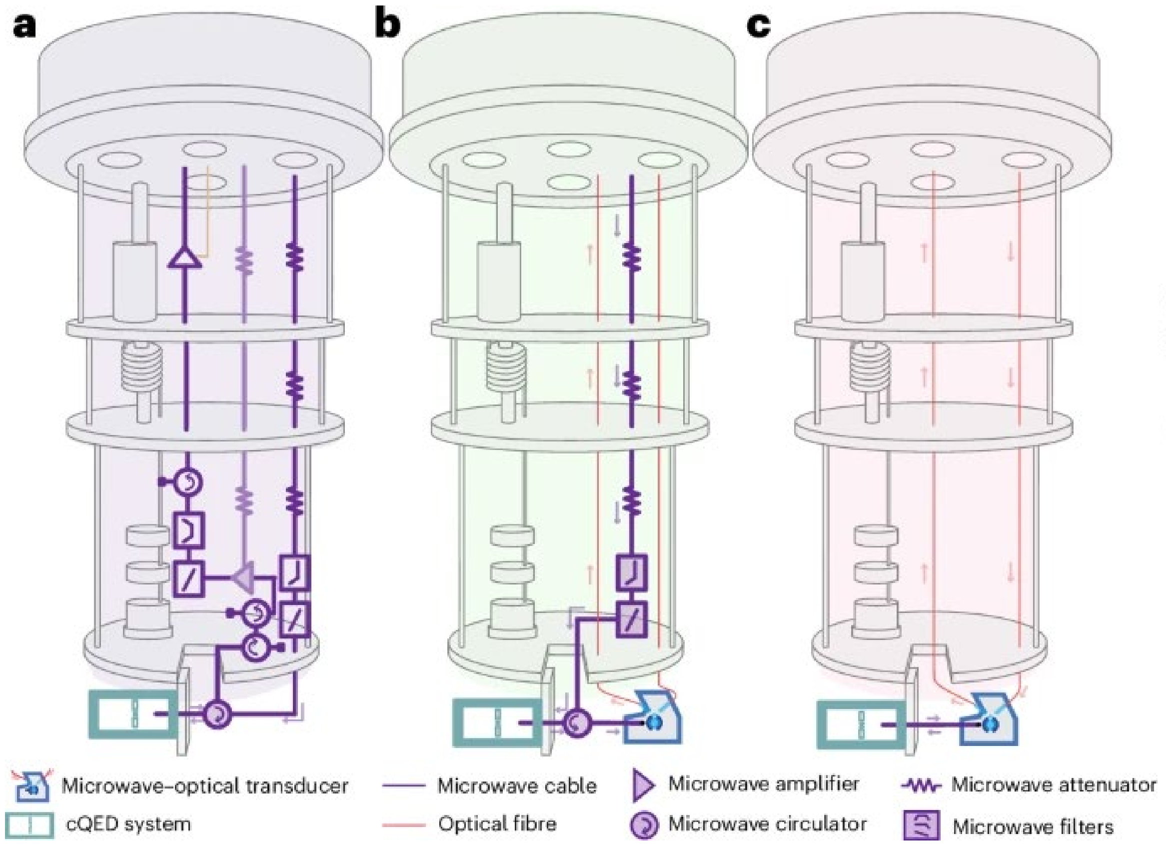

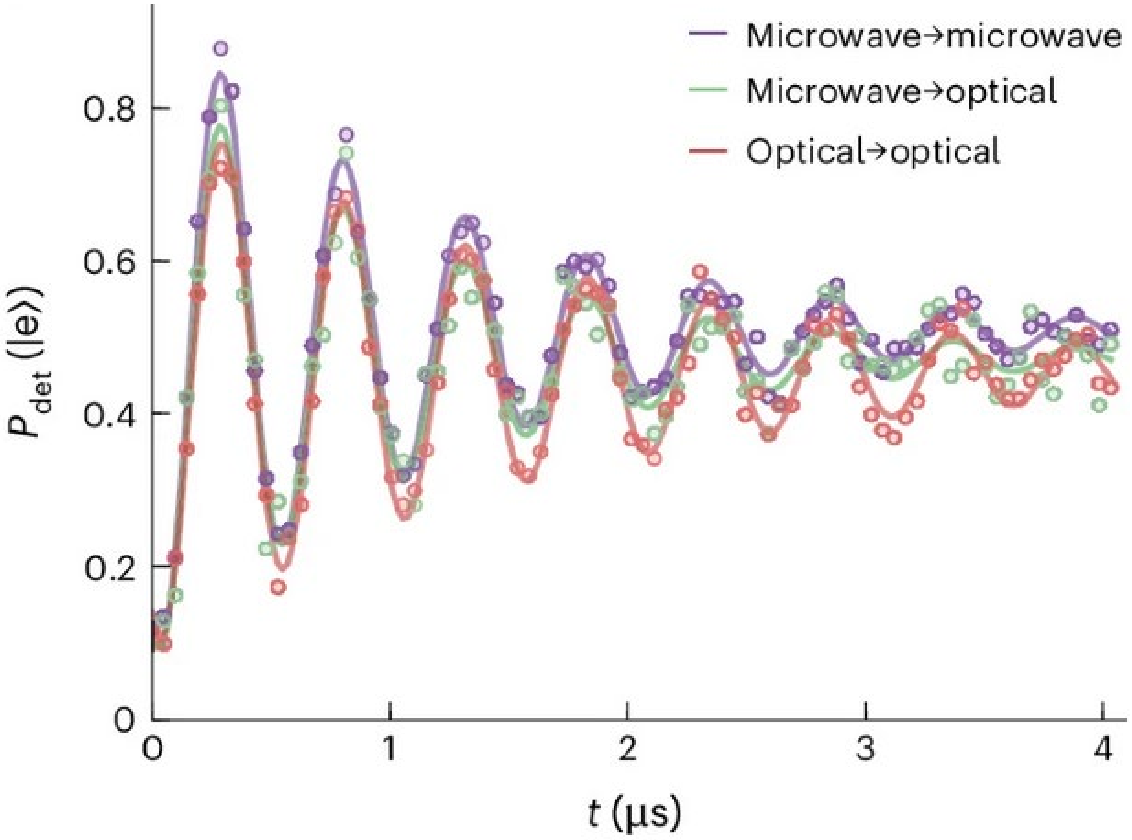

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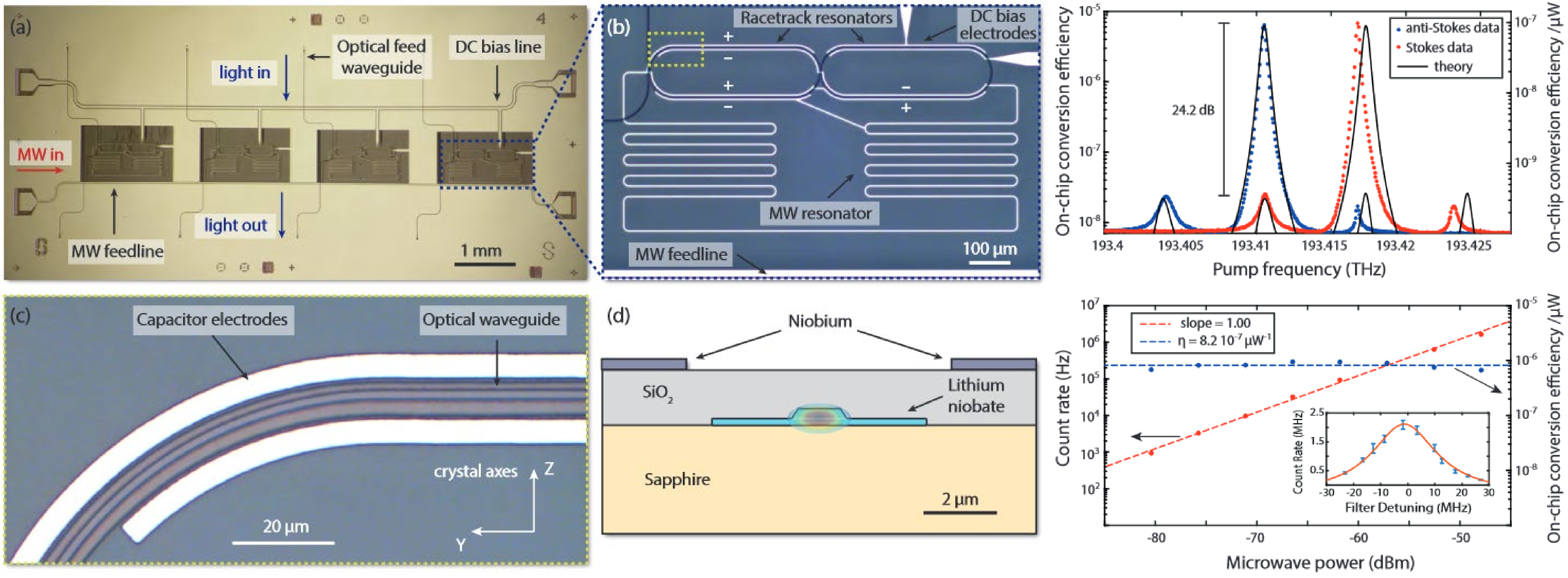
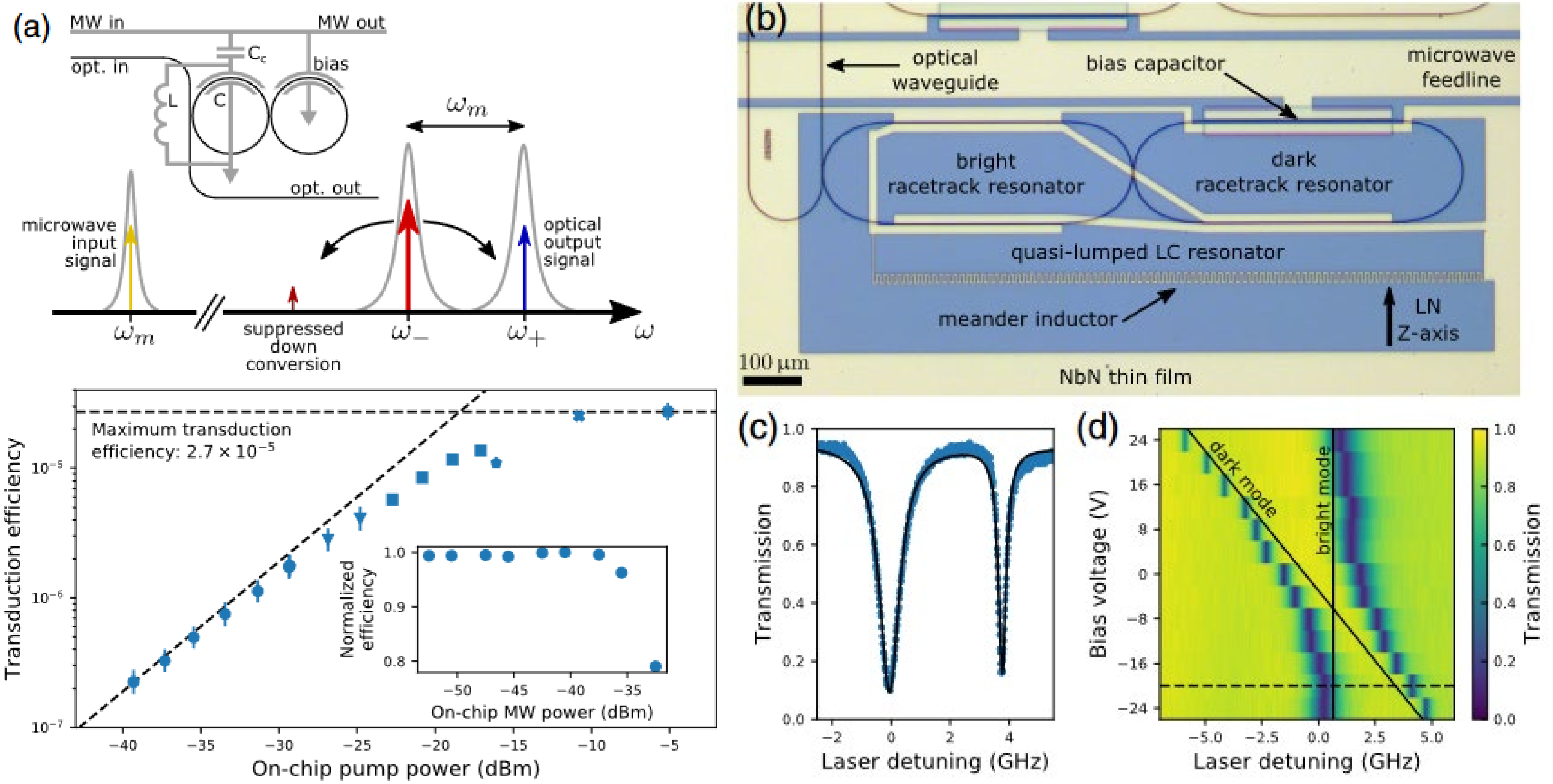
Lithium niobate integrated transducer (Stanford)



[1] T. P. McKenna et al., Cryogenic microwave-to-optical conversion using a triply resonant lithium-niobate-on-sapphire transducer, Optica, OPTICA **7**, 1737 (2020).

Lithium niobate integrated transducer (Harvard)



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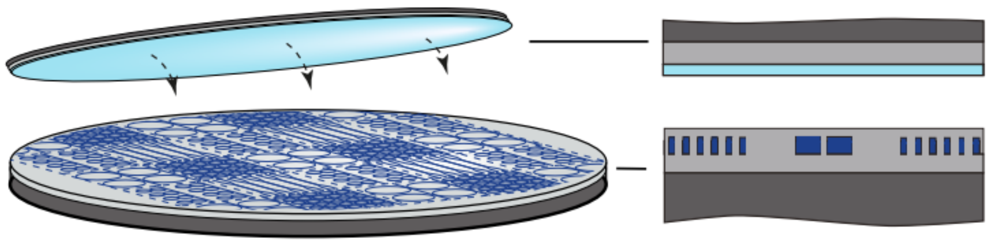
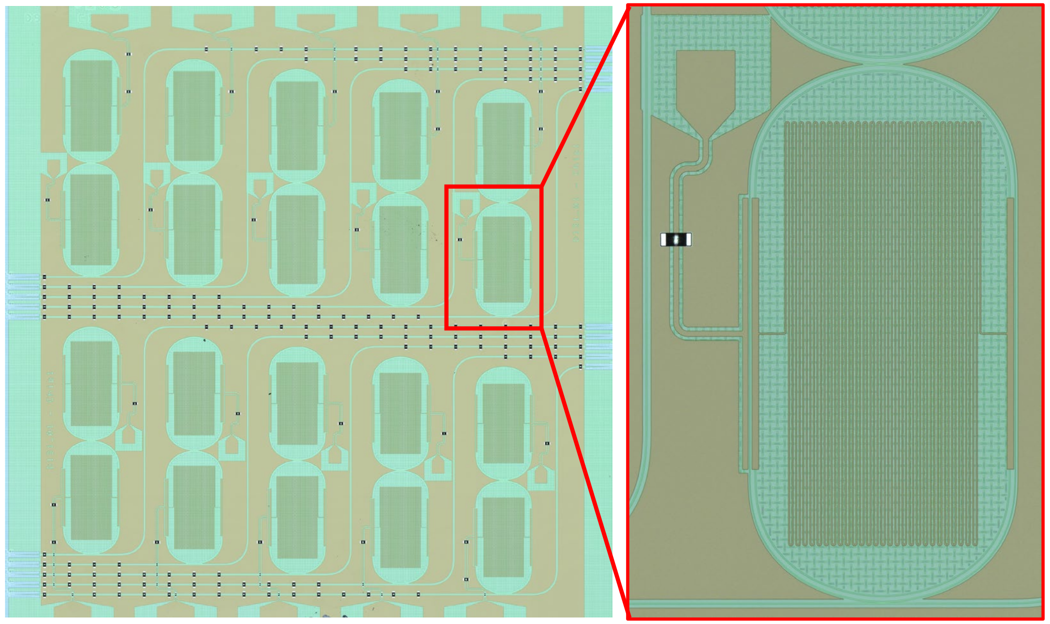
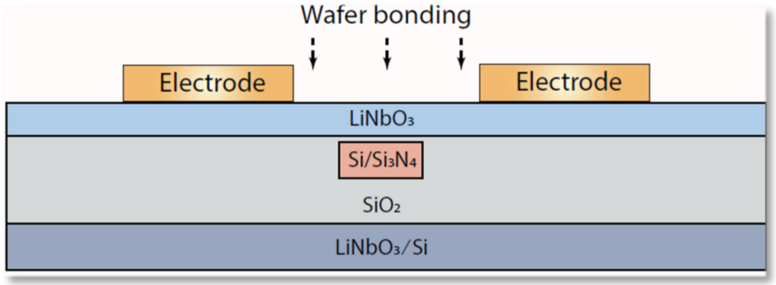
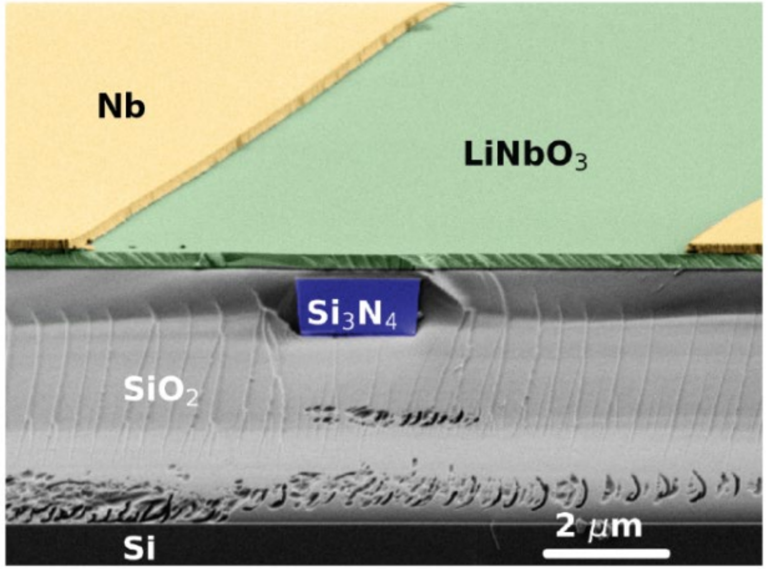
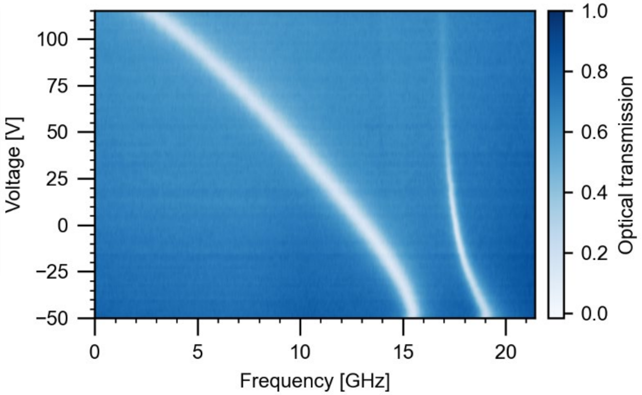
Lithium niobate integrated transducer (EPFL)

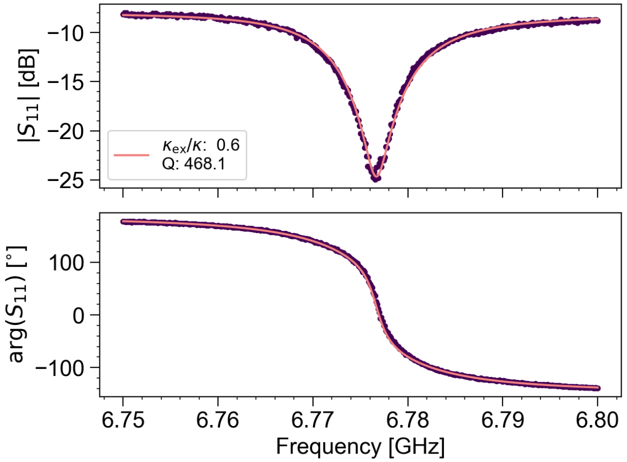












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Piezo-optomechanical photon-photon interface (EPFL)

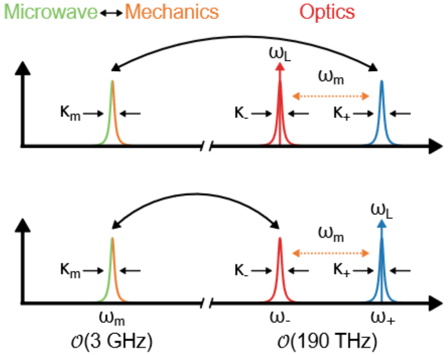
- Microwave transmission line piezoelectrically coupled to high-overtone bulk acoustic resonances
  - Entangled microwave-optical pair generation: large transduction bandwidth
  - Microwave photonics: pump-power handling
- Design simplicity
  - CMOS-compatible fabrication processes
  - No superconducting element required

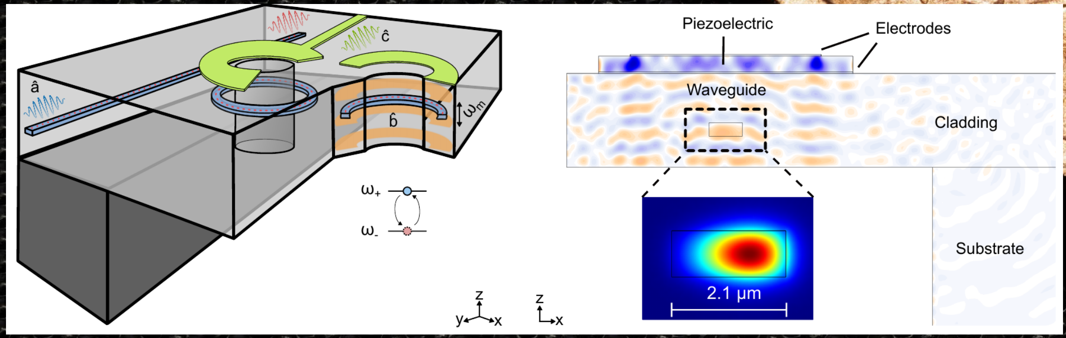
Direct conversion ("beam splitter")

$$\hat{H}_{\mathrm{OM}}^{(\mathrm{BS})} = -\hbar g \left(\delta\hat{a}^{\dagger}\hat{b} + \delta\hat{a}\hat{b}^{\dagger} \right)$$

Entanglement ("two-mode squeezing")

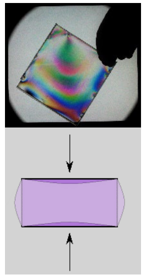
$$\hat{H}_{\mathrm{OM}}^{(\mathrm{TMS})} = -\hbar g \left(\delta\hat{a}\hat{b} + \delta\hat{a}^{\dagger}\hat{b}^{\dagger} \right)$$

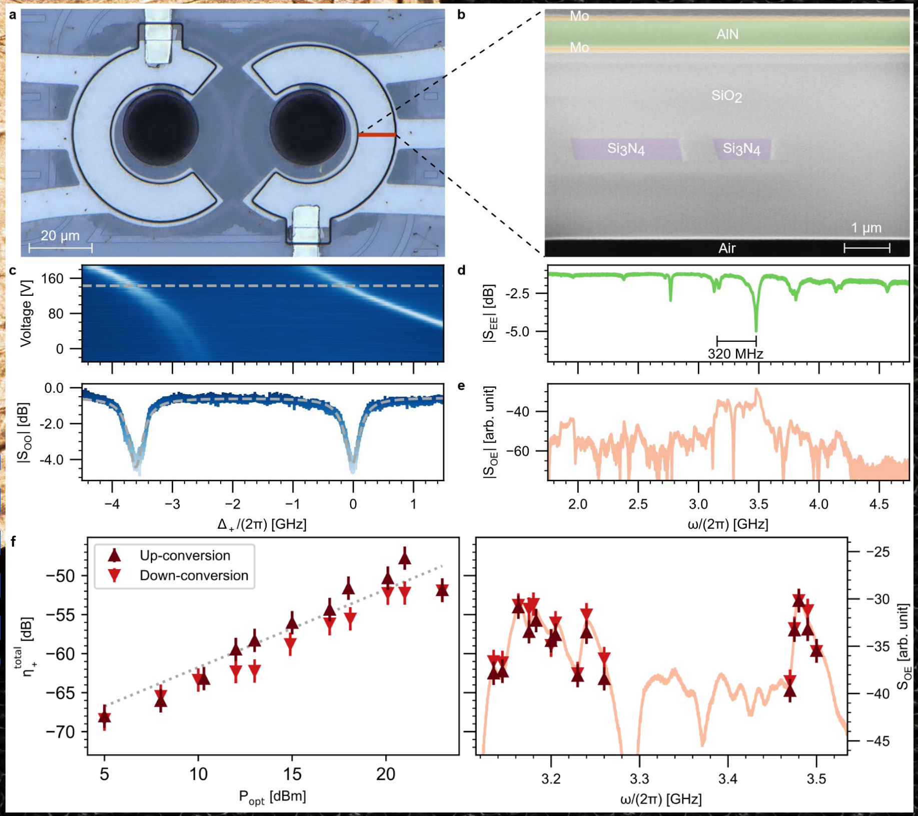




Photoelastic effect:

Moving boundaries:





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Microwave-optical transducers at cryogenic temperatures (EPFL)