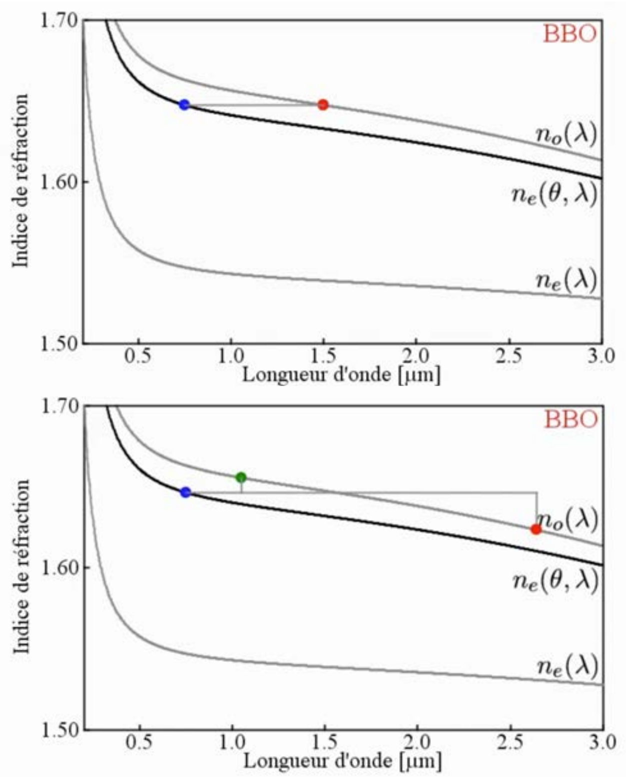
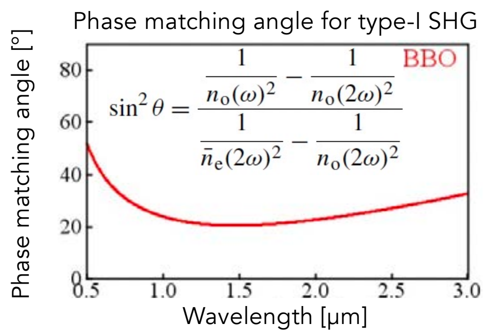
Colinear phase matching in DFG/SFG/SPDC



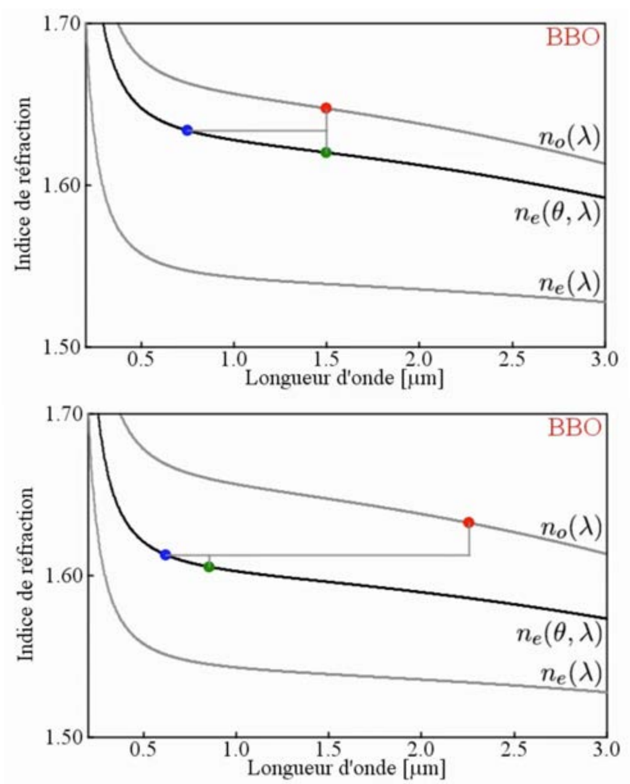
Ex.: Type I, negative crystal (oeo)

	Positive uniaxial ($n_e > n_0$)	Negative uniaxial ($n_e < n_0$)
Type I	$n_3^o\omega_3 = n_1^e\omega_1 + n_2^e\omega_2$	$\boxed{n_3^e\omega_3 = n_1^o\omega_1 + n_2^o\omega_2}$
Type II	$n_3^o\omega_3 = n_1^o\omega_1 + n_2^e\omega_2$	$n_3^e\omega_3 = n_1^e\omega_1 + n_2^o\omega_2$



Source: Manuel Joffre, Coursera

Colinear phase matching in DFG/SFG/SPDC

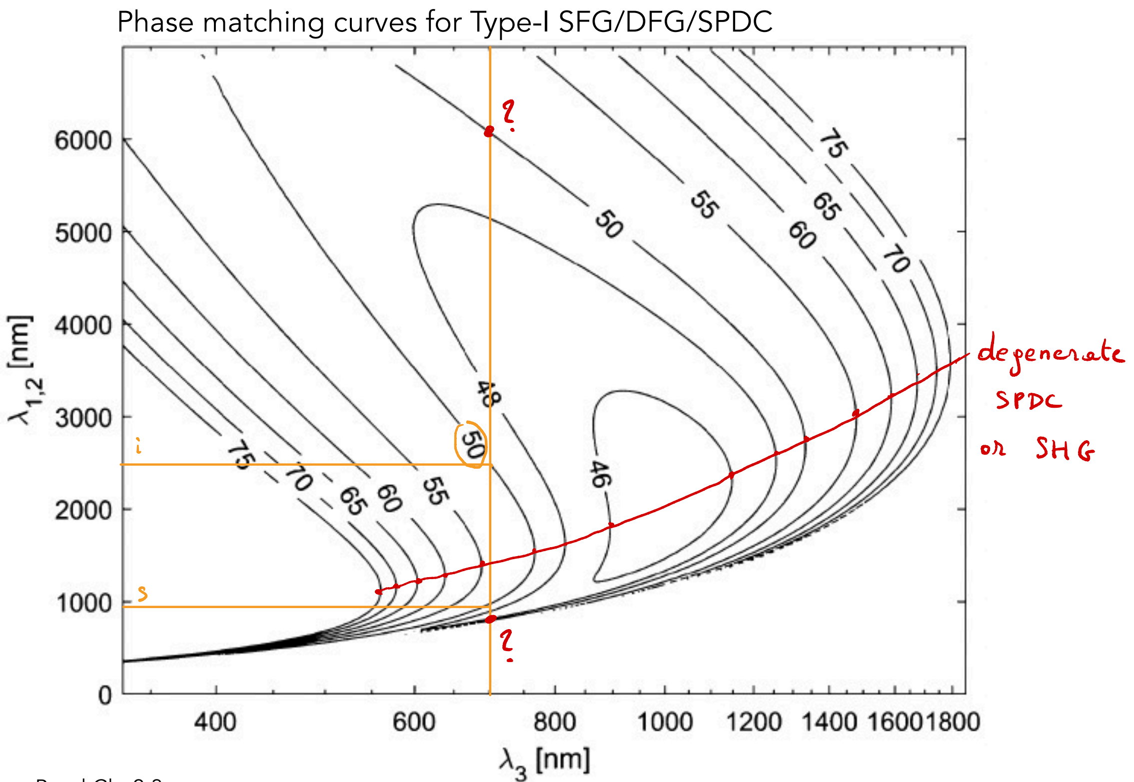


Type II, negative crystal (oee/eeo)

	Positive uniaxial ($n_e > n_0$)	Negative uniaxial ($n_e < n_0$)
Type I	$n_3^o\omega_3 = n_1^e\omega_1 + n_2^e\omega_2$	$n_3^e\omega_3 = n_1^o\omega_1 + n_2^o\omega_2$
Type II	$n_3^o\omega_3 = n_1^o\omega_1 + n_2^e\omega_2$	$\boxed{n_3^e\omega_3 = n_1^e\omega_1 + n_2^o\omega_2}$

Source: Manuel Joffre, Coursera

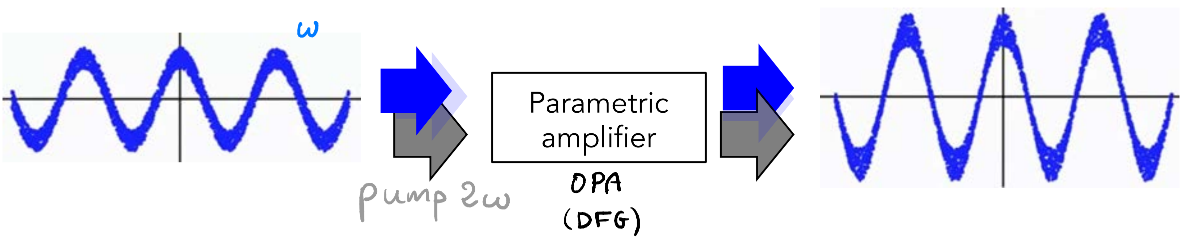
Colinear phase matching in DFG/SFG/SPDC



Source: Boyd Ch. 2.3

↳ highest frequency (pump for SPDC)

Degenerate DFG: squeezing



I) Degenerate type I DFG → phase sensitive amplifier.

Reminder : non-degenerate DFG, idler input = vacuum

then :
$$I_s(z) = I_s(0)\cosh^2(gz)$$
$$I_i(z) = I_s(0)\sinh^2(gz)$$

↳ even though the eqs. are phase-preserving, the resulting amplifier is phase insensitive

Degenerate case : $a_i = a_s = a(z)$

$$\frac{da}{dz} = i\xi a_p a^*(z) = g\, a^*(z) \qquad \text{where } \xi \propto \chi^{(2)}_{eff}$$

By choosing the origin of time, we can take $g \in \mathbb{R}$

$$\frac{d^2a}{dz^2} = g^2 a(z) \Rightarrow a(z) = A e^{gz} + B e^{-gz}$$

Initial conditions $\quad a(z=0) = A + B$

$$g\, a^*(0) = g(A-B) \Rightarrow a^*(0) = A - B$$

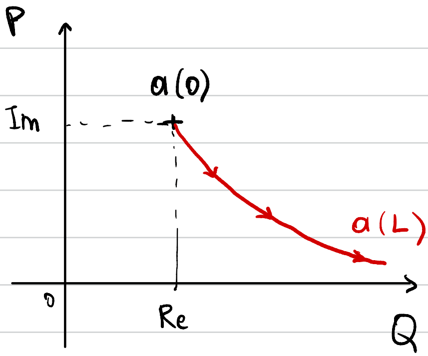
$$A = \frac{a(0)+a^*(0)}{2} = \mathrm{Re}[a(0)] \quad \text{and} \quad B = \frac{a(0)-a^*(0)}{2} = i\,\mathrm{Im}[a(0)]$$

optical quadratures

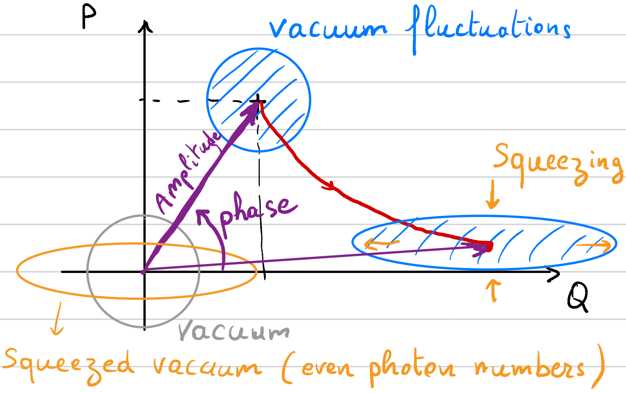
$$\boxed{a(z) = \underbrace{\mathrm{Re}[a(0)]}_{\text{"Q" is amplified}} e^{gz} + i\,\underbrace{\mathrm{Im}[a(0)]}_{\text{"P" is damped}} e^{-gz}}$$

Phase space representation

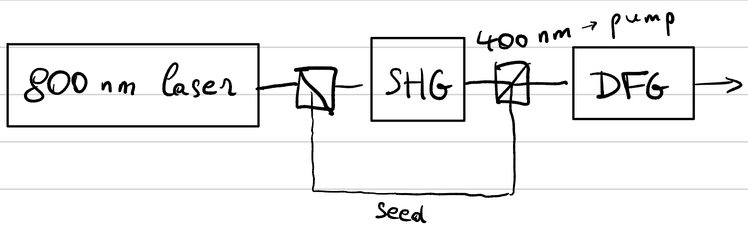
Classical



Quantum

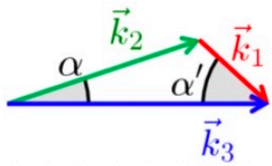


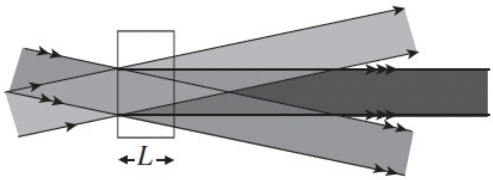
Experiment :



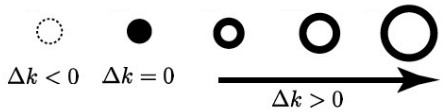
Noncollinear phase matching (for SPDC)

- So far, we only considered phase matching in the collinear geometry where all beams propagate in the same direction. Then phase matching reduces to a scalar equation about refractive indices.
- But all the *k*'s are vectors, so different geometries are possible, in particular for the case of parametric down conversion where signal and idler are produced at frequencies summing to that of the pump.

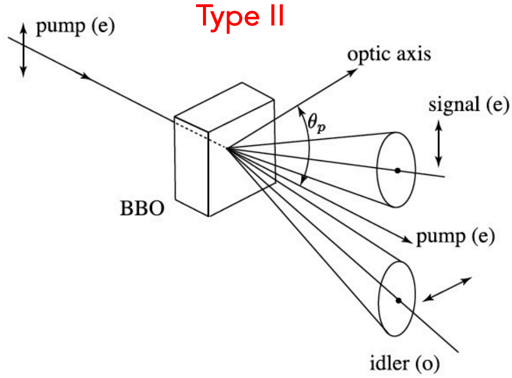




Type I



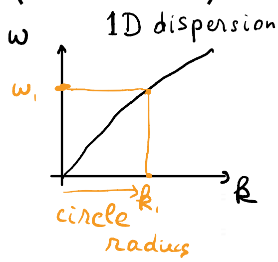
Type II

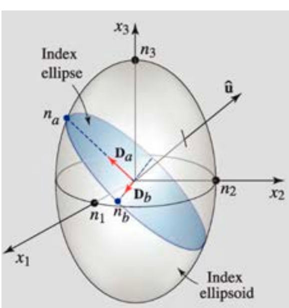


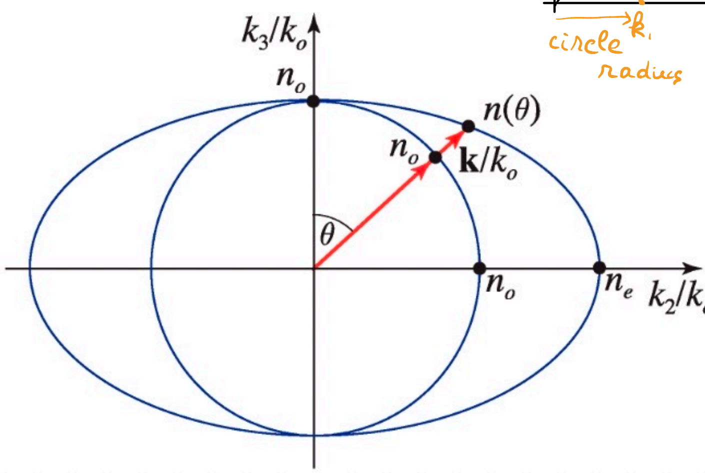
Source: Boyd Ch. 2.12

Dispersion relation in a uniaxial crystal (week 03)

$$\sum_{j=1,2,3}\frac{k_j^2}{k^2-n_j^2\,k_o^2}=1 \quad \Rightarrow \quad \boxed{(k^2-n_o^2k_o^2)\left(\frac{k_1^2+k_2^2}{n_e^2}+\frac{k_3^2}{n_o^2}-k_o^2\right)=0}$$







Index ellipsoid

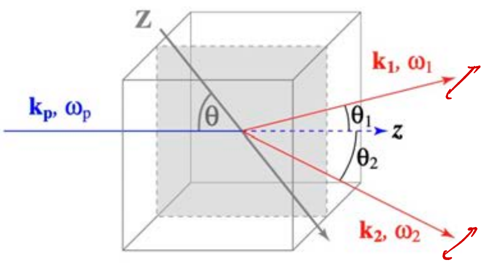
Dispersion ellipsoid → 2D dispersion drawn at fixed ω

(positive uniaxial)

↳ 2 shells for 2 polarizations

Saleh & Teich, Ch. 6.3

Type I SPDC non collinear phase matching

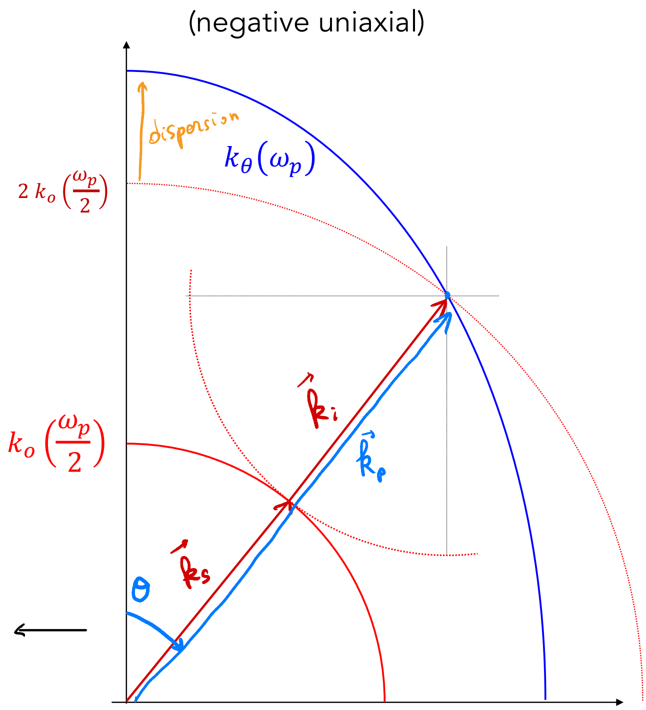


$$\omega_1 = \omega_2 = \frac{\omega_p}{2}$$

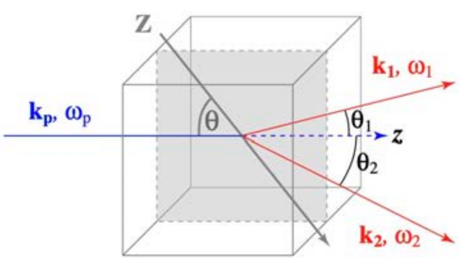
at degeneracy

Collinear phase matching :

$\theta_s = \theta_i = \theta_p$

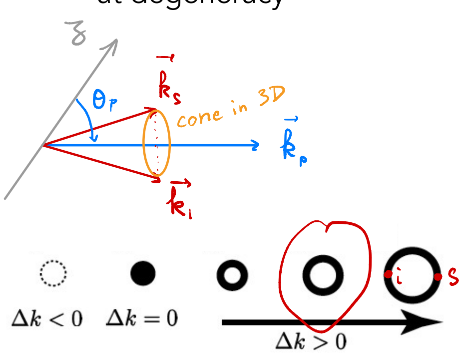


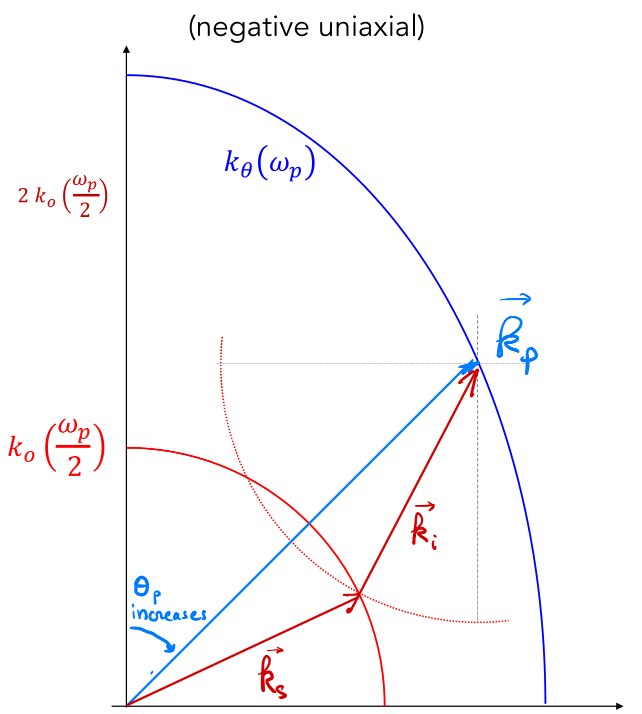
Type I SPDC non collinear phase matching



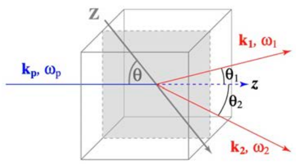
$$\omega_1 = \omega_2 = \frac{\omega_p}{2}$$

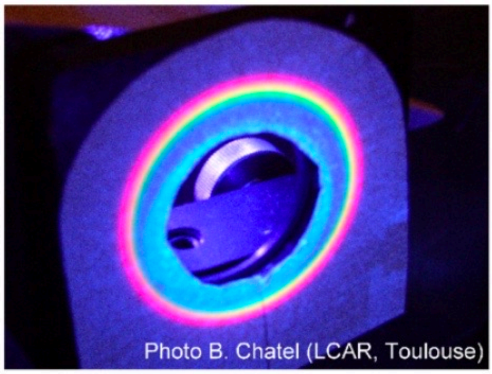
at degeneracy



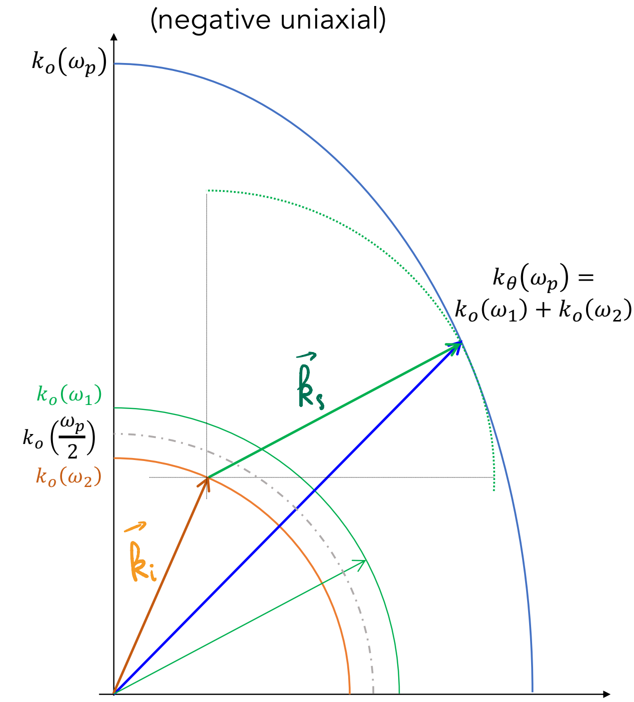


Type I SPDC – phase matching near degeneracy



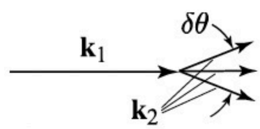


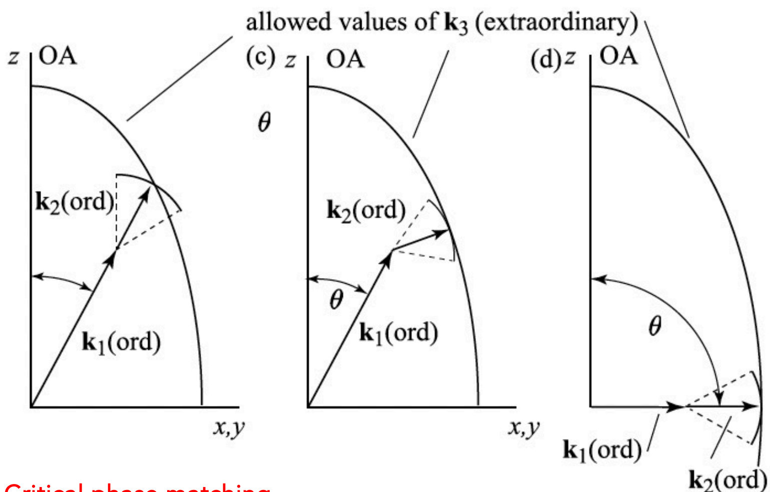
Source: Manuel Joffre, Coursera



Critical vs. noncritical phase matching

Example: we want to achieve efficient sum-frequency generation for a large angular distribution of k_2.





Critical phase matching
the phase mismatch scales approximately linearly with **δθ**.

Noncritical phase matching
the phase mismatch scales only quadratically with **δθ**

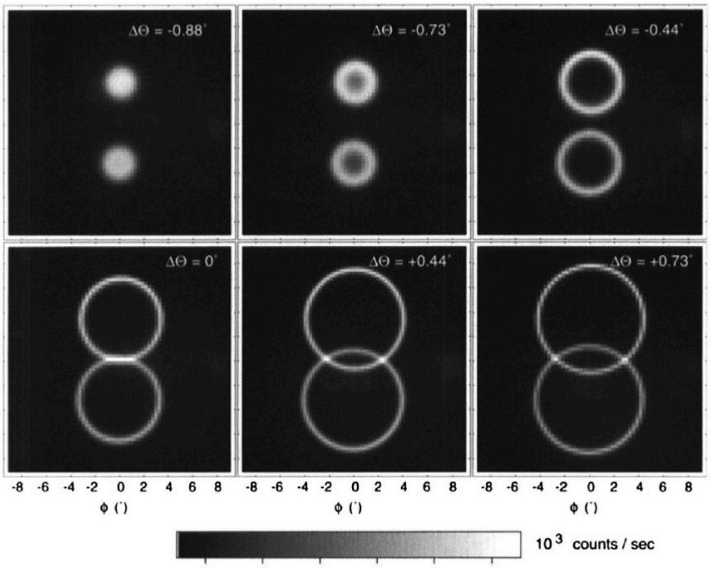
walk-off is eliminated for **θ** = 90°

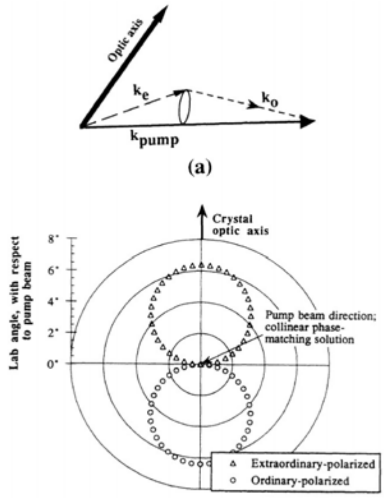
Temperature tuning

For some crystals, notably lithium niobate, the amount of birefringence is strongly temperature-dependent. As a result, it is possible to phase-match the mixing process by holding **θ** fixed at 90 degrees and varying the temperature of the crystal.

Source: Boyd Ch. 2.12

Non-colinear phase matching in SPDC – Type II





Nonclassical effects from spontaneous parametric down-conversion: Adventures in quantum wonderland

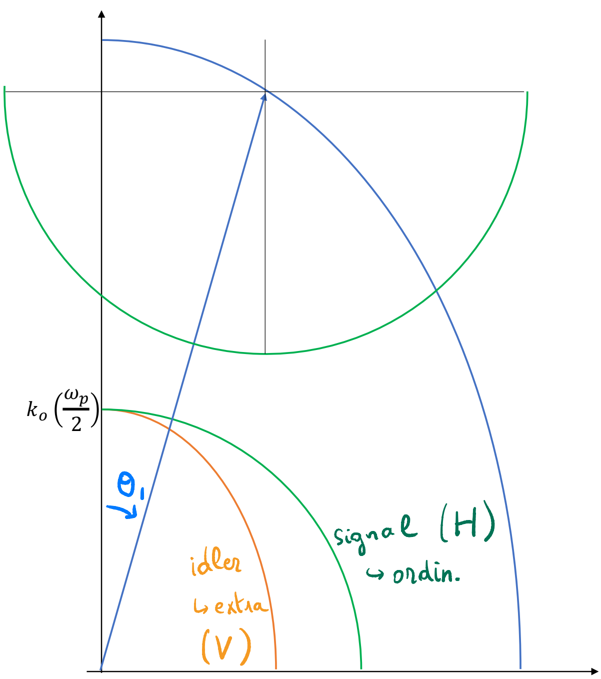
Kwiat, Paul Gregory, Ph.D.

University of California, Berkeley, 1993

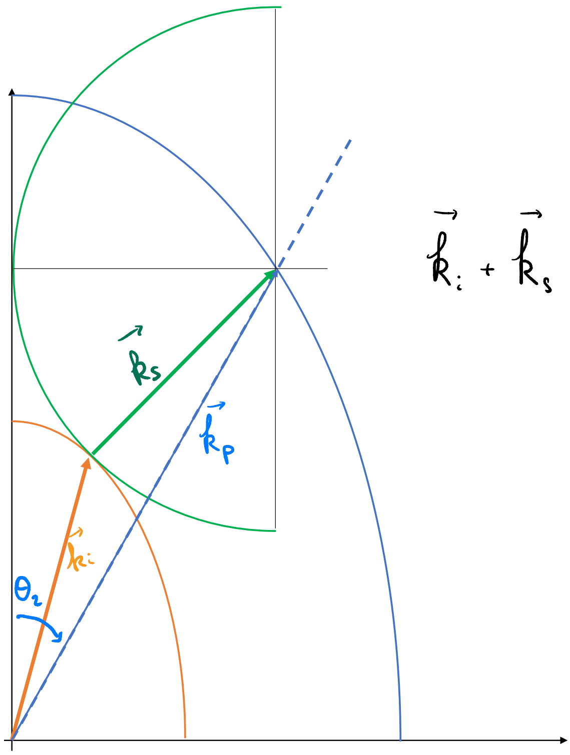
https://copilot.caltech.edu/documents/16795/kwiat-thesis.pdf

Proposal for a loophole-free Bell inequality experiment
Paul G. Kwiat, Philippe H. Eberhard, Aephraim M. Steinberg, and Raymond Y. Chiao
Phys. Rev. A 49, 3209 (1994)
https://journals.aps.org/pra/abstract/10.1103/PhysRevA.49.3209

Type II SPDC – phase matching near degeneracy

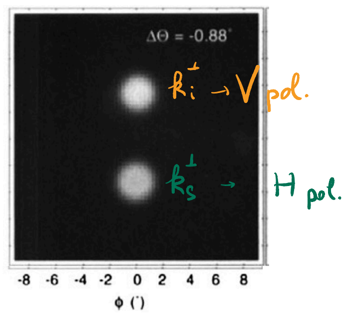


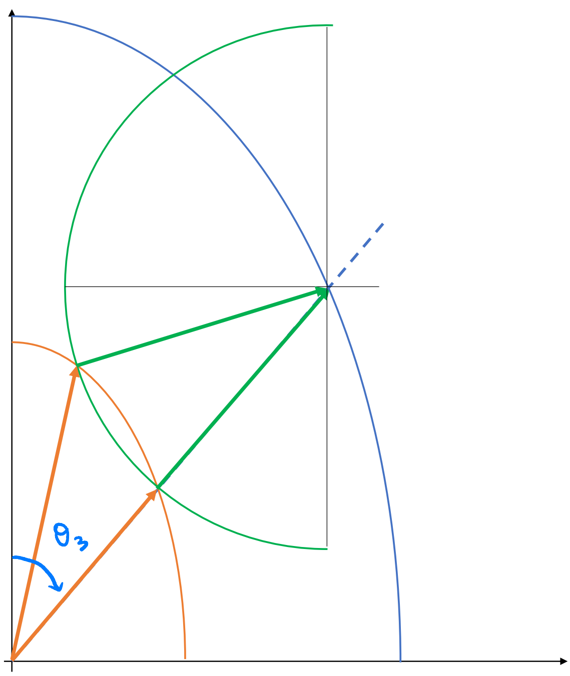
No phase matching possible for this θ_{Pump} because $|\vec{k}_s|+|\vec{k}_i| < |\vec{k}_p|$



$$\vec{k}_i + \vec{k}_s = \vec{k}_p \rightarrow \Delta\vec{k} = \vec{0}$$ phase matching.

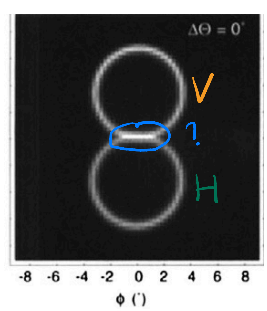
$\Delta\theta = -0.88°$

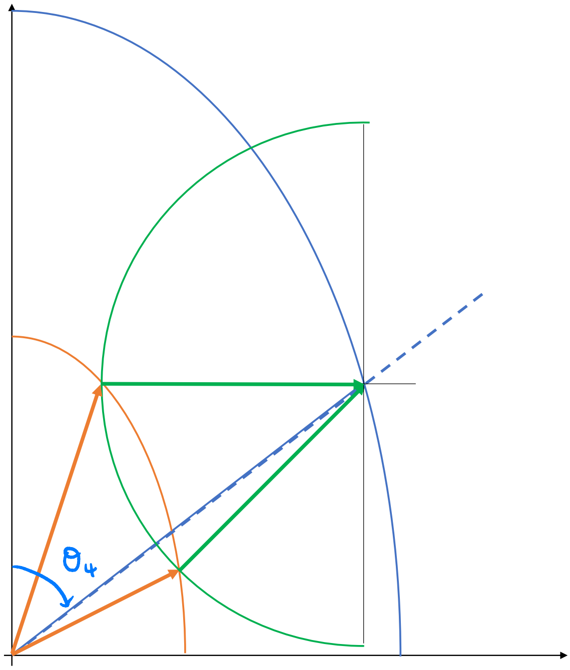


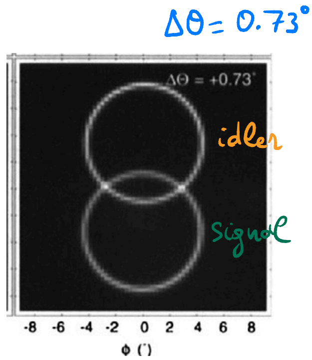


$\Delta\theta = \theta - \theta_3$

$\Delta\theta = 0°$







Entangled photons – limitations of nonlinear crystals

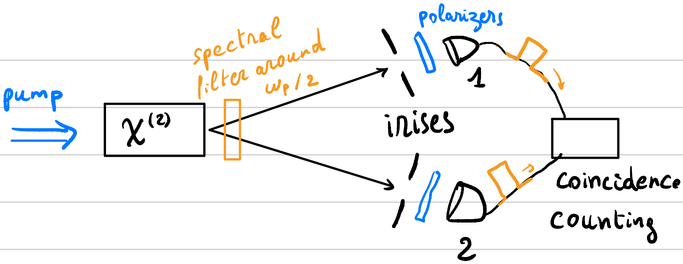
Bulk birefringent crystals

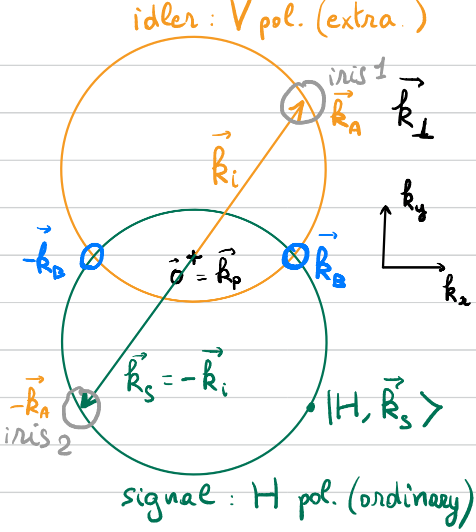
- Due to vectorial phase-matching, only a small fraction of the total twin photon flux can be used.
- Pump powers on the order of 100 mW are typically needed to reach twin photon fluxes on the order of kHz – MHz in a narrow collection cone.
- Beam walk-off due to birefringence causes additional distinguishability (in type II SPDC) and decreases efficiency.

Waveguides or fibers

- The *k*-vector takes only discrete values in the transverse direction (corresponding to a discrete number of guided modes), reducing the phase-matching condition to a one-dimensional equation along the propagation axis.
- Quasi-phase matching by periodic poling of the crystal often required
- The pump field is confined over long distances (high peak intensity) => higher efficiency (scaling as $1/A_{eff}$)
- The guided modes provide a natural basis of spatial modes for rigorous quantization

II) Type II SPDC : polarization entanglement





$$\boxed{|\Psi_{si}\rangle \propto \int_{\vec{k}_\perp \in \text{cone}} |H, \vec{k}_\perp\rangle \otimes |V, -\vec{k}_\perp\rangle}$$

pure state! (quantum superposition)

$$\neq \quad \hat{\rho}_{si} \propto \int_{\vec{k}_\perp} |H, \vec{k}_\perp\rangle |V, -\vec{k}_\perp\rangle \langle H, \vec{k}_\perp| \langle V, -\vec{k}_\perp|$$

↳ classical mixture

<u>Geometry A</u> : $|\tilde{\Psi}_{si}\rangle_A = |H, \vec{k}_A\rangle \otimes |V, -\vec{k}_A\rangle$ → separable postselected state

<u>Geometry B</u> : $|\tilde{\Psi}_{si}\rangle_B = \frac{1}{\sqrt{2}} \left(|H, \vec{k}_B\rangle |V, -\vec{k}_B\rangle + |H, -\vec{k}_B\rangle |V, \vec{k}_B\rangle \right)$

$$\simeq \frac{1}{\sqrt{2}} \left(|HV\rangle + |VH\rangle \right) \quad \text{entangled}$$

III) Connection between indistinguishability and entanglement

Consider a generic two-particle entangled state :

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \quad \text{where} \quad |\uparrow\downarrow\rangle = |\uparrow\rangle_1 \otimes |\downarrow\rangle_2$$

↳ If there is a way to distinguish between the two particles, we can write the state : $\boxed{|\tilde{\Psi}\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle \otimes |A\rangle_E - |\downarrow\uparrow\rangle \otimes |B\rangle_E \right)}$

↳ other degree of freedom (internal or external.)

↳ reduced state after tracing out "E".

We assume that $\langle A|B\rangle = 0$

If the measurement does not resolve the property $|A\rangle_E$ vs. $|B\rangle_E$, the reduced state is $\hat{\rho}_{red} = Tr_E(|\tilde{\Psi}\rangle\langle\tilde{\Psi}|) = \langle A|\tilde{\Psi}\rangle\langle\tilde{\Psi}|A\rangle + \langle B|\tilde{\Psi}\rangle\langle\tilde{\Psi}|B\rangle$

$$\boxed{= \frac{1}{2}\left(|\uparrow\downarrow\rangle\langle\uparrow\downarrow| + |\downarrow\uparrow\rangle\langle\downarrow\uparrow| \right)}$$

↳ classical mixture, not entangled.

On the contrary, if $|A\rangle_E = |B\rangle_B$, we have

$$|\tilde{\Psi}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) \otimes |A\rangle_E \qquad \text{(no correlation with environment)}$$

and $\hat{\rho}_{red}$ remains entangled.