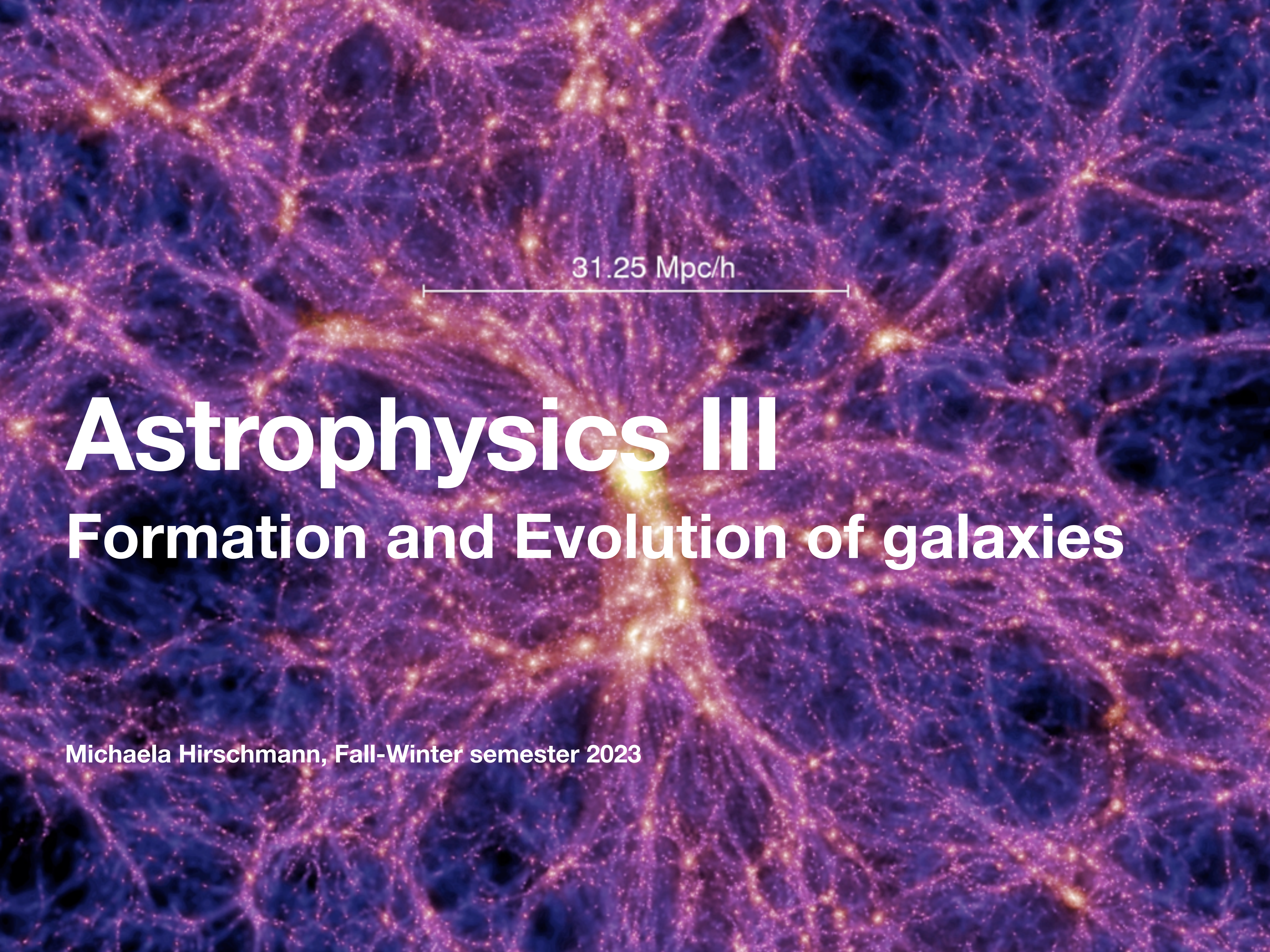


Astrophysics III

Formation and Evolution of galaxies

Michaela Hirschmann, Fall-Winter semester 2023

Lecture content and schedule

- *Chapter 1*: Introduction (galaxy definition, astronomical scales, observable quantities — repetition of Astro-I)
- *Chapter 2:* Brief review on stars
- *Chapter 3*: Radiation processes in galaxies and telescopes;
- *Chapter 4:* The Milky Way
- *Chapter 5*: The world of galaxies I
- *Chapter 6*: The world of galaxies II
- *Chapter 7:* Black holes and active galactic nuclei
- *Chapter 8:* Galaxies and their environment;
- *Chapter 9:* High-redshift galaxies

Part I: Observational basics & facts of galaxies first 7 lectures

- *Chapter 10*:
  - Cosmology in a nutshell; Linear structure formation in the early Universe
- *Chapter 11*:
  - Dark matter and the large-scale structure
  - Cosmological N-body simulations of dark matter
- *Chapter 12*: Populating dark matter halos with baryons: Semi-empirical & semi-analytical models
- *Chapter 13:* Modelling the evolution of gas in galaxies: Hydrodynamics
- *Chapter 14*: Gas cooling/heating and star formation
- *Chapter 15*: Stellar feedback processes
- *Chapter 16*: Black hole growth & AGN feedback processes
- *Chapter 17*: Modern simulations & future prospects

Part II: Theory & models of galaxy evolution processes second 7 lectures

Outline of this lecture

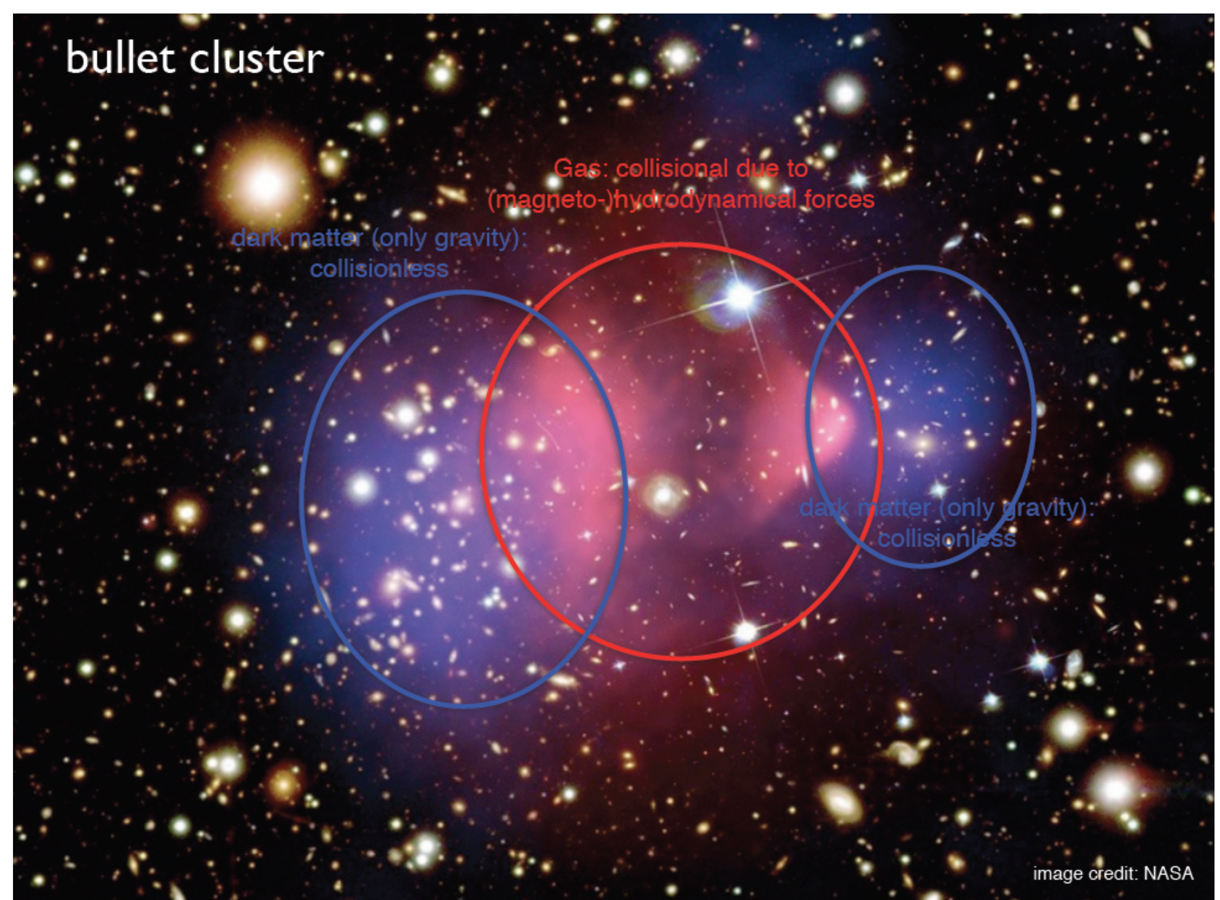
- Why hydrodynamics?
- Eulerian methods/mesh codes
  - Advection schemes
  - Riemann problems
  - Adaptive mesh refinement
- Lagrangian methods/SPH codes
  - SPH & Equations of motions
  - Modern SPH: Density-entropy formulation
- "Combined" methods: Moving-mesh scheme

The importance of modelling gas

- Everything we can observed/see is gas or is made from gas

image credit: NASA

- Need to follow gas dynamics
  - To form galaxies and stars in a spatially resolved fashion
  - To study the interstellar, intergalactic & intracluster medium in 3D
  - …e.g. to compare images of simulated with observed galaxies



What is fluid/hydrodynamics ?

- Study of the behaviour of fluids—liquids and gases—and their interactions via external forces.
- Focus on understanding the macroscopic properties of fluids, such as velocity, pressure, density, and temperature.
- Approximation that the individual gas/fluid elements are treated collectively as a continuous medium with well-defined macroscopic properties.
- Founded on the principles of conservation of mass, momentum, and energy —> the total mass, total momentum, and total energy of a fluid system are constant —> hydrodynamic equations
- Collisional Nature: gas particles in galaxies can interact collisions, pressure forces, and radiative processes. —> energy dissipation, mass redistribution, and shocks

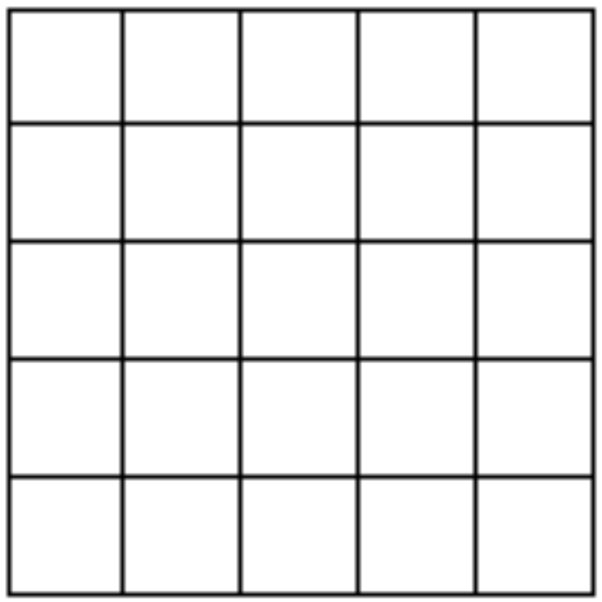
Advantages of hydrodynamic approach

- Several advantages compared to semi-analytics:
  - Interaction between DM and baryons is taken into account (without relying on approximations like adiabatic contraction)
  - Dynamics of the diffuse cooling gas is captured in full generality (no spherical symmetry or quasi-static evolution)
  - Once some sub-resolution schemes for feedback are adopted, hydro-sims can treat the subsequent evolution of the supernovae/AGN-driven winds fully self-consistently
  - Automatically account for morphological transitions during mergers, environmental processes etc.
- Hydrodynamics suitable for such complex problems, but there is no perfect numerical recipe for all hydrodynamic problems
- Depending on the problem of investigation, one may also include different other physics *(see e.g. Springel+10)* like
  - radiation-hydrodynamics (accounting for the interaction of photons with the gas)
  - magneto-hydrodynamics (accounting for the interaction of magnetic fields with the gas) and cosmic rays

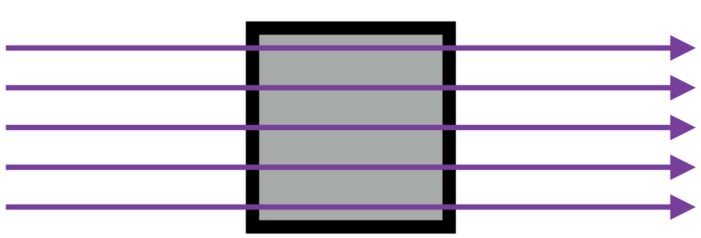
Eulerian versus Lagrangian methods

Eulerian/grid methods

discretise space
finite-volume scheme



use a grid fixed in space



Flow through fixed cell

Moving mesh

discretise space
finite-volume scheme

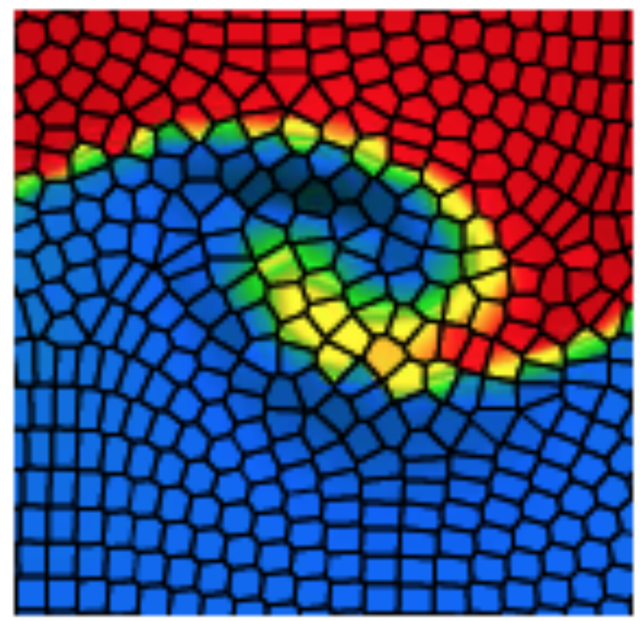
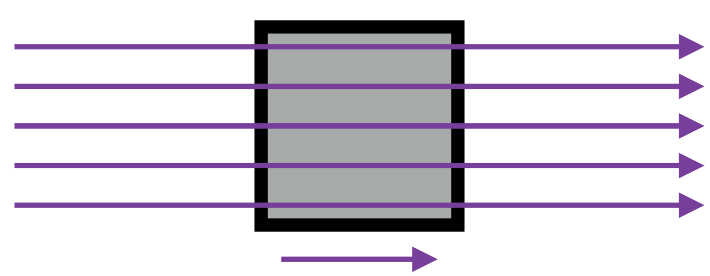


image credit: V. Springel

uses an unstructured mesh moving with the flow

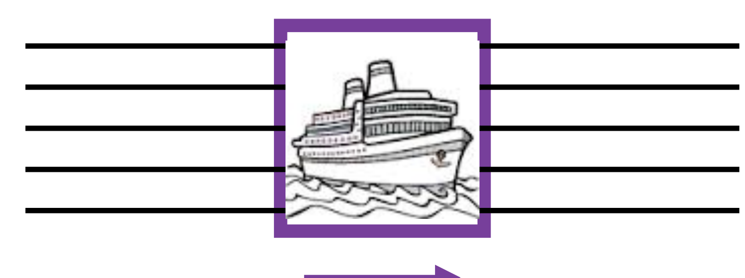


Flow through cell moving with flow

Lagrangian/SPH methods

discretise mass

use particles for the gas (like in N-body) which move with the flow



Fixed element moving with flow

Outline of this lecture

- Why hydrodynamics?

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  - Advection schemes
  - Riemann problems
  - Adaptive mesh refinement

- Lagrangian methods/SPH codes
  - SPH & Equations of motions
  - Modern SPH: Density-entropy formulation

- Combined methods: Moving-mesh scheme

The hydrodynamic equations

- Eulerian/Grid-based methods are based on the fluid-dynamical, so called Euler equations
- These equations are expressed in terms of conserved quantities:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

- Continuity equation, mass conservation

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) + \nabla P = \mathbf{0}$$

- Momentum equation, momentum conservation

$$\frac{\partial \rho e_{\rm tot}}{\partial t} + \nabla \cdot ((\rho e_{\rm tot} + P)\mathbf{u}) = 0$$

- First law of thermodynamics, energy conservation

$$e_{\rm tot} = e + \frac{u^2}{2}$$

where e = internal energy per unit mass

$$P = (\gamma - 1)\rho e$$

- Equation of state for an ideal, monoatomic gas with the polytropic index γ = 5/3

Basic methodology of a grid code

- Primitive variables: $\rho, \vec{u}, P$
- Conservative variables, density, momentum density and total energy density $q = (\rho, \rho\vec{u}, \rho e)$
- Standard approach: divide space into grid cells and store the cell-averaged conservative quantities at all the grid points
- Use re-construction schemes, which can take several neighbouring cells into account to reconstruct the field/distribution of any variable
- Advection: solve hydro-equations by computing the flux of mass, momentum and energy across grid cell boundaries/contact discontinuities, i.e. solve Riemann problem
- Calculate new cell-averaged conservative quantities and do re-constructions etc…

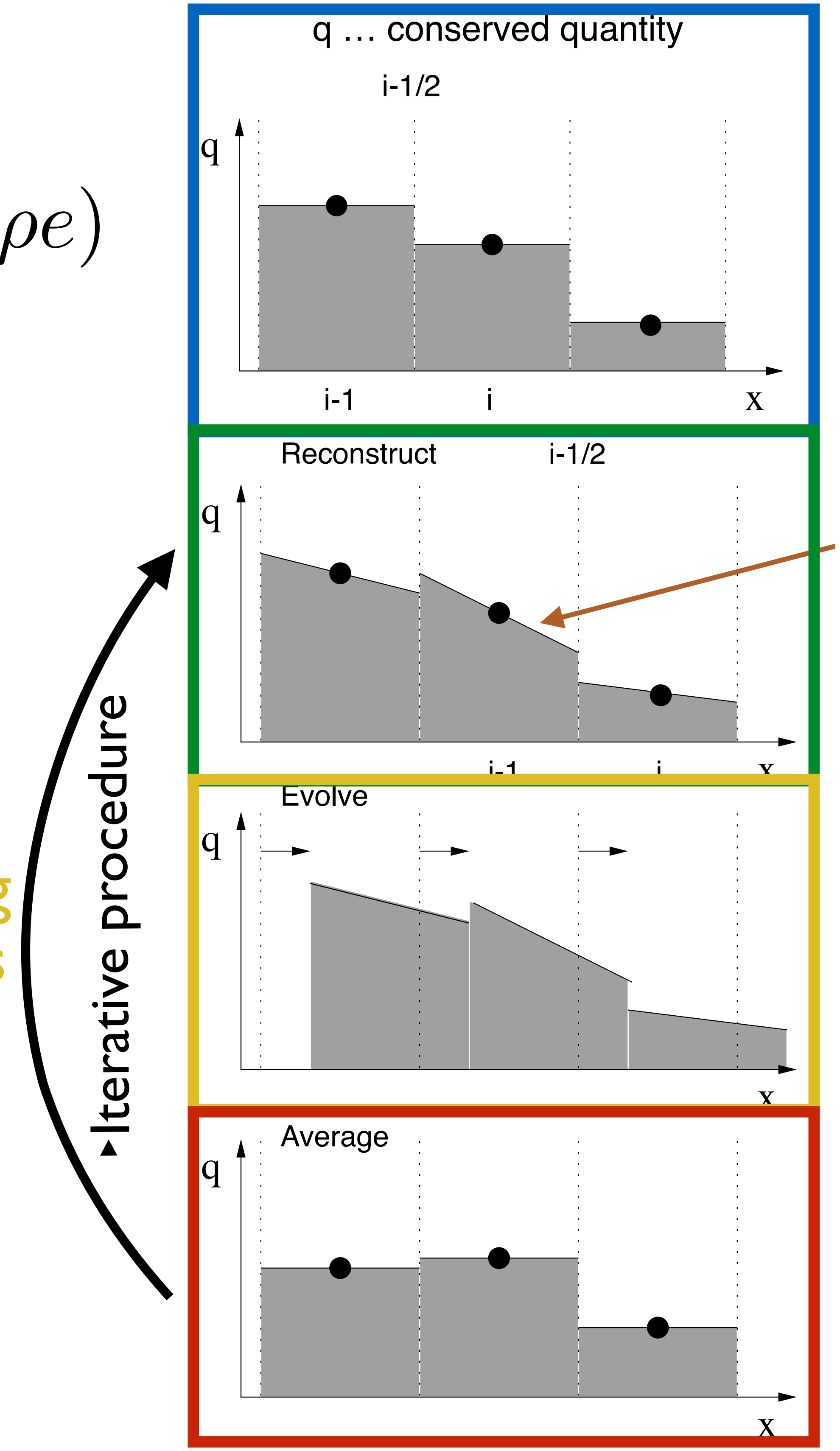


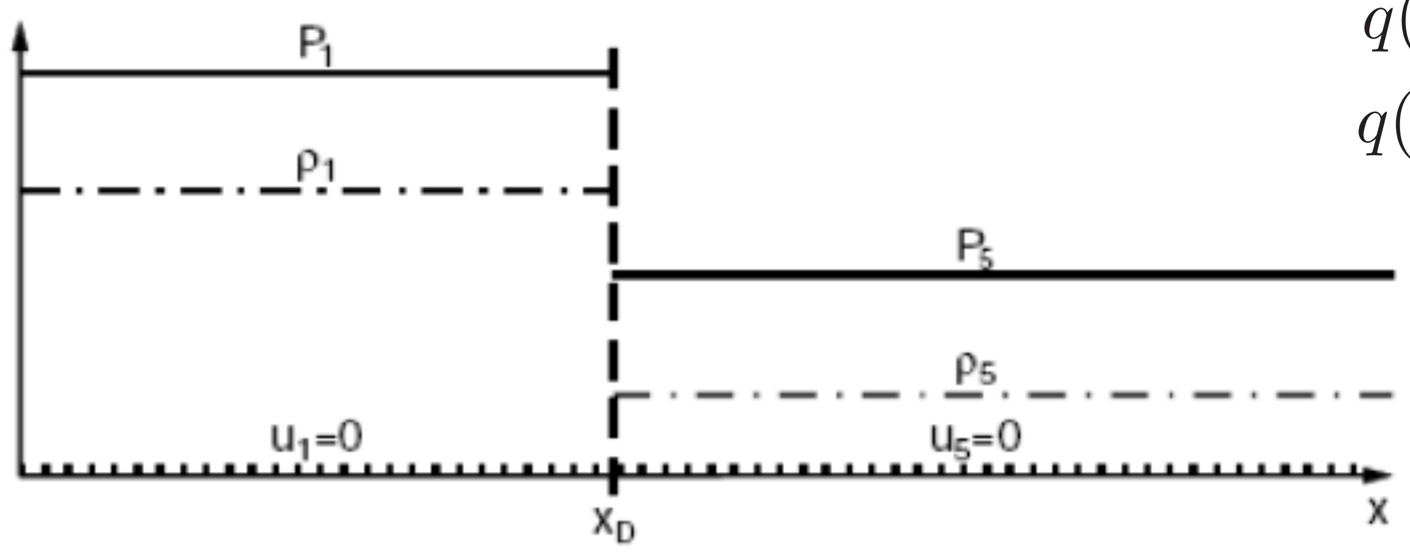
image credit: C.P. Dullemond

How to solve coupled hydro equations?

- With suitable advection scheme, we can solve a simple test case

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

- Hydrodynamic equations are, however, coupled —> No simple global decomposition possible
- To finally fully solve the hydro equations at each cell interface (and to capture shocks and contact discontinuities), we have to solve the "Riemann" problem
- Specific example for Riemann problem: shock tube



$$q(x, t = t_0) = q_{\mathrm{L}} \quad \text{for } x < x_0$$
$$q(x, t = t_0) = q_{\mathrm{R}} \quad \text{for } x > x_0$$

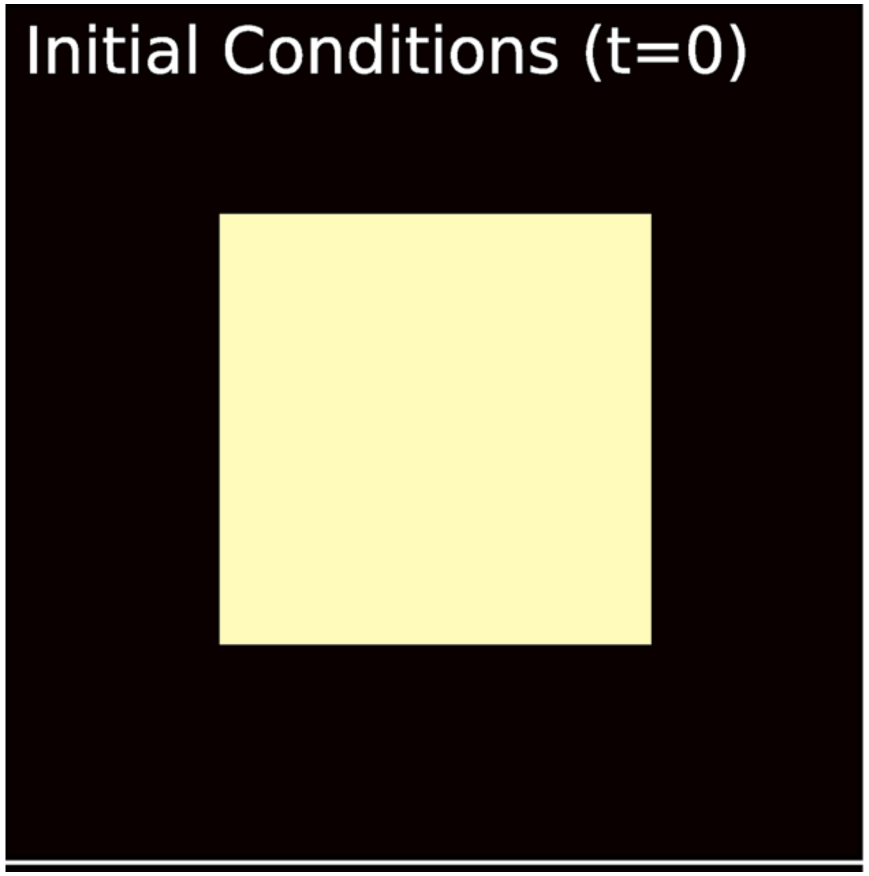
Riemann solvers

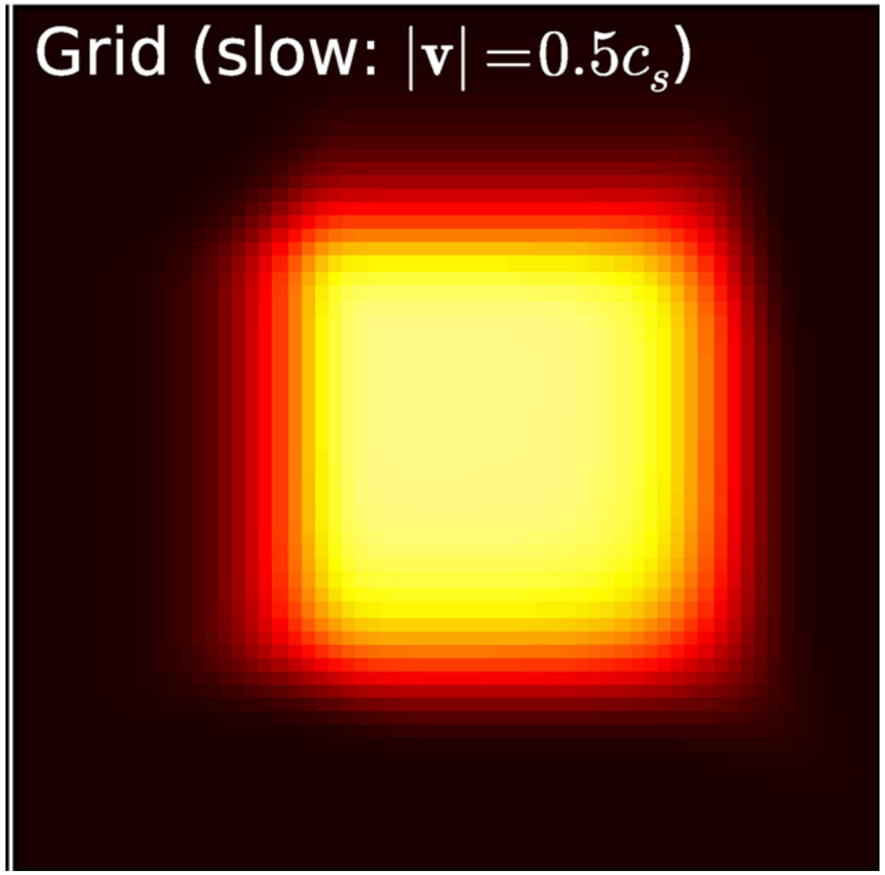
- There exists an exact solution for the general Riemann problem, can be found in textbooks (e.g. Courant & Friedrichs '48)
  - One can show that solutions are self-similar, i.e. they only depend on $q(x,t) = q(x - x_0/(t - t_0))$
  - Very time-consuming
  - Various approximate solutions, e.g.
    - Roe's linearised Riemann solver (Powell+99)
    - HLLE method (Harten, Lax, van Leer and Einfeldt)
    - HLLC method (Harten-Lax-van Leer-Contact)
- A more detailed description of these methods is outside the scope of this lecture…
- They all have their pro's and con's…

Grid codes in 3D

- So far, discussed everything in 1D, but galaxies are 3D
- Directionally split schemes
  - Apply 1D hydro-solver alternately along the different directions
  - Less memory needed
  - But spherical symmetry is less well preserved
- Unsplit schemes
  - Compute fluxes for all interfaces of cells
  - Update cell values once per time step
  - E.g. used in Ramses
- (Self-)gravity (hydro in combination with DM-N-body) can be simply added in the momentum equation

Success and Limitations



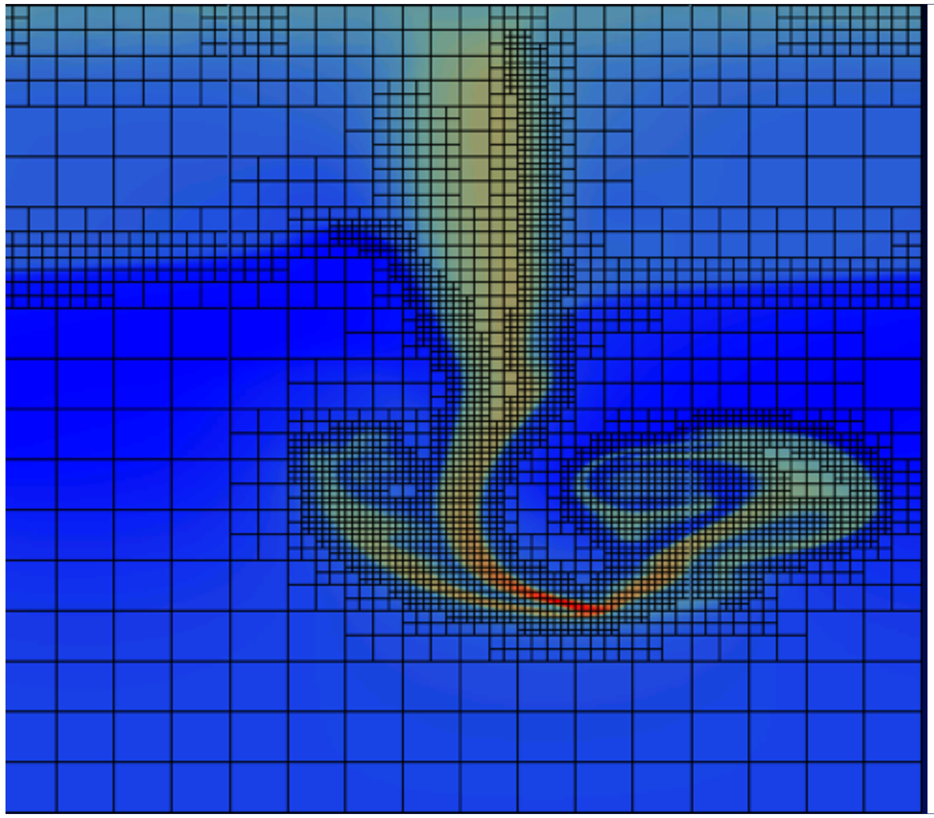
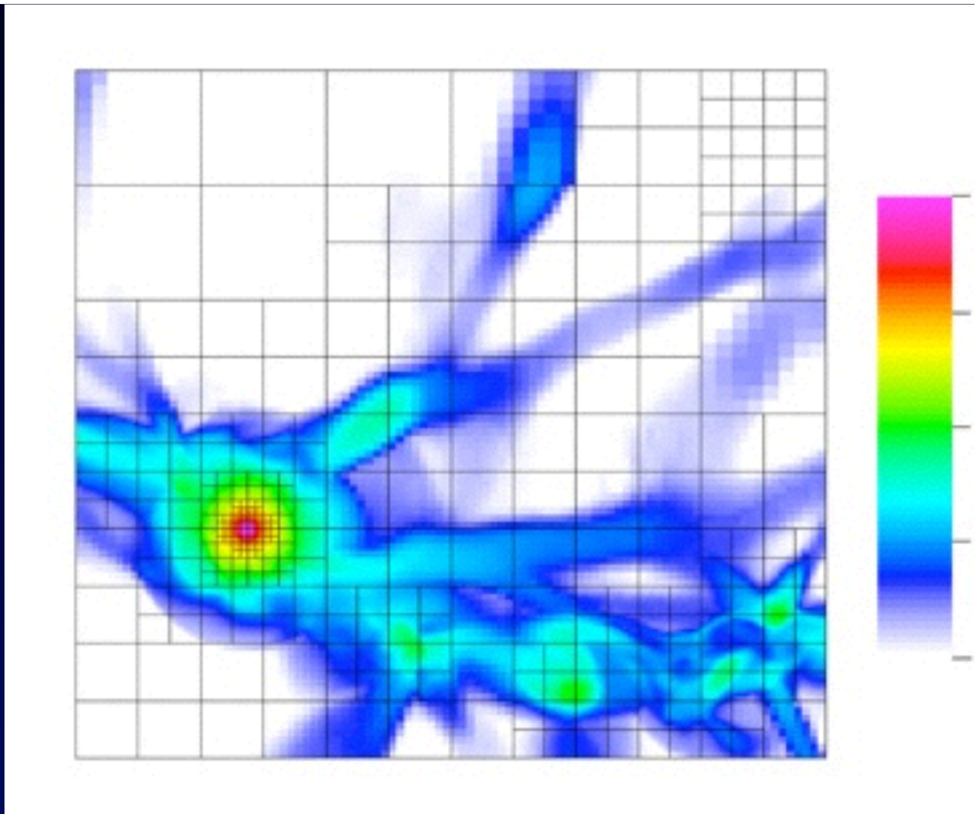
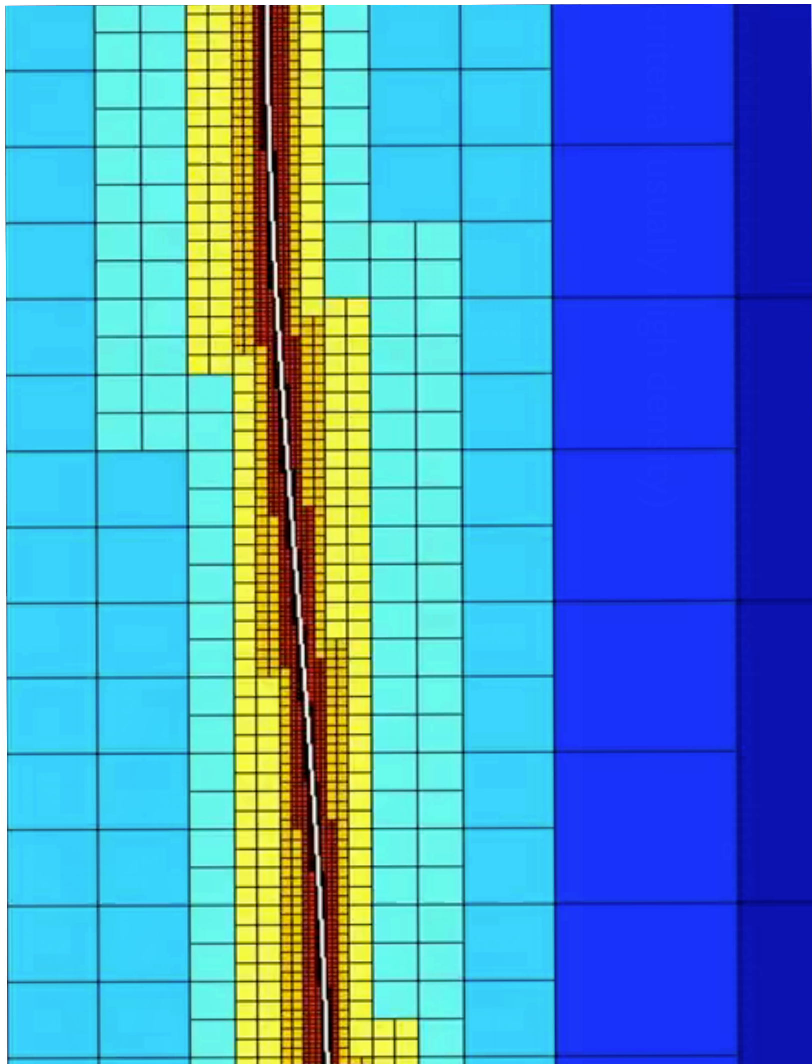


- Main strength is the accurate hydro,
- Automatically accounts for contact discontinuities, shocks etc.

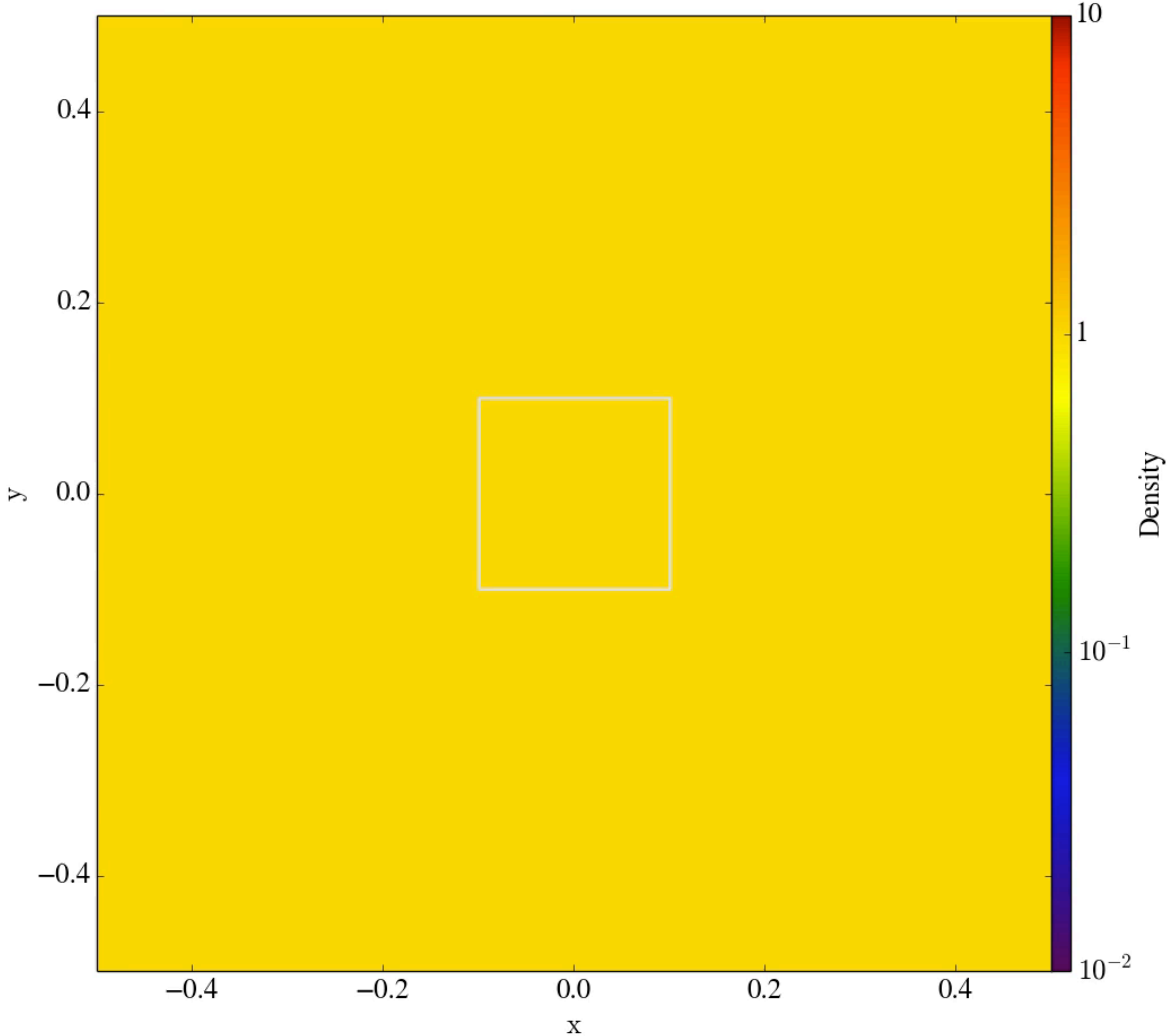
- One major limitation of grid codes is their spatial resolution when we use a fixed grid size, increasing the grid size globally is computationally very expensive
- Advection errors: somewhat diffusive (no exact energy conservation) —> unphysical forces
- No exact conservation of angular momentum
- No Galilean invariance (Galilean transformation to a different inertial system)

Adaptive mesh refinement

- A good possibility to overcome the limitation in spatial resolution is the adaptive mesh refinement technique:
    - the local resolution (i.e. number of grid cells) is adapted according to refinement criteria (typically high density, more cells at high density regions)

A test case for AMR: Sedov explosion



Summary: Eulerian methods on a grid

- Gas flow through fixed cells/grids
- Basic variables: density, momentum density and energy density
- Their evolution described by Euler equations —> conservation of mass, momentum and energy
- Transport of fluid flow through cells: reconstruction — advection (solving Riemann problem) — cell averaging
- Variable local resolution —> adaptive mesh refinement
- Different advantages and disadvantages…

Outline of this lecture

- Why hydrodynamics?

- Eulerian methods/mesh codes
  - Advection schemes
  - Riemann problems
  - Adaptive mesh refinement

- Lagrangian methods/SPH codes
  - SPH & Equations of motions
  - Modern SPH: Density-entropy formulation

- Combined methods: Moving-mesh scheme

The hydro-equations in Lagrangian form

- **Follow a gas element along its path** and see how it changes its direction of motion, its density & its pressure…
- To derive the corresponding Lagrangian form of the hydro equations, we need do introduce the co-moving convective derivative D_t as

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla}$$

- With that we can re-write the hydro-equations for an inviscous fluid:

$$\frac{d\rho}{dt} + \rho \vec{\nabla} \cdot \vec{u} = 0$$

- Continuity **equation,** a gas parcel changes its density when the gas motion converges

$$\frac{d\vec{u}}{dt} = -\frac{\vec{\nabla} P}{\rho}$$

- Momentum **equation,** a gas parcel will be accelerated due to a force which is the pressure gradient (+ grav. pot.)

$$\frac{de}{dt} = -\frac{P}{\rho} \vec{\nabla} \cdot \vec{u}$$

- First law of thermodynamics, the thermal energy of a gas parcel changes only as a result of adiabatic compression/expansion

where e = thermal energy per unit mass

$$P = (\gamma - 1)\rho e$$

- Equation of state for an ideal, monoatomic gas with the polytropic index γ = 5/3

‣ Exercise: conversion for momentum and energy equation

Classic continuity equation:

$$\frac{\partial \rho}{\partial t} + \underset{\sim}{\nabla} \cdot (\rho \underset{\sim}{u}) = 0$$

Co-moving derivative : $\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \underset{\sim}{u} \cdot \underset{\sim}{\nabla}$

"partial time derivative at fixed position changed, due to move of the fluid element"

$$\frac{\partial \rho}{\partial t} + \rho \underset{\sim}{\nabla} \cdot \underset{\sim}{u} + \underset{\sim}{u} \cdot \underset{\sim}{\nabla} \rho = 0$$

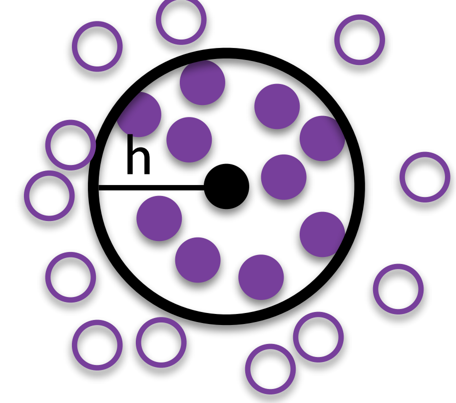
$= d\rho/dt$

$$\Rightarrow \frac{d\rho}{dt} + \rho \underset{\sim}{\nabla} \cdot \underset{\sim}{u} = 0$$

Continuity equation in Lagrangian form

Smoothed particle hydrodynamics

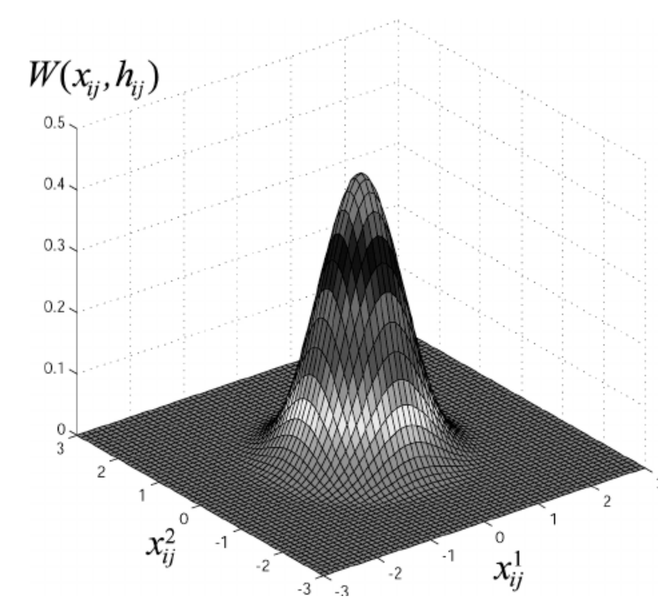
- Idea: Treat a hydrodynamic fluid in a completely mesh-free fashion —> Use a set of sampling particles to represent the fluid
- Technique to approximate the continuum dynamics of fluids through the use of particles which can be viewed as interpolation points
- The density of a particle in SPH is given by smoothing over nearest neighbour particle masses within a "Kernel"



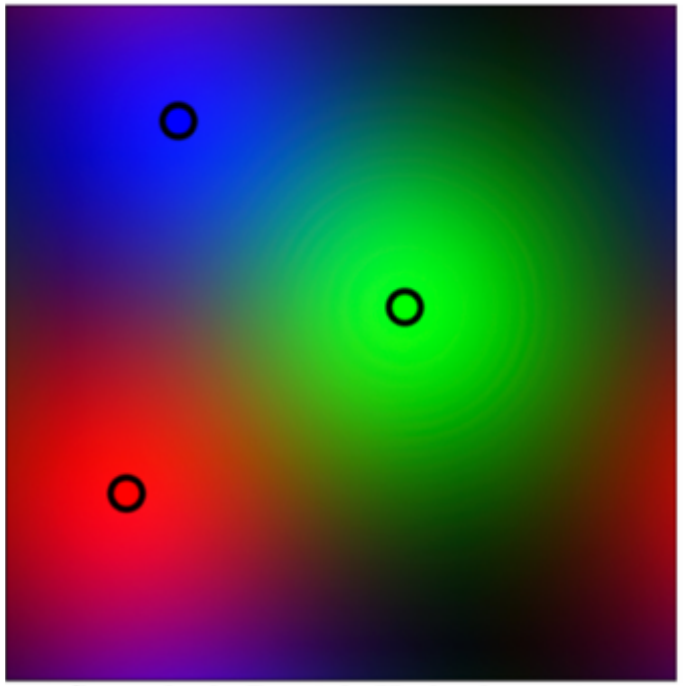
$$\rho(\vec{x}) = \sum_j m W(\vec{x} - \vec{x}_j, h)$$

- Kernel depends on inter-particle distance r=|$\mathbf{x}_i$-$\mathbf{x}_j$| and on "smoothing length" h
- Most commonly the cubic spline Kernel is used

$$W(r,h) = \frac{8}{\pi h^3} \begin{cases} 1 - 6(r/h)^2 + 6(r/h)^3 & 0 \le r/h \le 1/2 \\ 2(1 - r/h)^3 & 1/2 \le r/h \le 1 \\ 0 & 1 < r/h \end{cases}$$



Smoothed particle hydrodynamics



- For any field F, we can define a smoothed version

$$F_s(\vec{x}) = \sum_j \frac{m_j}{\rho_j} F_j W(\vec{x} - \vec{x_j}, h)$$

- The derivative of field F is given by

$$\vec{\nabla} F_s(\vec{x}) = \sum_j \frac{m_j}{\rho_j} F_j \vec{\nabla} W(\vec{x} - \vec{x_j}, h)$$

- Define constraints for the smoothing length h_i, e.g. requiring that the Kernel volume contains a constant mass for the estimated density

$$\rho_i h_i^3 = const. \propto M_{kernel} = N_{ngb} * m_i$$

- In principle, one could take these equations for **F** (replace with **P** or **u***), and insert these expressions into the Lagrangian fluid equations to get the equations of motions.

*Note that bold print means vector!

Entropy conserving equation of motion

- After some algebra, as demonstrated by Springel & Hernquist 2002, we obtain the EoM

$$\frac{d\vec{u}}{dt} = -\sum_j m_j \left(f_j \frac{P_j}{\rho_j^2} \vec{\nabla} W(\vec{x}_i - \vec{x}_j, h_j) + f_i \frac{P_i}{\rho_i^2} \vec{\nabla} W(\vec{x}_i - \vec{x}_j, h_i) \right)$$

- where the coefficients f_i are defined as $f_i = \left(1 + \frac{h_i}{3\rho_i} \frac{\partial \rho_i}{\partial h_i} \right)$

- In essence, we have transformed a complicated system of partial differential equations into a much simpler set of ordinary differential equations, we only have to solve one equation
- The continuity equation does not have to be evolved explicitly: density can be, at any point, calculated from the particle positions and masses
- The thermal energy and pressure can be derived from the density
- In these forms, the velocity (and thus, the positions dr/dt = u) of SPH particles can be integrated forward in time as in N-body simulations (e.g. using the leap-frog scheme)

Advantages of classic SPH

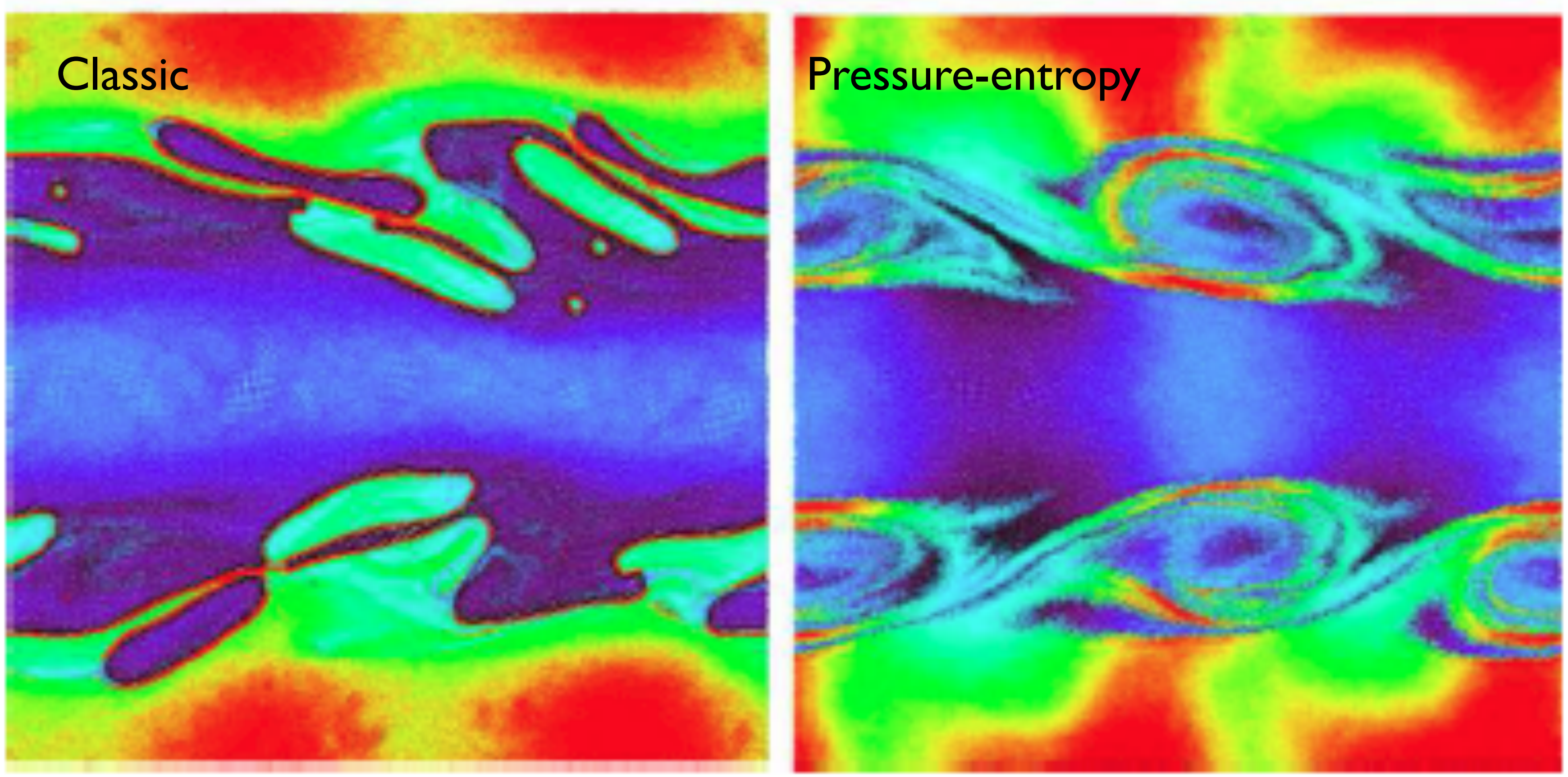
Advantages:

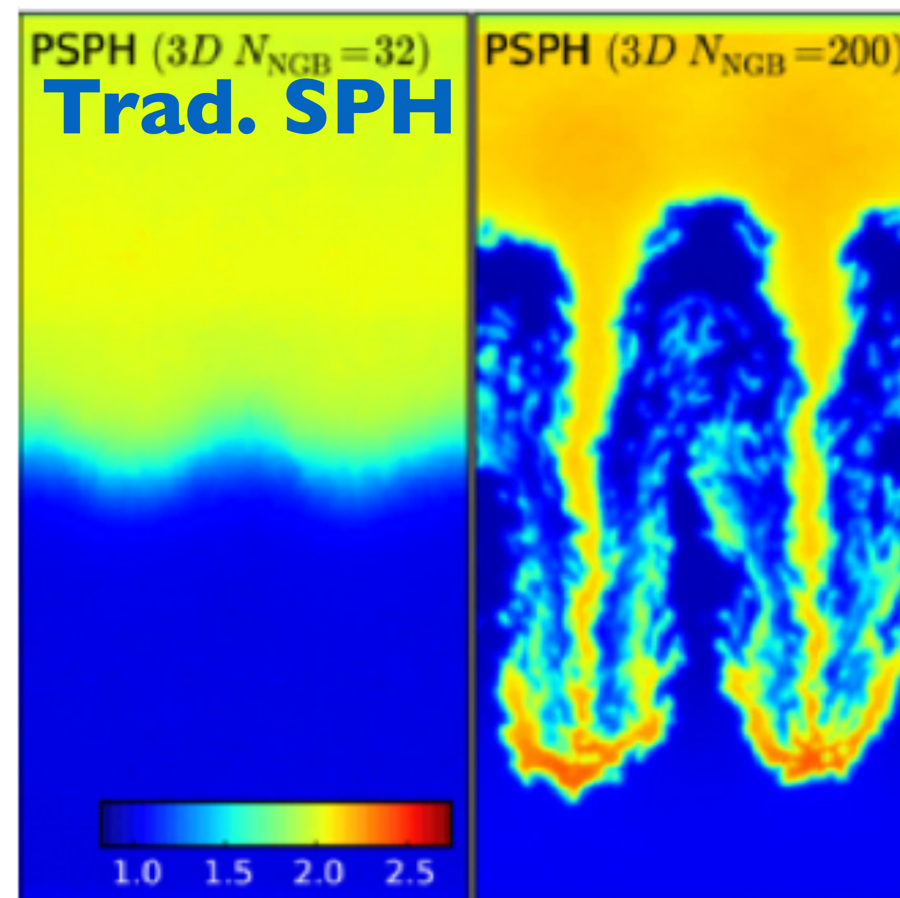
- Numerically rather robust
- Excellent conservation properties
- Lagrangian nature: trace flow and galilean invariant
- Good spatial resolution in high-density regions due to adaptive nature in density
- Couples straight forward to N-body gravity methods
- Intuitive for including sub-resolution physics

Problems of classic SPH

Short-comings:

- Poor description of fluid-mixing: dynamical instabilities/contact discontinuities (Rayleigh-Taylor, Kelvin-Helmholtz) —> modern SPH (smoothing over Pressure)
- Surface tension error: cold Kauffmann “blobs” form, not destroyable —> modern SPH
- Slow numerical convergence
- Compromised accuracy when modelling ISM and galaxy formation





Modern SPH Hydrotests

Summary: Smoothed Particle Hydro

- Gas flow described by particles moving with the flow
- Their evolution described by hydro equations in Langragian form —> conservation of mass, momentum and energy
- Particles are interpolation points, Kernel-weighted density, pressure etc.
- Key quantity: smoothing length, related to the Kernel volume
- Solve one equation of motion for u (integrate), and change hydro properties of gas particle (r, u, e) accordingly
- Modern SPH solves problems of contact discontinuities/fluid mixing (by smoothing over pressure/entropy instead of density)
- Automatic refinement on density/pressure, exactly Galilean invariant, conserves angular momentum (& entropy) exactly
- Somewhat less accurate hydro (contact disc.), slower convergence, but largely alleviated in modern SPH

Outline of this lecture

- Why hydrodynamics?

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  - Riemann problems
  - Adaptive mesh refinement

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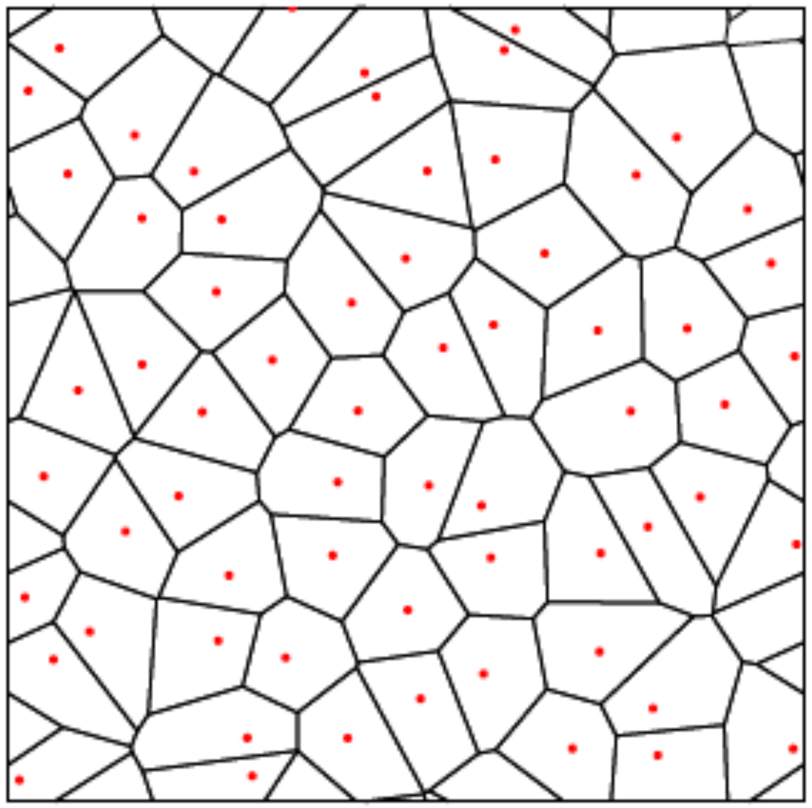
  - Modern SPH: Density-entropy formulation

- "Combined methods": Moving-mesh scheme

Moving mesh hydrodynamics

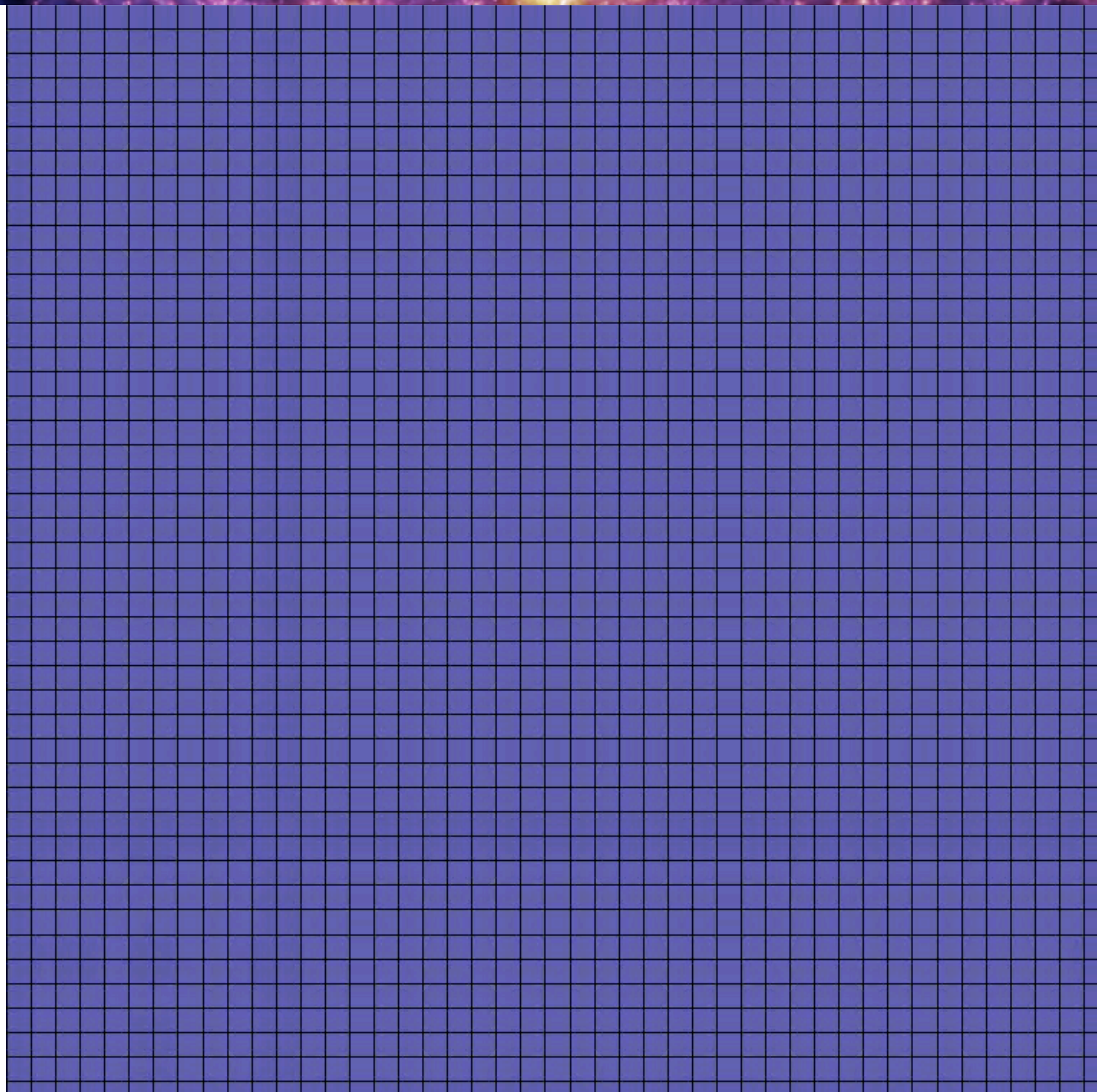
- To combine advantages of both Lagrangian and Eulerian codes: "moving mesh scheme", e.g. Arepo (Springel+10)
- Transform a set of points into an unstructured mesh using a Voronoi tessellation (cell size dependent on density, each particle represents a cell)
- Cells/points are allowed to move with the fluid like in a Lagrangian scheme
- Hydro-solver: based on a second-order unsplit Godunov scheme with an exact Riemann solver
- Transform updated cell average quantities back to particle scheme and move "particles" i.e. grid cells

For a given set of points, a Voronoi tessellation of space consists of non-overlapping cells around each of the sites such that each cell contains the region of space closer to it than any of the other sites. This definition holds both in 2D

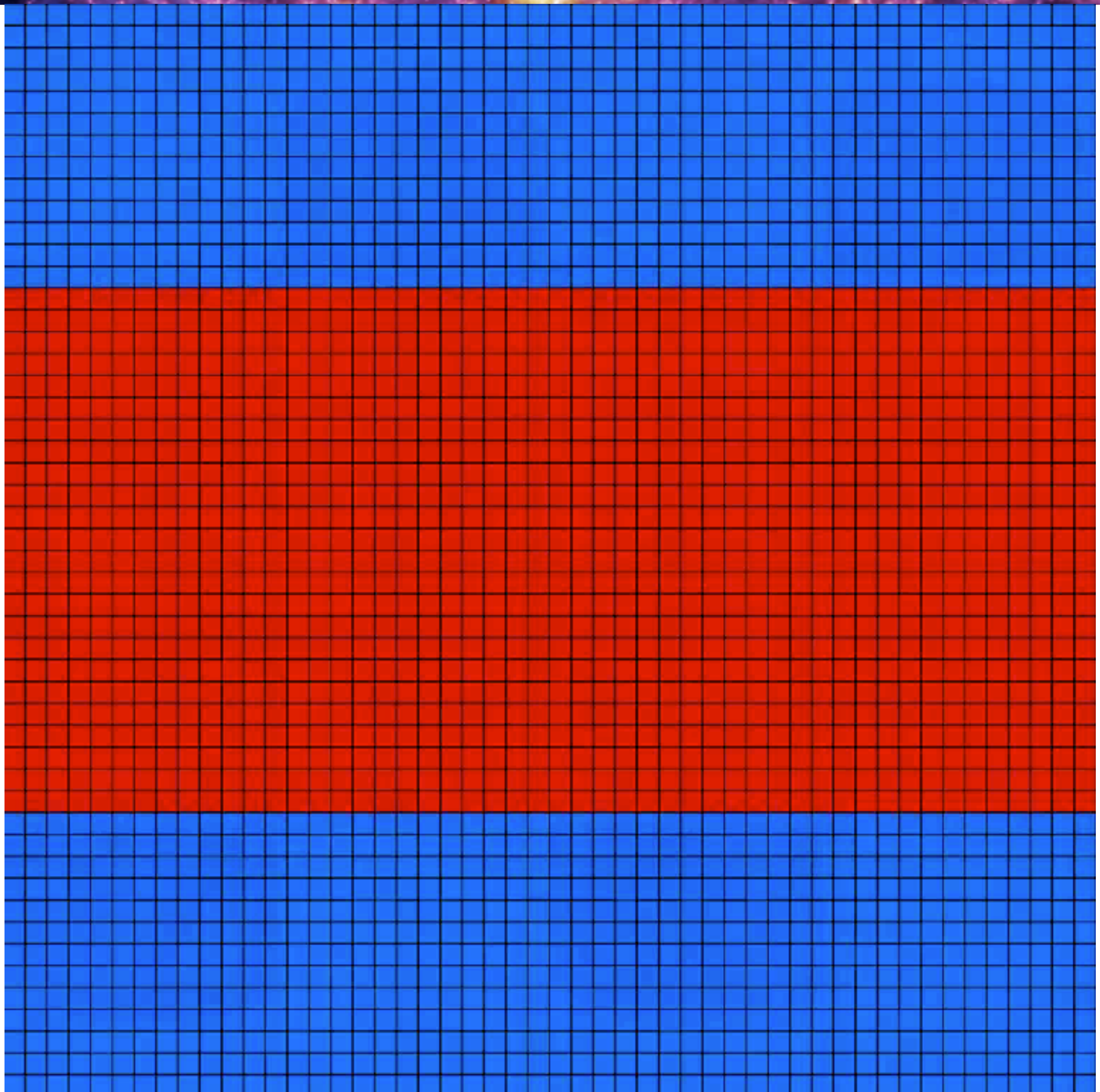


- **Galilean invariant, automatic refinement, good in capturing shocks and contact discontinuities**

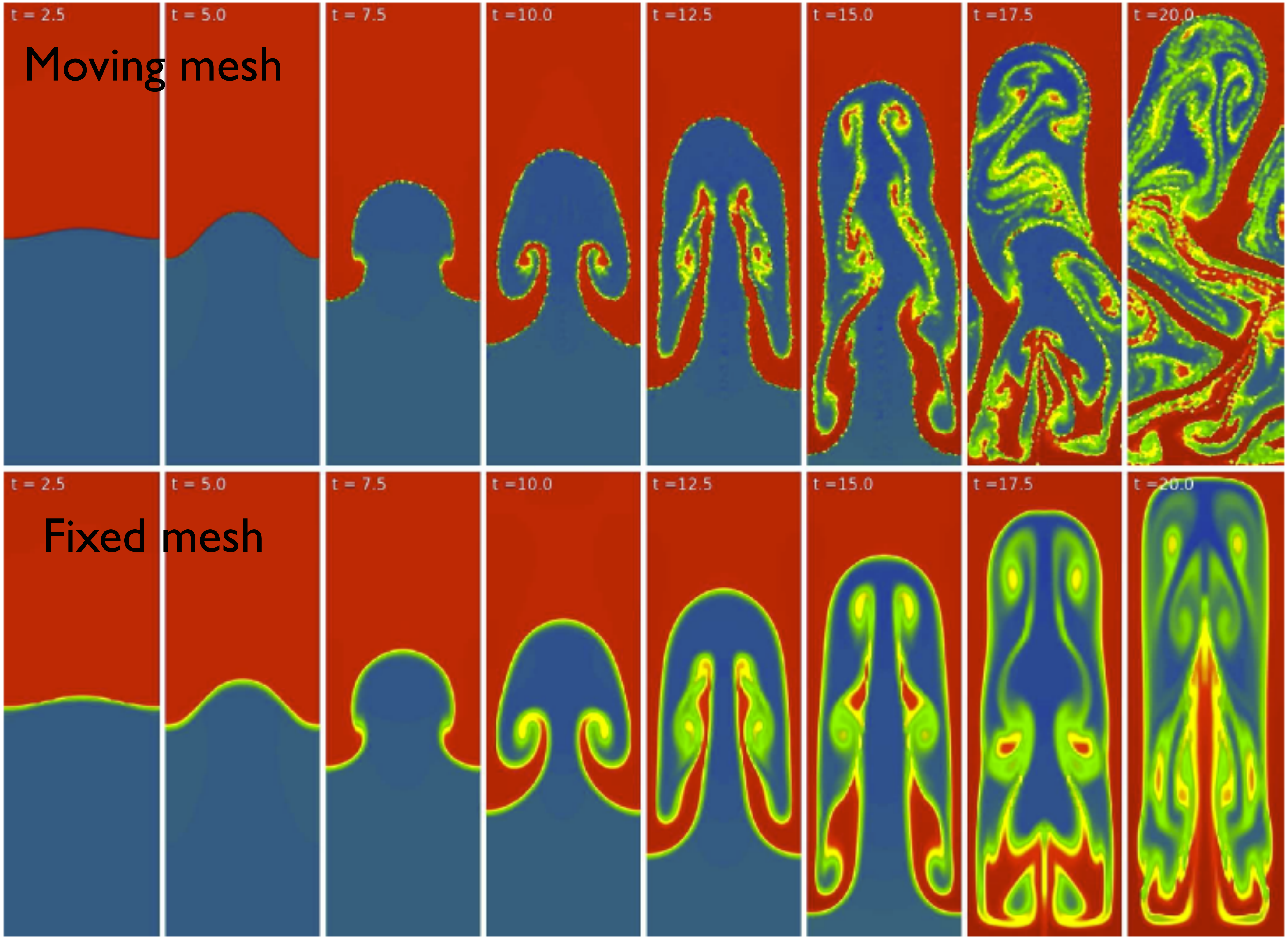
Moving mesh hydrodynamics



Moving mesh hydrodynamics



Moving mesh hydrodynamics

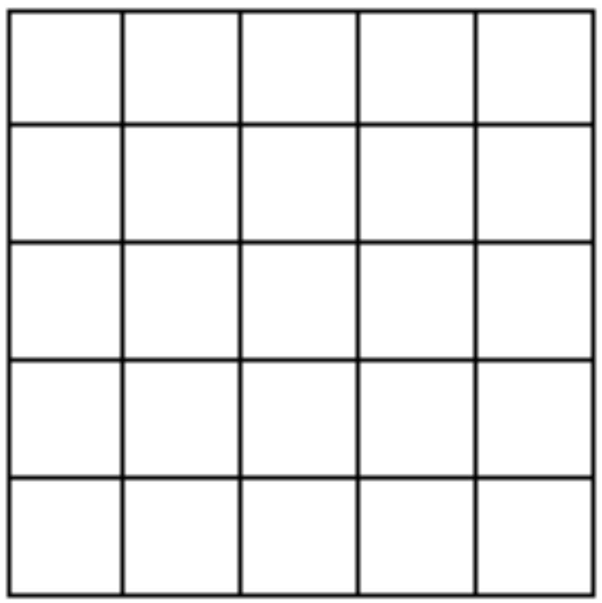


- Thanks to automatic refinement in Arepo, better performance for Rayleigh-Taylor instabilities

Summary: Eulerian vs Lagrangian methods

Eulerian methods

discretise space
finite-volume scheme



use a grid fixed in space

Pro

Accurate hydro

Con

Not galilean invariant
No conservation of angular momentum
Diffusive
Limited spatial resolution alleviated in AMR

Moving mesh

discretise space
finite-volume scheme

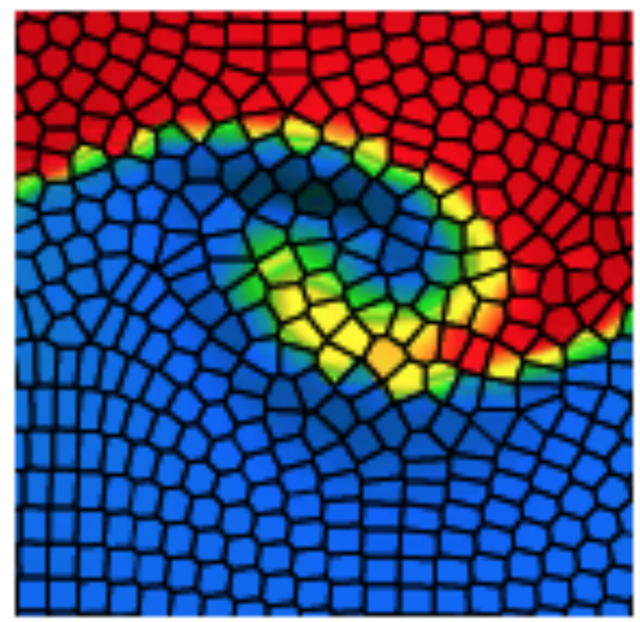


image credit: V. Springel

uses an unstructured mesh moving with the flow

Accurate hydro, automatic refinement on density, exactly Galilean invariant

Overhead ~30% for mesh construction

Lagrangian methods

discretise mass

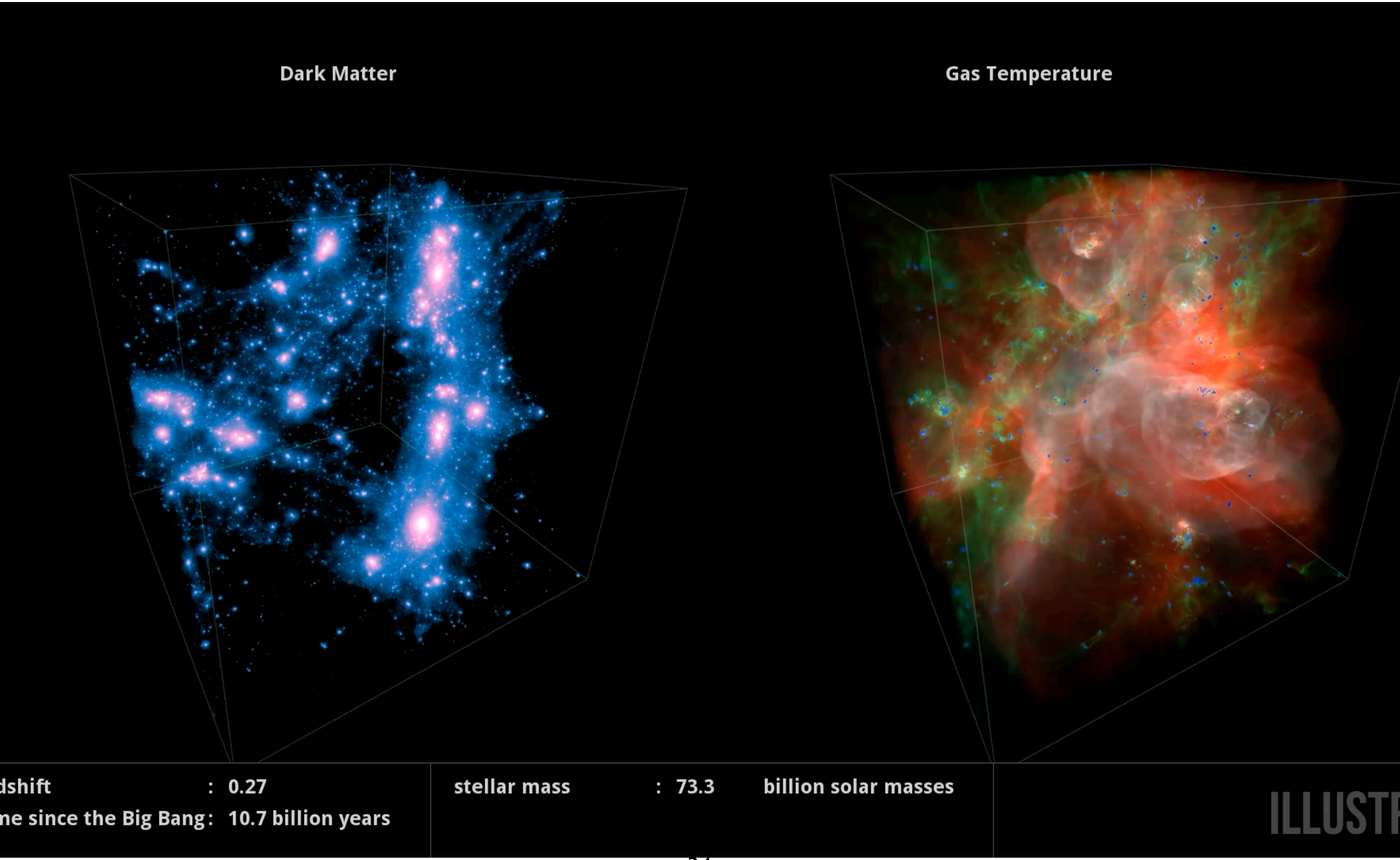
use particles for the gas (like in N-body) which move with the flow

Automatic refinement on density, exactly Galilean invariant, conserves angular momentum (& entropy) exactly

Somewhat less accurate hydro (contact disc.), slower convergence

But alleviated in modern SPH

Famous example simulation: Illustris



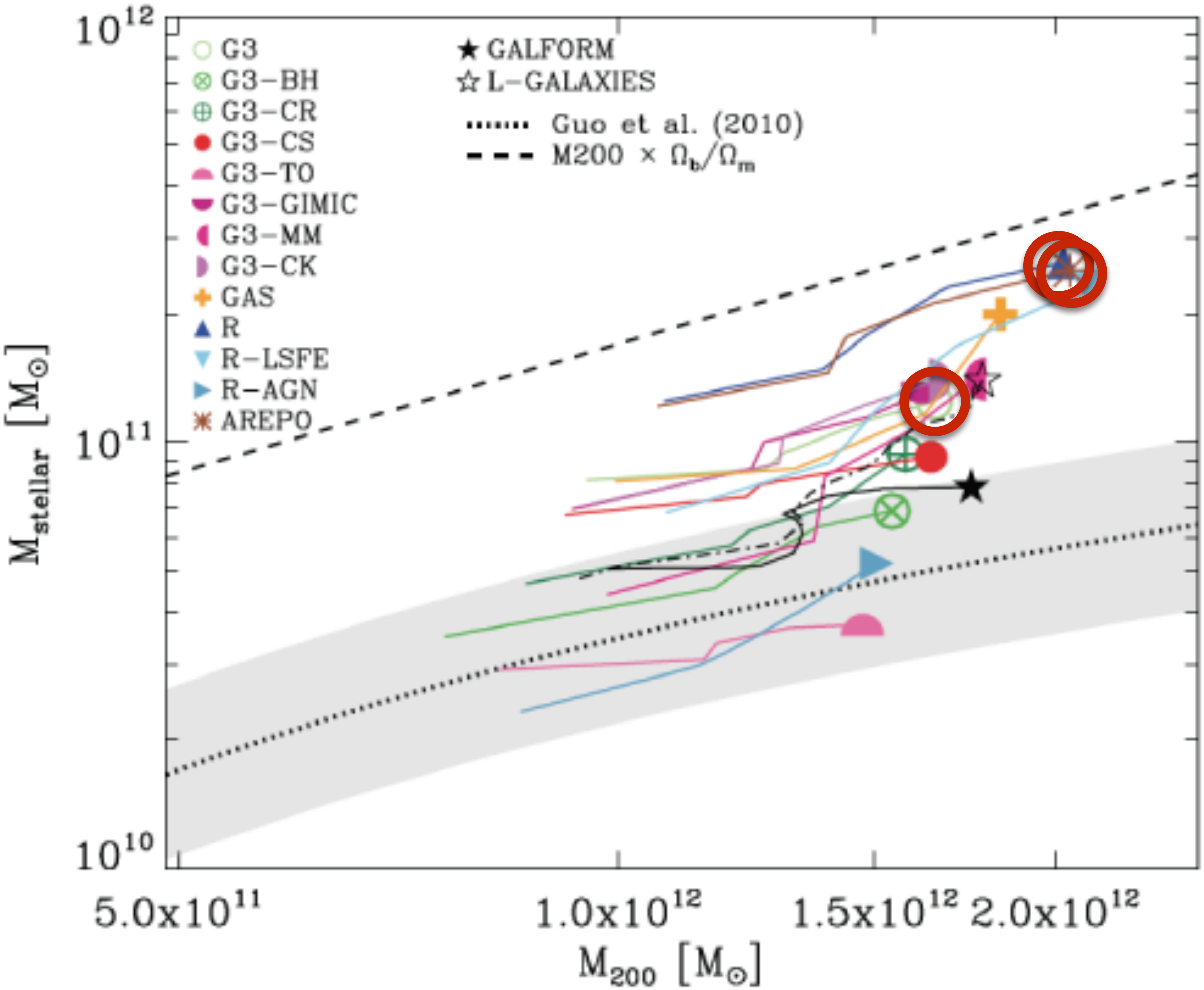
After…

having discussed these different and complex hydrodynamic schemes,

how relevant are they compared to other models for baryons (SF, feedback etc.)?

Relevance of hydro vs baryon physics

Aquila comparison project (Scannapieco et al.)



- Results from different hydrodynamic codes and feedback implementations show a great diversity
- Changes in the feedback model are more dramatic than from different hydrodynamic schemes!

Summary -- Chapter 13

- The evolution of gas in the Universe can be most accurately modelled via hydrodynamic simulations accounting for (self-)gravity
- Individual gas particles/cells are treated collectively as a continuous medium with well-defined macroscopic properties (density, pressure, velocity, & energy)
- System described by Euler equations (mass, momentum and energy conservation) in Eulerian and Lagrangian forms
- Different numerical schemes exist to solve these equations

  —> adaptive mesh, SPH, moving mesh [mesh-less] codes

  —> each of them has their advantages and disadvantages, but moving-mesh codes combine advantages

Up next…

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Chapters 1–9: Part I: Observational basics & facts of galaxies
first 7 lectures

Chapters 10–17: Part II: Theory & models of galaxy evolution processes
second 7 lectures