A visualization of the cosmic web, showing a complex network of filaments and clusters of galaxies. The filaments are represented by thin, glowing purple lines, while the clusters are denser regions of orange and yellow light. The background is a deep blue.

31.25 Mpc/h

Astrophysics III

Formation and Evolution of galaxies

Michaela Hirschmann, Fall-Winter semester 2024

Lecture content and schedule

- *Chapter 1:* Introduction (galaxy definition, astronomical scales, observable quantities — repetition of Astro-I)
- *Chapter 2:* Brief review on stars
- *Chapter 3:* Radiation processes in galaxies and telescopes;
- *Chapter 4:* The Milky Way
- *Chapter 5:* The world of galaxies I
- *Chapter 6:* The world of galaxies II
- *Chapter 7:* Black holes and active galactic nuclei
- *Chapter 8:* Galaxies and their environment;
- *Chapter 9:* High-redshift galaxies
- *Chapter 10:*
 - Cosmology in a nutshell; Linear structure formation in the early Universe
- *Chapter 11:*
 - Dark matter and the large-scale structure
 - Cosmological N-body simulations of dark matter
- *Chapter 12:* Populating dark matter halos with baryons: Semi-empirical & semi-analytical models
- *Chapter 13:* Modelling the evolution of gas in galaxies: Hydrodynamics
- *Chapter 14:* Gas cooling/heating and star formation
- *Chapter 15:* Stellar feedback processes
- *Chapter 16:* Black hole growth & AGN feedback processes
- *Chapter 17:* Modern simulations & future prospects

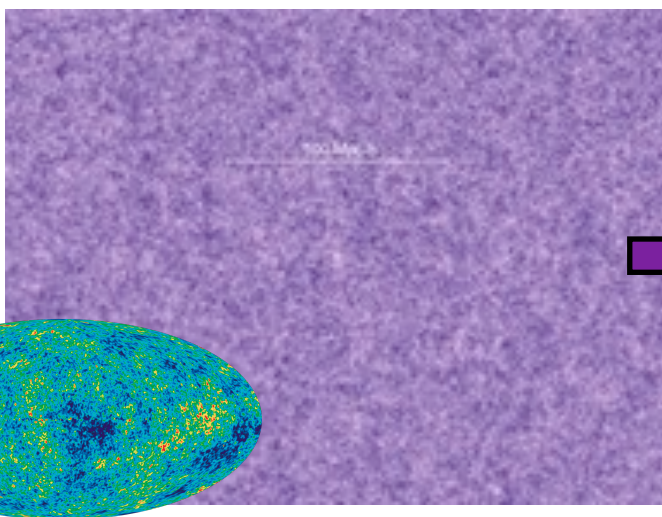
Part I:
Observational
basics & facts of
galaxies
first 7 lectures

Part II:
Theory & models
of
galaxy evolution
processes
second 7 lectures

What we learned from Chapter 10...

- **Power spectrum** after recombination can be derived from primordial power spectrum from inflationary theory combined with the transfer fct., normalisation set by CMB
- At high redshifts, density fluctuations grow in the linear regime
 - Can be described by linear perturbation theory
 - Using **Zel'dovich approximation**, position and velocity displacements can be estimated to lower z , e.g. $z \sim 50$
 - Use them as **initial conditions** for cosmological simulations

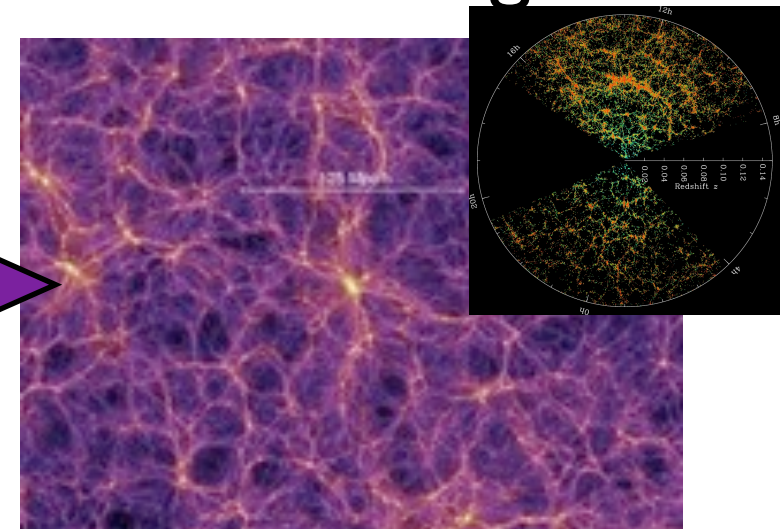
1. Generation of initial conditions



2. Running the simulation

On cosmological scales, *gravity* is the dominant mechanism

3. Creating mock catalogues



- As a first step, we simulate the evolution of **ONLY** dark matter

Outline of this lecture

- N-body simulations of dark matter
 - Equations of motions
 - Numerical effects and Softened gravity
 - Gravity algorithms and time integration
- Analysis and Results of dark matter simulations
 - Identification of dark matter halos
 - Halo mass function
 - Hierarchical structure formation & Merger trees
- Properties of dark matter halos
 - Universal Density profiles
 - Halo shape
 - Halo spin

Dynamics of a N-body system

- We assume that dark matter particles are only interacting via the gravitational force on an expanding background
- Collisionless dynamics: there are so many particles so that they do not scatter locally on each other, they just respond to their collective gravitational field $\Phi(\mathbf{x}, t)$

- Describe the N-body system in terms of the particle distribution function, which represents the number density of particles in phase space

$$f = f(\vec{x}, \vec{v}, t)$$

Collisionless
Boltzmann
equation (CBE)

- Poisson-Vlasov system

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \vec{x}} \cdot \vec{v} + \frac{\partial f}{\partial \vec{v}} \cdot \left(-\frac{\partial \Phi}{\partial \vec{x}} \right) = 0$$

$$\vec{\nabla}^2 \Phi(\vec{x}, t) = 4\pi G \int f(\vec{x}, \vec{v}, t) d\vec{v}$$

- CBE implies that phase space is conserved along each orbit (flow conserves mass, energy...), can be also applied to stars!
- The system of partial differential equations is difficult (impossible) to solve directly for non-trivial cases

Equations of motion of a N-body system

- The **N-body method** uses a finite set of particles to sample the underlying distribution function

➡ We discretise it in terms of N dark matter particles

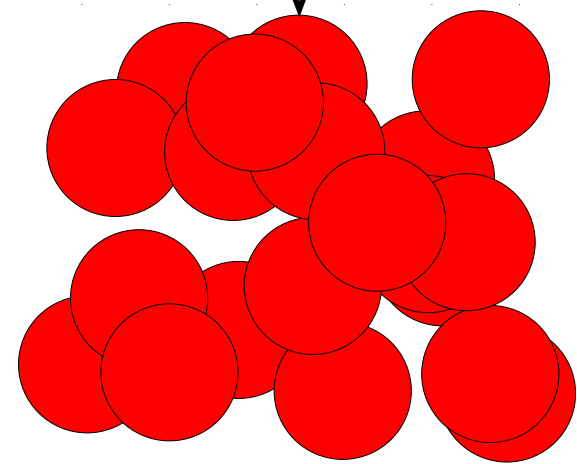
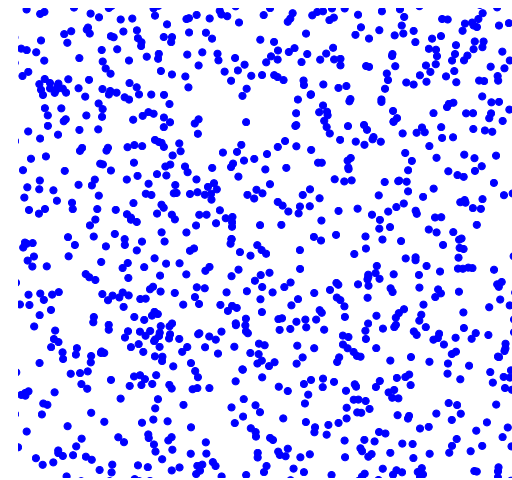
- **Equation of motions** (for the i-th particle) are a solution to the collisionless Boltzmann equation (now in co-moving units \mathbf{r}, \mathbf{u})

$$\frac{d\vec{r}_i}{dt} = \vec{u}_i, \quad \frac{d\vec{u}_i}{dt} + H(t)\vec{u}_i = -\frac{1}{a} \vec{\nabla} \Phi_i = \frac{1}{a} \frac{\vec{F}_i}{m_i}$$
$$\text{with } -\vec{\nabla} \Phi_i = \frac{\vec{F}_i}{m_i} = \sum_{i \neq j} \frac{m_j (\vec{r}_j - \vec{r}_i)}{|\vec{r}_j - \vec{r}_i|^3}$$

- Can be derived from combining the Euler & Poisson equation with the Friedmann equations
- **Main computational task** is calculating the gravitational forces on each particle i (for N particles: need to sum N^2 forces!)
- Based on the calculated gravitational force, the equations of motions have to be integrated in time → new position and velocity of particle i

Dark matter in a N-body system

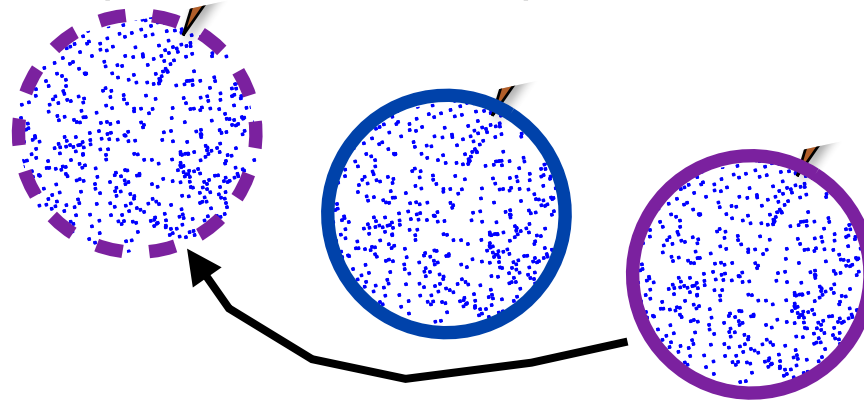
- Box with $B=50$ Mpc has a total mass of $\sim 10^{16} M_{\odot}$
→ $\sim 10^{71}$ WIMPs, not feasible to simulate
- Consider “macro” particles: with $N=1024^3$, in a box with $B=50$ Mpc, the simulation’s particle mass is $\sim 10^7 M_{\odot}$
 - One particle is representing $\sim 10^{60}$ WIMPs (or whatever particle)!!



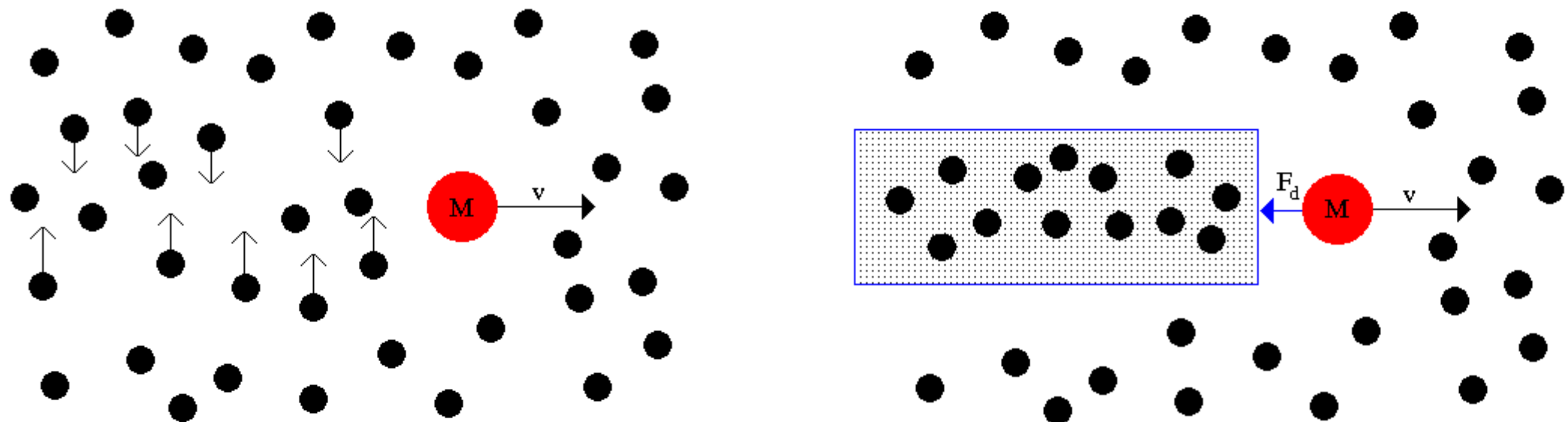
- To what extent does representing 10^{60} dark matter particles by one simulation particle have unwanted side effects?

Dynamical friction

- Particle collisions are unwanted when modelling a collisionless system
- Low resolution: Collision (Acceleration)



- High resolution: Dynamical friction (Deceleration)

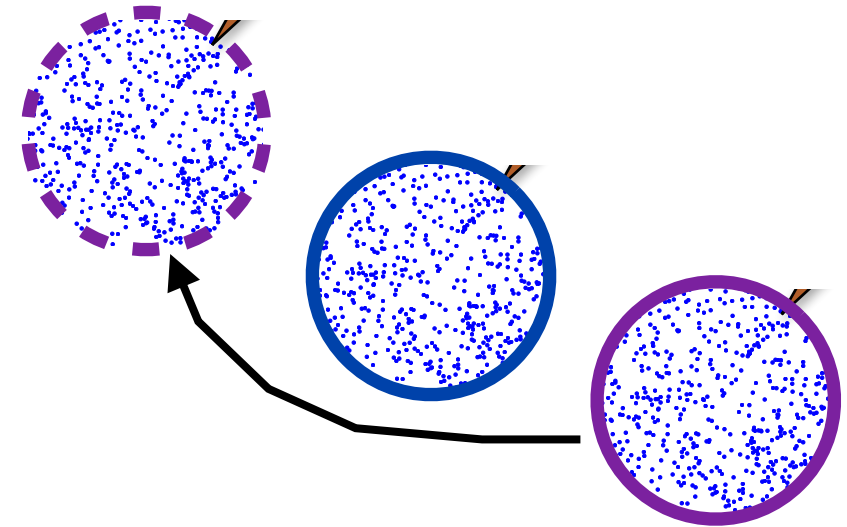


➡ Drag force should slow down particle M

Particle collisions can, however, enter the systems with “macro” particles due to numerical limitations, i.e a system of DM particles may get somewhat collisional

Gravitational softening

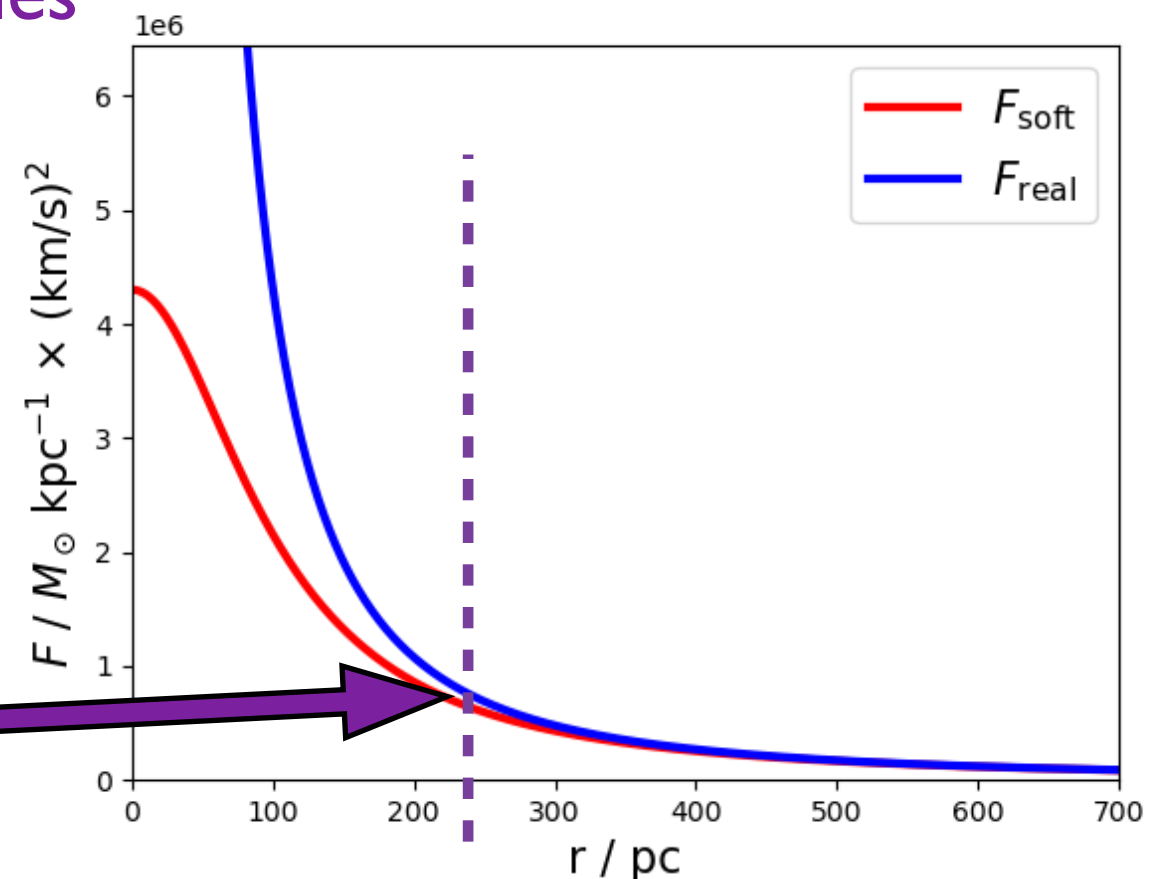
- How can we get rid of unwanted collisional effects (artificial acceleration for low-resolution)?
- To ensure collisionless nature of DM we have to
 - Prevent bound pairs



→ Need to **soften force law on small scales**

$$\frac{\vec{F}_i}{m_i} = \sum_{i \neq j} \frac{m_j (\vec{x}_j - \vec{x}_i)}{((\vec{x}_j - \vec{x}_i)^2 + \epsilon^2)^{3/2}}$$

gravitational softening



Conflicting requirements for simulations

- We want
 - large particle number N to have a small particles mass, i.e. high resolution (to capture small halos/galaxies and their physics)
 - large volume V (for good statistics and to capture rare objects like rich galaxy clusters)
- **Dynamic range problem** faced by cosmological simulations, at any given time, halos/galaxies exist on a large range of mass scales

→ We need

- Efficient self-gravity algorithms,
- Efficient time integration algorithms,
- Codes run in parallel (mpi) of super-computing clusters (HPC facilities) with nodes having large memory and efficient communication

Overview on gravity algorithms

- Gravity algorithms:

- Direct summation (PP-algorithm) $t \sim N^2 \rightarrow$ most accurate, but not competitive in cosmological runs since time-consuming

$$\frac{\vec{F}_i}{m_i} = \sum_{i \neq j} \frac{m_j (\vec{x}_j - \vec{x}_i)}{((\vec{x}_j - \vec{x}_i)^2 + \epsilon^2)^{3/2}}$$

- PM: Particle-mesh codes (hardly used alone nowadays)
- P³M: Particle-mesh combined with direct summation
- Tree codes (hardly used alone nowadays)
- Tree-PM: Combination of particle mesh and tree algorithm (e.g. used in Gadget)
- Reduction to NlogN scaling \rightarrow significant speed up!

Time steps and integration

- After calculating the force on each particle —> quantify the position & velocity in the next time step, i.e. **integrate the equations of motion over time**

$$\frac{d\vec{x}_i}{dt} = \vec{v}_i, \quad \frac{d\vec{v}_i}{dt} = \vec{a} = -\frac{1}{a_{\text{scale}}} \vec{\nabla} \Phi_i - H(t) \vec{v}_i$$

- **Accuracy strongly dependent on the size of the time step and on the integrator scheme used**
- **Integration scheme:** leap-frog: second-order accurate, symplectic nature)
- **Individual time steps:** Acceleration/velocity criterium: dynamical time-scale depends on environment (large in low-density regions, small in halo centers)

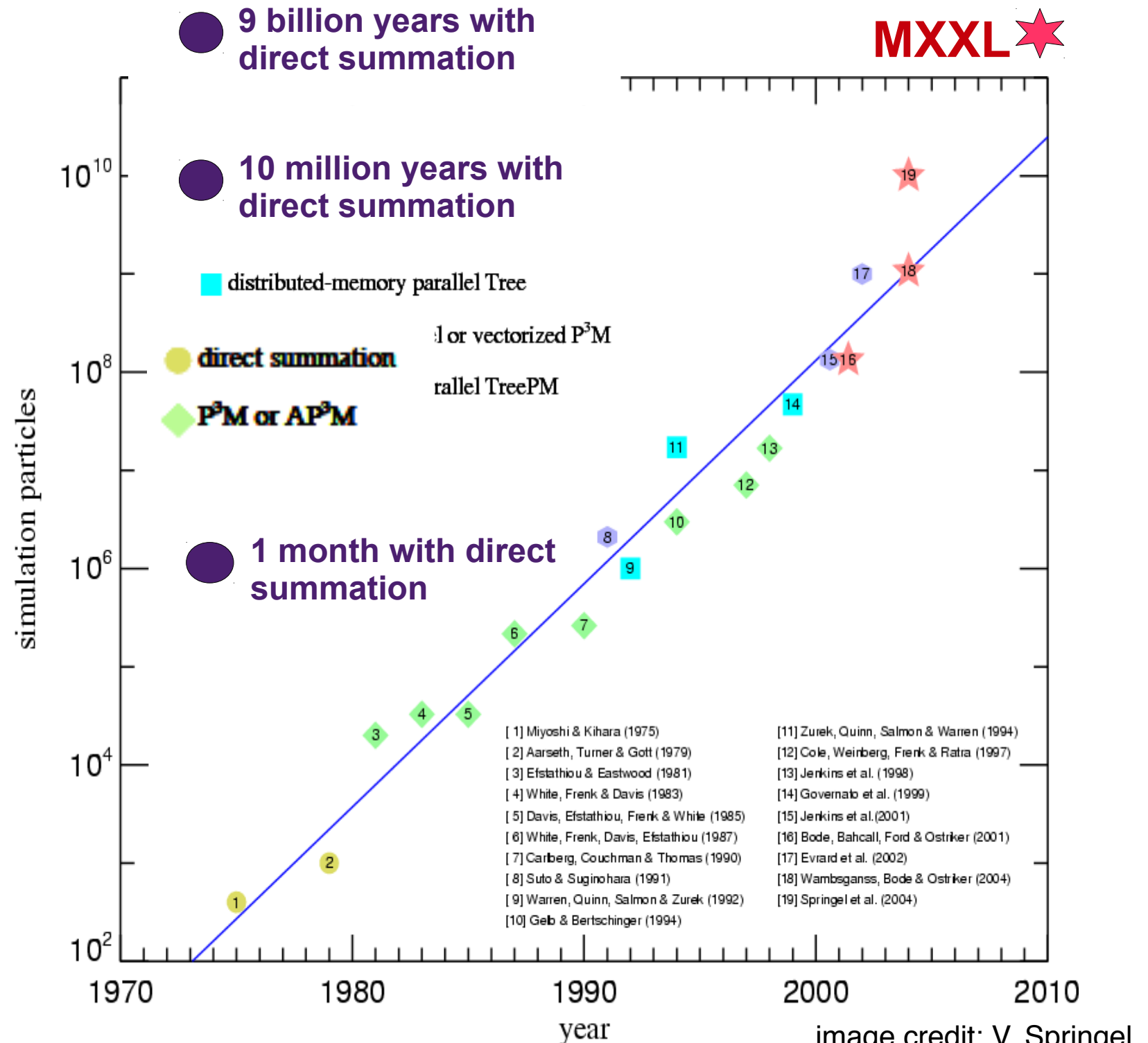
Size of DM simulations over time

- To further achieve better performance (large resolution at **a lot of computing power is necessary**, i.e. a reasonable method to distribute different tasks on several processors, to run them “in parallel”

- Computers double their speed every 18 month

- Particle number in simulation doubles every 16-17 month (Moore's law)

- Only possible if gravity algorithms scale close to N (or $N \log N$)



Example: Millennium XXL

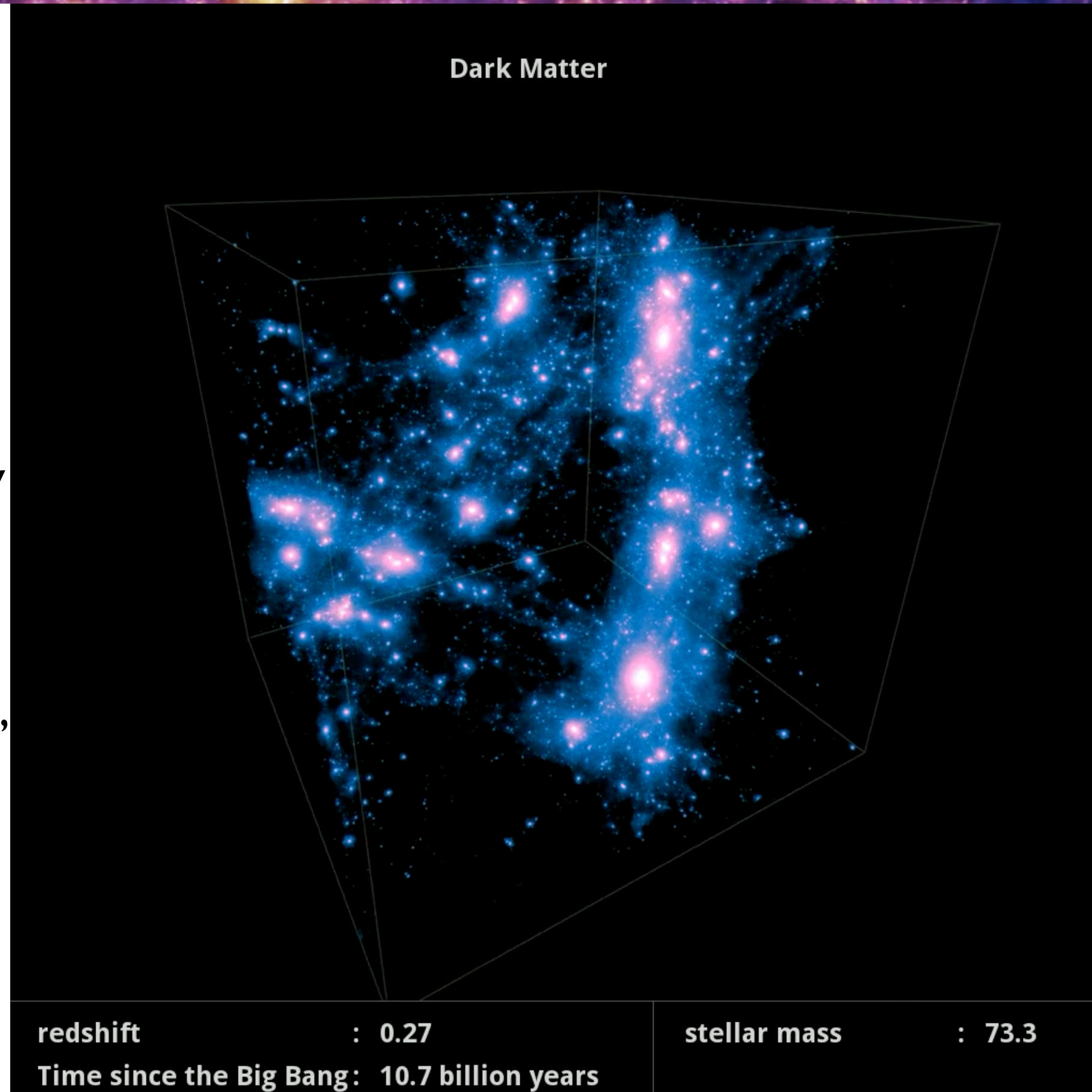
This simulation follows all 6720^3 DM particles in a cosmological box of side 4.1 Gpc (ran on 12,000 cores at Juelich SCC, mpi-parallel only)

The mass density field in the Millennium-XXL focusing on the most massive halo present in the simulation at $z=0$. Each inset zooms by a factor of 8 from the previous one; the side-length varies from 4.1 Gpc down to 8.1 Mpc. All these images are projections of a thin slice through the simulation of thickness 8 Mpc.

<http://galformod.mpa-garching.mpg.de/mxxlbrowser/>

Example: Illustris simulation

- Volume 106.5^3 Mpc^3
- Number of particles 1820^3 (DM)
 - DM particle mass $6.2e6 M_{\odot}$
 - Initial conditions based on WMAP9 cosmology
- Run on 8192 cores with 19Mio CPU hours (~3month)
- But was run with baryons, pure DM run would be much, much faster!
- **On one core it would have run for 2000 years!**



Summary — Dark matter simulations



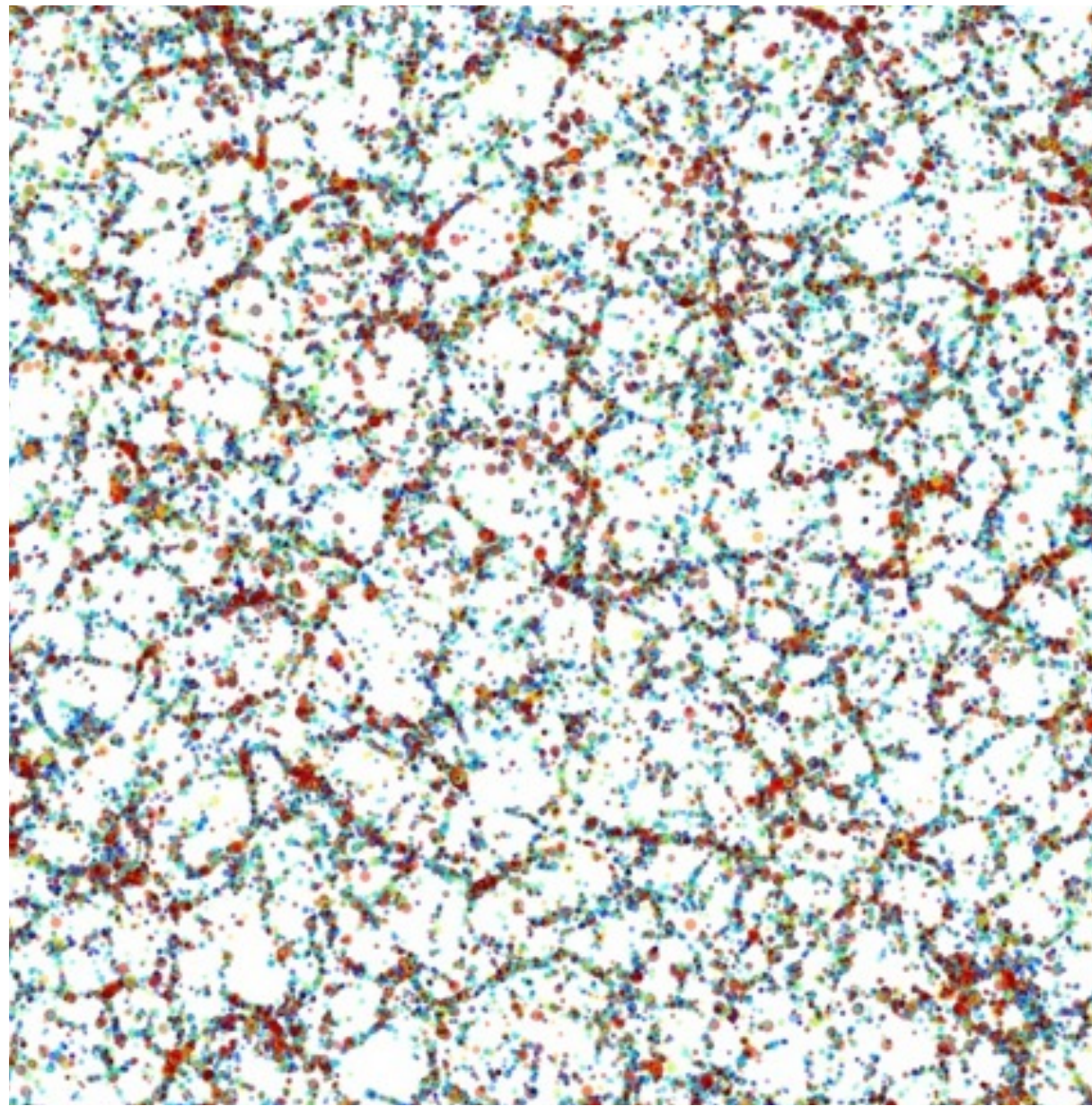
- a system of DM particles can be described by the collisionless Boltzmann equation —> impossible to solve
- represent matter by particles, DM are collisionless particles (only gravitational interaction)
- represent (part of) the Universe by a (usually) periodic 'box' in an expanding space-time
- if we know initial conditions (positions & velocities), we can solve Newton's equations for each particle
- to preserve collisionless nature of DM —> force softening
- to run N-body simulations with large volume and high resolution: efficient gravity solvers and time integration with highly parallel codes

Outline of this lecture

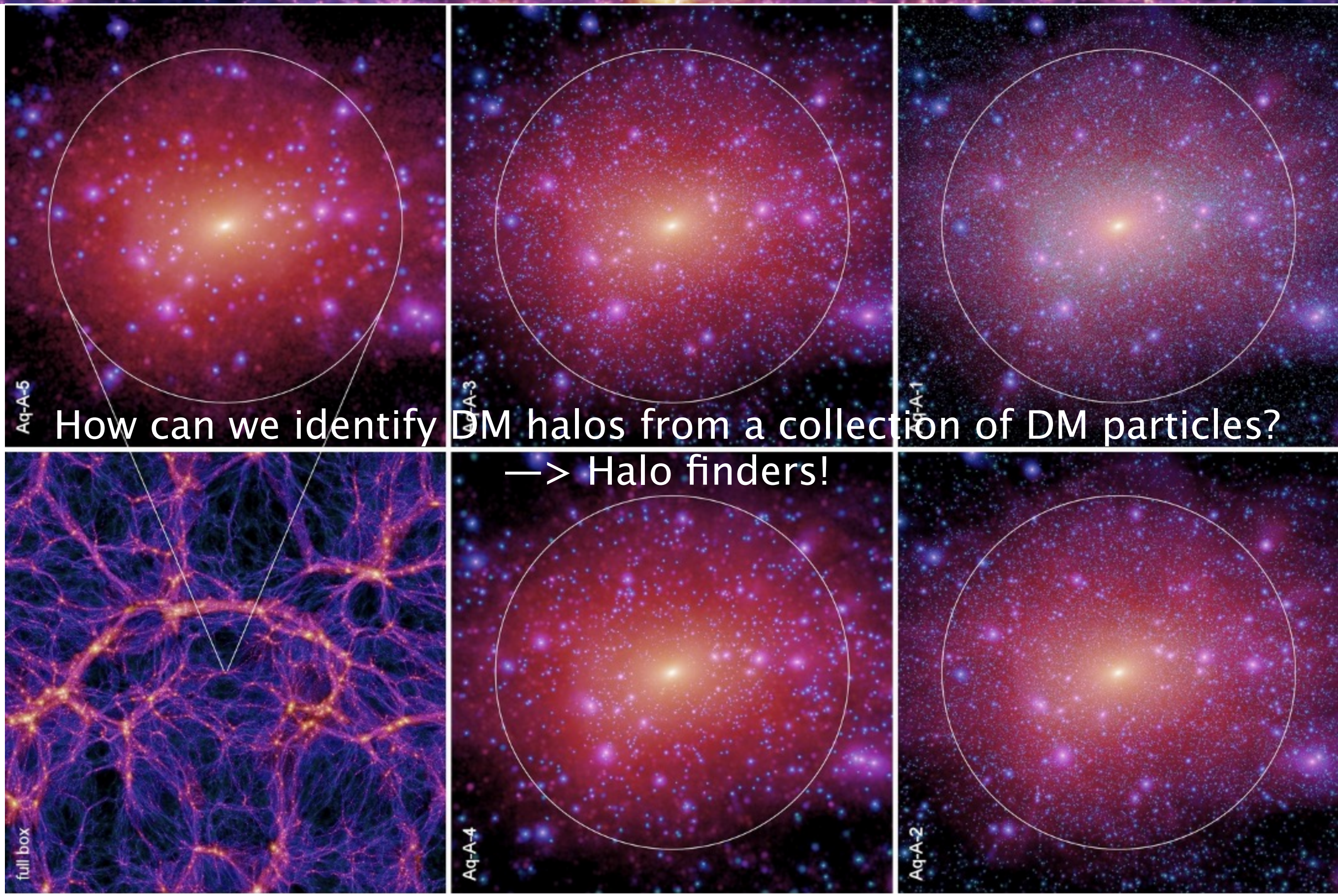
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Cosmological simulations

- Up to now:
 - Initial condition construction
 - Codes to evolve dark matter motions driven by gravity
- But we have information only on dark matter particles and not on dark matter halos (virialised objects, within which galaxies evolve)

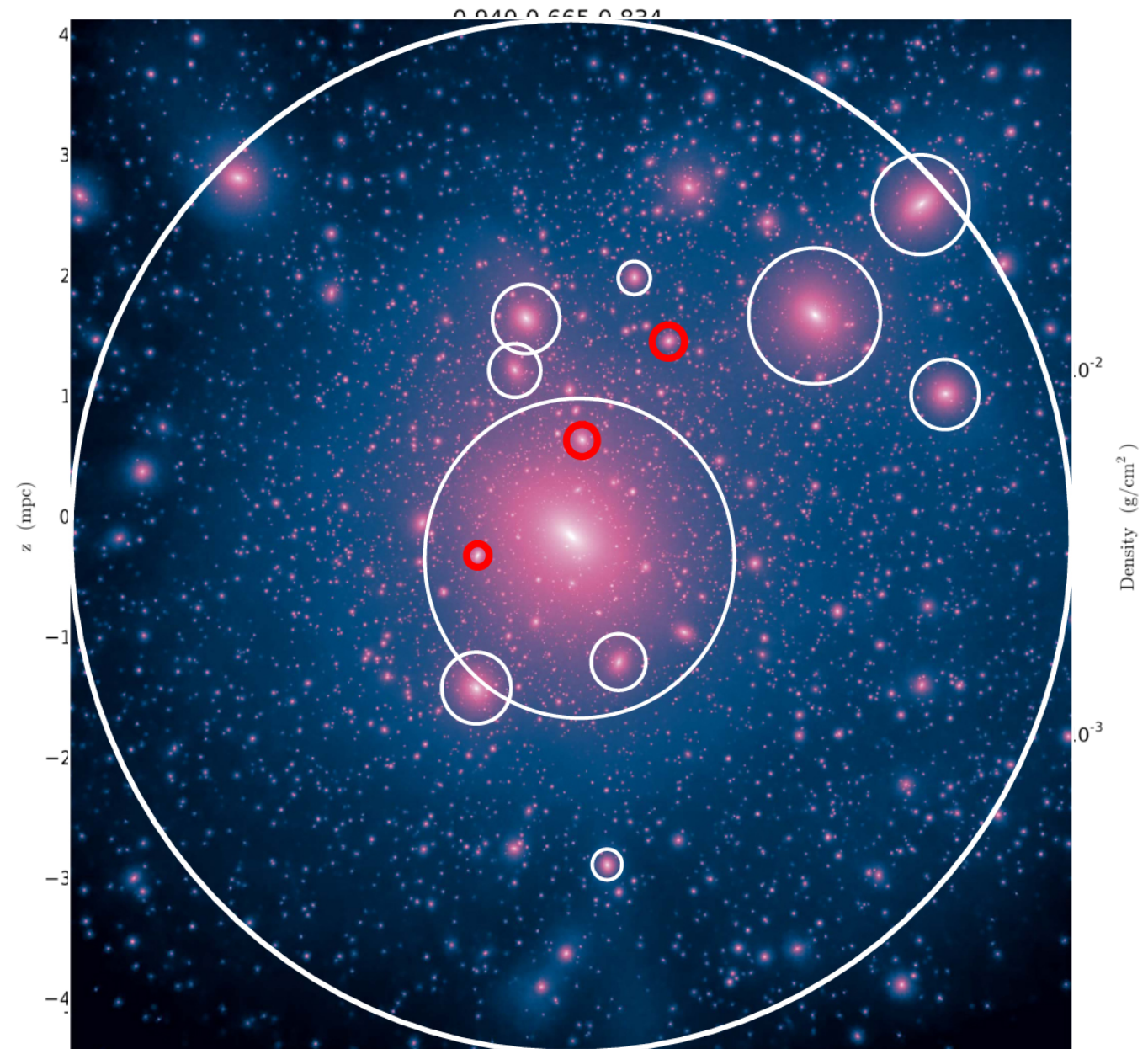


Example: Millennium simulation



Halo finders for large simulations

- More and better algorithms have been developed with time
- Most important ones are listed using **Positions (& velocities) of particles** :
 - 1985: **FOF**
 - 1991: DENMAX
 - 1995: adaptive FOF
 - 1997: **BDM**
 - 1998: HOP
 - 1999: hierarchical FOF
 - 2001: **SKID**
 - 2001: **SUBFIND**
 - 2006: **6DFOF**
 - 2009: AHF
 - 2010: **ROCKSTAR**



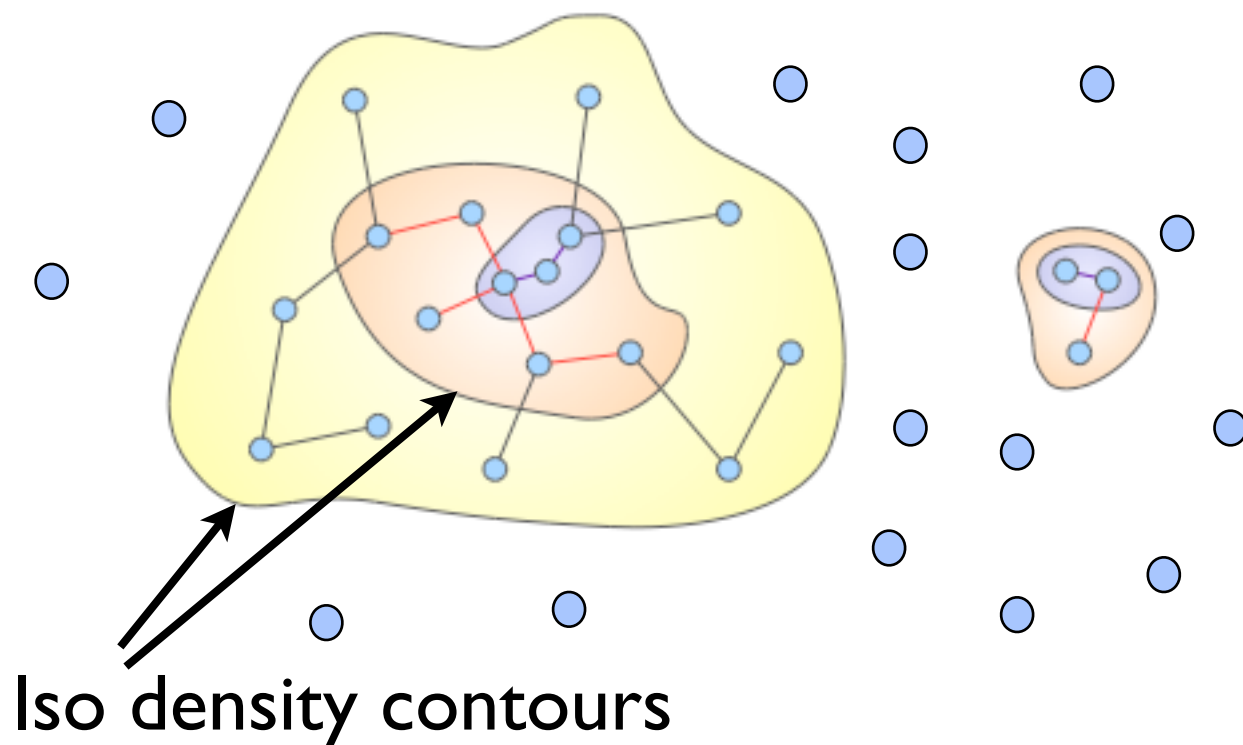
Map simulations into observables
(galaxies are typically defined as stars/gas within 1/10 of R_{vir})

Friends-of-friends (FOF)

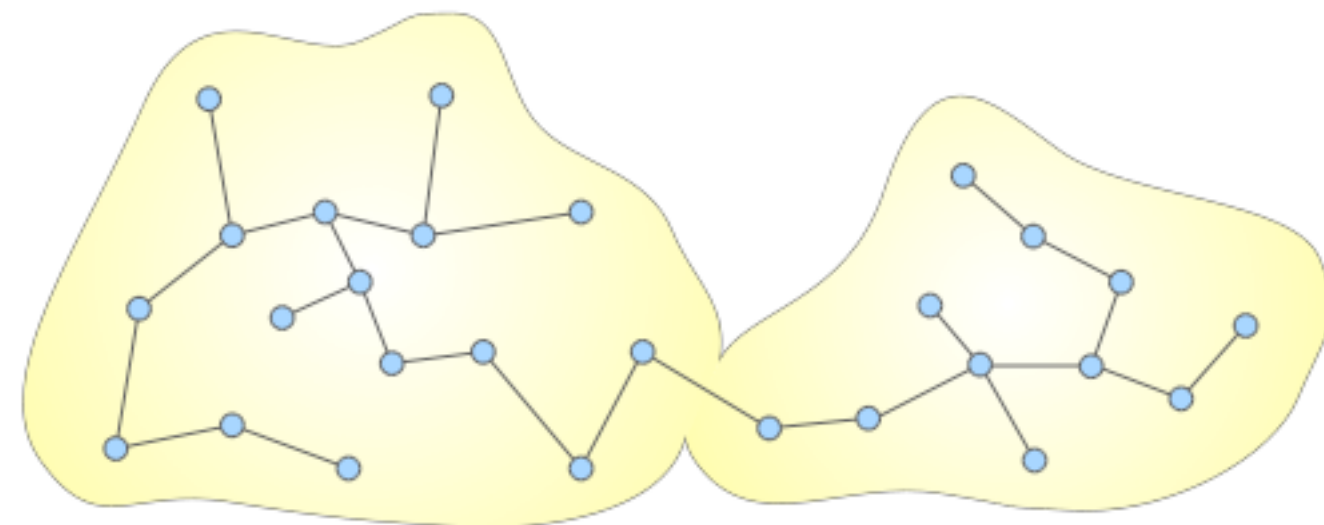
- Considering only particle position to group spatially close particles together using a “linking length” b

$$|\vec{x}_i - \vec{x}_j| < b\Delta x = bB/N^{1/3}, \quad B = \text{Boxsize}, \quad N = \# \text{ of particles}$$

Different linking length



FOF bridge



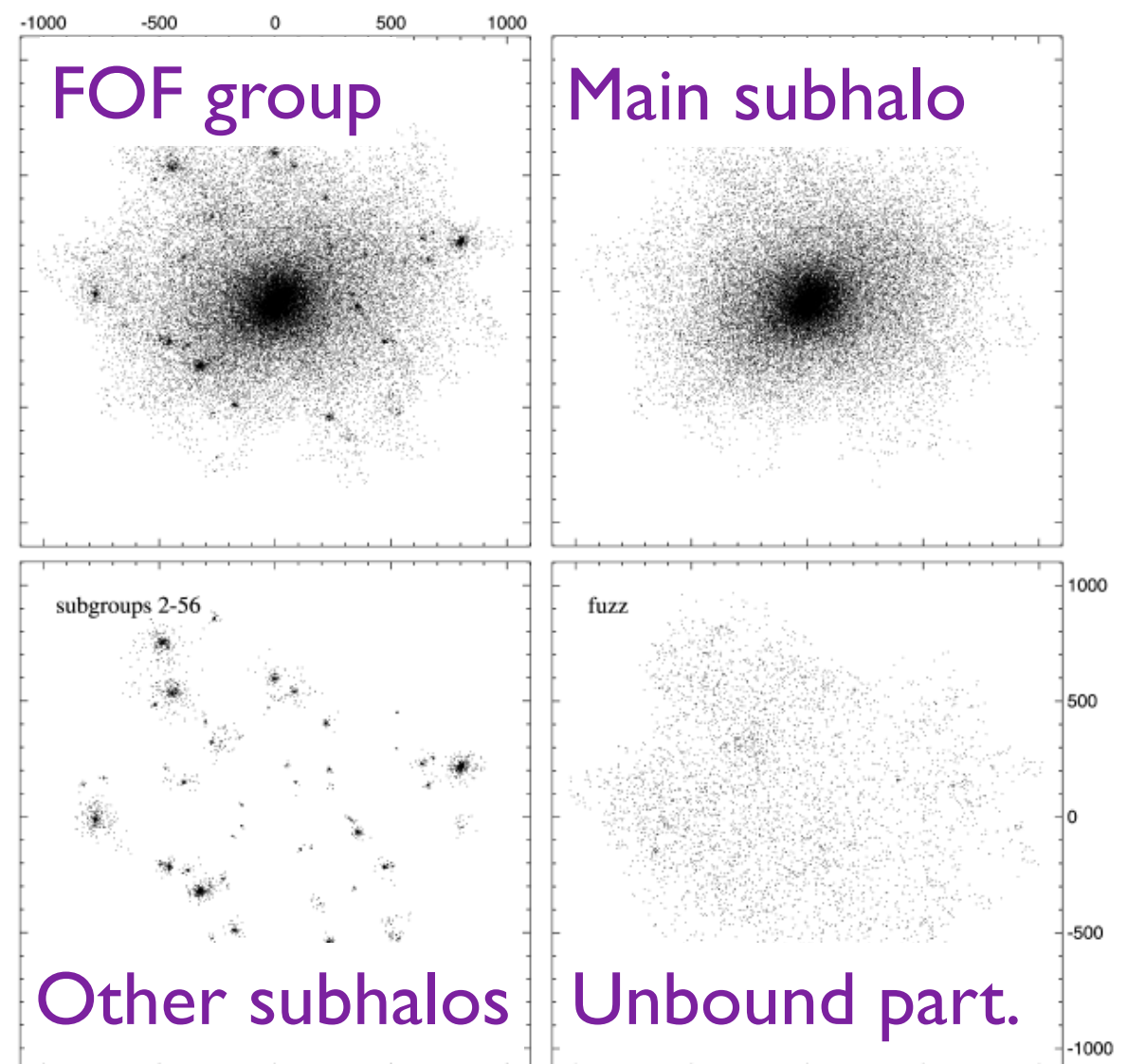
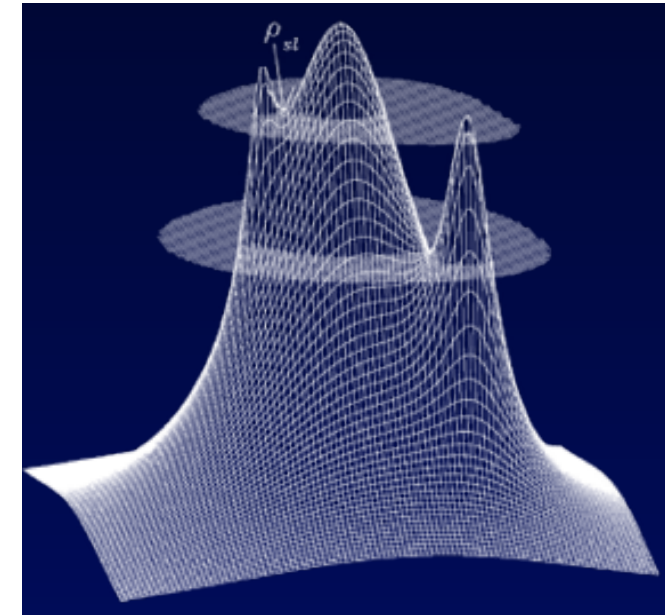
FOF groups cannot intersect: if two objects come close enough (distance smaller than the linking length), they “combine” via a linking bridge

- Advantages:** fast, arbitrary halo shapes
- Disadvantages:** no subhalos, danger of linking bridges

SUBFIND

- Identifies locally overdense, gravitationally bound regions within an input parent halo (FOF halo) using positions and velocities (3D+3D)
- Assign a density to the particles in the parent-FOF halo (“kernel” interpolation over the nearest neighbours)
- Identify locally overdense regions (areas enclosed by an isodensity contour that traverses a saddle point)
- Remove unbound particles

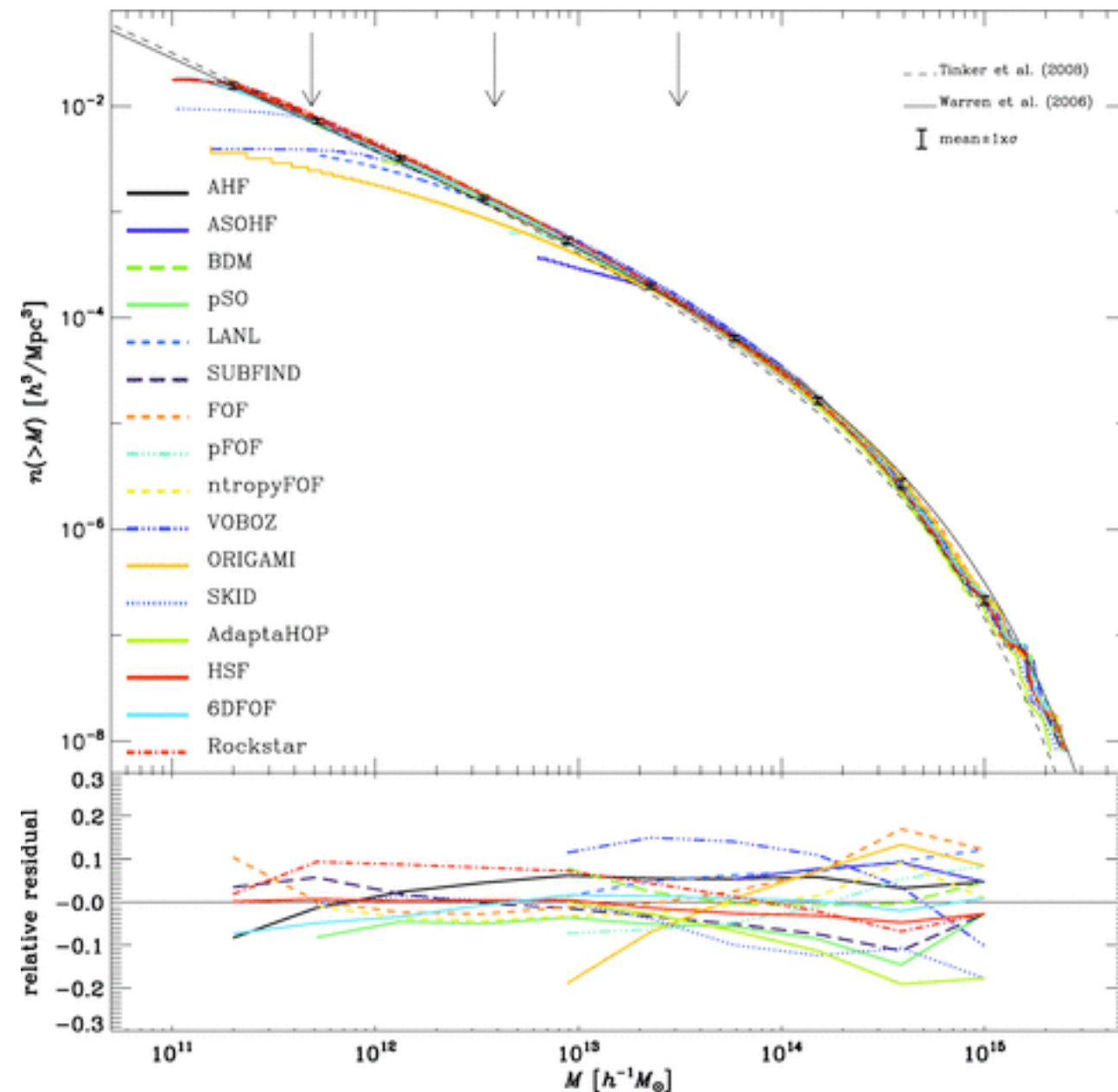
$$v_i > v_{\text{esc}}(r_i) \propto \sqrt{\Phi(r_i)}$$



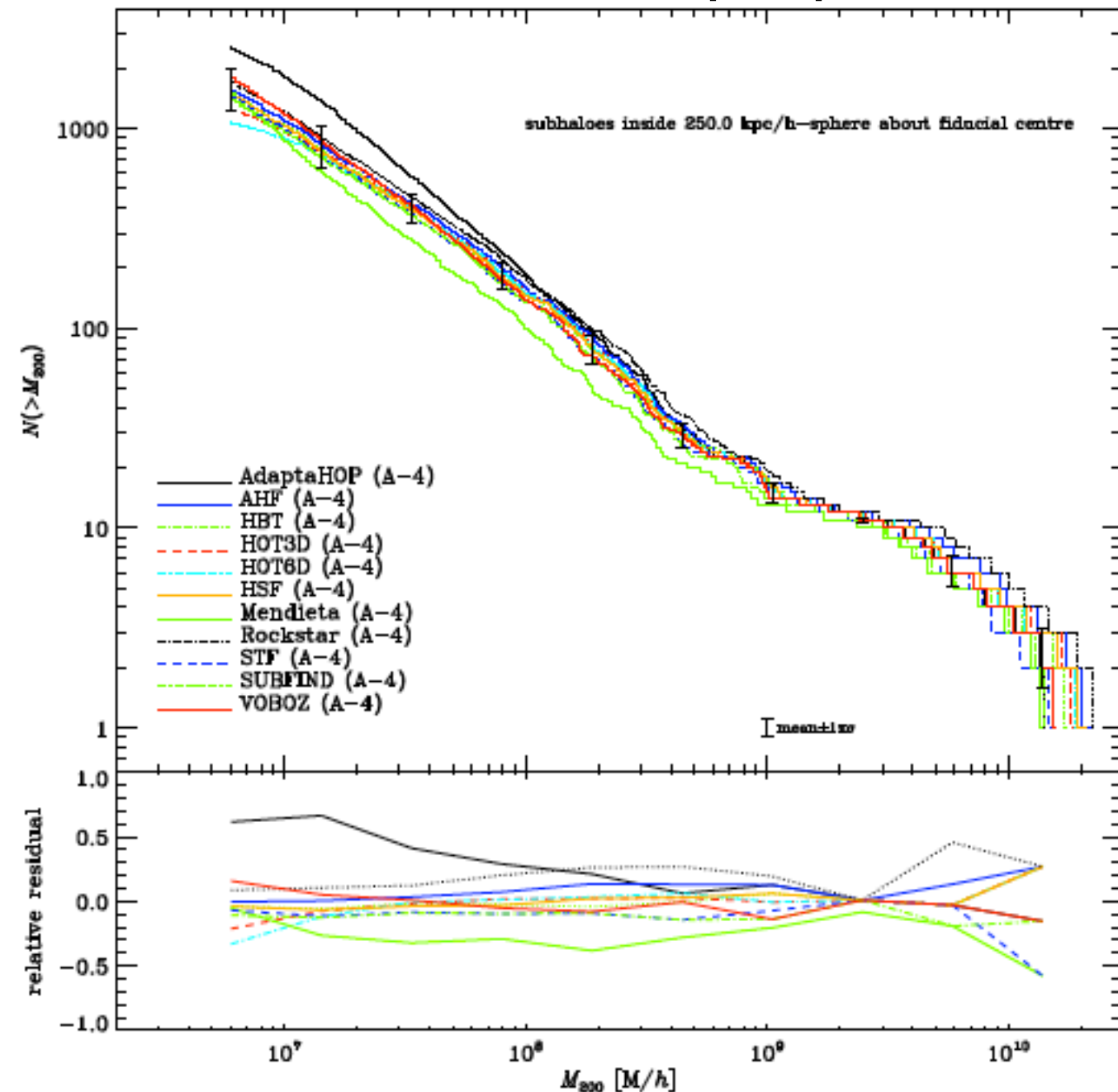
Halo mass function for different halo finders

- Big comparison projects (Knebe et al. Papers!)
- Even if different halo finders do not all identify same objects, in general, rather good agreement

Halo mass function



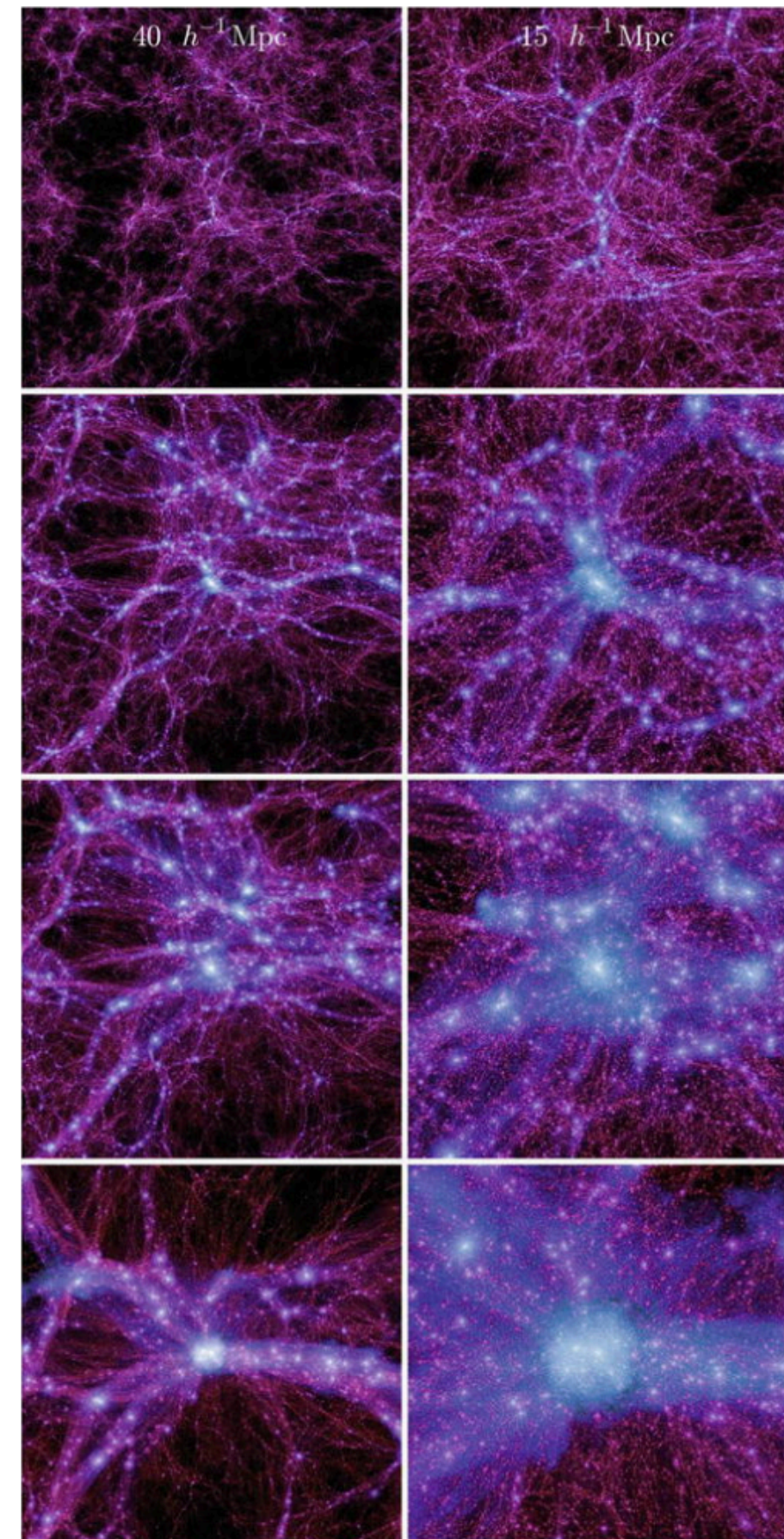
Subhaloes inside a 250kpc sphere



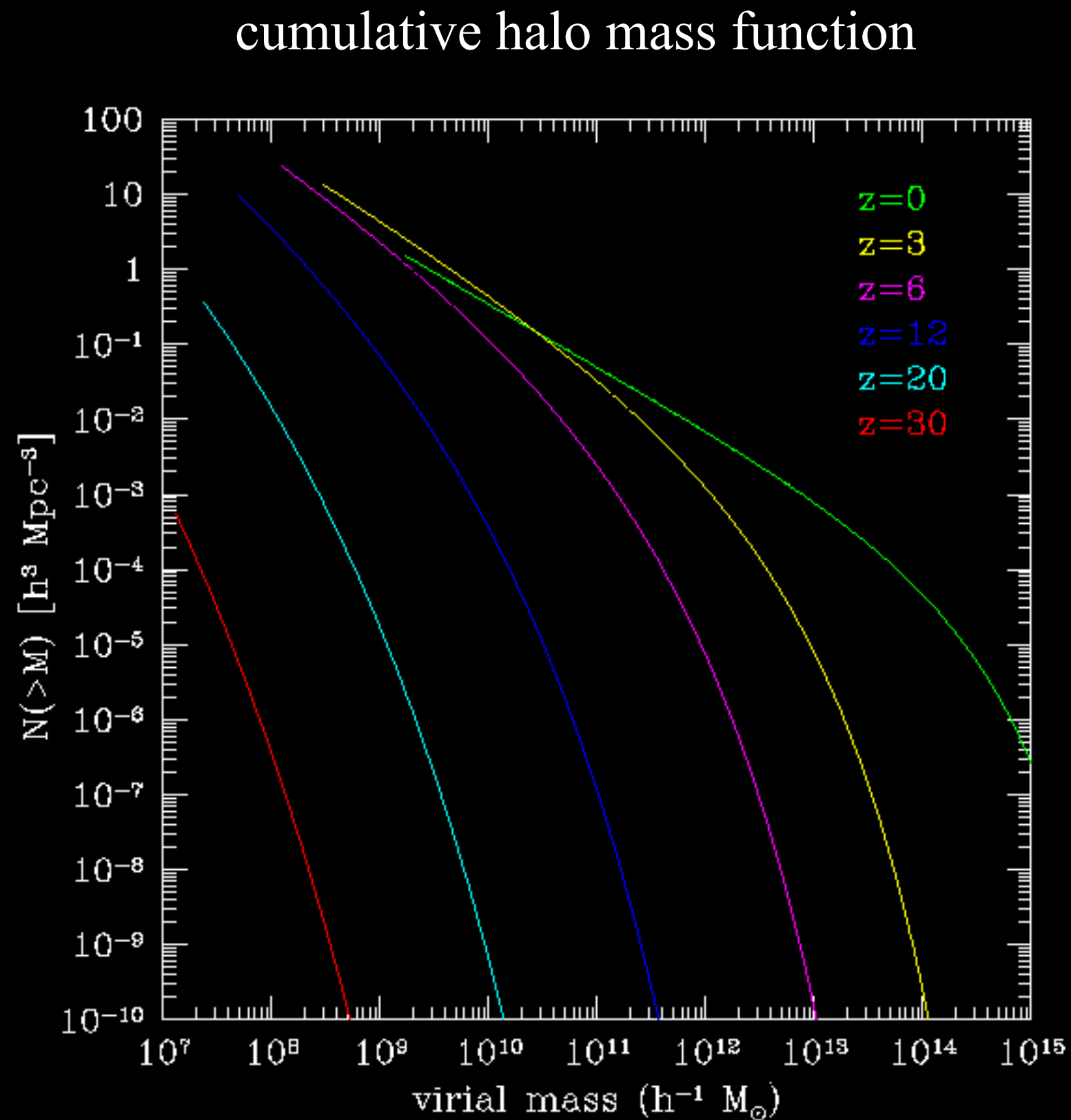
Hierarchical structure formation

What's the evolution of halos/
structure over time?

- DM forms walls, filaments & halos.
- filaments form at intersection of walls,
- halos form at intersection of filaments
- size of the largest object increases with time
- many more small objects than large ones
- small objects form first, then merge together into larger ones for cold dark matter —> “bottom-up fashion”

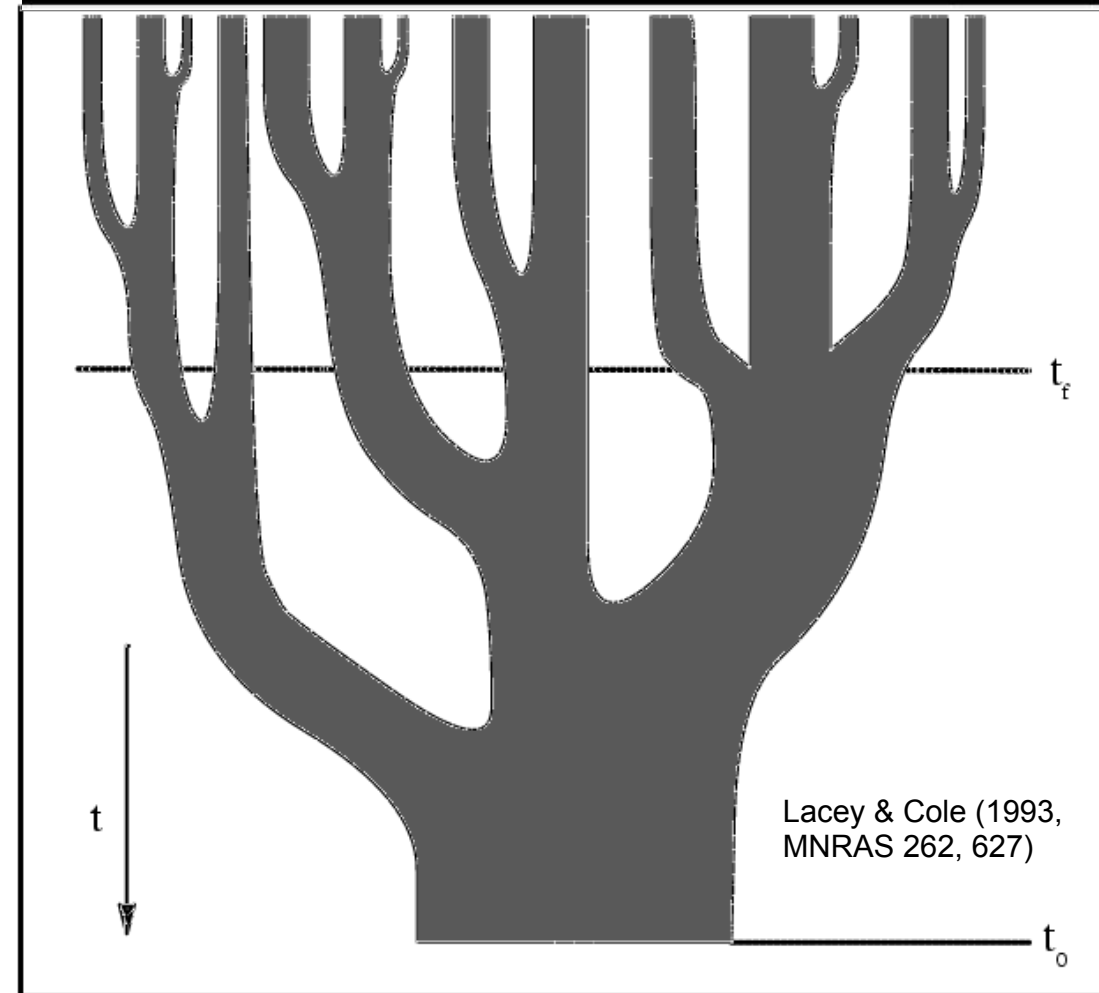
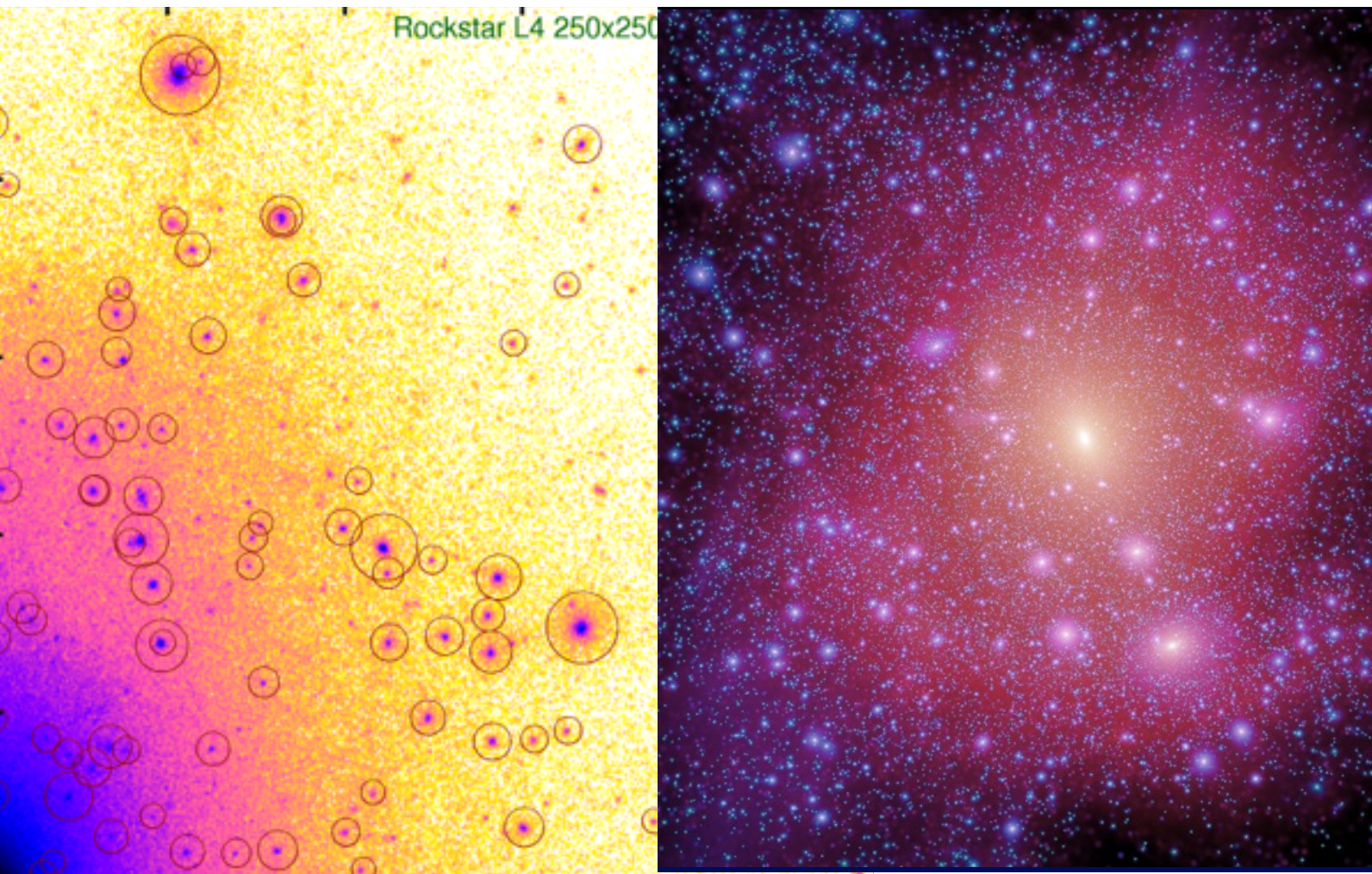
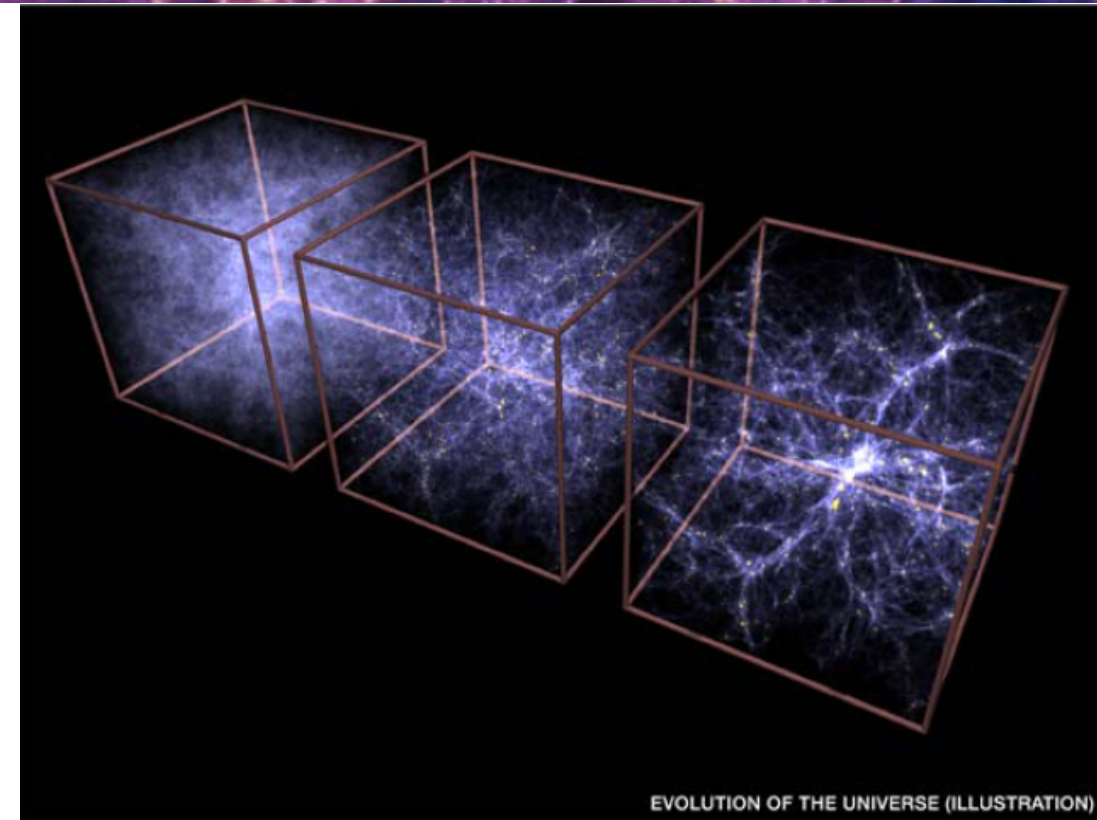


Evolution of the halo mass function



Merger trees of DM halos

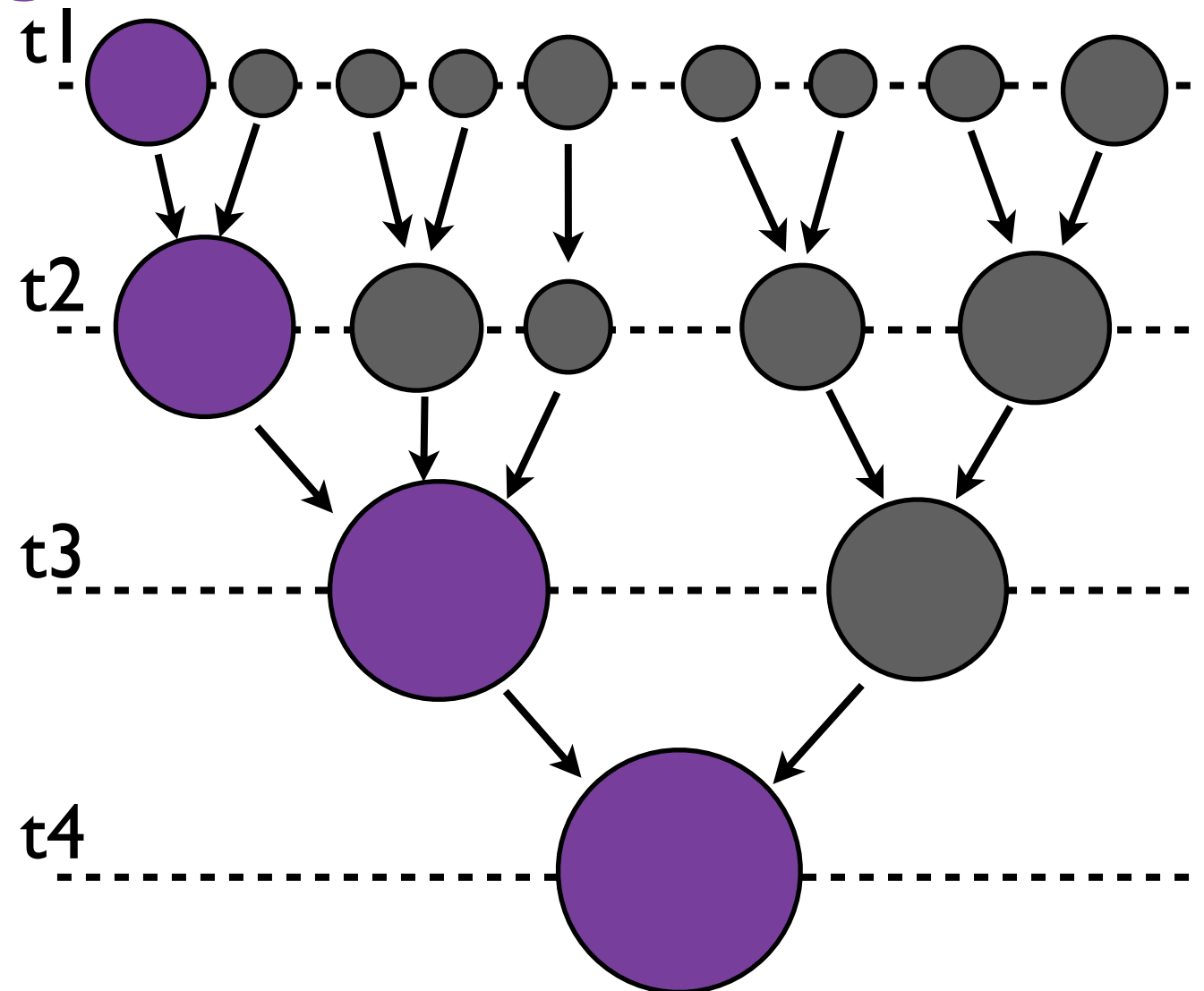
- ✓ Identify, halos, subhalos, subsubhalos and
- ➡ Connect them over time



Merger trees of DM halos

- Link halos through time
- Identify descendants in later snapshots (or progenitors in earlier snapshots)
- Use particle ID: for given halo identify all haloes in later snapshot that contain its particles
- Descendent is the halo with the largest amount of matching particles
- When subhalos merge, their particles move into the “main” halo
- Structure formation is HIERARCHICAL

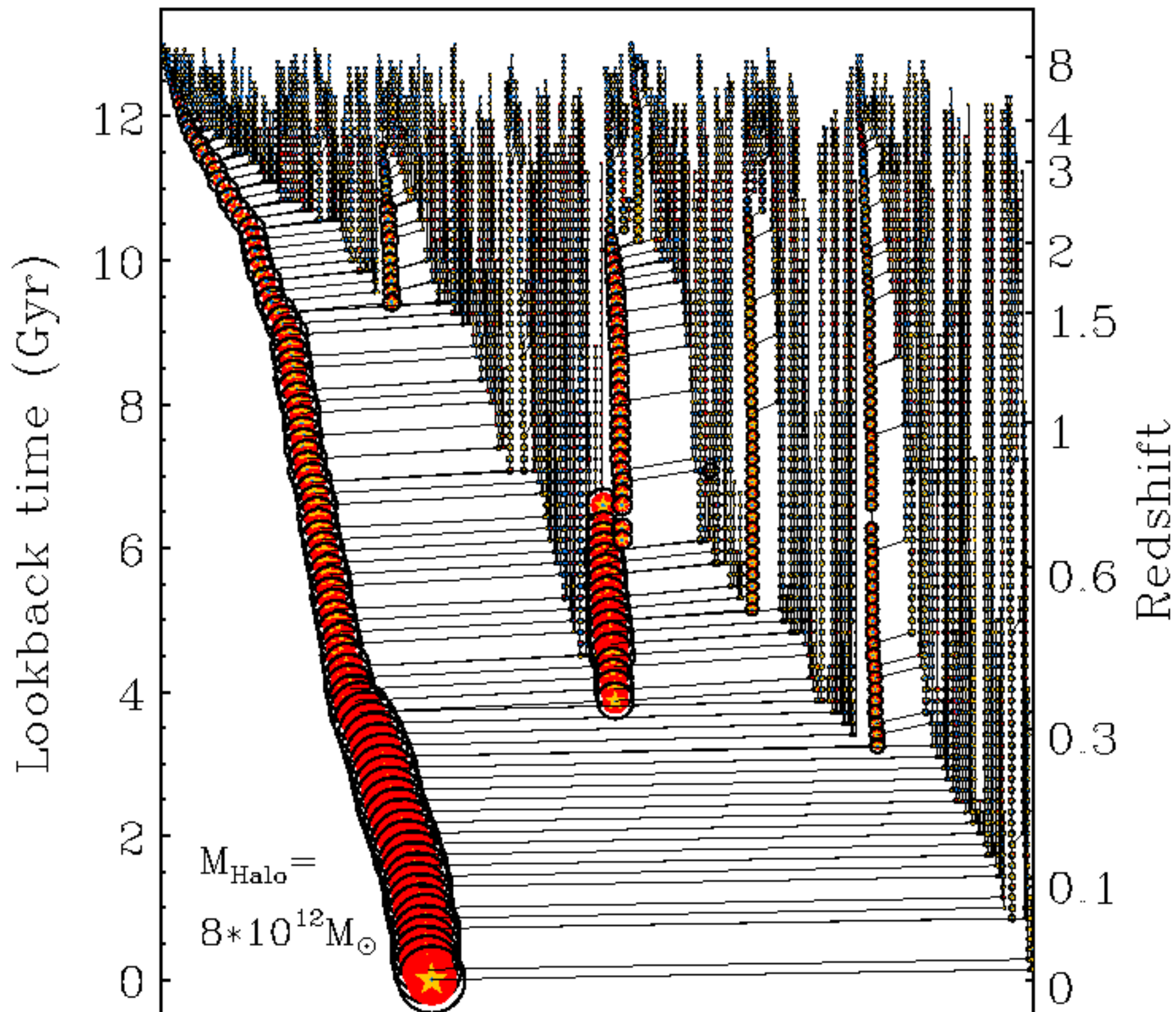
Main branch, consisting of the most massive progenitors



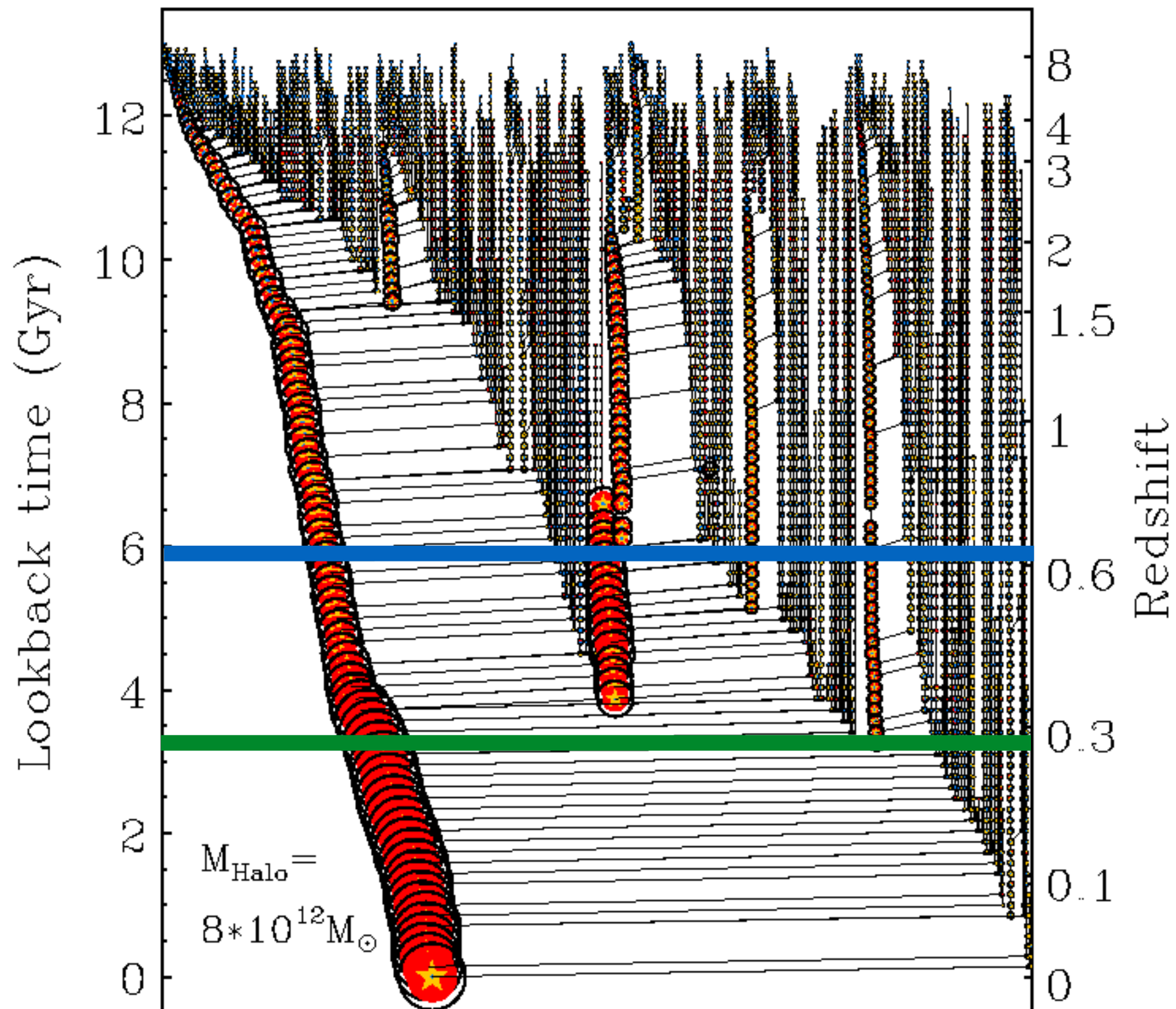
Formation time: half of the final DM mass is accumulated in all progs

Assembly time: half of the final DM mass is assembled in the main prog

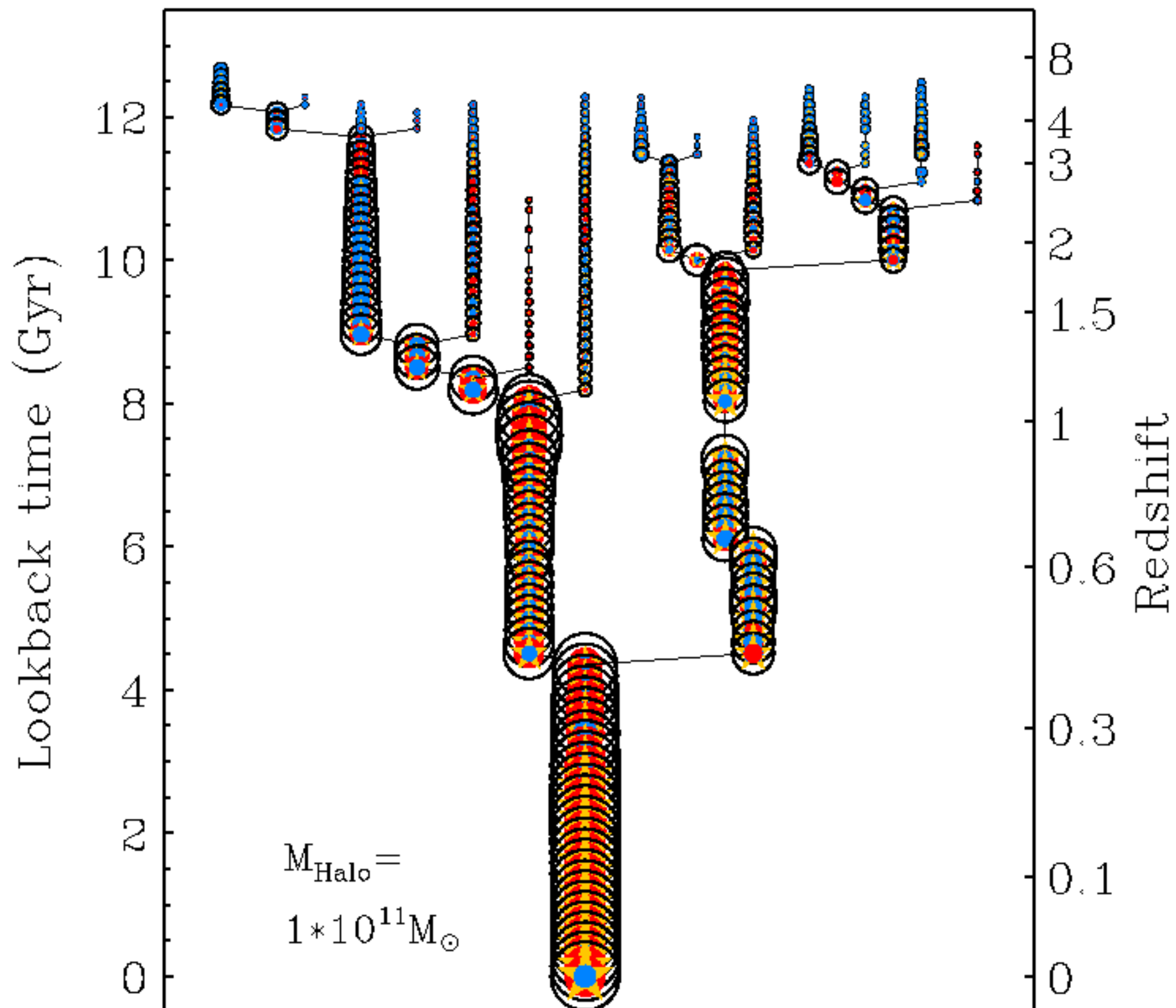
Merger trees from N-body simulations



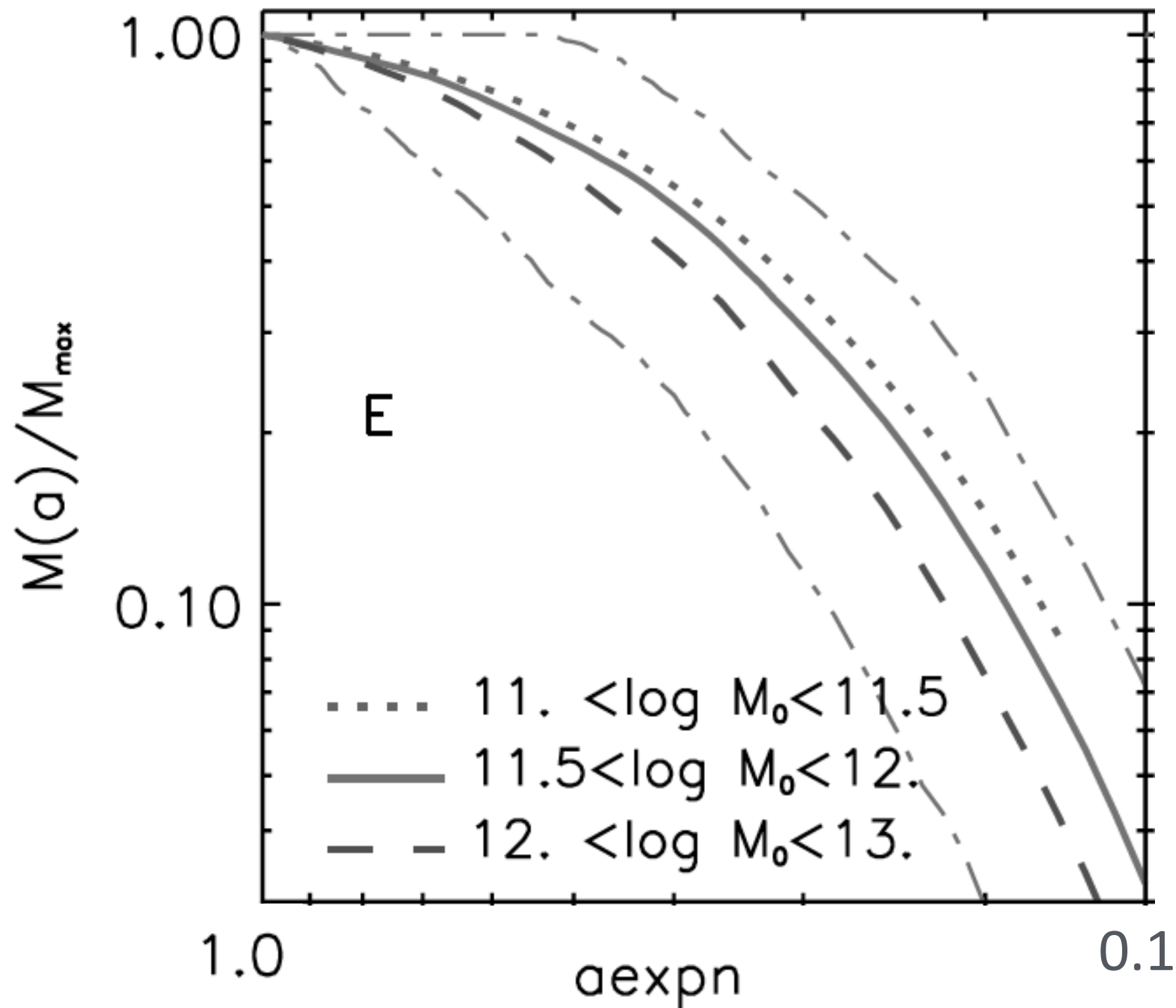
Assembly and formation times



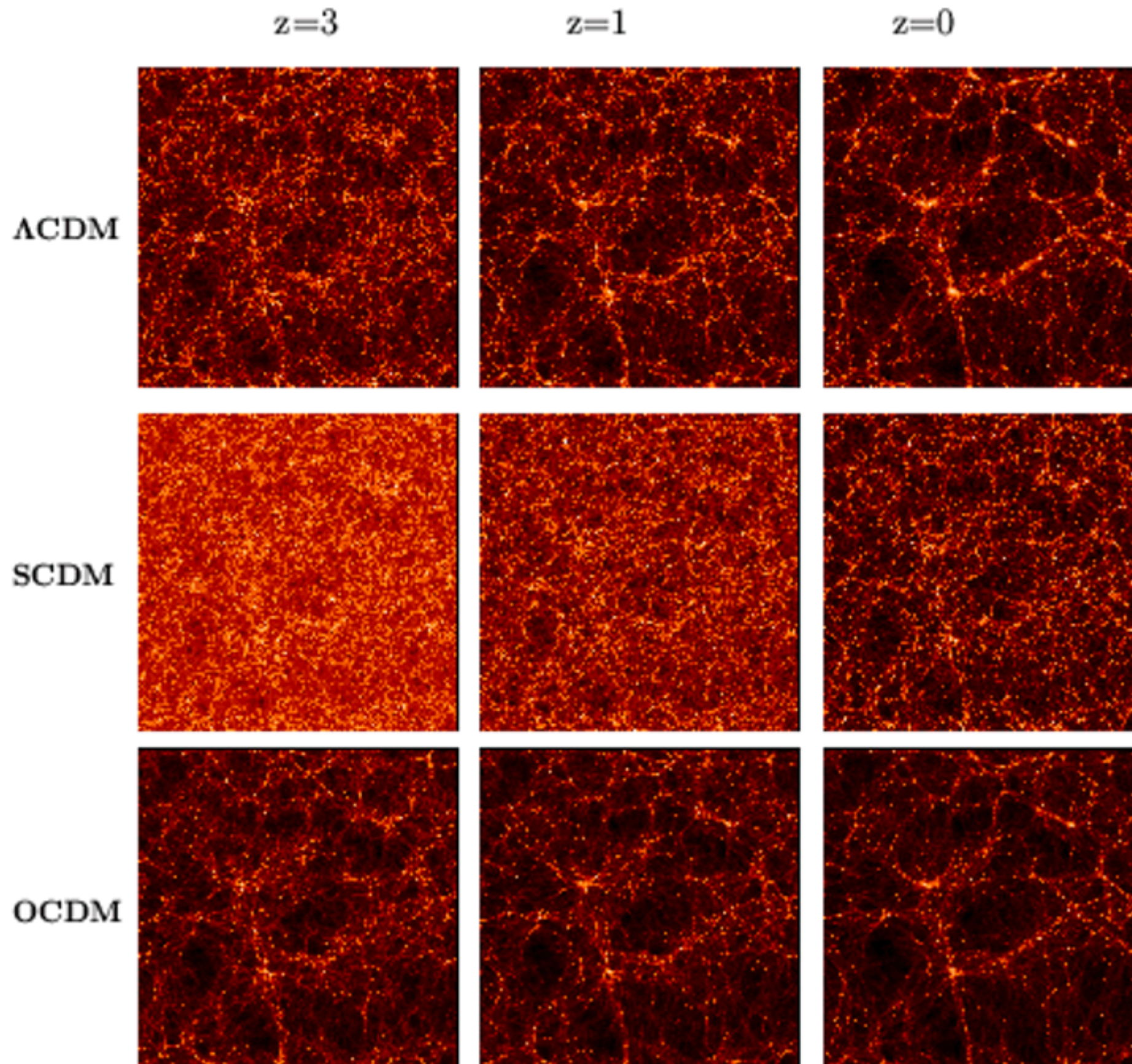
Merger trees from N-body simulations



Assembly histories for DM halos



Structure formation in different cosmologies

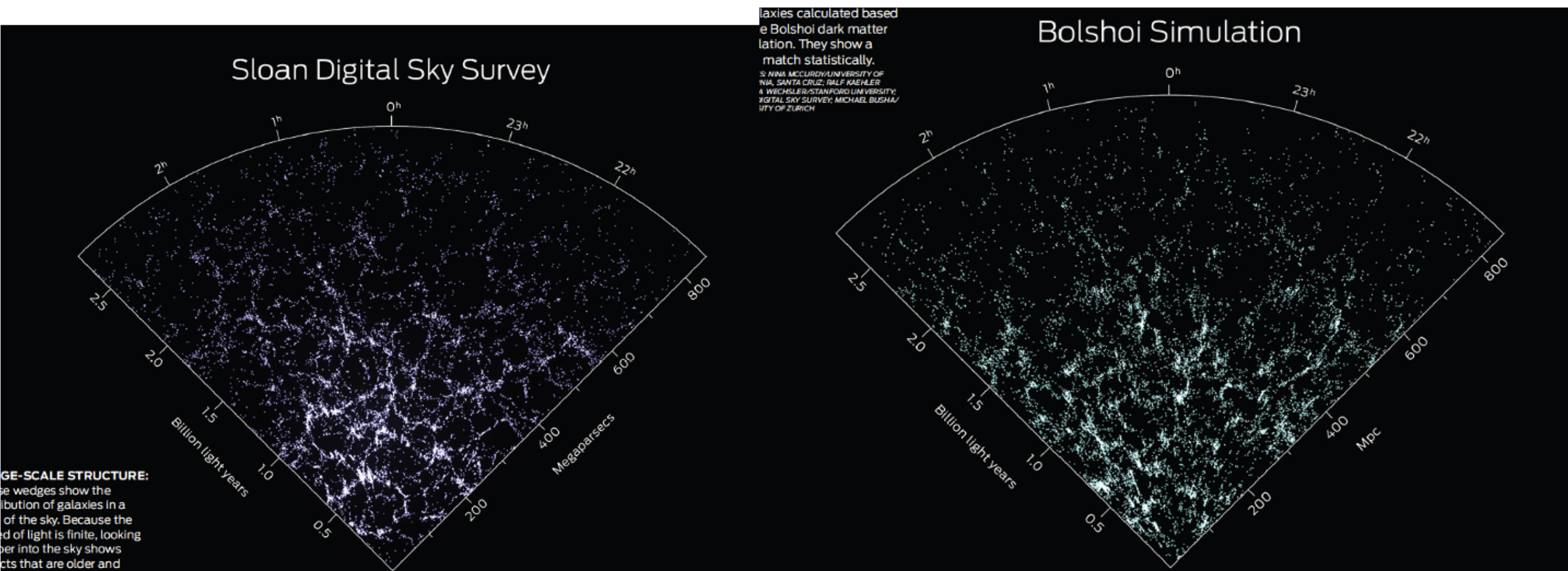


evolution of
large-scale
structure
strongly
depends on
cosmological
model

Virgo collaboration

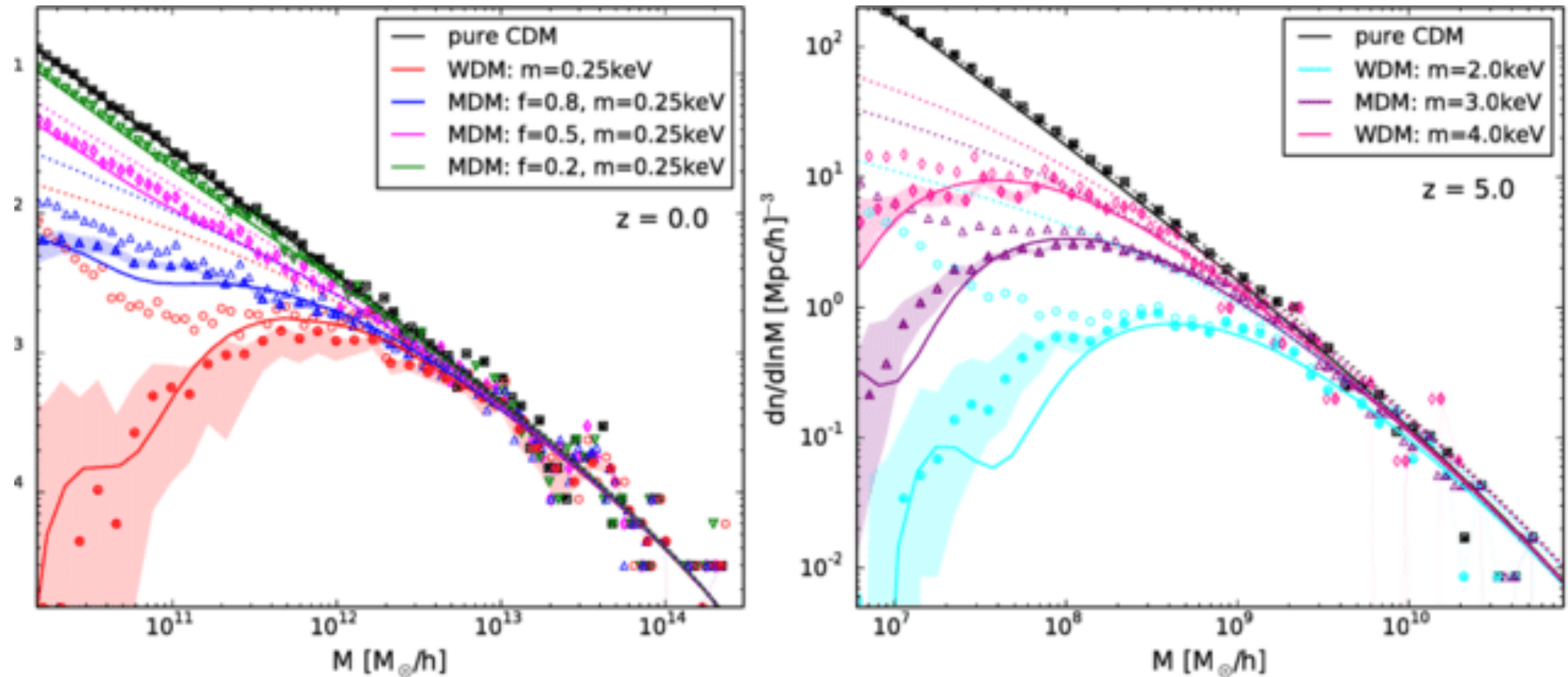
Large-scale structure in SDSS & simulation

LambdaCDM provides a perfect match to the observed large-scale structure



see also Nature paper, Springel et al. 2005

Halo functions for cold & warm dark matter



In WDM scenarios (lower rest-mass energy and/or higher fraction in mixed scenarios), number density of low-mass halos suppressed

Summary — Results of DM simulations

- **DM is responsible for setting the large-scale structure!**
- **Halo finder** are necessary to group particles together and to identify virialized halos and sub-halos
 - Often: Central halo with subhalos around
 - Hierarchical evolution of the halo mass function
- Merger trees/mass assembly histories can be constructed linking halos over time
- In a Lambda-cold dark matter Universe, **DM assembles in a hierarchical fashion consistent with the observed large-scale structure**
 - low-mass DM halos assemble their mass faster/earlier than massive halos

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Dark matter halo properties

- Having constructed halo catalogues and identified the dark matter particles belonging (bound) to one halo, we can compute and analyse:
 - halo shape \longrightarrow triaxial
 - halo density profile \longrightarrow universal
 - halo's rotation curves and maximum circular velocity (max of $\sim GM/R$)
 - halo's spin
- In next lectures: look at galaxies in halos and compute mock observations

Navarro-Frenk-and-White profiles

- Numerical simulations showed that virialized halos have a **universal density profile, a so called NFW profile** (Navarro, Frenk & White 1995, 1997)



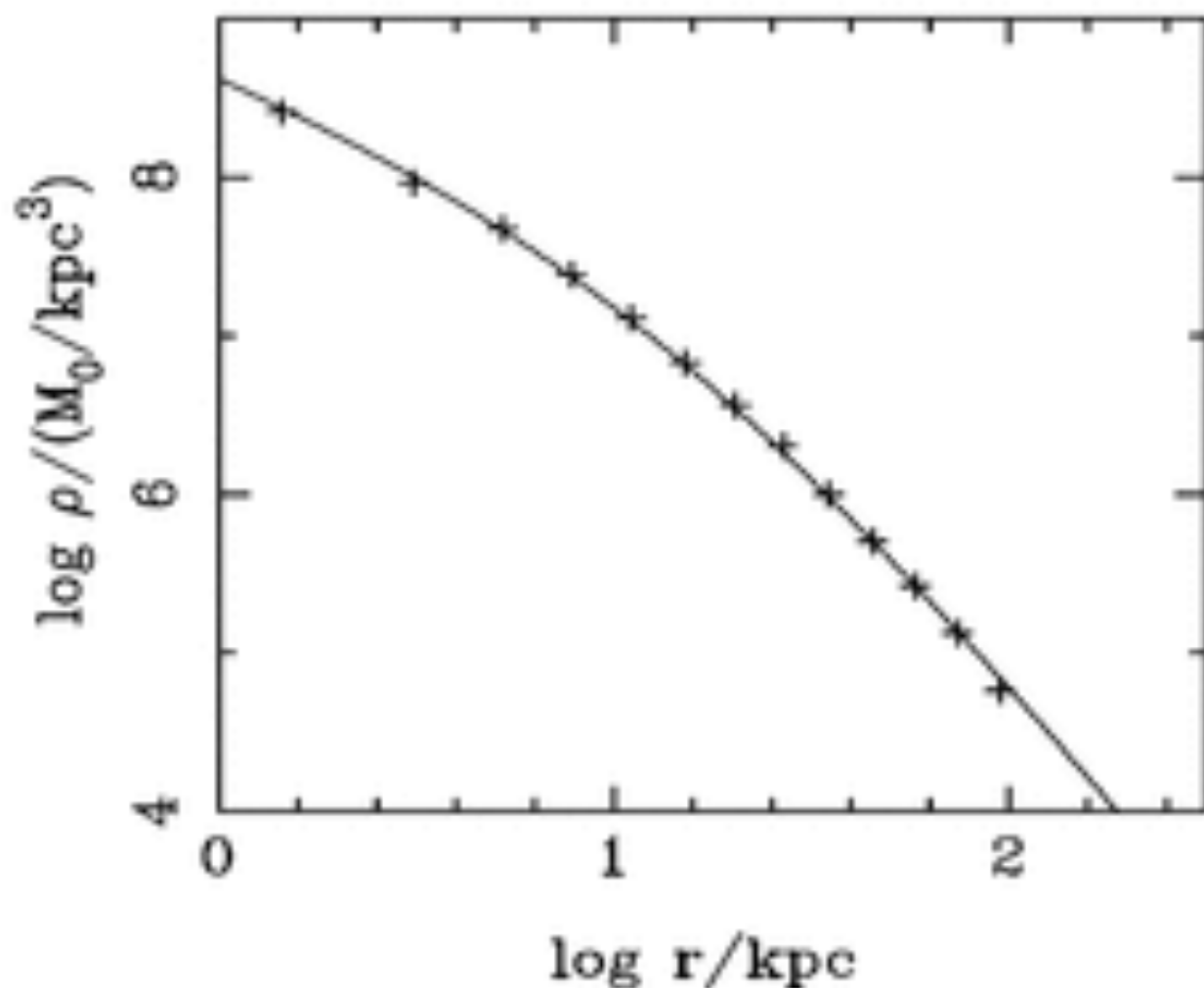
Universal density profiles

- Numerical simulations showed that virialized halos have a **universal density profile, a so called NFW profile** (Navarro, Frenk & White 1995, 1997)

$$\rho(r) = \rho_{\text{crit}} \frac{\delta_{\text{char}}}{r/r_s (1 + r/r_s)^2}$$

$$\delta_{\text{char}} = \frac{\Delta_h}{3} \frac{c^3}{\ln(1+c) - c/(1+c)}$$

$$c = r_h/r_s$$



- r_s is the scale radius (where $\rho(r)$ changes from $\sim r^{-1}$ to $\sim r^{-3}$)
- δ_{char} the characteristic overdensity (dep. on c)
- c is the concentration parameter (dep. on mass and redshift)
- For any cosmology, redshift and mass, the **NFW profile is completely characterised by its concentration parameter c and scale length r_s**

Universal density profiles

- For any cosmology, redshift and mass, the NFW profile is completely characterised by its concentration parameter c and scale length r_s

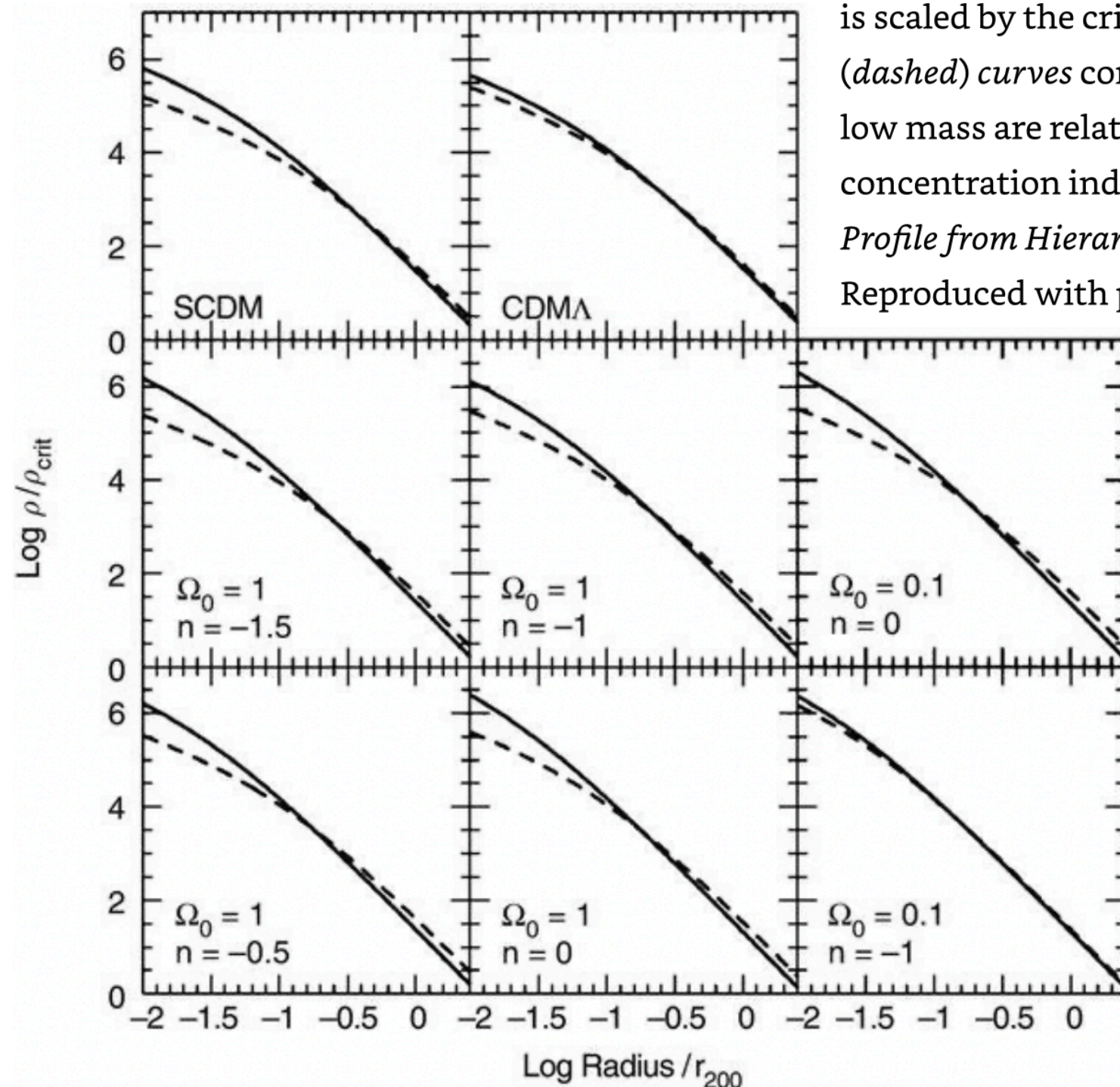


Fig. 7.18 The density profiles from Fig. 7.17, but now the density is scaled by the critical density, and the radius scaled by r_{200} . Solid (dashed) curves correspond to halos of low (high) mass—thus, halos of low mass are relatively denser close to the center, and they have a higher concentration index c . Source: J.F. Navarro et al. 1997, *A Universal Density Profile from Hierarchical Clustering*, ApJ 490, 493, p. 497, Fig. 3. ©AAS. Reproduced with permission

While there is a general consensus on the universal nature of DM halo density profiles, there are exceptions, especially in the inner regions of galaxies, that continue to be subjects of active research and debate.

Halo angular momentum

- Dark matter halos acquire most of their angular momentum $J = V_{\text{max}} * M * R$ (circular motion) from tidal torques close to “the turnaround” point
- Define a **dimensionless spin parameter**

$$\lambda = \frac{J}{\sqrt{2} M_{\text{vir}} V_{\text{vir}} R_{\text{vir}}}$$

- N-body simulations show that for halos formed through hierarchical clustering the **median spin value, $\lambda_{\text{med}} \sim 0.04-0.05$, is almost independent of**
 - halo mass
 - redshift
 - cosmological parameters

$\Rightarrow \lambda$ measures the dynamical importance of *ordered rotational* motion, v_{rot} relative to *total velocity dispersion*, σ required for dynamical support.

$\lambda \ll 1 \Rightarrow v_{\text{rot}} \ll \sigma, \Rightarrow$ most K.E. in random motions $\Rightarrow \sim$ spherical.

$\lambda \sim 1 \Rightarrow v_{\text{rot}} \sim \sigma, \Rightarrow$ most K.E. in circular motions $\Rightarrow \sim$ disk.

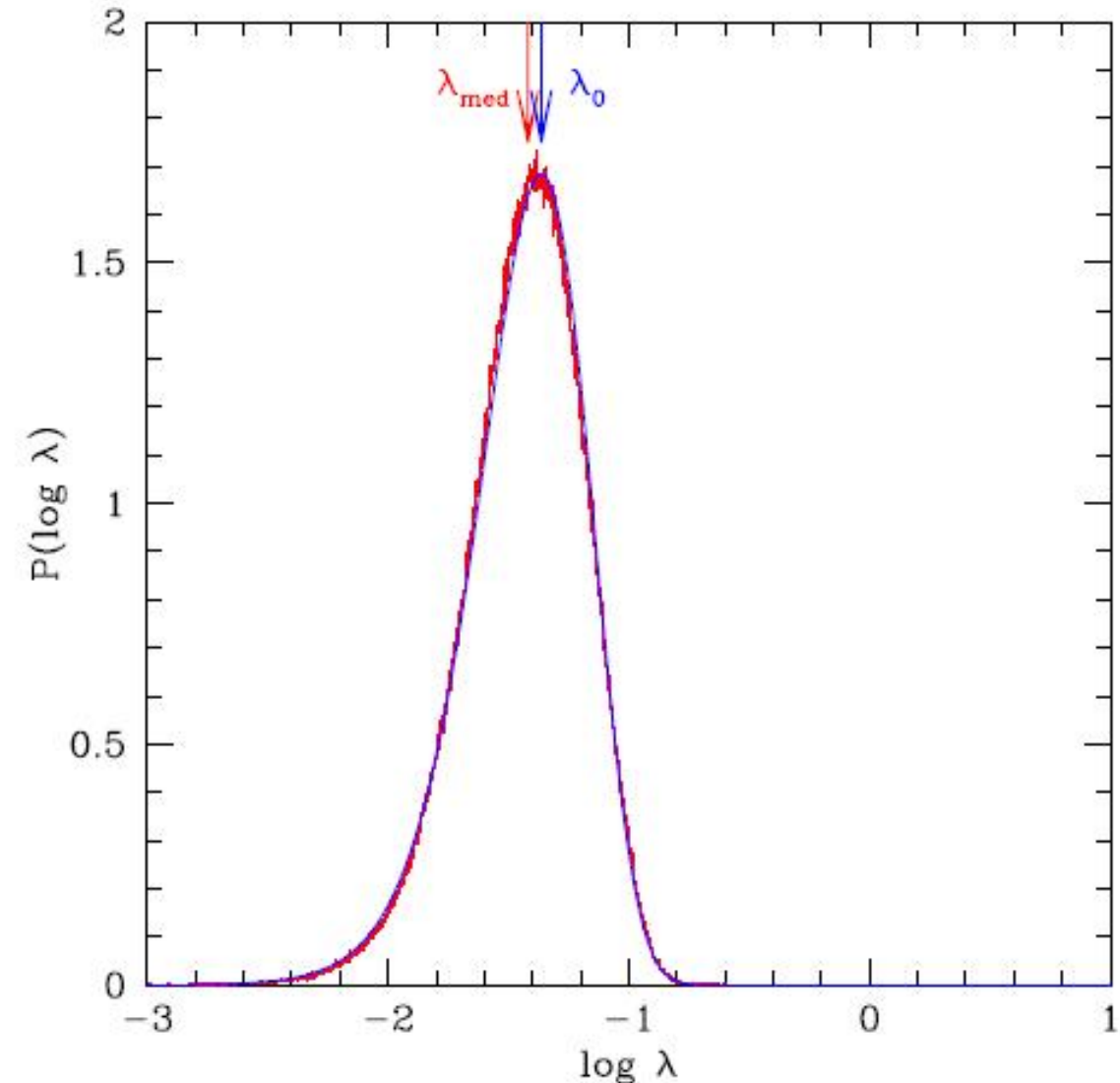
Halo angular momentum

- The **distribution of the spin** is well fit by a lognormal distribution

$$P(\lambda)d\lambda = \frac{1}{\sqrt{2\pi}\sigma_\lambda} \exp \left[-\frac{(\ln\lambda + \ln\lambda_{\text{med}})^2}{2\sigma_\lambda^2} \right] \frac{d\lambda}{\lambda}$$

where $\sigma_\lambda \equiv \sigma(\ln) = 0.5 - 0.6$

- **DM halos are typically rotating slowly** and supported by random motions (rotation supported systems have instead $\lambda \sim 1$)



Bett et al 2007

Many other interesting halo properties, like their triaxial shapes
You can read more about them in [Griffin+16](#), [Klypin+12](#) and [Lovell+12](#)...

Summary — Lecture 7

- **Structure formation in the non-linear regime has to be modelled by numerical N-body simulations**
- Dynamics of dark matter can be followed by modelling a collisionless fluid only interacting gravitationally using N-body methods
- Based on cosmological initial conditions, large and well-resolved DM N-body simulations have been able to run thanks to several improvements
- Halo finder are necessary to group particles together and to identify virialized halos and sub-halos
- Merger trees/histories can be constructed
- In a CDM Universe
 - **DM is responsible for setting the large-scale structure!**
 - **DM assembles in a hierarchical fashion consistent with the observed large-scale structure**
- Universal halo properties:
 - Universal halo profile (NFW)
 - Constant mean spin parameter

Failure of LambdaCDM?

- **Large scales:** Good match between observations and N-body simulations

- **Small scales:** **Some discrepancies**

Observations:

- Shallow density cores
- Few satellites/low-mass gals
- Low total satellite masses

Simulations of CDM:

- Steep density cores
- Many satellites/low-mass gals
- Higher total satellite masses

• Possible solutions:

- Cosmology/CDM is wrong?
- Observations are wrong?
- Simulations are wrong?

- ▶ **Maybe, but not much**
- ▶ **Better telescopes**
- ▶ **Missing physics, no gas dissipative processes so far!!**

Up next...

- *Lecture 1 (Repetition of Astro-I and Astro-II):*
 - Introduction (galaxy definition, astronomical scales, observable quantities)
 - Brief review on stars
- *Lecture 2:*
 - Radiation processes in galaxies and telescopes;
 - The Milky Way
- *Lecture 3: The world of galaxies I*
- *Lecture 4:*
 - The world of galaxies II;
 - Black holes and active galactic nuclei
- *Lecture 5:*
 - Galaxies and their environment;
 - High-redshift galaxies
- *Lecture 6:*
 - Cosmology in a nutshell
 - Linear structure formation in the early Universe
- *Lecture 7:*
 - Dark matter and the large-scale structure
 - Cosmological N-body simulations of dark matter
- *Lecture 8: Populating dark matter halos with baryons: Semi-empirical & semi-analytical models*
- *Lecture 9: Modelling the evolution of gas in galaxies: Hydrodynamics*
- *Lecture 10: Gas cooling/heating and star formation*
- *Lecture 11: Stellar feedback processes*
- *Lecture 12: Black hole growth and AGN feedback processes*
- *Lecture 13: Success and challenges of modern simulations*
- *Lecture 14: Future prospects and mock exam*

Part I: Observational basics & facts of galaxies

Part II: Theory & models of galaxy evolution processes