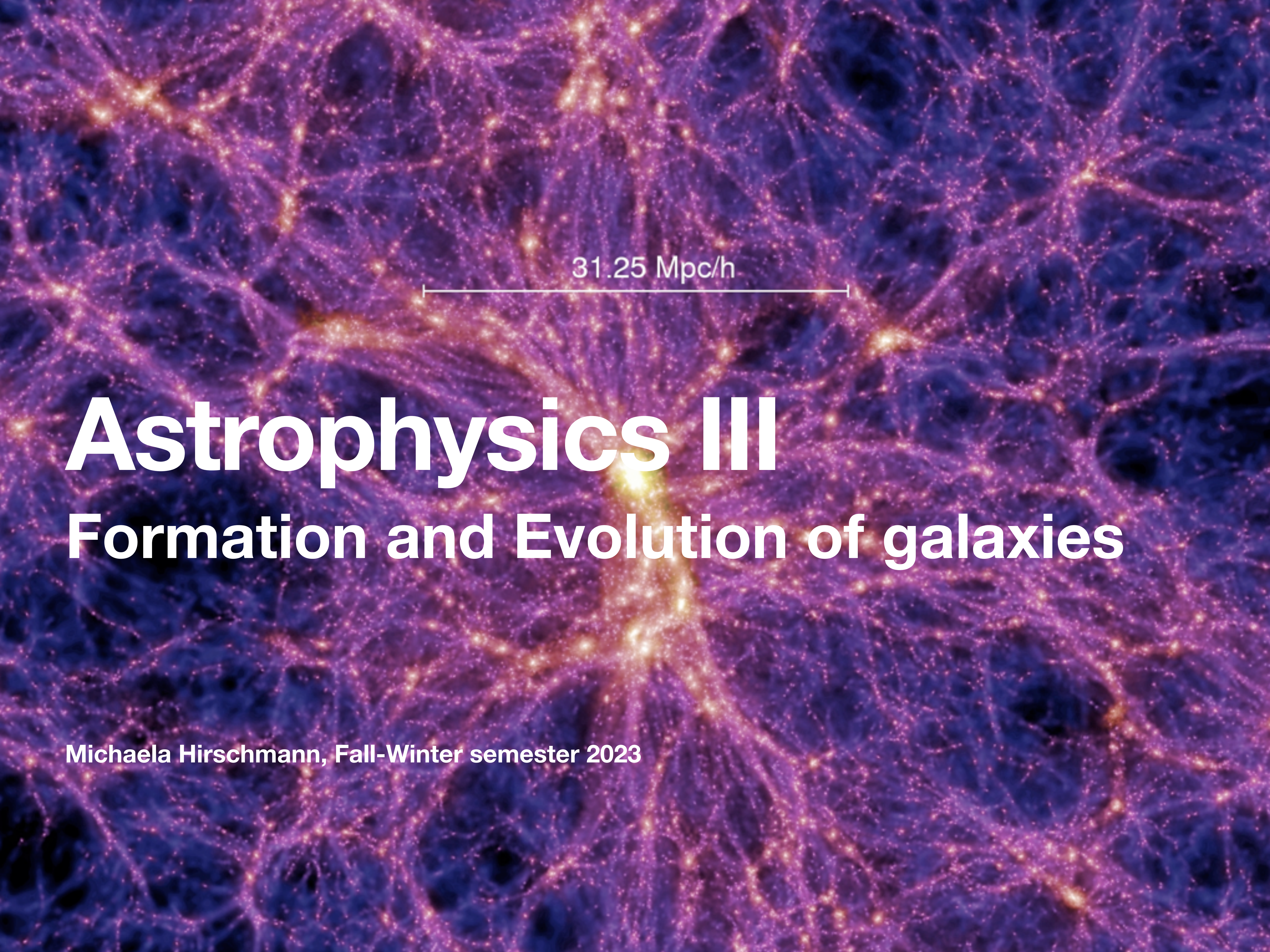


Astrophysics III

Formation and Evolution of galaxies

Michaela Hirschmann, Fall-Winter semester 2023

Lecture content and schedule

- *Chapter 1*: Introduction (galaxy definition, astronomical scales, observable quantities — repetition of Astro-I)
- *Chapter 2:* Brief review on stars
- *Chapter 3*: Radiation processes in galaxies and telescopes;
- *Chapter 4:* The Milky Way
- *Chapter 5*: The world of galaxies I
- *Chapter 6*: The world of galaxies II
- *Chapter 7:* Black holes and active galactic nuclei
- *Chapter 8:* Galaxies and their environment;
- *Chapter 9:* High-redshift galaxies
- *Chapter 10*:
  - Cosmology in a nutshell; Linear structure formation in the early Universe
- *Chapter 11*:
  - Dark matter and the large-scale structure
  - Cosmological N-body simulations of dark matter
- *Chapter 12*: Populating dark matter halos with baryons: Semi-empirical & semi-analytical models
- *Chapter 13:* Modelling the evolution of gas in galaxies: Hydrodynamics
- *Chapter 14*: Gas cooling/heating and star formation
- *Chapter 15*: Stellar feedback processes
- *Chapter 16*: Black hole growth & AGN feedback processes
- *Chapter 17*: Modern simulations & future prospects

Part I: Observational basics & facts of galaxies first 7 lectures

Part II: Theory & models of galaxy evolution processes second 7 lectures

Outline of this lecture

- Cosmology in a nutshell
  - Motivation
  - The cosmological principle
  - Robertson-Walker Metric and Friedmann equation
  - The age of the Universe
  - Cosmological parameters
- The inhomogeneous Universe
  - Linear perturbation theory
  - Density fluctuations, power spectrum & transfer function
  - Non-linear growth: The spherical collapse model
- Generating initial conditions for cosmological simulations
  - Particles in a simulation box and in zoom-in simulations

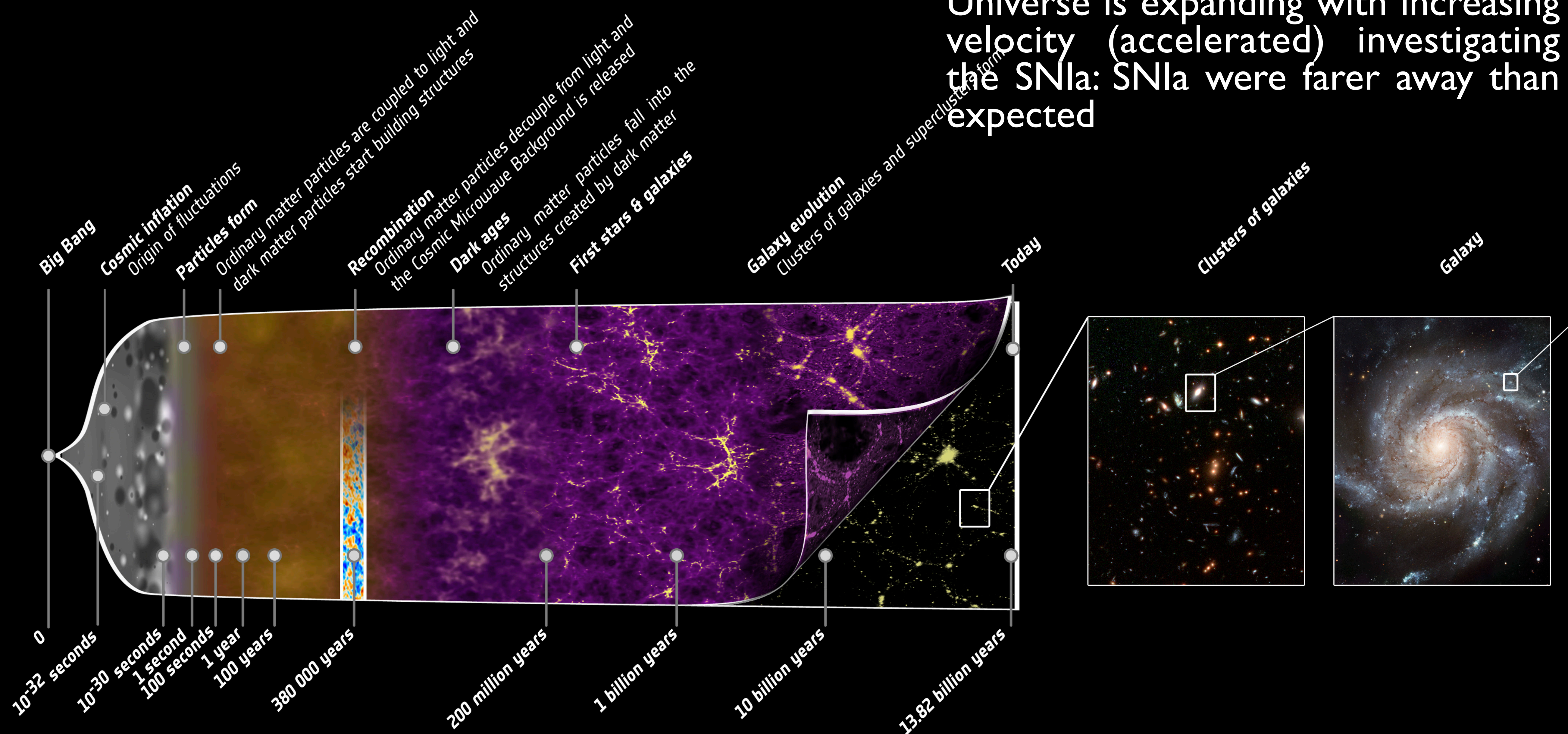
What is cosmology?

- The study of the origin, overall structure, composition, evolution and future of our Universe!
- Dedicated course to observational cosmology given by Prof Kneib in the summer term
- In this course: only very, very basics — needed to describe the evolution of galaxies in our Universe

Evolution of the Universe in a nutshell

Quantum fluctuations+inflation (period of exponential expansion) gave rise to density inhomogeneities, in agreement with CMB

- 1998: Perlmutter, Schmidt & Riess: Universe is expanding with increasing velocity (accelerated) investigating the SNIa: SNIa were farer away than expected



https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_history_of_Universe

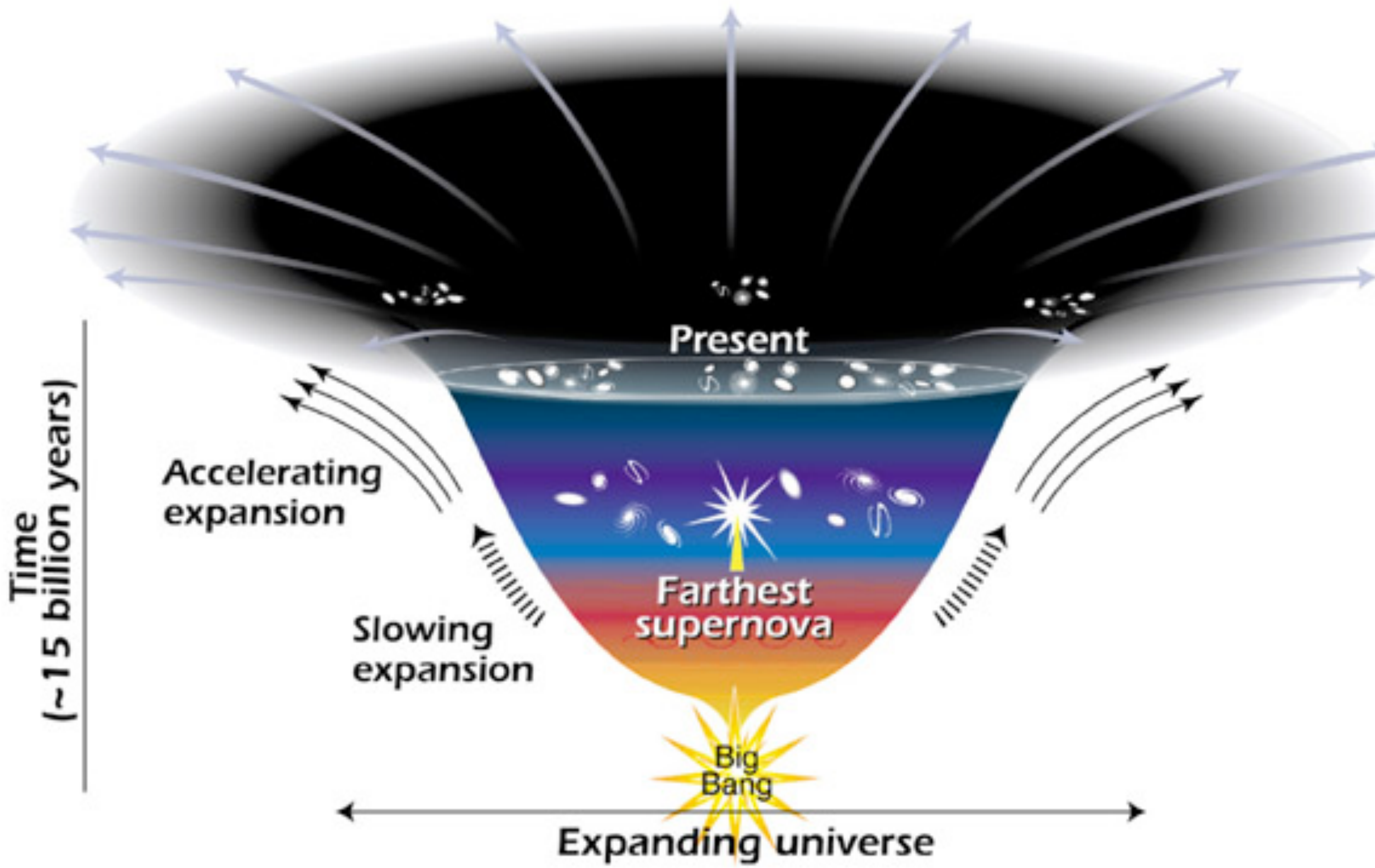
Expansion of the Universe

- Cosmological redshift: direct consequence of Hubble expansion, we can derive a connection to the scale factor a

$$z_{\cos} = \frac{\lambda_o - \lambda_e}{\lambda_e} \qquad 1 + z_{\cos} = 1/a \qquad a(t) = R(t)/R(t_0)$$

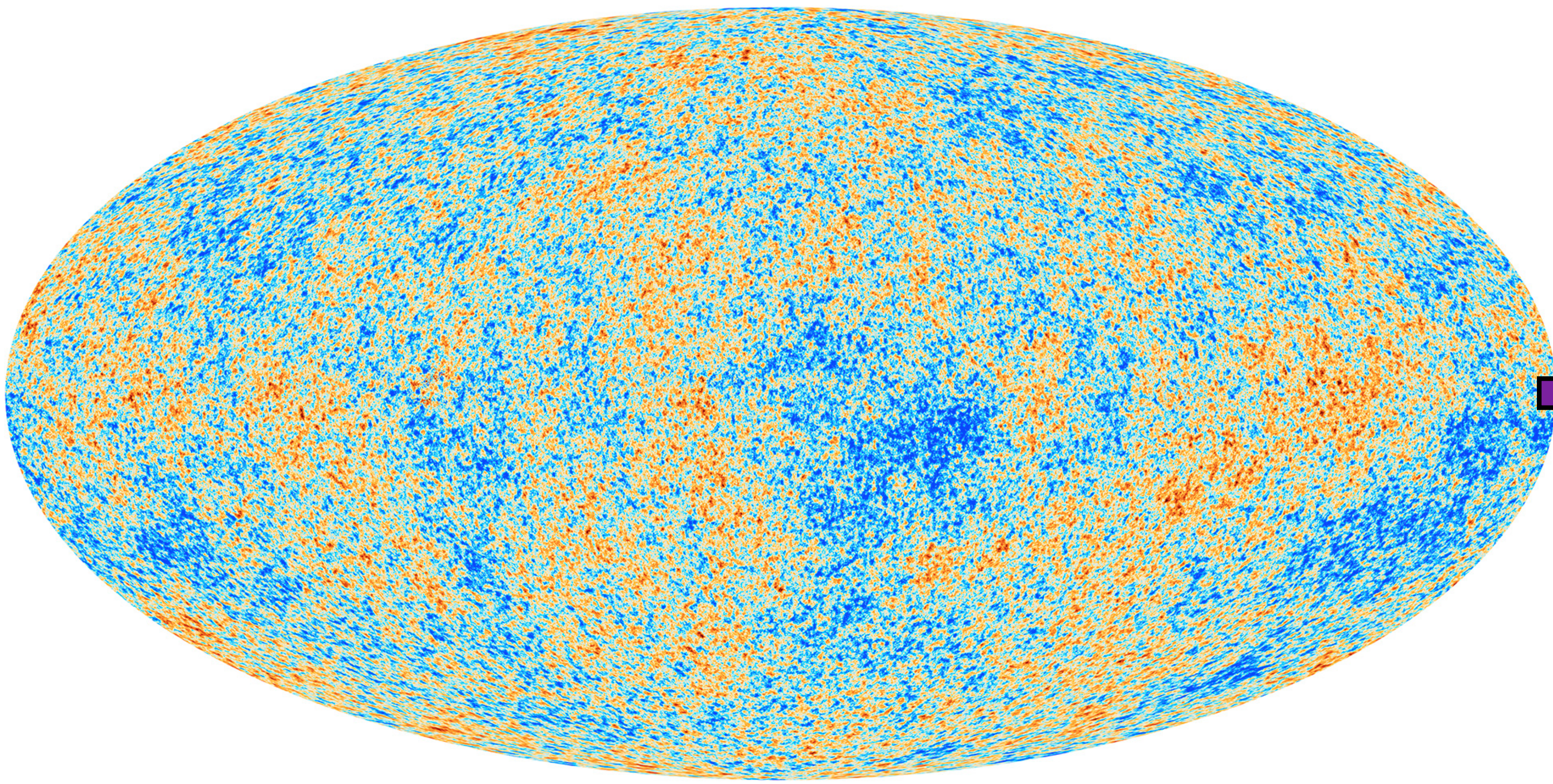
$$a(z=0) = 1$$

- 1998: Perlmutter, Schmidt & Riess: Universe is expanding with increasing velocity (accelerated) investigating the SNIa:
- SNIa were farer away than expected in a EdS Universe meaning the expansion of the universe must have been accelerated
- Attributed to "dark energy"



Cosmic microwave background

- With time, Universe expands, cools and density drops: photons could propagate freely to “us” from a last scattering surface (inheriting the blackbody spectrum) at ~380,000 after Big Bang
- Small temperature fluctuations (~10^{-5}), fluctuations very smooth
- On large scales: Universe is very uniform



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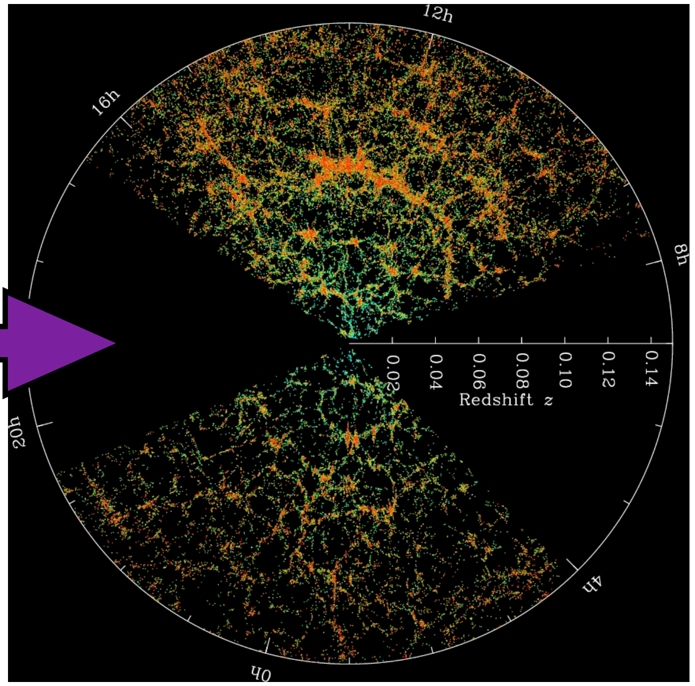


- We need **theory** to understand how initial small density fluctuations emerged into the large-scale structure observed today
- For that, we need a cosmological back ground.

Cosmological principle

- Cosmological principle: on sufficiently large scales, the Universe is homogenous and isotropic
  - the observed properties of the Universe are isotropic, i.e. independent of direction
  - our position is by no means preferred to any other
  - the Universe is, thus, isotropic around all its points: it's homogeneous

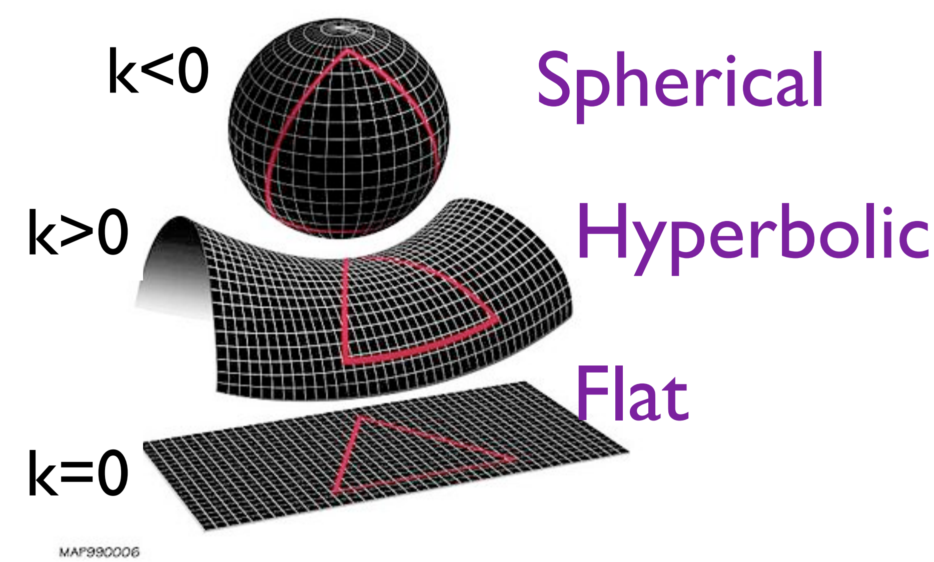
Robertson - Walker metric

- Geometric properties of a homogeneous and isotropic and potentially non-static universe are described by the 4D-space-time Robertson-Walker metric, specified by the *scale factor a(t)* and the *curvature k*.

$$ds^2 = c^2 dt^2 - dl^2 = c^2 dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

what is a metric?

- specifies the distance between two points in space
- depends on the *geometry* of the space
- what are some familiar examples?
- no change of coordinates can change one metric into another

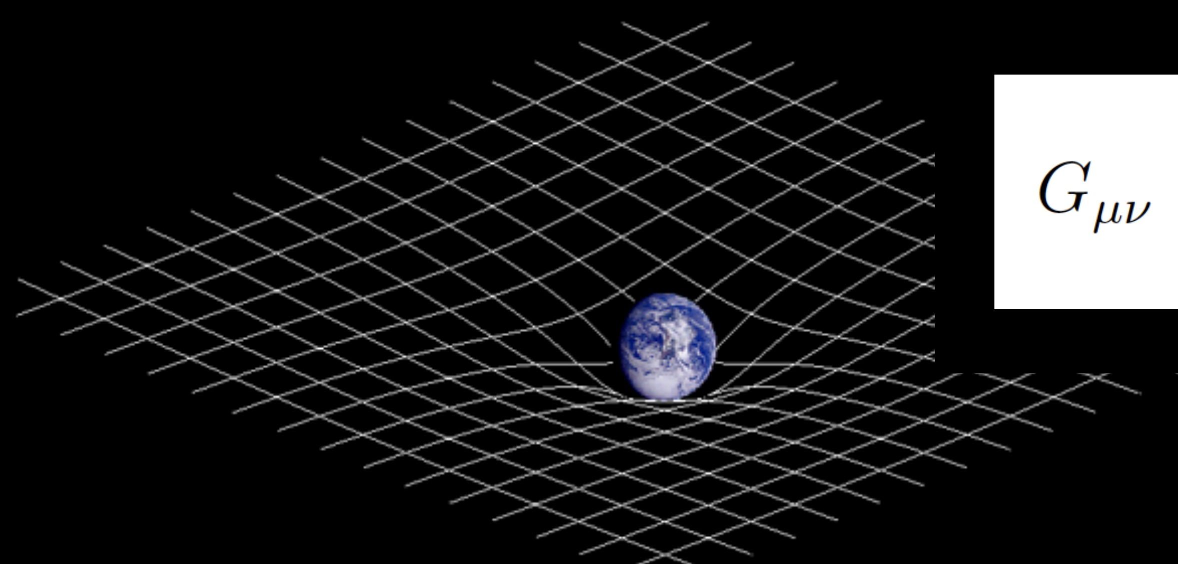


From general relativity to cosmology

- Cosmology is based on Einstein's theory of general relativity according to which the space-time geometry is determined by the matter distribution in the Universe

two sentence General Relativity*

- mass-energy tells space-time how to curve
- the curvature of space-time tells mass-energy how to move



Einstein equation

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

curvature metric mass and energy

* due to John Wheeler

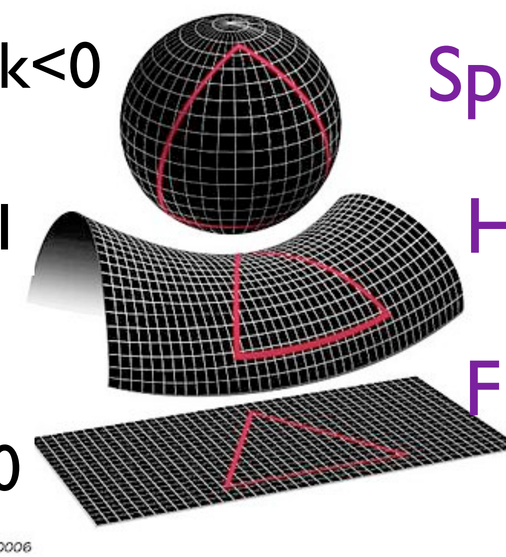
The dynamics of the Universe

- Dynamics of the space-time metric is reduced to dynamics of a(t)
- To obtain an expression for a(t) for any given k and matter/energy content (being isotropic and homogeneous):

➡Combination of Einstein field equation (GR) and the Robertson-Walker metric (based on isotropy and homogeneity) results in the Friedmann-Lemaitre equations

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3P/c^2\right) + \frac{\Lambda c^2}{3}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$



- Energy conservation for Einstein equ. leads to a third useful "adiabatic" equation

$$\frac{\dot{\rho}}{\rho} = -3\left(1 + \frac{P}{\rho}\right)\frac{\dot{a}}{a}$$

Hubble function and Critical density

- Hubble function H(t) is defined as the relative expansion rate:

$$H(t) = \frac{\dot{a}}{a}, \;\; H_0 = H(a=1) = 100 \; h \; \mathrm{km/s/Mpc}$$

- h is the dimensionless Hubble parameter (historical origin since H was long not known exactly)

- The critical density of the Universe is given by

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}, \;\; \rho_{c0} = \rho_c(t_0) = \frac{3H_0^2}{8\pi G}$$

  - If k = 0 and Λ = 0, this is the minimum density, under which the Universe would collapse under its own gravity

Evolution of the Hubble function

- Densities are often expressed in terms of critical densities, i.e. as dimensionless matter/radiation density parameters

$$\Omega_{m/r} = \frac{\rho_{m/r}(t)}{\rho_c(t)}, \; \Omega_{m/r,0} = \Omega_{m/r}(t_0) = \frac{\rho_{m/r}(t_0)}{\rho_{c0}}$$

- Dim.less density parameter for Lambda:

$$\Omega_\Lambda(t) = \frac{\Lambda c^2}{3H^2(t)}, \; \Omega_{\Lambda 0} = \Omega_\Lambda(t_0) = \frac{\Lambda c^2}{3H_0^2}$$

- Substituting these density parameters into the spatial Friedmann equation yields the evolution of the Hubble fct H

$$H^2(a) = H_0^2 \left(\frac{\Omega_{r0}}{a^4} + \frac{\Omega_{m0}}{a^3} + \Omega_{\Lambda 0} - \frac{kc^2}{a^2 H_0^2} \right) = H_0^2 E(a)^2$$

- Relative importance of Ω's change with time (radiation, matter and DE-dominated phases)

Evolution of radiation, matter, DE densities

- Relative importance of Ω's change with time (radiation, matter and DE-dominated phases)

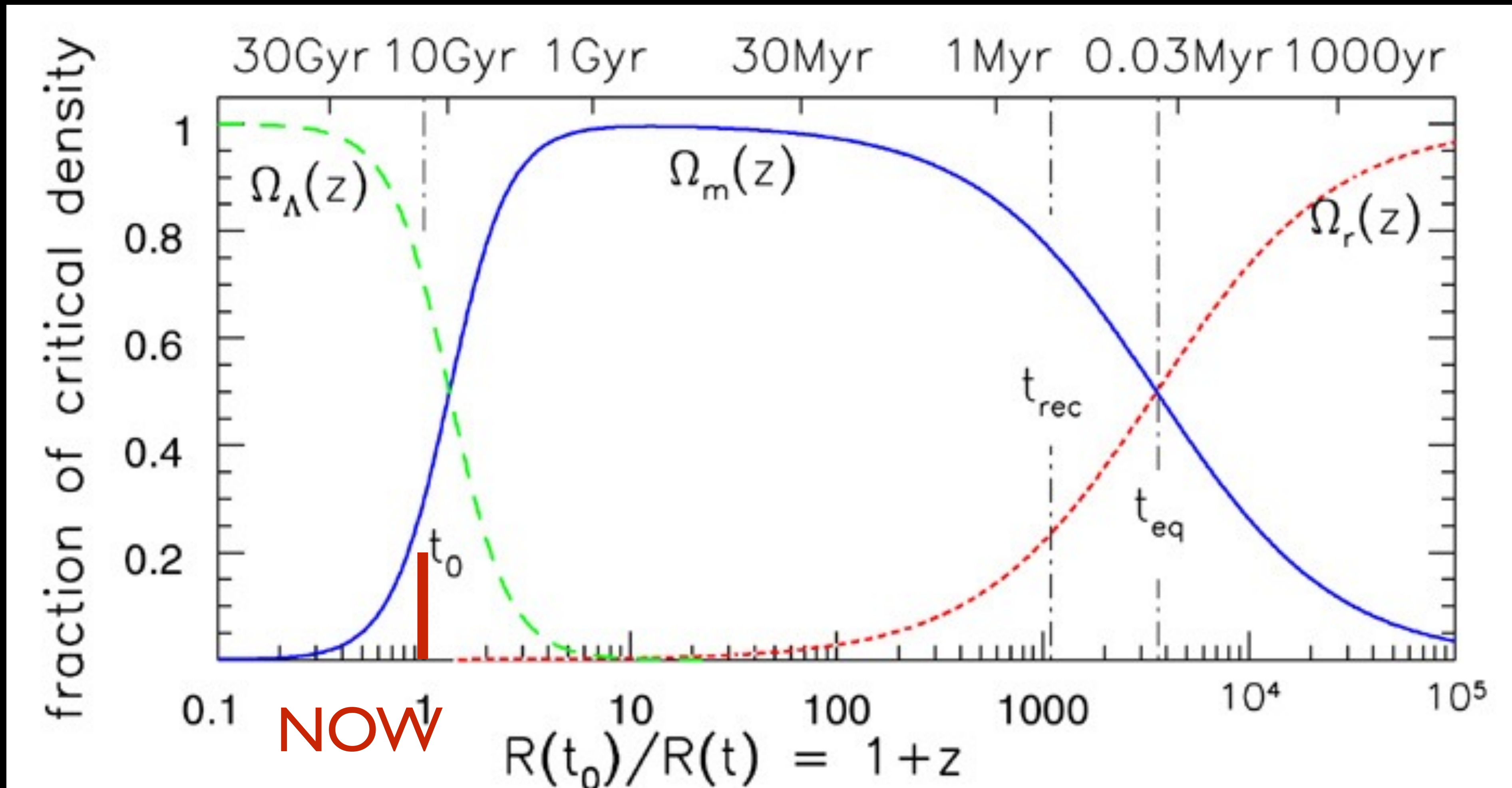
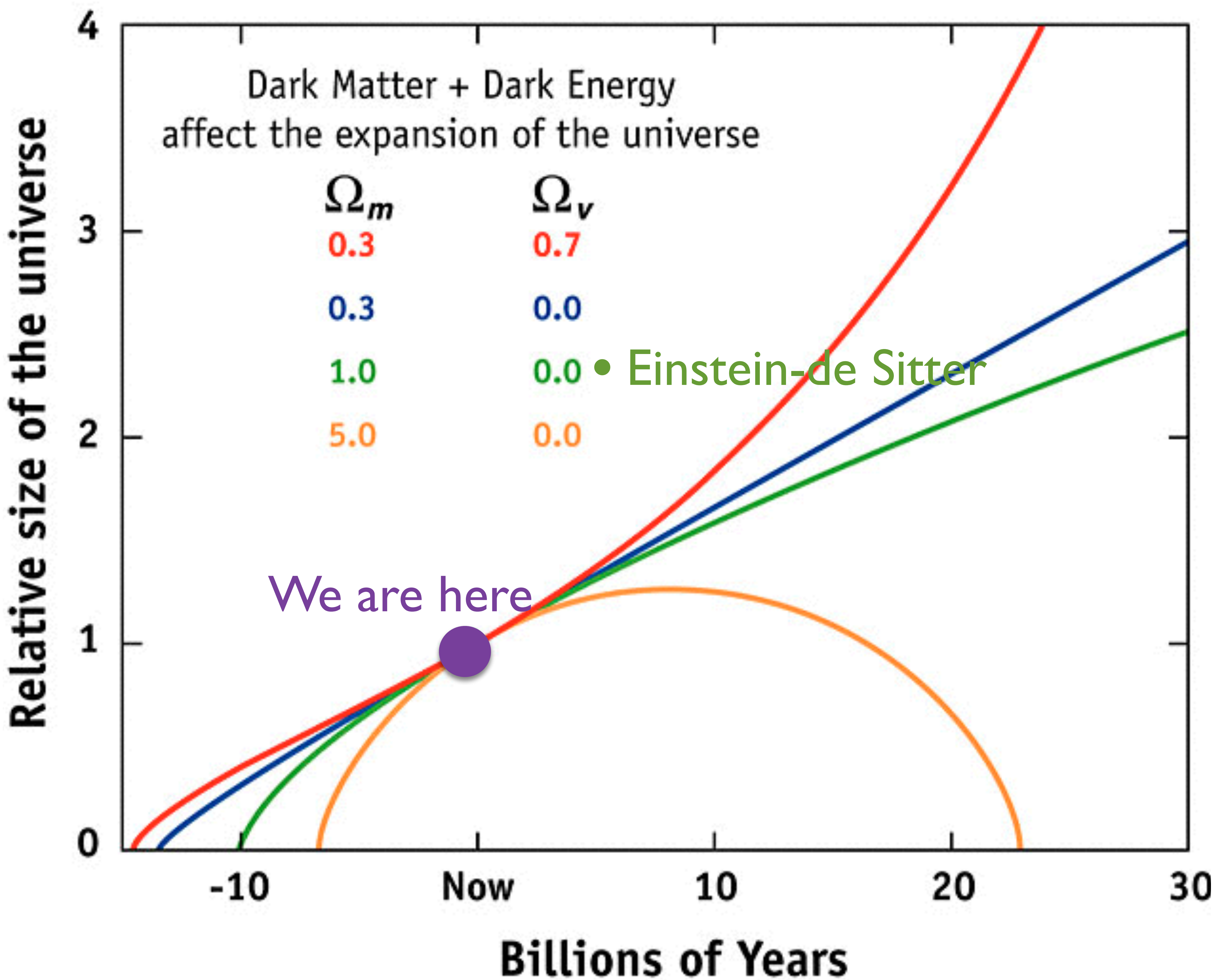


Fig 8.7 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

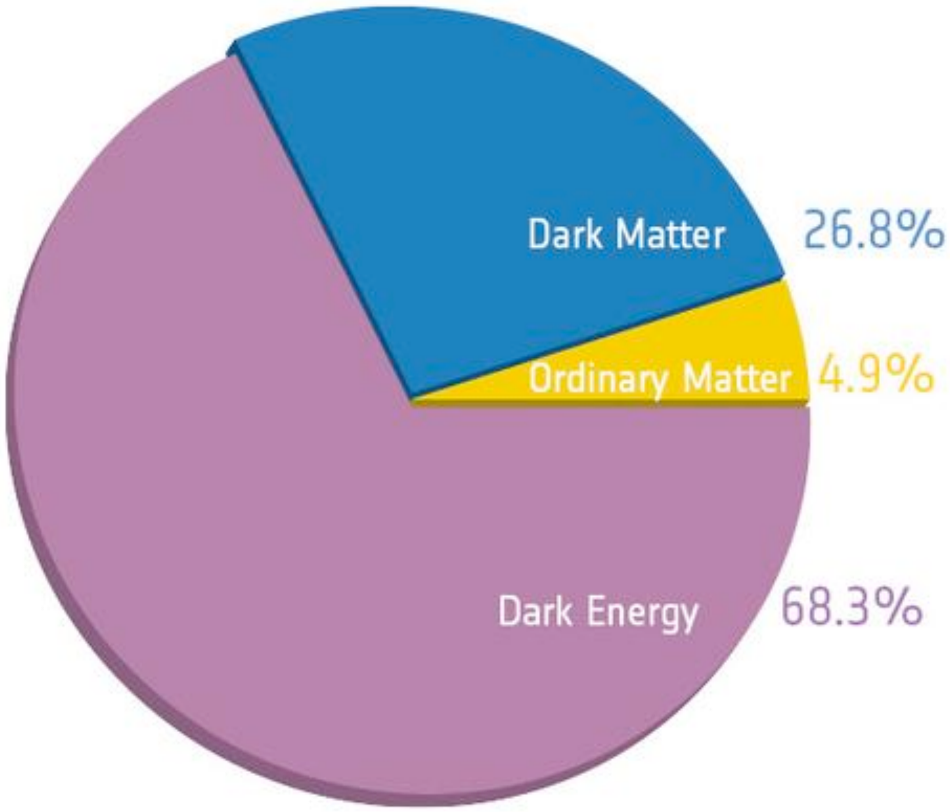
Age and size evolution of the Universe



Planck cosmology

- Combination of CMB, SNe and clusters lead to accurate measurements of our cosmological parameters

- Era of "precision cosmology" thanks to Planck measurements:



$\Omega_m = 0.309 = \Omega_{dm} + \Omega_{bar} = 0.259 + 0.048$ **~84% of all matter is dark**

$\Omega_\Lambda = 0.691$ **~70% of the energy density is "dark"**

$\Omega_r \sim 1e\text{-}5$

$\Omega_k \sim 0$ Our Universe is spatially flat

$h = 0.678$ The Hubble time is 13.8 Gyr

$\sigma_8 = 0.823$

We are living in a DARK Universe

what is the dark energy?

- cosmological constant

  constant energy density in the vacuum of space; acts as repulsive force, physical origin and why its energy density is precisely tuned unclear

- quintessence

  class of theoretical models with a dynamic and evolving form of dark energy —> energy density varies over time and space; possibly described by a scalar field (a new fundamental field)

- modified gravity

  attempt to explain the accelerated expansion without invoking dark energy —> modify Einstein's general relativity at cosmological scales, altering the gravitational interactions on large scales.

What is dark matter?

we don't know what dark matter is, but we know what it must do (and not do):

- interact via gravity in the same way as normal matter
- not interact via the strong or EM force
- it cannot radiate energy so it is dissipationless and collisionless
- Candidates:
  - sterile neutrino (standard model)
  - supersymmetric (SUSY) Lightest Supersymmetric Partner particle (LSP) – e.g. neutralino (partner of photon, Z boson, or Higgs) – example of a Weakly Interacting Massive Particle (WIMP)
  - axions (symmetry breaking)
- mass of the dark matter particle largely determines how much kinetic energy it has – more massive particles move more slowly (cold)

Basics of cosmology — Summary

- Cosmological principle: on large scales, Universe is homogeneous and isotropic
- Combining cosmological principle, Robertson-Walker metric and GR results in the Friedmann equations, which are describing the dynamics of the Universe
- Observations indicate that we are living in flat Universe with ~30% of matter and 70% of dark energy
- Dark matter should interact via gravitational forces, not via strong/el-mag force —> dissipation less and collision less…
- Initial (quantum) density fluctuations, which got (most likely) amplified via inflation, grow further driven by gravity, seeds of the present-day large-scale structure

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The inhomogeneous Universe

- There are pronounced structures in the Universe from stars to galaxy clusters (on comparably small scales) ➞ How did they form?
- Initial density fluctuations (enlarged by inflation) and action of gravity
- To describe small density fluctuations in the linear regime, we can use Newtonian perturbation theory for a self-gravitating fluid (for simplicity no Lambda, for the non-linear evolution see lecture 7)
- Continuity equation (Mass cons.)

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

- Euler equation (Mom. cons.)

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{\vec{\nabla} P}{\rho} - \vec{\nabla}\Phi$$

- Poisson equation

$$\vec{\nabla}^2 \Phi = 4\pi G \rho$$

Linear perturbation theory

- Assume small perturbations ρ and v:

$$\rho(\vec{x},t) = \rho_0(t) + \delta\rho(\vec{x},t) \text{ and } \vec{v}(\vec{x},t) = \vec{v}_0(t) + \delta\vec{v}(\vec{x},t)$$

- Insert into continuity equation, neglect 2n-order terms:

$$\frac{\partial(\rho_0 + \delta\rho)}{\partial t} + \vec{\nabla} \cdot (\rho_0\vec{v}_0 + \delta\rho\vec{v}_0 + \rho_0\delta\vec{v}) = 0$$

- Homogeneous quantities fulfil continuity equation as well

$$\frac{\partial\rho_0}{\partial t} = -\rho_0\vec{\nabla} \cdot \vec{v}_0$$

- Insert this into equation above results in:

$$\frac{\partial\delta\rho}{\partial t} + \vec{v}_0 \cdot \vec{\nabla}\delta\rho + \rho_0\vec{\nabla} \cdot \delta\vec{v} + \delta\rho\vec{\nabla} \cdot \vec{v}_0 = 0$$

- Define density contrast $\mathfrak{D} = \dfrac{\delta\rho}{\rho_0}$ << 1 in linear regime

- We can re-write the equation to

$$\dot{\mathfrak{D}} + \vec{v}_0 \cdot \vec{\nabla}\mathfrak{D} + \vec{\nabla} \cdot \delta\vec{v} = 0$$

Linear perturbation theory

- Similarly, Euler and Poisson equations yield in:

$$\frac{\partial \delta\vec{v}}{\partial t} + (\delta\vec{v} \cdot \vec{\nabla})\vec{v}_0 + (\vec{v}_0 \cdot \vec{\nabla})\delta\vec{v} = -\frac{\vec{\nabla}\delta P}{\rho_0} - \vec{\nabla}\delta\Phi$$

$$\vec{\nabla}^2\delta\Phi = 4\pi G\rho_0\mathfrak{D}$$

- Now switch from **x** and **v** to co-moving coordinates **r** and **u**:

$$\vec{x} = a \cdot \vec{r} \qquad \vec{v}_0 = \dot{a}\vec{r} \qquad \delta\vec{v} = \vec{u}(= a\dot{\vec{r}})$$

- …with co-moving spatial and time derivatives of

$$\vec{\nabla}_x \to \frac{\vec{\nabla}_r}{a}, \quad \left.\frac{\partial}{\partial t}\right|_x \to \left.\frac{\partial}{\partial t}\right|_r - \frac{\dot{a}}{a}\vec{r} \cdot \vec{\nabla}_r$$

- With that, we can simplify the perturbation equations to
  - Continuity equation $\quad \dot{\mathfrak{D}} + \frac{\vec{\nabla} \cdot \vec{u}}{a} = 0$
  - Euler equation $\quad \dot{\vec{u}} + H\vec{u} = -\frac{\vec{\nabla}\delta P}{a\rho_0} - \frac{\vec{\nabla}\delta\Phi}{a}$
  - Poisson equation $\quad \vec{\nabla}^2\delta\Phi = 4\pi G\rho_0 a^2\mathfrak{D}$

Exercises!

Growth equation

- Combining the divergence of the Euler equation with time derivative of the continuity and the Poisson equation yields in

$$\ddot{\mathfrak{D}} + 2H\dot{\mathfrak{D}} = \left(4\pi G\rho_0\mathfrak{D} + \frac{c_s^2\vec{\nabla}^2\mathfrak{D}}{a^2}\right)$$

Do analytic calculations in exercises!

$$\text{where } P = c_s^2\rho$$

- "Growth equation", decaying mode negligible after some time, different solutions for different cosmologies

- In general solved by the growth function F(a):

$$\mathfrak{D}(r,a) = \mathfrak{D}_0(r)F(a)$$

- By definition mean of the density contrast vanishes:

$$\langle\mathfrak{D}(a)\rangle = \left\langle\frac{\rho-\rho_0}{\rho_0}\right\rangle = 0$$

Fourier decomposition of the density field

- In the linear regime (and in a flat Universe), expand perturbation fields in some suitable mode functions, e.g. plane waves, and perturbation field can be represented by its Fourier transform D'

$$\mathfrak{D}(\vec{r},t) = \int \frac{d^3k}{(2\pi)^3} \mathfrak{D}'(\vec{k},t) e^{-i\vec{k}\cdot\vec{r}}$$

- With the Fourier transform we get growth equation in Fourier space

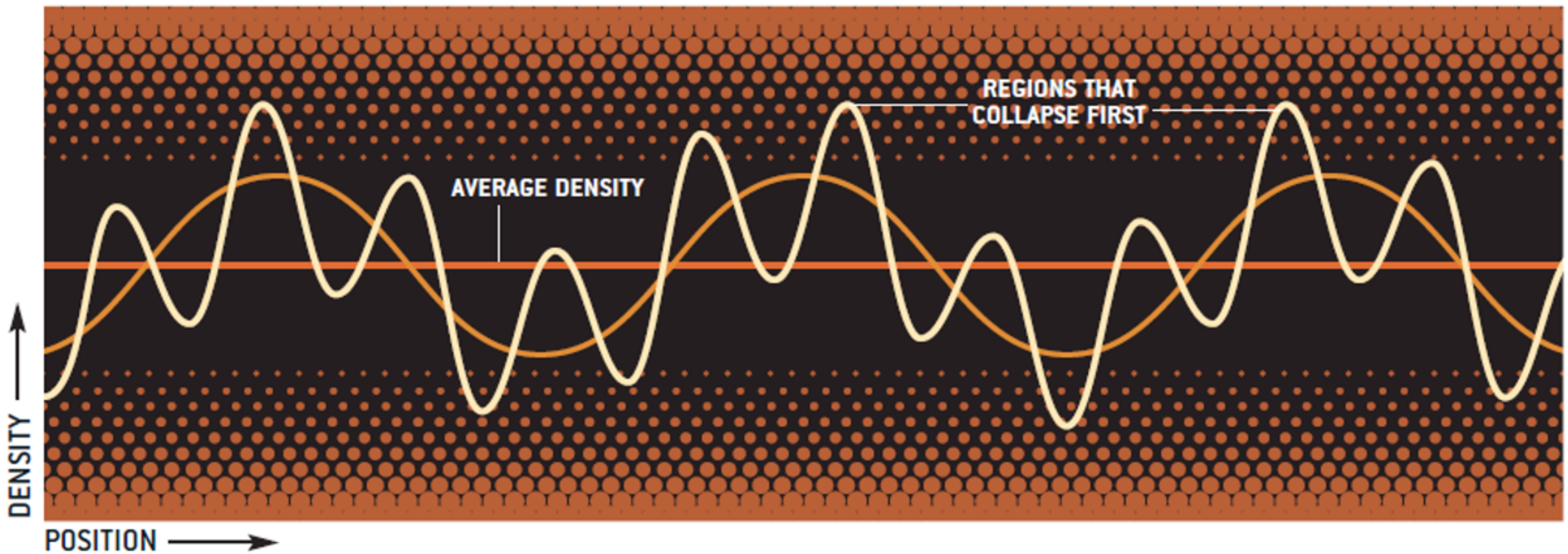
$$\ddot{\mathfrak{D}}' + 2H\dot{\mathfrak{D}}' = \mathfrak{D}' \left(4\pi G\rho_0 - \frac{c_s^2 k^2}{a^2} \right)$$

- Since the perturbation equation is linear in Fourier space, each Fourier mode satisfies the growth equation separately, i.e. each mode grows independently of all the others

Fourier decomposition of the density field

GALACTIC DENSITY VARIATIONS

DENSITY VARIATIONS in the pregalactic universe followed a pattern that facilitated the formation of protogalaxies. The variations were composed of waves of various wavelengths in a pattern that music connoisseurs will recognize as "pink noise." (Indeed, they originated as sound waves in the primordial plasma.) A small wave was superimposed on a slightly larger wave, which was superimposed on an even larger wave, and so on. Therefore, the highest density occurred over the smallest regions. These regions collapsed first and became the building blocks for larger structures. —*G.K. and F.v.d.B.*



- Since the perturbation equation is linear in Fourier space, each Fourier mode satisfies the growth equation separately, i.e. each mode grows independently of all the others

Power spectrum

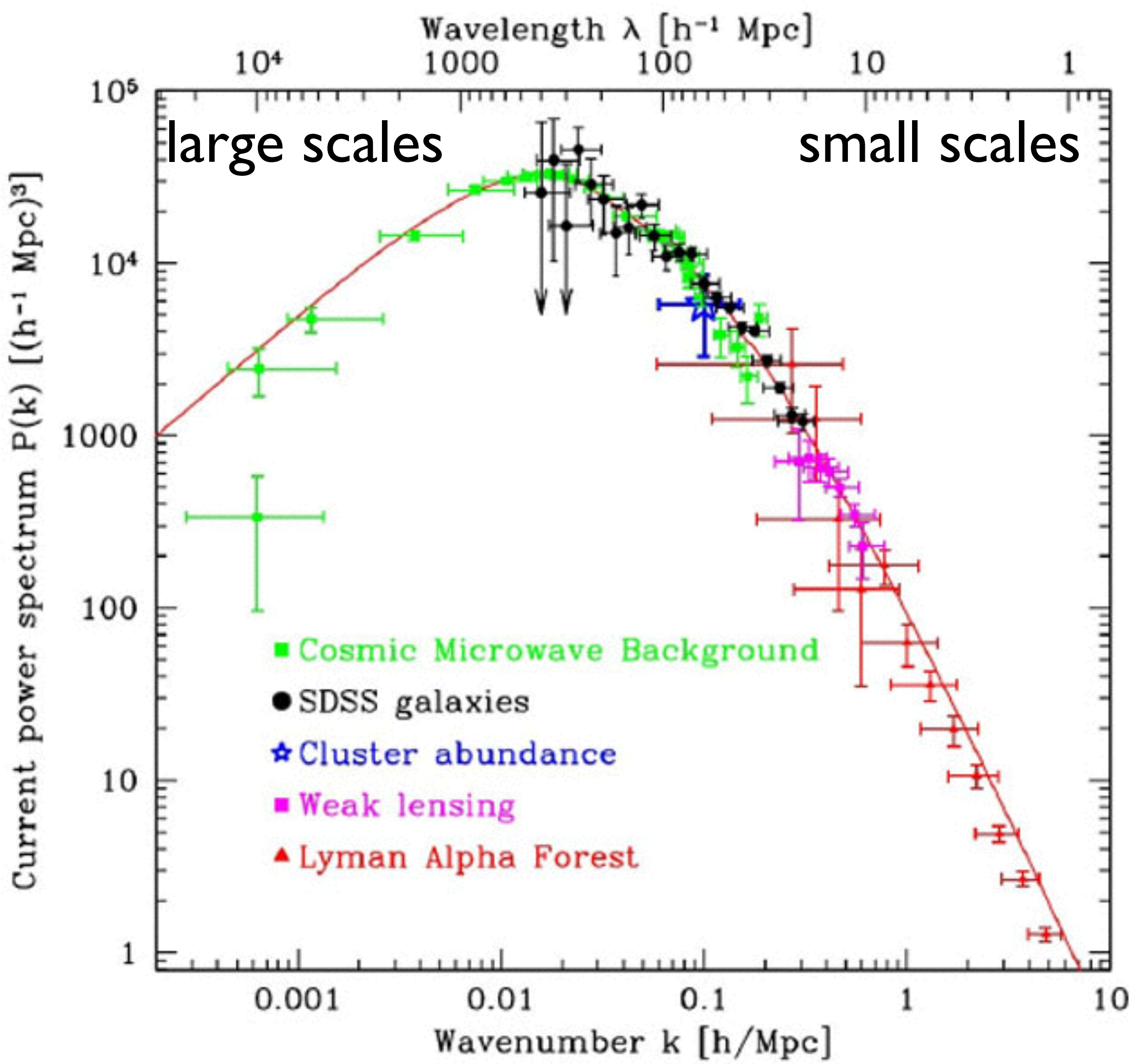
- A convenient way to specify the cosmic density field is given by the so called "power spectrum", a probability distribution function for different modes, defined by the variance of $\mathfrak{D}'$, the density contrast in Fourier space

$$P(k) = \langle|\mathfrak{D}'(\vec{k})|^2\rangle$$

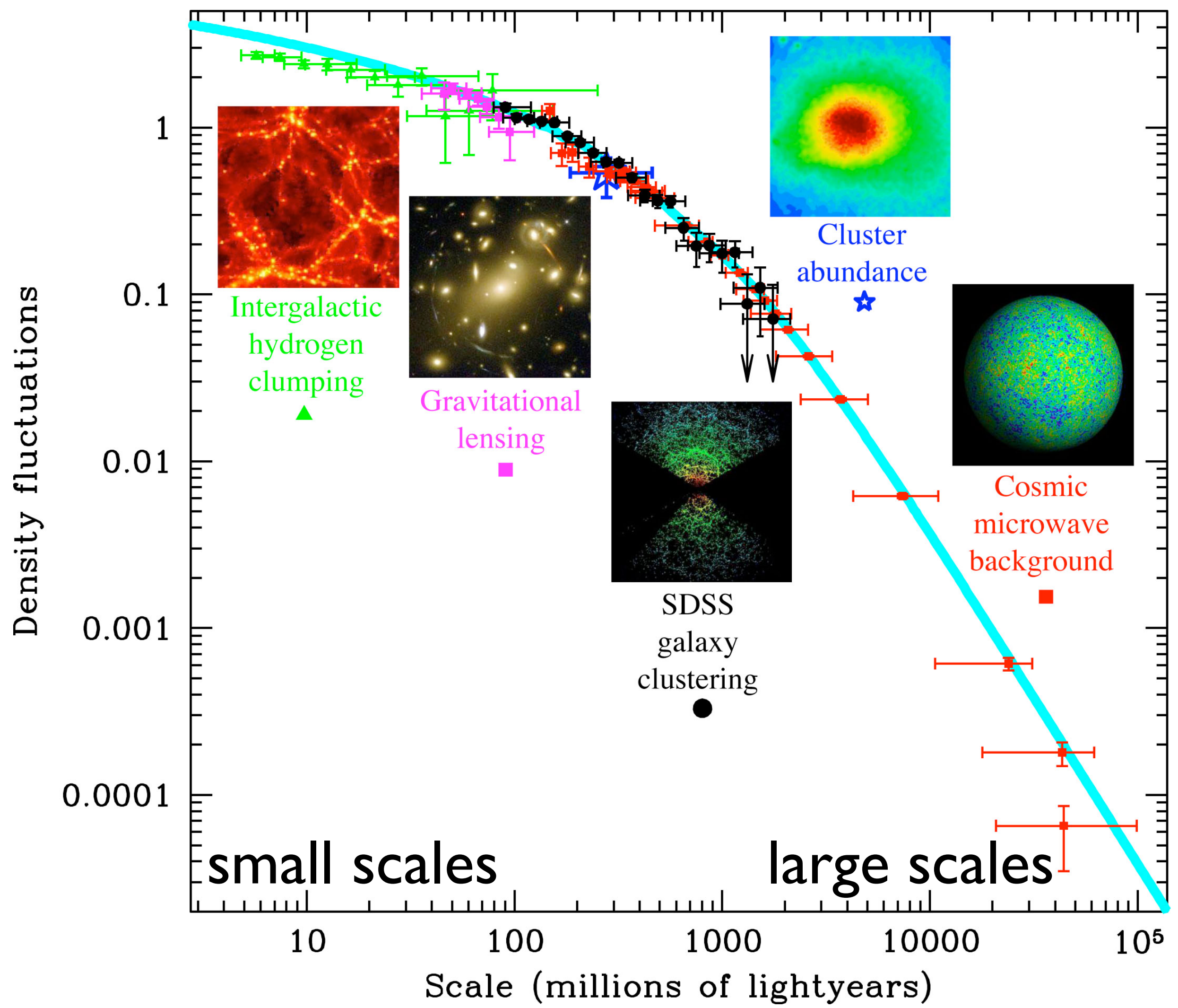
- Statistical properties of density perturbations at different spatial scales: P(k) characterizes the amplitude of these fluctuations at different spatial scales, or equivalently, at different wavelengths or wave numbers

Power spectrum

fourier space



real space



- inflation predicts a primordial power spectrum $P(k) \sim k^n$ that is very close to scale free (n=1) and Gaussian
- deviates on small scales from today's Universe

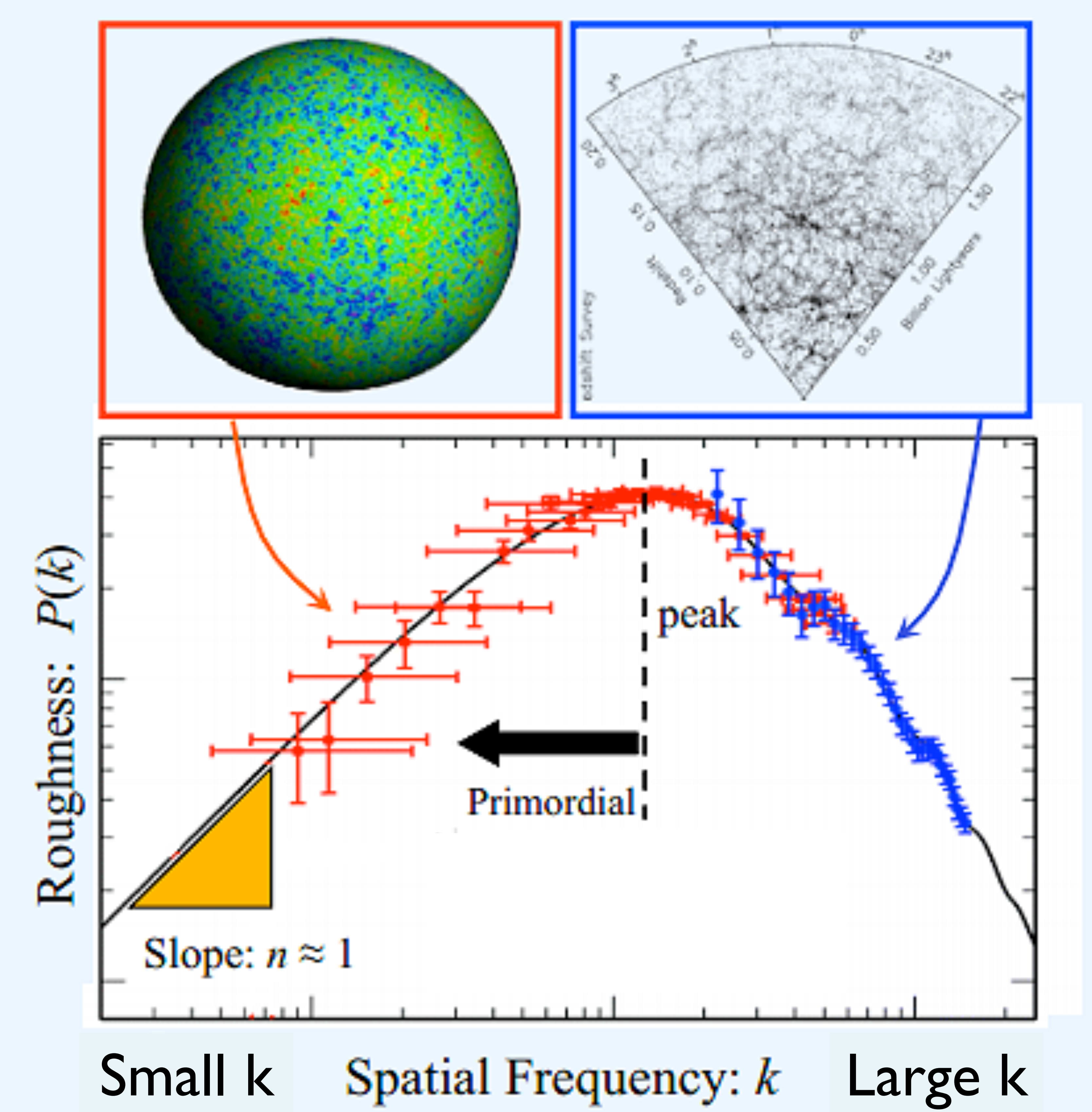
Power spectrum

- Power spectrum has a primordial component

$$P_i(k) = Ak^n$$ with n~1

predicted by inflation theory

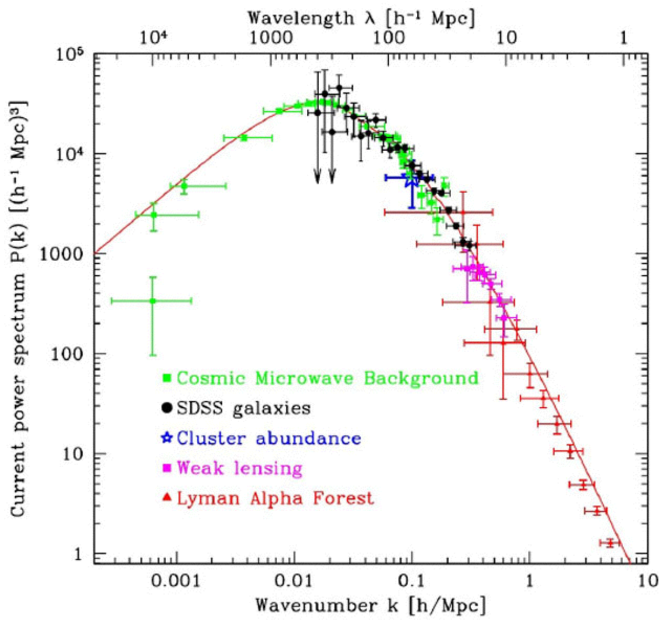
- and a "processed" one…



(a) Power Spectrum

Power spectrum

How do we get the observed P(k)?



- Get P(k) from inflation with n~1

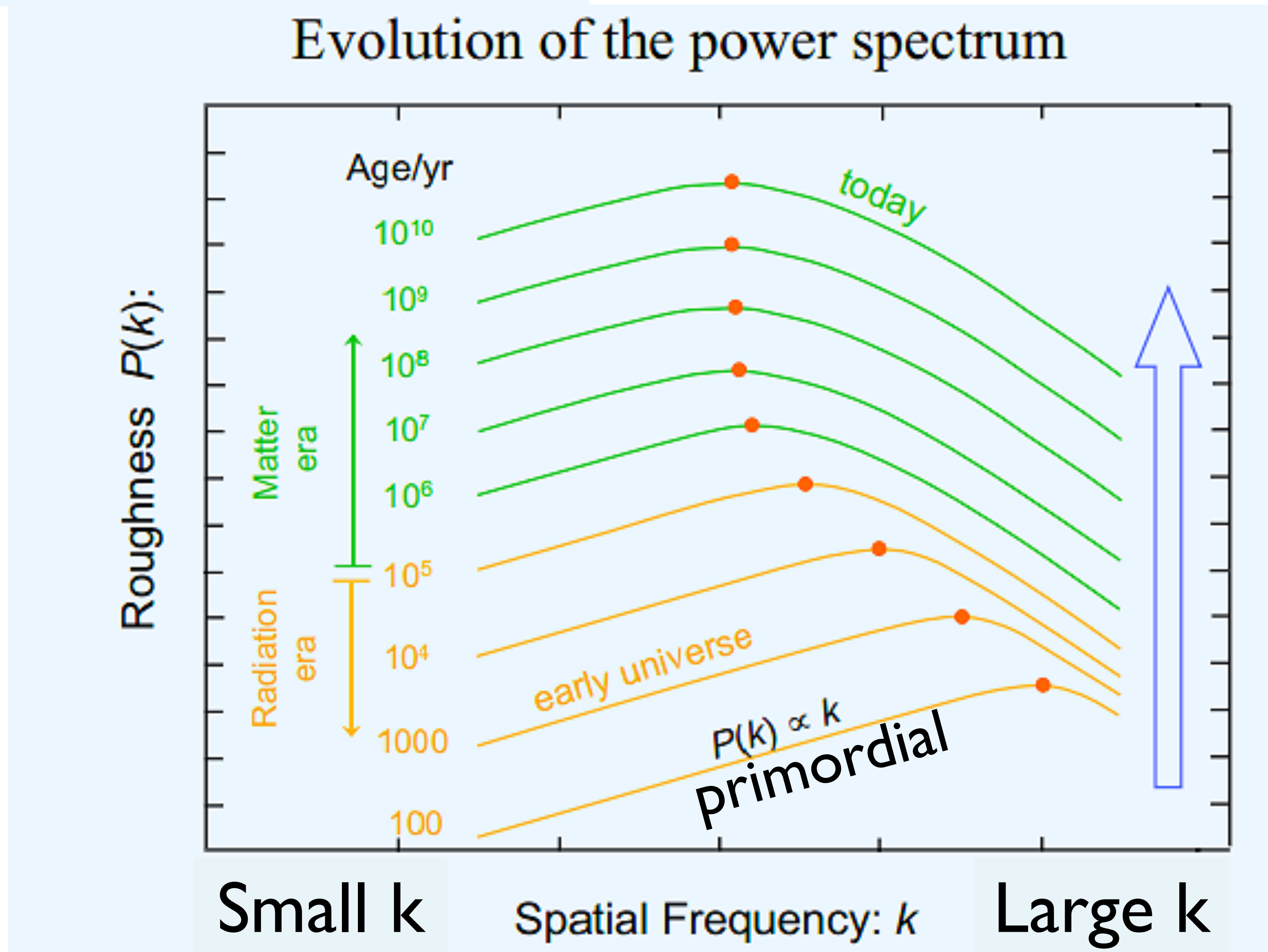
$$P_i(k) = Ak^n$$

- Such a spectrum matches CMB constraints on large scales, but not on smaller
- Many complex processes change $P_i(k)$ before and during recombination (linear regime) due to coupling of radiation and matter
  - Silk damping for baryons
  - Free streaming damping for DM
  - Different growth for DM and baryonic matter

➡ Can change the Power spectrum on smaller scales

Evolution of the Power spectrum

- Merely by including these effects of the transition between a radiation-dominated and matter-dominated universe, the CDM cosmology roughly explains the differing spectral indices of these two regimes as in observations.



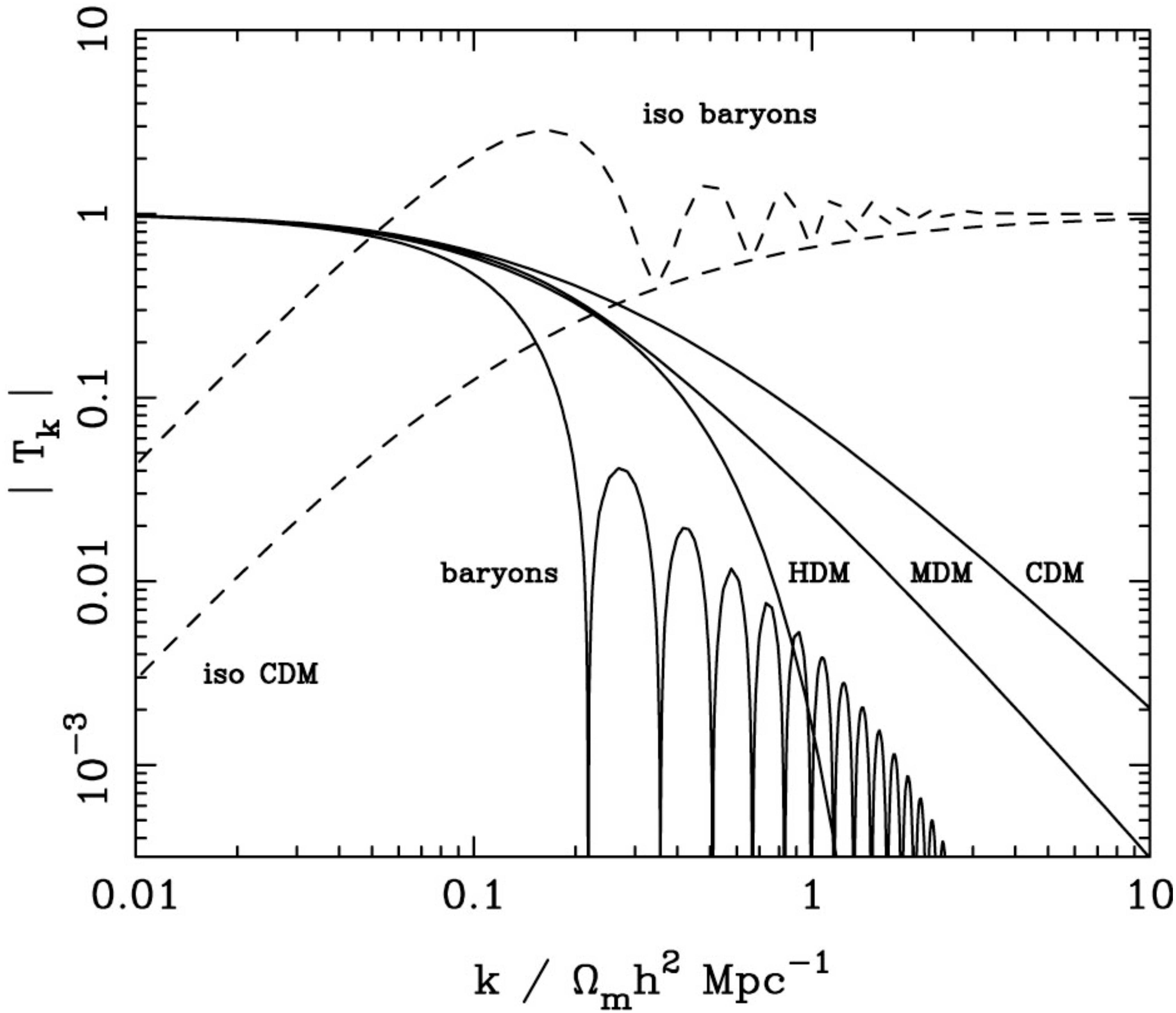
Power spectrum and transfer function

- To account for such processes before and during recombination, the initial power spectrum is transferred to post-recombination time by the Transfer function T(k) to compute a power spectrum at any later time after t_0

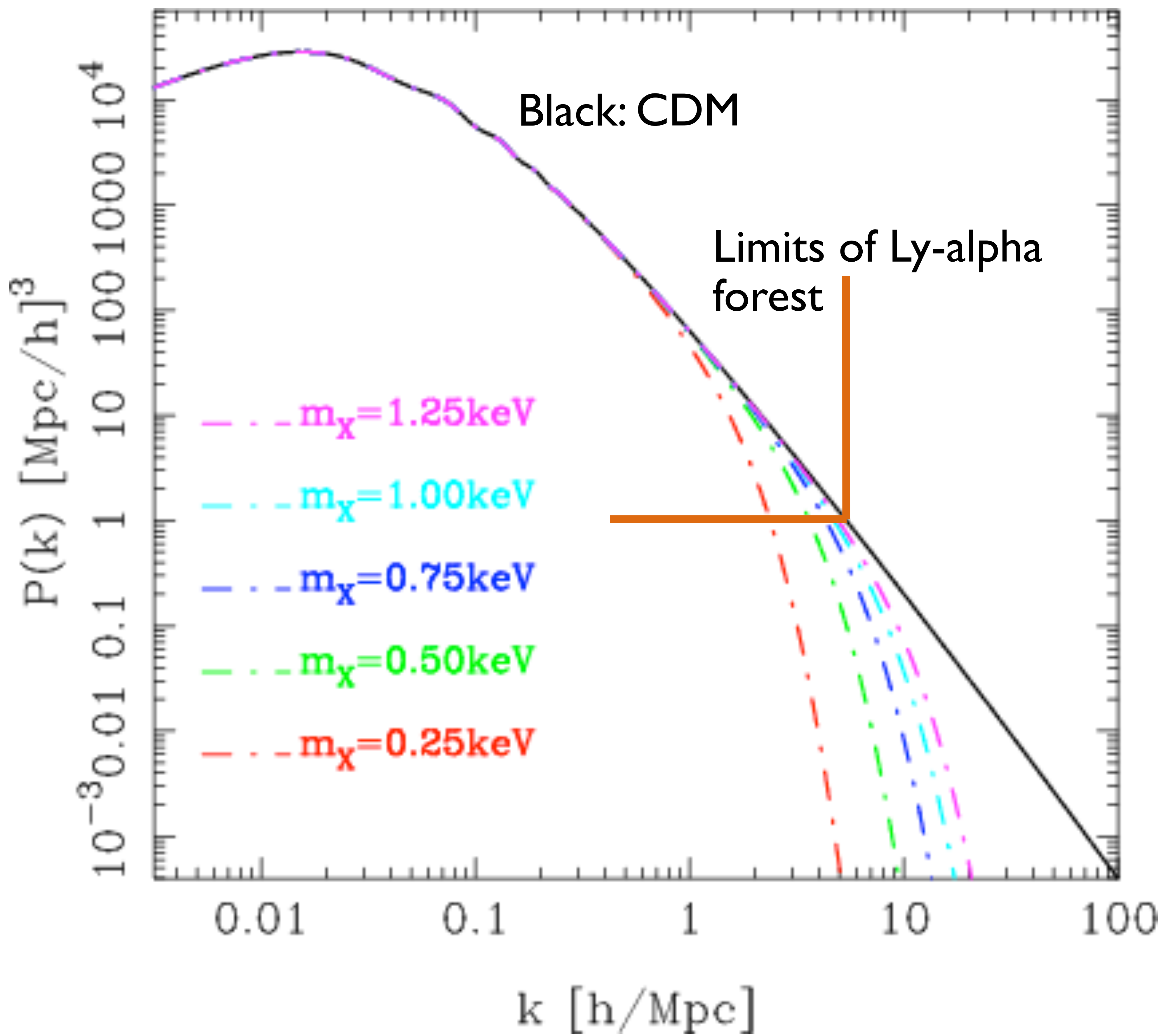
$$P(k) = T^2(k) P_i(k)$$

- Transfer functions are calculated based on cosmological parameters, the temperature of the CMB radiation and the mass of dark matter particles (different for cold/warm/hot DM) etc.
- Absolute normalisation is not (yet) predicted by theory, thus normalisation is taken from observed spectrum (CMB)
- Power spectrum and Transfer fct. can be computed using CAMB: http://lambda.gsfc.nasa.gov/toolbox/tb_camb_form.cfm

Transfer function



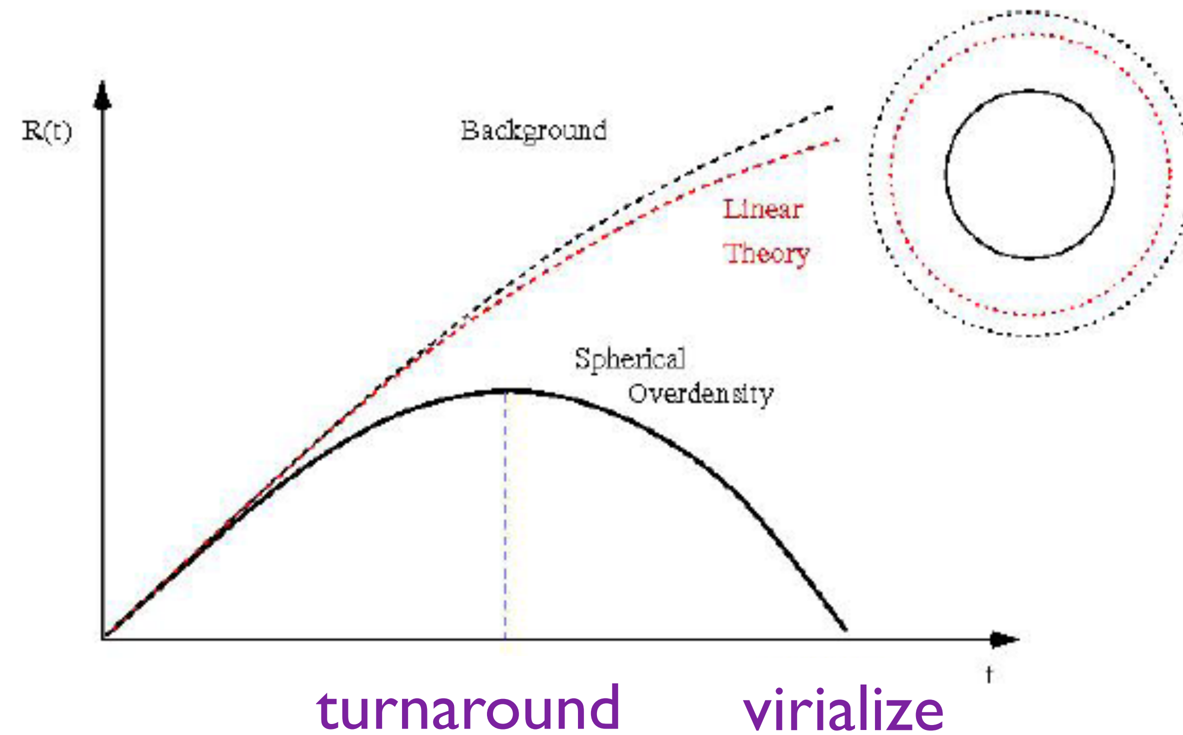
Processed power spectrum



Beyond linear theory: Spherical collapse

- Linear theory is strictly restricted to small density perturbations with 𝔇<<1 before and around recombination time.
- For larger density perturbations, let's first consider a simple analytical, non-linear model assuming spherical symmetry
- A spherical over-dense region can be treated as a separate universe and its radius as a function of time is given by solving the Friedmann's equations.
- Should collapse under its own gravity forming a bound, virialised object
- For a given mass, the spherical overdensity can be related to the density of the unperturbed universe

$$\rho_{\rm sph}/\rho_{\rm background} = (r_{\rm background}/r_{\rm sph})^3$$



- so that at turnaround:

$$\rho_{\rm sph}/\rho_{\rm background} = (r_{\rm background}(t_{\rm turn})/r_{\rm sph}(t_{\rm turn}))^3$$

$$= (3\pi/4)^2 \approx 5.56 \text{ for } \Omega_{\rm m} = 1$$

The spherical collapse model

- Assume system composed of DM: it cannot dissipate away its energy —> potential energy is being converted into kinetic energy

- Energy conservation at the turnaround point:

$$T_{\rm vir} + W_{\rm vir} = W_{\rm turnaround}$$

- Virial theorem:

$$2T_{\rm vir} + W_{\rm vir} = 0,$$

- This implies:

$$W_{\rm vir} = 2W_{\rm turnaround} \qquad \text{or} \qquad r_{\rm vir} = r_{\rm turnaround}/2$$

as $W \propto GM^2/r$.

- The density contrast in virial equilibrium becomes:

$$\Delta_{\rm vir} = \rho_{\rm sph}/\rho_{\rm background} = \left(r_{\rm background}(t_{\rm vir})/r_{\rm vir}\right)^3$$

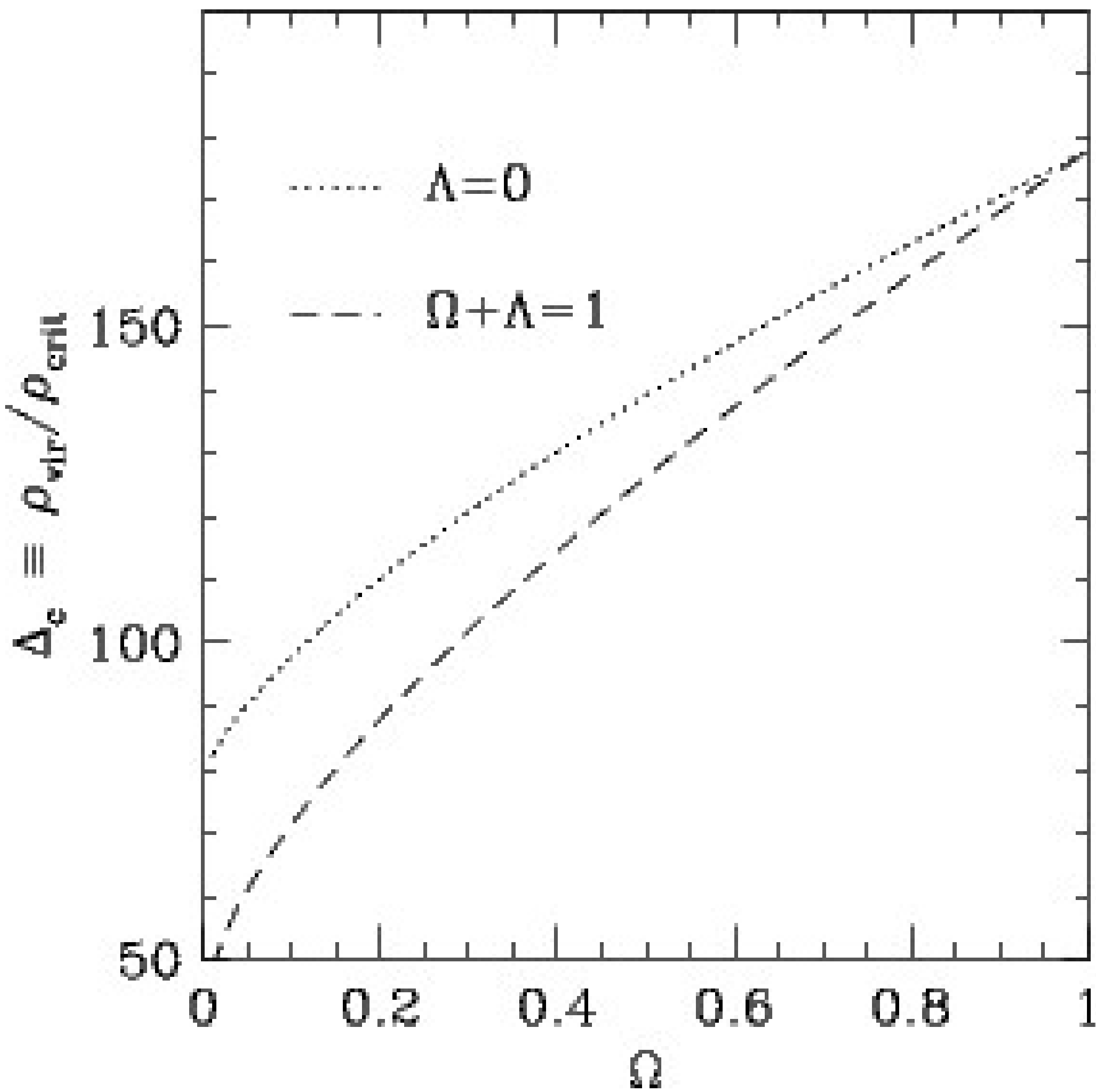
$$= 18\pi^2 \approx 178 \text{ for } \Omega_{\rm m} = 1$$

Virial mass

$$\boxed{M = \frac{4\pi}{3} r_{200}^3 \, 200 \, \rho_{\rm cr}(z)}$$

The spherical collapse model

- This calculation can be easily repeated for any other cosmology, the virial overdensity is always 100-200 $\rho_{\rm crit}$



- In reality, spherical overdensity doesn't apply, and virialisation is never complete, but these are useful guides to keep in mind before turning to numerical simulations

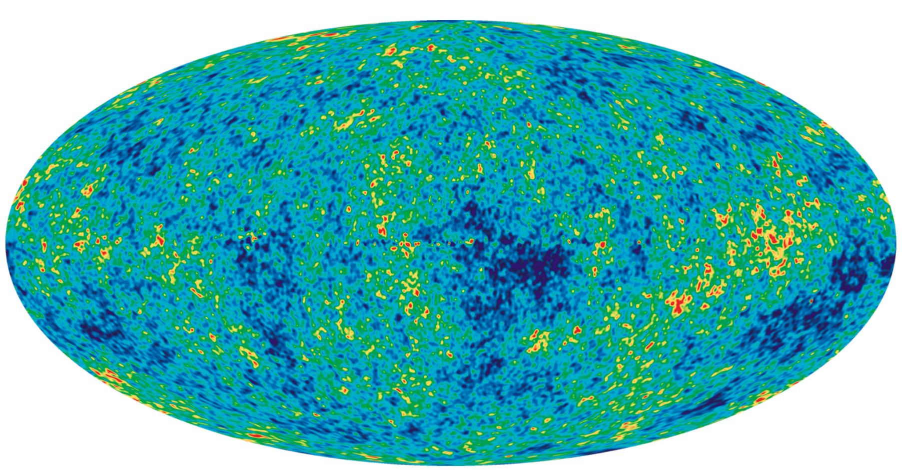
Structure formation — summary

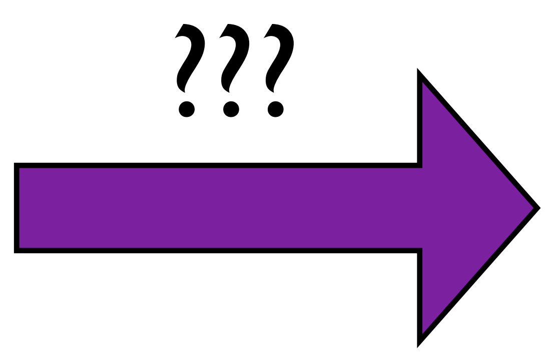
- Quantum fluctuations thought to have been inflated to larger-scale density fluctuations shortly after BB
- Competition between pressure of expansion & attraction of gravity
- Over-densities become more overdense, under-densities become more underdense
- Early Universe: structure formation in the linear regime can be described by Newtonian perturbation theory
- Power spectrum: distribution of amplitude of density fluctuation as a function of scales (often in Fourier space)
- Transfer function: describes how density fluctuations in the early universe evolve over time (e.g., Silk damping, Free streaming etc.)
- Beyond linearity: Eventually, in some places, gravity ‘wins’ and a ‘gravitationally bound’ object forms (“dark matter halo”) —> non-linear regime: simplest form: spherical collapse model;

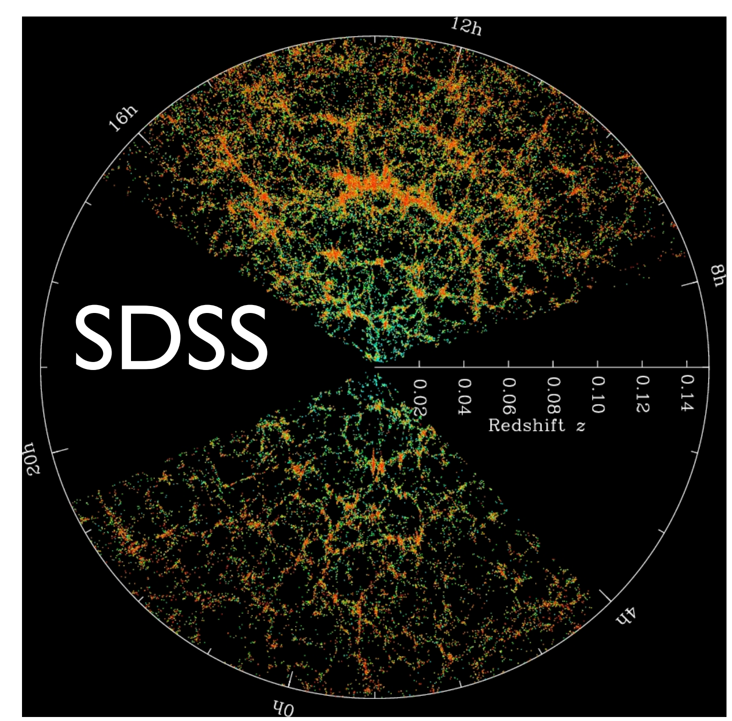
Structure formation in the non-linear regime

- As the density perturbations become larger, the linear approximation breaks down, and non-linear effects become significant.
- To study the later stages of structure formation and the detailed properties of galaxies and clusters, more sophisticated numerical simulations, such as N-body simulations (for dark matter), are required.
- For simplicity, consider only dark matter for now (as it dominates the matter content, and thus, structure formation)

CMB







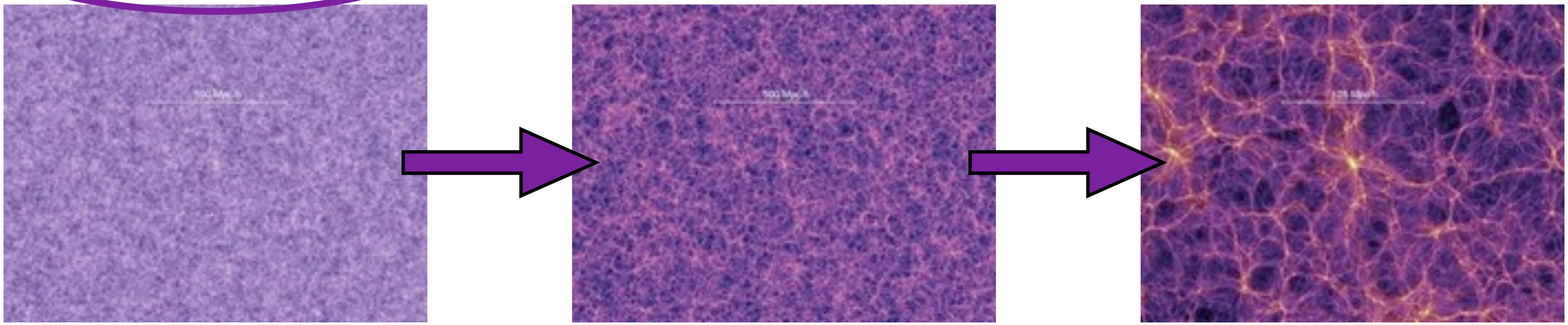
Outline of this lecture

- Cosmology in a nutshell
  - Motivation
  - The cosmological principle
  - Robertson-Walker Metric and Friedmann equation
  - The age of the Universe
  - Cosmological parameters
- The inhomogeneous Universe
  - Linear perturbation theory
  - Density fluctuations, power spectrum & transfer function
  - Non-linear growth: The spherical collapse model
- Generating initial conditions for cosmological simulations

N-body codes

- represent matter by particles
- represent the Universe by a (usually) periodic 'box'
- expanding space-time
- if we know initial conditions (positions & velocities), we can solve Newton's equations for each particle —> density field

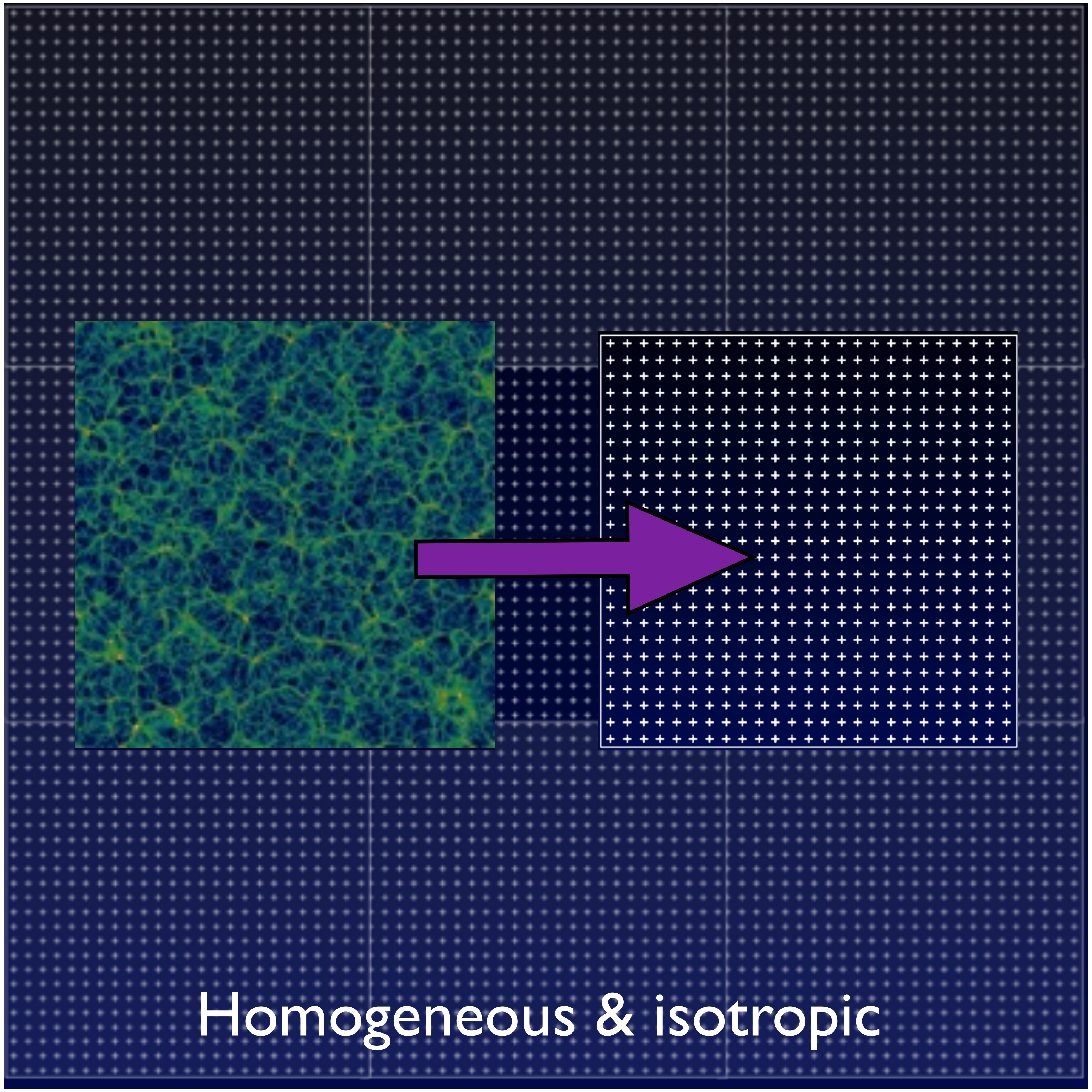
1. Generation of initial conditions?
2. Running the simulation
3. Creating mock catalogues



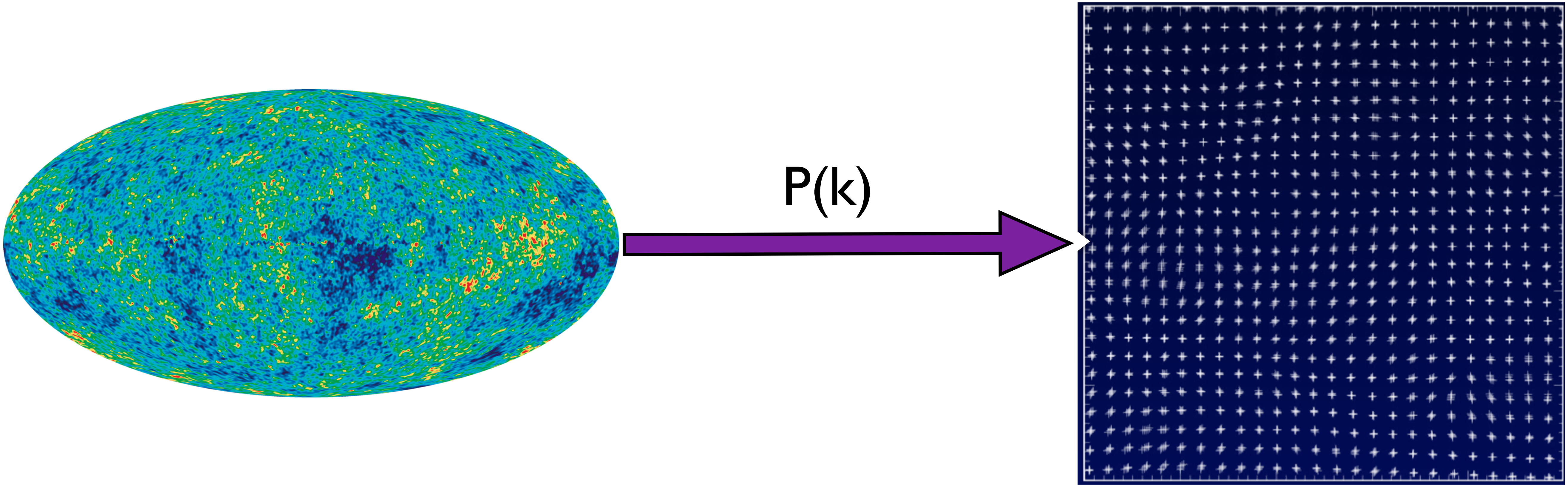
Generation of initial conditions

- On large scales, density is isotropic and homogenous
- Discretise initial density field into uniformly distributed particle grid

➡Assume periodic boundary conditions



Generation of initial conditions



Perturbed density field

- Universe shows small density fluctuations already at high z
- How to convert CMB into density fluctuations?

➡Use power spectrum to describe density fluctuations

$$P(k) = \langle |\mathfrak{D}'(k)|^2 \rangle$$

How do we get P(k)? —> taking advantage of the Transfer function!

The Zel'dovich approximation I

- Given the power spectrum at some time around recombination, how to impose a spectrum of fluctuations **after recombination** on a particle grid, or i.a.w. how to get the displacements from an initial uniform particle distribution? —> Zel'dovich approximation for pressure less fluid

- Using perturbation theory we have derived (in co-moving units)

$$\dot{\mathfrak{D}} + \frac{\vec{\nabla} \cdot \vec{u}}{a} = 0$$

$$\dot{\vec{u}} + H\vec{u} = -\cancel{\frac{\vec{\nabla}\delta P}{a\rho_0}} - \frac{\vec{\nabla}\delta\Phi}{a} \qquad \ddot{\mathfrak{D}} + 2H\dot{\mathfrak{D}} = \left(4\pi G\rho_0\mathfrak{D} + \cancel{\frac{c_s^2\vec{\nabla}^2\mathfrak{D}}{a^2}}\right)$$

$$\vec{\nabla}^2\delta\Phi = 4\pi G\rho_0 a^2\mathfrak{D}$$

- Given that all fluctuations were small at early times, assume that at more recent epochs only the growing mode has a significant amplitude

$$\mathfrak{D}(r, a) = \mathfrak{D}_i(r)F(a)$$

- After some algebra (get expression for Phi(F), integrate Euler and Continuity and growth equations) get position displacements

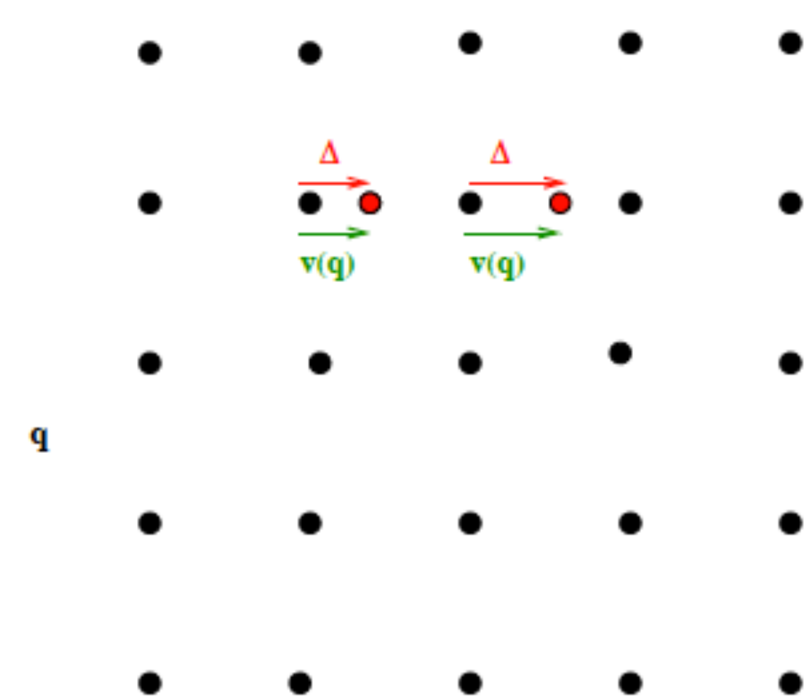
The Zel'dovich approximation II

- Get the position displacement from the unperturbed initial position x_i (in comoving space):

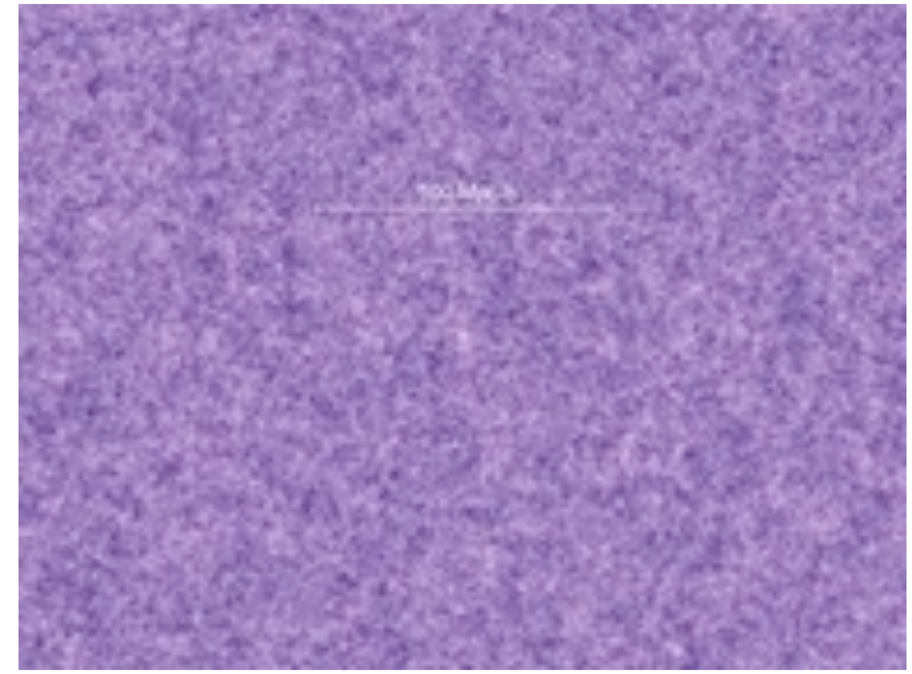
$$\vec{\psi} = \vec{r} - \vec{r}_i = -\frac{F\vec{\nabla}\Phi_i}{4\pi G\rho_i} = -\frac{a\vec{\nabla}\Phi(a)}{4\pi G\rho_i}$$

- To displace particles on grid go to Fourier space using the power spectrum
- Displacement in k space: $\vec{\psi}'_k = \frac{i\vec{k}}{k^2}\mathfrak{D}'_k \qquad \mathfrak{D}'_k \propto \sqrt{P(k)}$

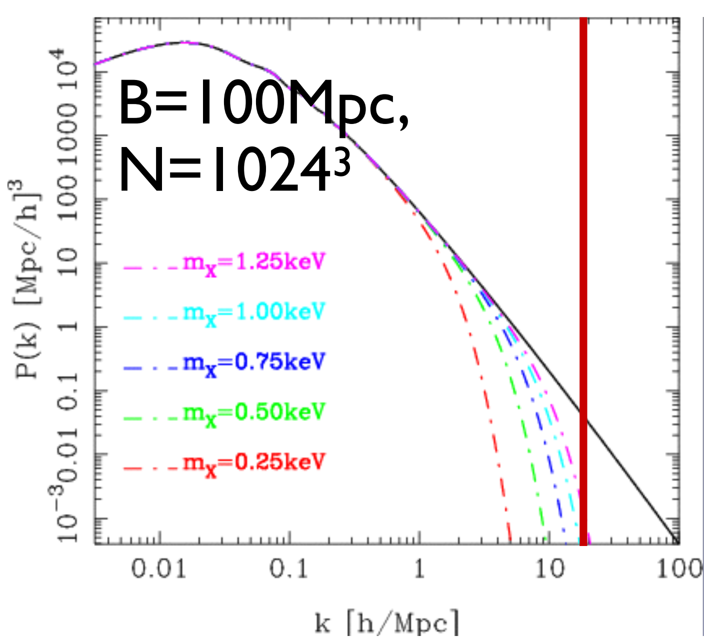
- The Zel'dovich approximation (applicable to a pressureless fluid, like DM) can be used to extrapolate the evolution of structures into the regime where displacements are no longer small

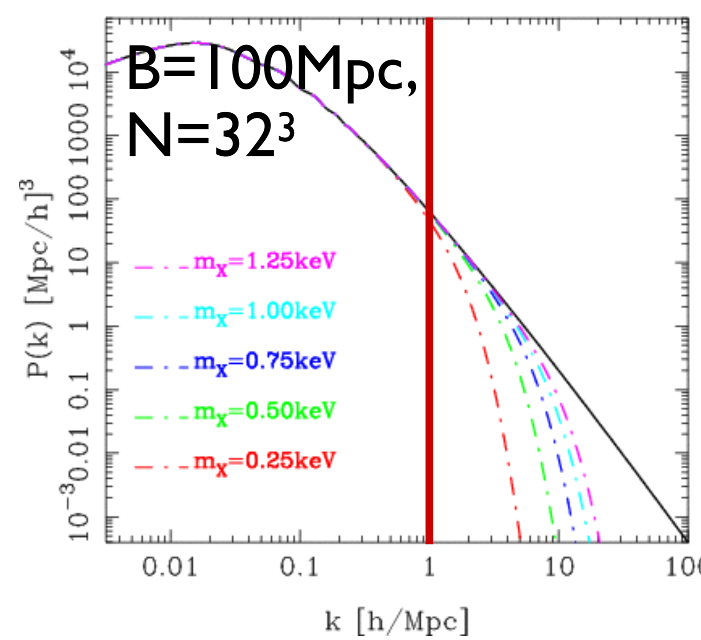


Free parameters and limitations of ICs



- Free parameters (interwoven):
  - Cosmology ΛCDM
  - Box length B
  - Number of particles N
  - Starting redshift z_{ini}
- Initial redshift constraints:
  - not too early (integration of numerical noise) and
  - not too late (shell crossing may occur not taken into account)
- Wavenumber limitation: N/B^3 has to be large enough to capture small scales





Initial conditions for simulations — Summary

- Large scales: Universe is homogenous and isotropic: place DM particles on a uniform grid in a box with periodic boundaries and in an expanding space-time
- Small-scale density fluctuations based on
  - Power spectrum and transfer function
  - Zel'dovich approximation for a pressureless fluid such as dark matter

    ➡displacement of particles from a uniform distribution dependent on P(k) at redshifts ~50-100
- Free parameters of cosmological simulations: Box size, particle number, initial redshift and the cosmological model

Up next…

- *Chapter 1*: Introduction (galaxy definition, astronomical scales, observable quantities — repetition of Astro-I)
- *Chapter 2:* Brief review on stars
- *Chapter 3*: Radiation processes in galaxies and telescopes;
- *Chapter 4:* The Milky Way
- *Chapter 5*: The world of galaxies I
- *Chapter 6*: The world of galaxies II
- *Chapter 7:* Black holes and active galactic nuclei
- *Chapter 8:* Galaxies and their environment;
- *Chapter 9:* High-redshift galaxies
- *Chapter 10*:
    - Cosmology in a nutshell; Linear structure formation in the early Universe
- *Chapter 11*:
    - Dark matter and the large-scale structure
    - Cosmological N-body simulations of dark matter
- *Chapter 12*: Populating dark matter halos with baryons: Semi-empirical & semi-analytical models
- *Chapter 13:* Modelling the evolution of gas in galaxies: Hydrodynamics
- *Chapter 14*: Gas cooling/heating and star formation
- *Chapter 15*: Stellar feedback processes
- *Chapter 16*: Black hole growth & AGN feedback processes
- *Chapter 17*: Modern simulations & future prospects

Part I: Observational basics & facts of galaxies
first 7 lectures

Part II: Theory & models of galaxy evolution processes
second 7 lectures