Solid state systems for quantum information, Correction 5

Assistants : franco.depalma@epfl.ch, filippo.ferrari@epfl.ch

Exercise 1 : The DC SQUID

The DC SQUID is largely used in the case of superconducting qubits. This SQUID consists of two JJs in parallel forming a loop. An external flux can thread the loop and generate a DC current that flows through the junctions, changing their dynamics.

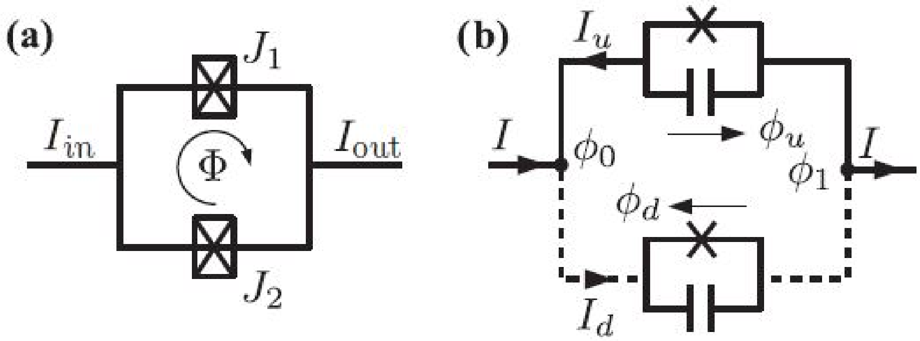

Figure 1: (a) A DC-SQUID is a device consisting on two junctions in parallel, threaded by some magnetic flux. (b) A more detailed version of the circuit must take into account the capacitive and inductive energy of the junctions if the pair is unbalanced.

1. Find the Lagrangian of the system.
   Hint: Assume that out and in going currents are the same. Take the notation $\phi_\pm = \frac{1}{2}(\phi_d \pm \phi_u)$. Assume $\dot{\Phi} = 0$ and that the junctions are identical.
2. Find the associated Hamiltonian. Why can this device be used to design "tunable" qubits?

Solution 1 :

1. We write $\phi_\pm = \frac{1}{2}(\phi_d \pm \phi_u)$ and $I = I_d - I_u$. We first express the branch fluxes in terms of node fluxes: $\phi_u = \phi_1 - \phi_0$ and $\phi_d = \Phi + \phi_0 - \phi_1$ (following the sign convention given in the figure). We then deduce that $\phi_+ = -\frac{\Phi}{2}$ and $\phi_- = \frac{\Phi}{2} + (\phi_0 - \phi_1)$. We now write the Langrangian which has contributions of both capacitances, two JJs non linearities and the flowing current contribution:

$$\mathcal{L} = \frac{1}{2}C_J\dot{\phi}_d^2 - \frac{1}{2}C_J\dot{\phi}_u^2 - E_J\cos\left(\frac{\phi_u}{\varphi_0}\right) - E_J\cos\left(\frac{\phi_d}{\varphi_0}\right) + I_d\phi_d - I_u\phi_u. \tag{1}$$

By assuming $\dot{\Phi} = 0$ and using the trigonometric identity $\cos(a)+\cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$, we get:

$$\mathcal{L} = C_J\dot{\phi}_-^2 + 2E_J\cos\left(\frac{\phi_d+\phi_u}{2\varphi_0}\right)\cos\left(\frac{\phi_d-\phi_u}{2\varphi_0}\right) - I\phi_-$$
$$\mathcal{L} = C_J\dot{\phi}_-^2 + 2E_J\cos\left(\frac{\Phi}{2\varphi_0}\right)\cos\left(\frac{\phi_-}{\varphi_0}\right) - I\phi_-.$$

2. The associated Hamiltonian is then easily found:

$$\frac{Q_-^2}{4C_J} + I\phi_- - 2E_J\cos\left(\frac{\Phi}{2\varphi_0}\right)\cos\left(\frac{\phi_-}{\varphi_0}\right). \tag{2}$$

We can write our new effective Jospehson energy as $E_{J,eff} = 2E_J\cos\left(\frac{\Phi}{2\varphi_0}\right)$, which is a Jospehson energy dependant on the external flux Φ. So by changing an external magnetic field (delivered by an outside coil or on chip flux line), it is possible to tune the Josephson energy (or non linear inductance), which in turn tunes the resonance frequency of the device.